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FACULTY OF BIOMEDICAL ENGINEERING
Department of Biomedical Technology

Design of the Multicompartment Mathematical Model of the Lungs

Navrh multikompartmentoveho matematickeho modelu plic

Master thesis

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Title: **Design of the Multicompartment Mathematical Model of the Lungs**

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Instructions for processing:

Realize an experimental measurement on a five-compartment mathematical model of the respiratory system. Analyze a ventilatory data, pressure distribution etc. for the measurement with different added dead space into the structure of the model.

Design and develop a mathematical model corresponding to the five-compartment experimental model of the respiratory system. Realize mathematical simulations of the physically measured experiments.

Compare and discuss results of physical measurements and mathematical modelling. Analyze the effect of the dead space upon the intrapulmonary and ventilatory parameters.

References:

[1] K Horsfield, G Dart, DE Olson, etc. , Models of the human bronchial tree, Journal of applied physiology, ročník 31, číslo 2, 1971, 207-217 s.

[2] J. JANE PILLOW, MALCOLM H. WILKINSON, HEATHER L. NEIL and C. ANDREW RAMSDEN, In Vitro Performance Characteristics of High-Frequency Oscillatory Ventilators , Am. J. Respir. Crit. Care Med. , ročník 164, číslo 6, 2001, Září, 1019-1024 s.

[3] Weibel ER, morphometry of the human lung, ed. 1st, Springer Verlag, 1963, ISBN XXX

Validity of assignment until date: **20.08.2018**

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In Kladno, 20.02.2017

Declaration

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Acknowledgement

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Abstract

Mechanical ventilation or artificial lung ventilation is an invasive procedure which is used to assist or replace spontaneous breathing. In positive pressure ventilation, air is pushed into the trachea and through the respiratory system causing an overstretching of the airways and the alveoli, leading to ventilator induced lung injury (VILI). It is therefore necessary to monitor the pressure distribution in a patient's respiratory system so as to avoid such injuries.

This thesis describes a five compartment mathematical model of the respiratory system, it describes an experimental measurement on the five compartment physical model and analyses the ventilatory data such as pressure distribution for the measurement with different added dead space into the structure of the model. The mathematical model shall be useful in simulating the performance of a ventilator under different ventilatory regimes and to analyse the pressure distribution in each compartment under changing parameters of the ventilator such as PEEP and I: E ratio.

For the purpose of writing, the process was split into two parts, physical model and mathematical model. The first part describes the physical five compartment model and the experimental measurement done and analysis of the pressure distribution in each compartment of the model.

The second part describes the design and development of the mathematical model corresponding to the physical model of the respiratory system and the realization of mathematical simulations of the physically measured experiments.

Keywords

Five-compartment Respiratory model, Mathematical model, Dead space, Pressure distribution;

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List of Acronyms

Acronym	Meaning
CMV	Continuous Mandatory Ventilation
VC	Volume Controlled
PEEP	Positive End Expiratory Pressure
I:E ratio	Inspiratory Expiratory phase time ratio
BPM	Breaths per minute
V_T / TV	Tidal Volume
$\dot{V}$	Minute Volume
MM	Mathematical Model
DS	Deadspace
HFV	High Frequency Ventilation
HFOV	High Frequency Oscillatory Ventilation
PCO_2	Partial Pressure of Carbon Dioxide

CMV (Continuous Mandatory Ventilation) is a mode of the ventilator, wherein a breath of fixed volume or pressure is delivered to the patient irrespective of the effort.

VC (Volume Controlled) is a mode of control of the ventilatory regime, wherein the breaths delivered are dependent on the set Tidal Volume, irrespective of the pressure required to deliver it.

PEEP (Positive End Expiratory Pressure) is a therapist selected pressure level for the patient airway at the end of expiration.

I: E ratio (Inspiratory Expiratory phase time ratio) is the ratio of inspiratory interval to expiratory interval of a mandatory breath.

BPM (Breaths per Minute) is the number of breaths to be delivered in a minute. It represents the total respiratory rate of the patient.

V_T (Tidal Volume) is the depth of breathing or the volume of gas inspired or expired during each respiratory cycle.

$\dot{V}$ (Minute Volume) is the Volume of gas that is exchanged per minute during quiet breathing.

DS (Dead Space) is the volume of gas that is inhaled but does not take part in the gas exchange.

HFV (High Frequency Ventilation) is a method of ventilation in which a patient is provided ventilation at frequencies much higher than respiration rate.

PCO_2 (Partial Pressure of Carbon Dioxide) is a measure of relative concentration of gas in air or fluid.

List of Symbols

Symbol	Unit	Meaning
V_{AC}, V	Volts	Voltage
R	Ohms	Resistance
L	Henry	Inductance
C	Farad	Capacitance
m_a	$kg.m^{-4}$	Acoustic Weight
c_a	$m^4.s^2.kg^{-1}$	Acoustic Compliance
r_a	$N.s.m^{-5}$	Acoustic Resistance
ρ_0	$kg.m^{-3}$	Air Density
l	m	Length of tube
S	m^2	Cross-section Area of tube
V	m^3	Volume of Demijohn
c_0	$m.s^{-1}$	Velocity of propogation
μ	$N.s.m^{-2}$	Dynamic air viscocity
R_t	m	Radius of tube

1. Introduction

Mechanical Ventilation or Artificial Ventilation can be defined as a way to replace or assist in the spontaneous breathing, usually carried out by a ventilator. It is an invasive procedure as it involves the penetration of an endotracheal through the skin or the mouth. There are two types of ventilation 1.Positive Pressure ventilation where the air is pushed inside the trachea and 2.Negative Pressure ventilation where air is sucked into the lungs. Most commonly Positive Pressure Ventilation is used in clinical practices and most of the mechanical ventilators available in the market operate on the principal of pushing the air into the trachea. There are many modes of ventilation as the technology has evolved and amongst the most basic mode of ventilation is the CMV (Continuous Mandatory Ventilation) mode. In the CMV mode of ventilation, the ventilator provides a set amount of volume (or pressure) at a fixed rate in a minute to the patient irrespective of the effort made by the patient. In this thesis, the Volume Controlled- Continuous Mandatory Ventilation mode has been used to obtain experimental results.

In a human being, there are two lungs, one lying on each side of the midline in the thoracic cavity. They are cone shaped and have an apex, a base, a tip, costal surface and medial surface. The right lung is divided into three distinct lobes: upper, middle and lower. The left lung is smaller because the heart occupies space left of the midline. It is divided into only two lobes: upper and lower. The division between the lobes are called fissures. The lungs are also composed of the bronchi and smaller air passages, alveoli, connective tissue, blood vessels, lymph vessels and nerves, all embedded in an elastic connective tissue matrix. Each lobe is made up of a large number of lobules.

