Report on the bachelor thesis "Graph theory and quantum structures"
(the author of the thesis: **Václav Voráček)**

The thesis consists of 7 chapters. The first chapter (Introduction) indicates the link of quantum mechanics with the algebraic notions to be studied. It is argued why the standard event structure, the Boolean algebra, is not suitable for the depiction of the events of a quantum experiment. It is thus seen how orthomodular structures come into existence. The notion of a quantum state naturally occurs.

The second chapter (Hypergraphs) introduces the notion of a hypergraph and it reviews its basic properties. The stage is thus prepared for the representations of orthomodular structures (quantum logics) by hypergraphs.

The third chapter (Groups) deals with the notion of a group in a certain connection with systems of linear equations. A system of linear equations is associated with a hypergraph. This approach is fundamental as far as the results of the thesis are concerned—see the remarkable Theorem 3.5.

The fourth chapter (Orthoalgebras) introduces the first type of "quantum logics"—the orthoalgebras. The measures (resp., the states) are investigated, and an explicit relation with hypergraphs is shown. An important result lies in Theorem 4.14 that formulates a nontrivial estimate for a Greechie diagram to allow for a state. The results of Prop 4.17 and Theorem 4.20 are already valuable novelties in the theory of quantum logics.

In chapter five (Orthomodular posets) and chapter six (Orthomodular lattices), the author extends his interest in group-valued measures to more specific logics. He again succeeds in finding the estimates for the existence of group-valued measures. In the end, the author speculates on the numerical side of the problem pursued and reveals a program on the matter that matches the results of the other authors.

Let me comment more in detail on the technical qualities of the thesis. The author took up a rather combinatorially difficult question on constructing a rather exotic, at first sight, types of logics. However, these questions are rather important for understanding the intrinsic properties of quantum logics. Moreover, the measures that range in more general groups than the reals deserve to be studied not only for the reason of "intellectual curiosity", though this is obviously sufficient in theoretical mathematics when the results are interesting, but also for more "practical" reasons when one has to deal with "non-linear-space-valued measures".

The style of the thesis is fairly good; occasional clerical mistakes do not affect the overall high impression. The references illuminate well the state of the art. A note on specific states (two-valued states, Jauch-Piron states, etc.) could be included, too, but, understandably, the text might become too cumbersome.

I highly value this bachelor thesis. The author significantly contributed to the hitherto known results (the main results of the thesis have been published). His mastery of the combination of the techniques of graph theory, linear algebra, and orthocomplemented structures strongly impresses. The constructions of the logic without the group-valued measures, the highlight of the thesis, are mature pieces of advanced mathematics. Mr. Voráček proved an extraordinary talent and should be advised to remain in the area of applied mathematics and theoretical branches of engineering.

A conclusion: The bachelor thesis of V. Voráček meets the highest standard of Ph.D. thesis at FEE CTU in Prague. My assessment is very positive, and I evaluate the thesis as **A (excellent)**.

31. 5. 2019 Prof. RNDr. Pavel Pták, DrSc.