A five-compartment model of the respiratory system represents structure of the human respiratory system in its simplified form. The human respiratory system though much complex, it can be represented by such a model for experimental purpose to analyse the pressure and flow distribution in the respiratory system. A mathematical model of such a physical model can be used to simulate the performance of the lung ventilator during artificial respiration. Such a model takes into consideration the airway resistances and compliances and models them mathematically to represent the human respiratory system to its closest assumptions. Using simulation platforms such as Simulink, a mathematical model of the respiratory system can be used to compare the data obtained by physical experiments to check the effectiveness of the simulation.

Training on simulation platforms has become an absolute necessity in this era. With the introduction of microprocessors, the simulators have become more and more sophisticated. Today the simulators designed to provide anaesthesia training and training to intensive care personnel have become very sophisticated and provide quite an extensive and in depth control over real-life situation. This is an area, which shows tremendous potential for expansion and promise to affect the future of medical education.

This thesis comprises of two parts, the physical model and the mathematical model. In the first part describing the physical model, the construction of the five compartment physical model, the apparatus for data acquisition and the parameters of each compartment of the model are described. Along with it, the different ventilatory regimens used to get experimental results regarding pressure distribution in each compartment has been thoroughly discussed. The experimental results of the different ventilatory regimens are obtained using National Instruments'LabView software and this data is further analysed using MathWorks MATLAB software.

The second part describing the mathematical model describes the relation between a respiratory system and an acoustic system and also how an electro-acoustic analogy can be used to use electric laws to solve acoustic or mechanical systems. This part also describes the construction of a mathematical model in National Instruments' SIMULINK, corresponding to the physical model of the respiratory system described in the first part. This part also describes some of the parameters of the respiratory system that change during the artificial Lung Ventilation and the parameters considered in the mathematical model which affect the efficiency of the model.

In further sections is the description of experiments done by introducing the dead space into the physical model and the change in pressure distribution in different compartments due to the introduction of dead space. Also, there is a description of the behaviour of the mathematical model relating to the addition of dead space in the physical model and the outcomes verified by the mathematical model.

In a separate experiment, the distribution of pressure in the different compartments of the model under High Frequency Oscillatory Ventilation was observed and the results have been discussed briefly.

Furthermore, an experiment on PCO_2 and gas pressure analysis in a single compartment model has been discussed briefly to understand the dependency of PCO_2 on Tidal and Minute Volume.

The aim of this thesis is to develop a mathematical model corresponding to the five-compartment physical model and to analyze various ventilatory data such as pressure distribution in every compartment of the model under different regimes of the ventilator. The mathematical model shall be useful in simulating the performance of a ventilator under different ventilatory regimes and to analyse the pressure distribution in each compartment under changing parameters of the ventilator such as PEEP and I: E ratio.

2. State of the art for modelling

A Mathematical Model (MM) of volume controlled ventilation plays an important role in correcting and developing by means of visualization, the performance of ventilator that provides artificial breath during patient's treatment. The MM of mechanical ventilation with various numbers of compartments and changes in parameters has been proposed in many scientific studies.[1]-[4]. Interest in the mathematical modelling of a respiratory system has been shown by studies involved in focusing on the details of the anatomy of the respiratory system, by considering the lungs as a series of small resistances and capacitances and converting the MM using Laplace.[5] . Furthermore, the pressure signals for inspiratory and expiratory activities have been modelled by quadratic equations and an exponential equation.[6]

Clare E, Barkalow introduced the lung simulator[7] for simulating spontaneous breathing and for testing of ventilatory devices used with spontaneously breathing patients, he invented a pneumatic lung analog and a kit for modifying training test lungs. The lung analog is a prototype of PneuView lung simulator from Michigan Instruments Inc.

In the age of computers, it has become possible to quickly change between different simulation settings, Meyer designed a respiratory servo system controlled by a micro-computer, using a solenoid valve assembly and a hydraulically operated cylinder-piston. This allowed a good flexibility in selecting breathing patterns and allowing implementation of complex sequences of breathing pattern. [8]

These models may have some very distinct advantages which help in diagnosing some respiratory diseases and suggesting treatment manoeuvres by studying the effectiveness of change in various parameters however they could not mimic the exact functioning of a VCV signal and its characteristics. This study involves in simulating the behaviour of VCV signal in a five-compartment model of the lung and analysing the behaviour of pressure distribution in each compartment and the effect of changes in physical parameters of the pathways and the compartments.

Another unique characteristic of this study is that it is based on a five-compartment model of the respiratory system which is quite different to the studies so far as long as the number of compartments are concerned, most of the models of the respiratory system are based on a single or two compartments of the respiratory system which oversimplifies the physiology of the lungs and does not take into consideration the complexity of the structure. According to the morphology of the human lungs, it is divided into five different compartments and hence for this study it is necessary to have a model which can visualize the pressure and flow distribution in all the five compartments of the respiratory system.

3. Measurements of the Physical model.

3.1 Apparatus

A Five Compartment model of the respiratory system consists of a respiratory circuit of the following:

[1] CareFusion Avea Ventilator

[2] Central Airways printed on a 3D printer

[3] 5 Demijohns forming the five compartment of the lungs

[4] Silicon tubes connecting the central airways to the demijohns

Figure 1: CareFusion Avea ventilator

The Care Fusion Avea is the ventilator that provides positive pressure ventilation to the respiratory circuit and is operated in CMV-VC mode (Continuous Mandatory Ventilation – Volume controlled) for the purpose of the experiment.

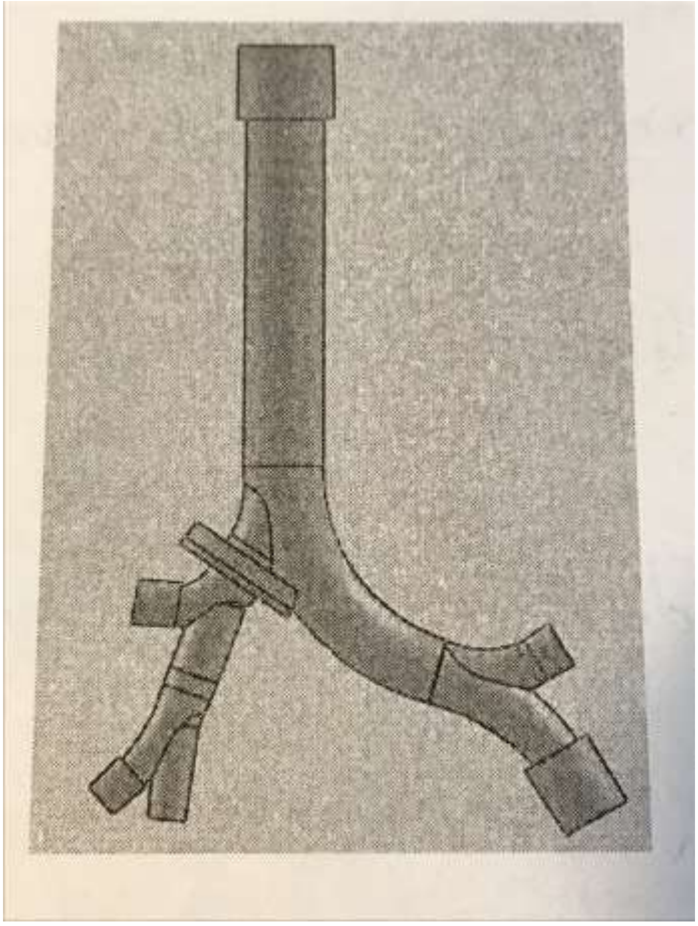


Figure 2: A model of the central airways

Using the Horsefield morphological description of the respiratory system [9], the geometrical dimensions of the central airways were modelled in Solidworks software and a model of it was printed using a 3D printer. The scheme of the central airways is depicted in the image.

This model is based on data that was obtained by measurements of a resin cast of a human bronchial tree. The preparation of the cast [10] and its description is discussed elsewhere.[11][12]

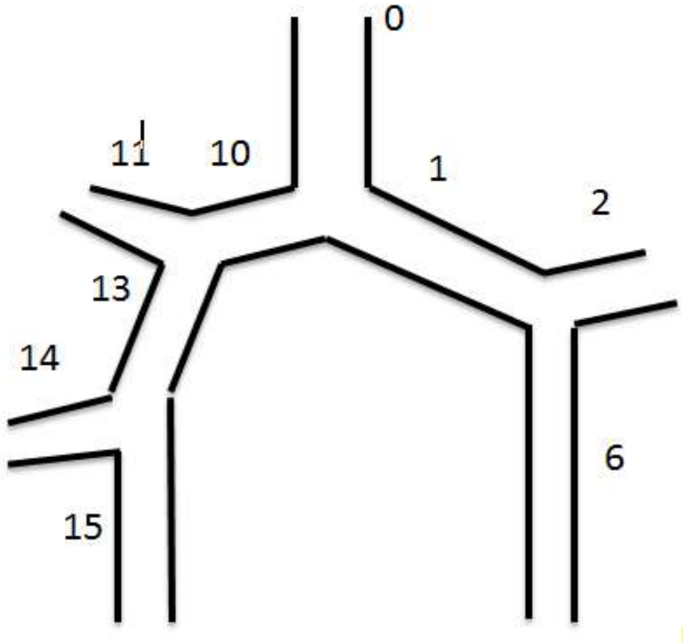


Figure 3: Outline of Central Airways

Airway No	Diameter (mm)	Length (mm)	Angle (°)
0	16	100	0
1	12	50	73
2	7.5	16	48
6	8	11	44
10	11.1	22	35
11	7.3	15.6	63
13	8.9	26	15
14	5.2	21	61
15	6.4	8	15

Table 1: Geometrical dimensions of the central airways of the laboratory model [9]

According to Horsefield morphological model [9], the alveolar space in the model was distributed into five compartments. The volume of the lung lobes with their compliance and abbreviations are summarized in the table below.

Compartment	Abbreviation	Compliance [L/kPa]	Volume (Litres)
Left Lower	LD	0.35	35
Left upper	LH	0.25	25
Right Lower	PD	0.25	25
Right Upper	PH	0.25	25
Right Middle	PS	0.10	10

Table 2: compartments of the model

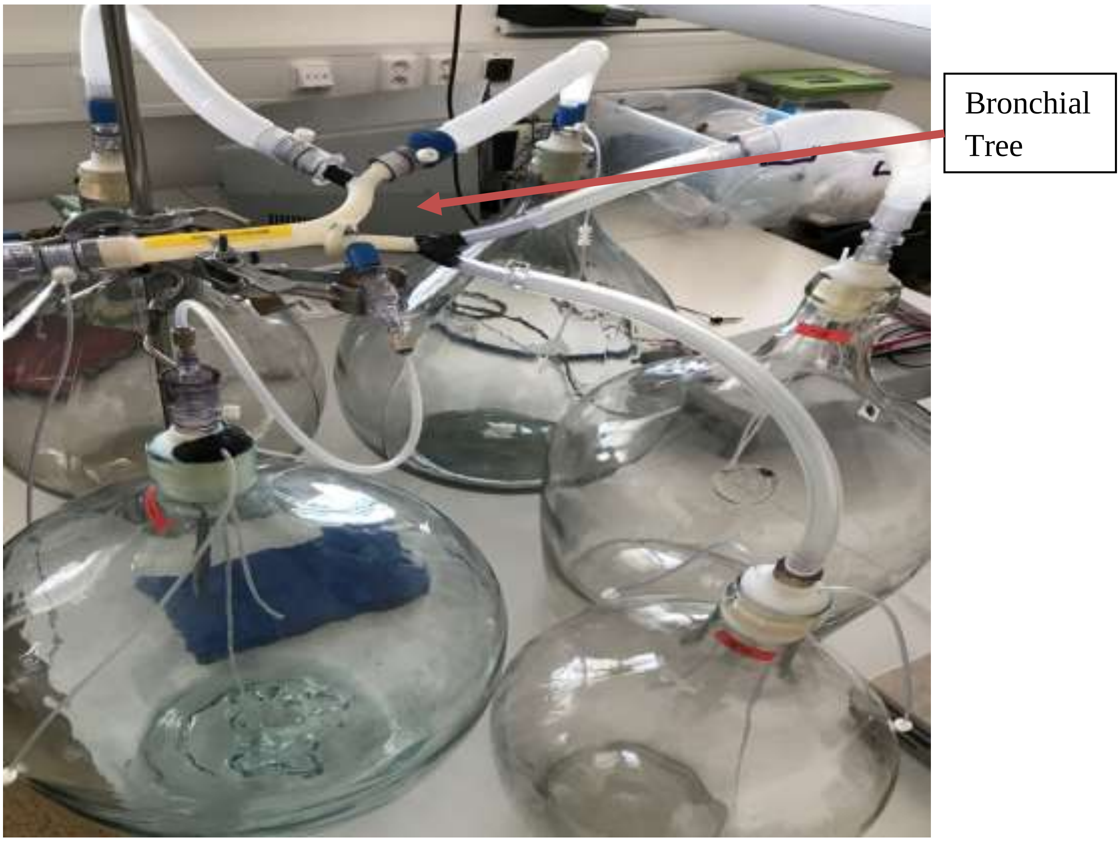


Figure 4: Demijohns representing compartments of the lungs

Figure 5: Five compartment model of the respiratory system

3.2 Methods and Measurement

The ventilatory circuit consists of a positive pressure ventilator, a 3D printed model of the central airways connecting the rigid demijohns via silicon tubes.

Pressure data was acquired at six different channels, five at each demijohn and one at the input of flow from the ventilator to the central airways.

Every demijohn of the ventilatory circuit has a piezoelectric pressure sensor inserted inside it. The pressure inside each demijohn was measured by interfacing sensors with a special device developed to measure pressures from ten inputs simultaneously in range of 0-6.5 kPa. This device consists of ten pressure sensors 26PC01 (Honeywell, USA) and a multifunction data acquisition board Ni USB – 6009 (Texas Instruments, USA). Each of the six channels was calibrated in advance to the experiment and the offset of every channel was compensated.

The Sampling rate of this device is 100 Hz. Six different regimes of the ventilator were used over a period of 2 minutes each and hence approximately 12 minutes of pressure data was obtained at each channel corresponding to every demijohn in the ventilatory circuit and the pressure data at the input to the respiratory circuit.

The pressure data is acquired using National Instruments' LabView software by interface with the Data acquisition board and then this data is imported in MathWorks' MATLAB for further analysis.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4
B	500	12	1:1	8
C	500	12	1:2	4
D	500	12	1:2	8
E	500	12	1:3	4
F	500	12	1:3	8

Table 3: Regimes of the ventilator

The pressure distribution during different regimes was analysed in individual demijohns to observe the changes in PEEP and I:E ratio.

Figure 6: 'Chobotnice', device to measure pressure at 10 inputs

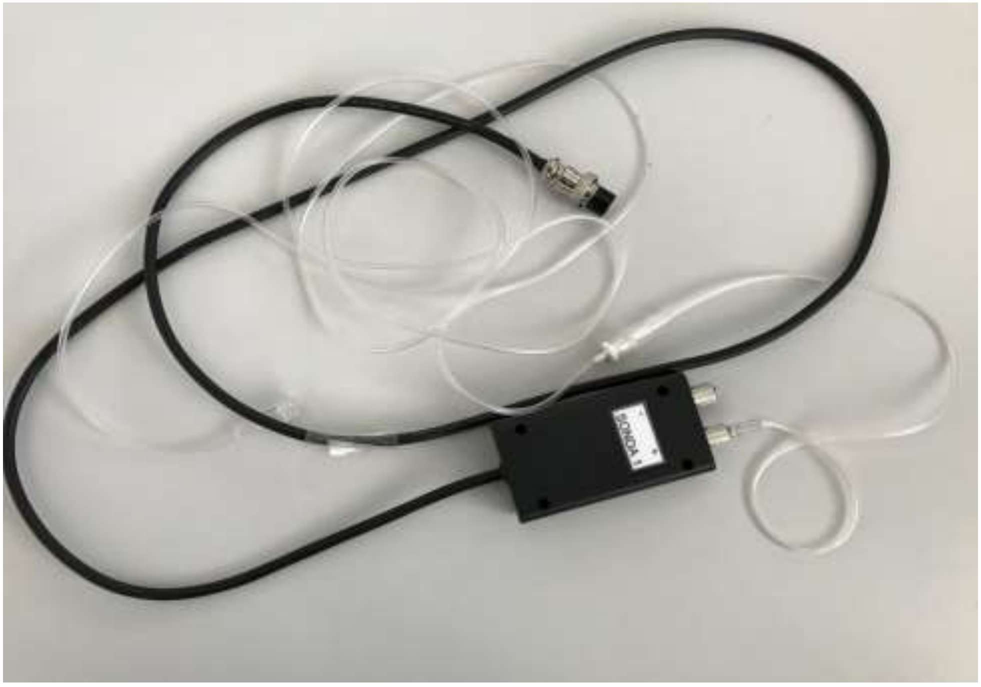


Figure 7: Pressure sensor

3.3 Results

3.3.1 Comparison for Regime 'A' in the left upper 'LH' Demijohn with the pressure at the input to the respiratory circuit.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4

Table 4: Regime 'A' of the ventilator

Compartment	Abbreviation	Volume (Litres)
Left upper	LH	25

Table 5: Left Upper 'LH' compartment of the model

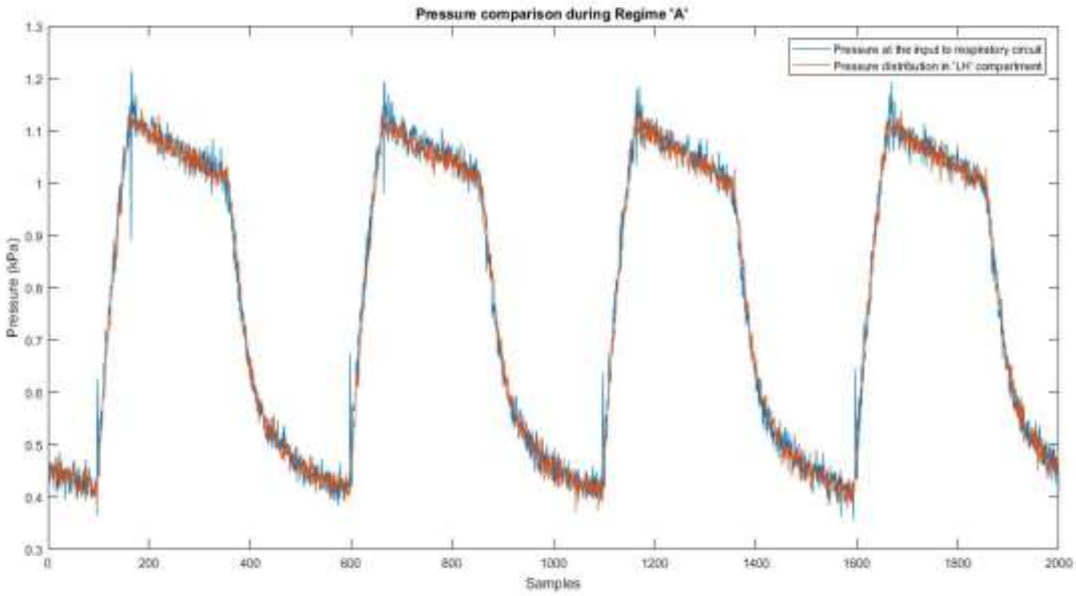


Figure 8: Regime 'A' comparison between pressure in 'LH' compartment and at the input of respiratory circuit

The blue pressure curve depicts the pressure at the input to the respiratory circuit and the red curve depicts the pressure distribution in the Left Upper compartment of the five compartment model. From the graph it can be concluded that the pressure distribution is quite stabilized inside the compartment as the pressure values are almost the same.

3.3.2 Difference in PEEP for Left lower ‘LD’ compartment.

To observe the difference in PEEP levels inside the Left Lower compartment, the pressure distribution inside the ‘LD’ compartment can be observed for Regime ‘C’ and regime ‘D’.

The PEEP during Regime ‘C’ is 4cm H_2O and PEEP during regime ‘D’ is 8cm H_2O, this can be observed graphically.

Compartment	Abbreviation	Volume (Litres)
Left Lower	LD	35

Table 6: Left Lower compartment 'LD' of the model

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
C	500	12	1:2	4
D	500	12	1:2	8

Table 7: Regimes of the Ventilator, Regime 'C' and 'D'

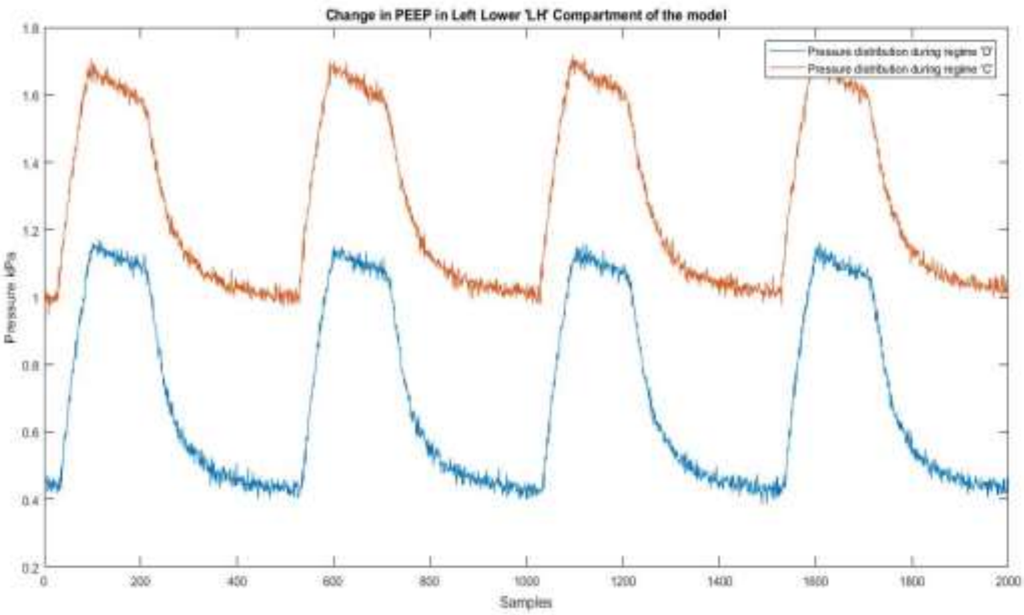


Figure 9: PEEP comparison in 'LD' compartment

The blue curve depicts pressure distribution when the PEEP is set to 4cm H2O and the red curve depicts the pressure distribution when the PEEP is set to 8cm H2O.

3.3.3 Difference in I: E ratio for Right middle ‘PS’ compartment.

To observe the difference in I:E ratio levels inside the Left Lower compartment, the pressure distribution inside the ‘PS’ compartment can be observed for Regime ‘A’ and regime ‘E’.

The I:E ratio for the flow inside the compartment during regime ‘A’ is 1:1 i.e. the inspiratory period and expiratory period is the same, in this case 2.5seconds each for the inspiratory and expiratory periods. During regime ‘E’, I: E ratio for the flow is 1:3 i.e. the inspiratory period is 1.25 seconds and the expiratory period is 3.75 seconds.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4
E	500	12	1:3	4

Table 8: Regimes of the ventilator, Regime'A' and 'E'

Compartment	Abbreviation	Volume (Litres)
Right Middle	PS	10

Table 9: Right middle 'PS' compartment of the model

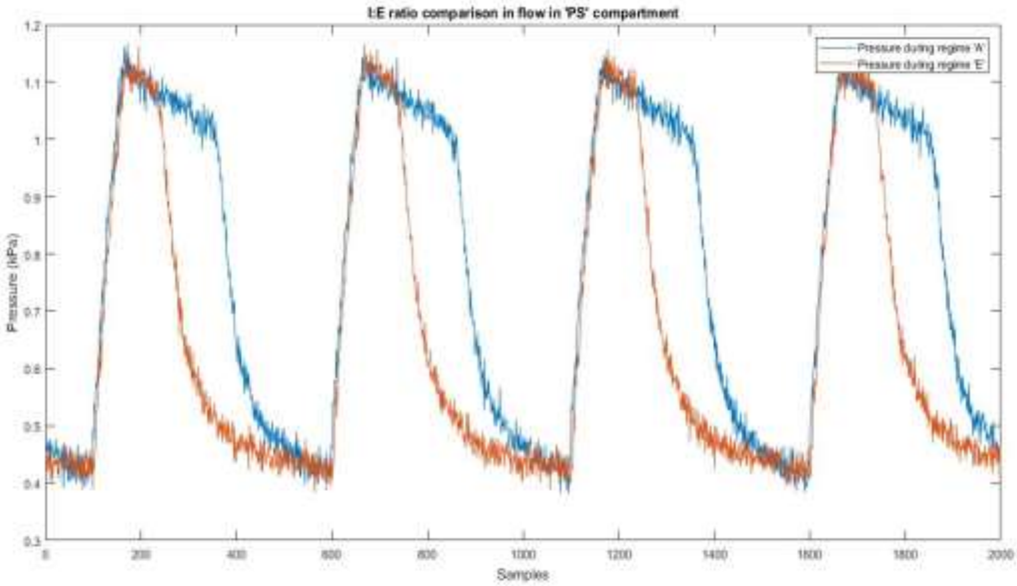


Figure 10: I:E comparison in 'PS' compartment

To blue curve depicts the flow inside ‘PS’ compartment during regime ‘A’ when I:E ratio is 1:1 and the red curve depicts the flow during regime ‘E’ when I:E ratio is 1:3. From the curves it can be observed that the blue curve has an equal inspiratory and expiratory phase, whereas the red curve has a smaller inspiratory phase that the expiratory phase indicating that the set volume is delivered to the respiratory model in a very short period of time ($1/4^{th}$ of the entire breath cycle) and a longer amount of time is for the expiration of the inspired breath.

3.4 Conclusions

From the graphs above, it can be safely concluded that the pressure distribution inside every demijohn of the respiratory model is homogenous. The pressure inside the demijohns is stabilized and has reached a point where the pressure at the input to the respiratory circuit and the pressure in every demijohn follow a very similar trend.

Moreover, from the pressure curves above, it can be seen that under different ventilatory regimes the pressure distribution curve differs in its shape and size and behaves according to the characteristic parameters of the set regime.

The changes in the PEEP and I:E ratio are quite evident from the results and in each regime, the pressure curves show a different shape based on the set PEEP and I:E ratio. In every demijohn of the circuit, the PEEP and I:E ratio changes affects the pressure distribution in a similar manner hence it can be concluded that resistances present in the circuit do not affect the flow in a major form. The change in I:E ratio produces a change in time trend of the pressure curve, which is as expected. At the same time the change of PEEP causes a change in amplitude of the pressure curve, also as expected.

The pressure amplitudes measured in different demijohns with differing compliances show no significant differences from each other and confirm to a previously published result [13].

4. Mathematical Model

This Section describes the construction of the Mathematical model in National Instruments' SIMULINK software corresponding to the five compartment physical model of the respiratory system.

This section also describes the co-relation between a respiratory system (with some simplifications) and an acoustic system and the use of electro-acoustic analogy to use electric laws to solve acoustic or mechanical systems.

The analysis on similarity between the results obtained experimentally on the physical model and the results obtained by the mathematical simulation is done in this section to show the accuracy of the model.

A mathematical model of the mechanical ventilator is a demonstration of the behaviour of the mechanical ventilator during artificial ventilation. The purpose of this thesis is to create and simulate a mathematical model (MM) of a volume controlled ventilator (VCV). This MM represents some important controlled parameters such as Positive End Expiratory Pressure (PEEP) and Inspiratory to Expiratory ratio (I: E ratio). The mathematical model of the respiratory system created for VCV is constructed for simulation by using the SIMULINK software package of MathWorks' MATLAB platform. The obtained simulator of a mechanical ventilator can represent the pressure signal of a continuous mandatory ventilation of volume controlled ventilation as a complete cycle of respiration and a continuous waveform. The simulator can reflect the change in parameters such as peak inspiratory pressure (PIP), PEEP and I: E ratio which can be visualised by pressure vs. time waveforms. The modelled simulator provides a simple environment to demonstrate the functioning of a mechanical ventilator for testing and monitoring VCV signal and other parameters during artificial ventilation. Furthermore, the MM can be used for training and visualisation purposes.

4.1 Simulation Method

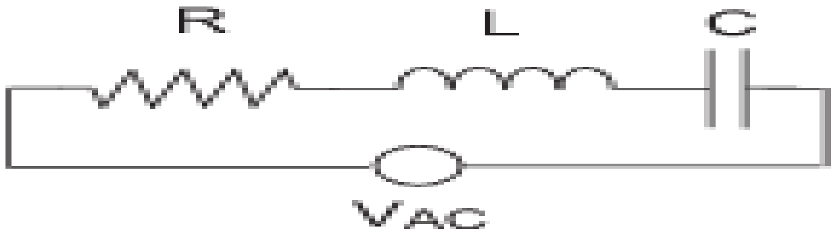
4.1.1 General Description

Two of the main programs used by engineers for simulation are LabView and Matlab. LabView is a virtual instruments platform used for automatic control and measuring technology, it simplifies the development procedures for process control and software testing. LabView has now become one of the major instruments in measuring and testing of technologies and instruments. It is possible to use LabView to acquire data easily from the pressure sensors from each compartment of the model due to LabView's strong hardware interface capability, however the limited toolboxes in LabView restricts the usage for a large application program. In our case, for complex application and large data processing and a complicated control algorithm, LabView is not the most adequate. Conversely, Matlab has very strong computing functions and a stable algorithm library. It has highly precise and efficient toolboxes in almost all engineering and computing fields. But it lacks in data communication, interfacing and net communication. Since both the software, Matlab and LabView have contrasting specialities; we can combine both for an efficient modelling of respiratory system. We use LabView for data acquisition and interface with the pressure sensors measuring pressure from the compartments of the lungs and Matlab for programming and simulating the physical model of the respiratory system mathematically. The data collected from the lung compartments by LabView can be sent to Matlab for purpose of computing and simulation, the processed data can then again be transferred back to LabView for output control and display.

The Simulink package in Matlab can further use the data acquired and can be used to model virtually the five compartment model of the respiratory system. Simulink can be used to simulate the exact behaviour of the pressure signal under the influence of various parameters such as compliance and resistance in the model and provide an accurate representation of the pressure distribution in the model.

4.1.2 Simple Model Simulation

It is possible to formulate a mathematical system that is to be modelled in several ways. Choosing the best-form mathematical model allows the simulation to execute faster and accurate. For example, consider a simple series RLC circuit.



According to Kirchhoff's law, the voltage drop across this circuit is equal to the sum of the voltage drop across each element of the circuit.

$V_{AC} = V_R + V_L + V_C$

Using Ohm's law to solve for the voltage across each element of the circuit, the equation for this circuit can be written as,

$$V_{Ac} = R_i + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

It is possible to model this system in Simulink, by solving for either the resistor voltage or inductor voltage. Choosing the appropriate system affects the structure of the model, its response and performance.

For the model, solving the RLC series using resistor voltage,

$$V_R = R_i$$

So,

$$R_i = V_{AC} - L\frac{di}{dt} - \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt \qquad (1)$$

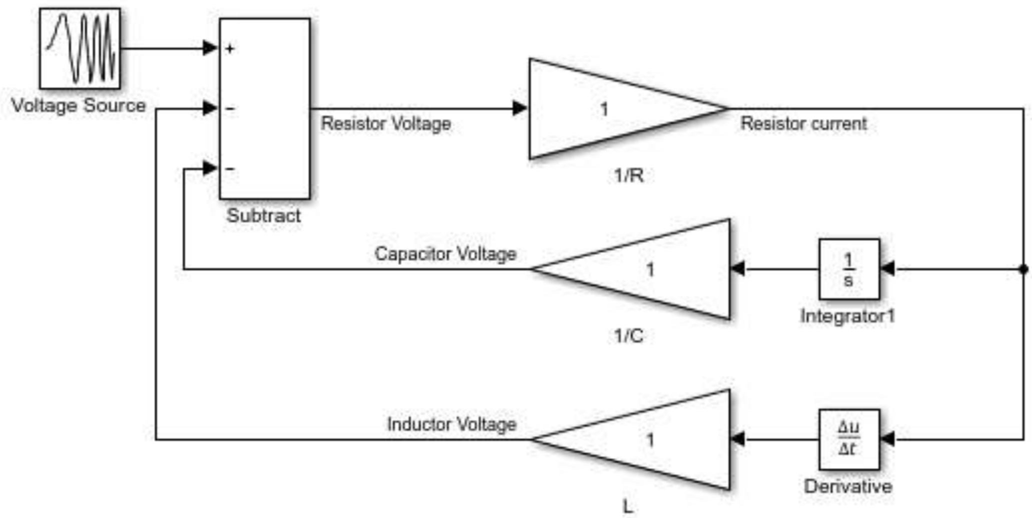


Figure 11: Series RLC circuit: Formulated to solve for resisor current

The above diagram shows the equation (1) modelled in Simulink. The resistor voltage is the sum of the voltage source, the capacitor voltage, and the inductor voltage. Here, we need the current in the circuit to calculate the capacitor and inductor voltages. To calculate the current, multiply the resistor voltage by a gain of 1/R. Calculate the capacitor voltage by integrating the current and multiplying by a gain of 1/C. Calculate the inductor voltage by taking the derivative of the current and multiplying by a gain of L.

A respiratory system (with some simplifications) can be considered as an acoustic system. Here we simplify the above describes system using the electro-acoustic analogy.

4.1.3 Electro-acoustic analogy

Lot of Variation in regimes for artificial lung ventilation (ALV) has been introduced in the history so far. The effectiveness of each regime is dependent on a lot of parameters including mechanical parameters of the respiratory system.

Many parameters of the respiratory system change during ALV and it affects the efficiency of ALV.

One of the parameters affecting the ALV efficiency is Airway resistance; it directly determines the airway flow during peak pressure.

Electro acoustic analogy is the relation between electric and acoustic circuits. It allows using electric laws to solve acoustic or mechanical systems.

Electro-Acoustic Analogy	
Electrical System	**Acoustic system**
Voltage [V]	Pressure [P]
Current [i]	Volume velocity [W]
Charge [q]	Volume shift
Inductor – Inductance [L]	Acoustic Inertor – Acoustic weight [m_a]
Resistor- Resistance [R]	Acoustic Resistor – Acoustic resistance [r_a]
Capacitor – Capacity [C]	Elastor – Acoustic compliance [c_a]

Table 10: Electro-Acoustic analogy

Respiratory system (with some simplifications) can be considered as an acoustic system.

To describe an acoustic system, the geometrical parameters of the acoustic system are necessary (in our case, the demijohns or the five compartments).

It is possible to compute the acoustic elements of the demijohns, if we know the geometrical dimensions, according to following equations.

- $m_a = \frac{\rho_0}{S} l$ [$kg.m^{-4}$; $kg.m^{-3}$, m, m^{-2}]–{Ac. Weight}

- $c_a = \frac{V}{\rho_0 c_0^2}$ [$m^4.s^2.kg^{-1}$; m^3, $kg.m^{-3}$, $(m.s^{-1})^2$]–{Ac. Compliance}

- $r_a = \frac{8\mu l}{\pi R_t^4}$ [N.s.m^{-5}; N.s.m^{-2}, m, (m)^4]–{Ac. Resistance}

ρ_0 = air density (kg.m^{-3}) = 1.293 kg.m^{-3}

l = length of the tube (m)

S = Cross-section of the tube (m^2)

V = Volume of the demijohn (m^3)

c_0 = Velocity of propogation (m.s^{-1}) = 340 m.s^{-1}

μ = dynamic air viscocity (N.s.m^{-2}) = 1.84x10^{-5} N.s.m^{-2}

R_t = Radius of the tube (m)

So, upon calculating the constants,

- $m_a = \frac{1.293}{S} l$ kg.m^{-4} - { ac. Weight}

- $c_a = \frac{V}{149470.8}$ m^4.s^2.kg^{-1} – { ac. Compliance}

- $r_a = \frac{(4.68\ x\ 10^{-5})}{R_t^4} l$ N.s.m^{-5} – {ac. Resistance}

4.1.4 Geometric Dimensions of the acoustic system

Now, the geometric dimensions of the acoustic system are as follows,

Demijohn	V- volume (l)	L - length of tube (cm)	S - cross section area (cm^2)	R_t – Radius of tube (cm)
LD	35	40	3.14	1
LH	25	40	3.14	1
PD	25	36	0.64	0.45
PH	25	24	0.64	0.45
PS	10	33	0.64	0.45

Table 11: Geometrical dimensions of demijohns

From the geometric dimensions, the acoustic elements for each demijohn are as follows,

LH (Left Upper)	
r_a	1.872×10^{3} $N.s.m^{-5}$
c_a	16.67×10^{-8} $m^{4}.s^{2}.kg^{-1}$
m_a	1.64×10^{3} $kg.m^{-4}$

Table 12: Acoustic elements for Left Upper demijohn

LD (Left Lower)	
r_a	1.872×10^{3} $N.s.m^{-5}$
c_a	23.33×10^{-8} $m^{4}.s^{2}.kg^{-1}$
m_a	1.64×10^{3} $kg.m^{-4}$

Table 13: Acoustic elements for Left Lower demijohn

PD (Right Lower)	
r_a	41 x 10^3 N.s.m^{-5}
c_a	16.67 x 10^{-8} $m^4.s^2.kg^{-1}$
m_a	7.2 x 10^3 kg.m^{-4}

Table 14: Acoustic elements for Right Lower demijohn

PH (Right Upper)	
r_a	27.4 x 10^3 N.s.m^{-5}
c_a	16.67 x 10^{-8} $m^4.s^2.kg^{-1}$
m_a	4.8 x 10^3 kg.m^{-4}

Table 15: Acoustic elements for Right Upper demijohn

PS (Right Middle)	
r_a	37.6 x 10^3 N.s.m^{-5}
c_a	6.67 x 10^{-8} $m^4.s^2.kg^{-1}$
m_a	6.66 x 10^3 kg.m^{-4}

Table 16: Acoustic elements for Right Middle demijohn

Now, going back to equation (1) and using electro-acoustic analogy

$$R_i = V_{AC} - L\frac{di}{dt} - \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

So,

$$r_a w = P_{AC} - m_a\frac{dw}{dt} - \frac{1}{c_a}\int_{-\infty}^{t} w(t)\,dt \text{---------------------------------------(2)}$$

For the mathematical modeling these acoustic parameters of every compartment will be utilized.

4.1.5 Mathematical Model

The equation (2) can be modeled in simulink as follows,

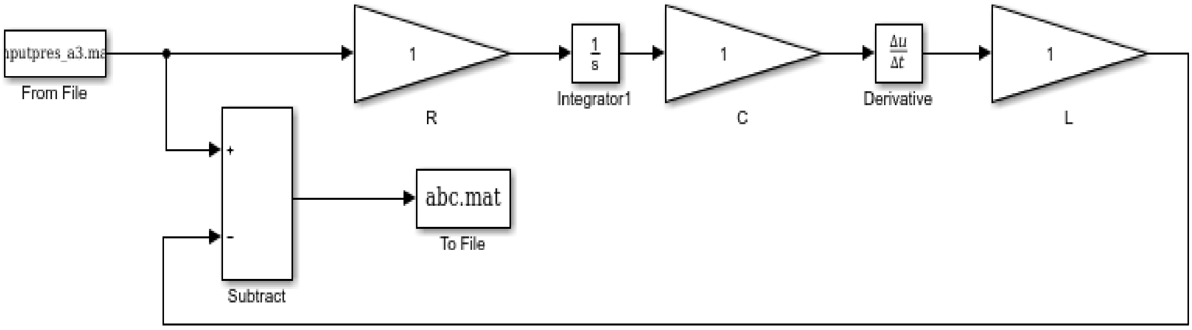


Figure 12: Mathematical Model for one lobe

The figure (2) depicts the mathematical model for a single lobe of the respiratory model, a Simulink model consisting of five lobes of the respiratory system can be modelled as follows.

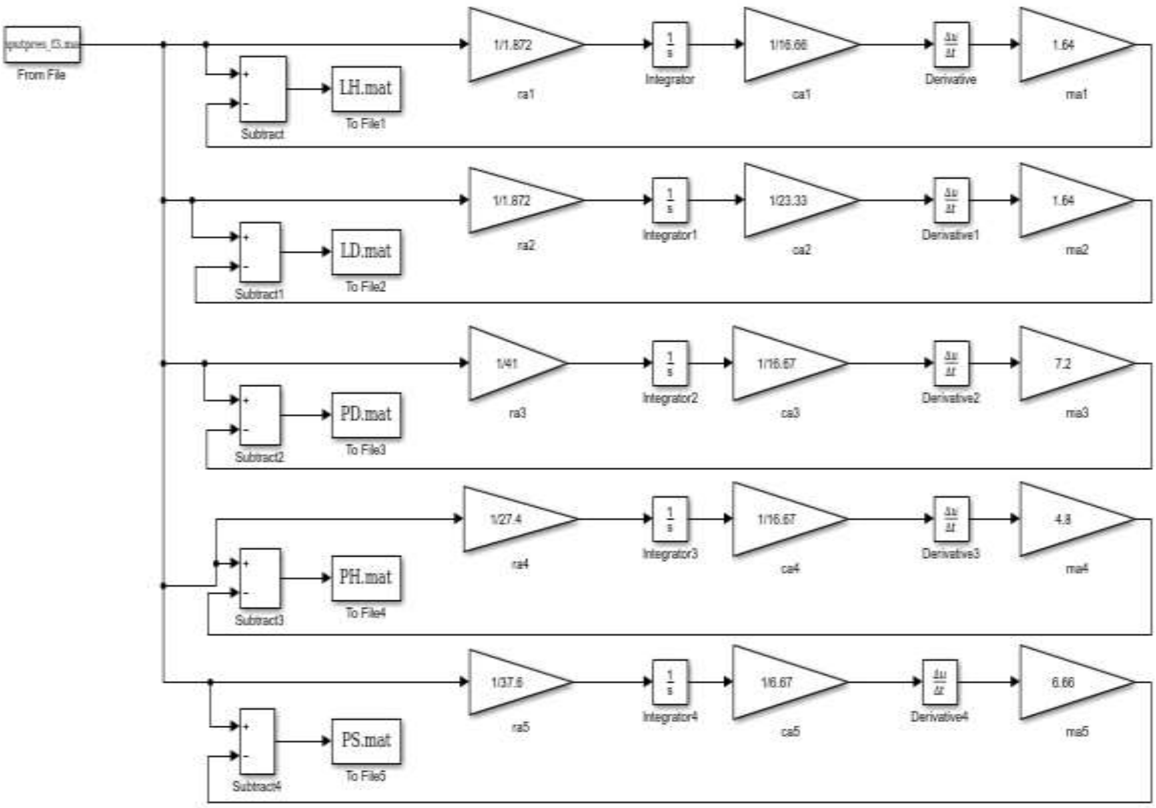


Figure 13: Five compartment mathematical model

4.2 Simulation performance

4.2.1 Input Signal

There was a very large amount of pressure data for the input to the model and equally large amount of data for the output at every lobe in the model; hence the data was compressed for the signal to a length which can be run in Simulink.

For the purpose of the analysis only the regiment 'A' was used i.e. 500 ml volume at 12 BPM at an I:E ratio of 1:1 and PEEP at 4cm H_2O.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4

Table 17: Regimes of the experiment, Regime'A'

Just to begin with, the input pressure signal to the respiratory pathway at the sixth sensor was compressed to 1:20th of its original size hence having to deal with only pressure values 1/20th times its original size.

Using the code,

```
>>load ('input_pres')                          input_pres     1593000x1
>> input_presA= input_pres (1:235000);         input_presA    235000x1
>>input_presA1=input_presA (1:20:235000);      input_presA1  11750x1
>>input_presA2=input_presA1 (1:2000);          input_presA2   2000x1
```

This input_presA2 signal was used as an input pressure signal to the model which is not too large to run the simulation. The final pressure values to be used to run the simulation looked like as follows:

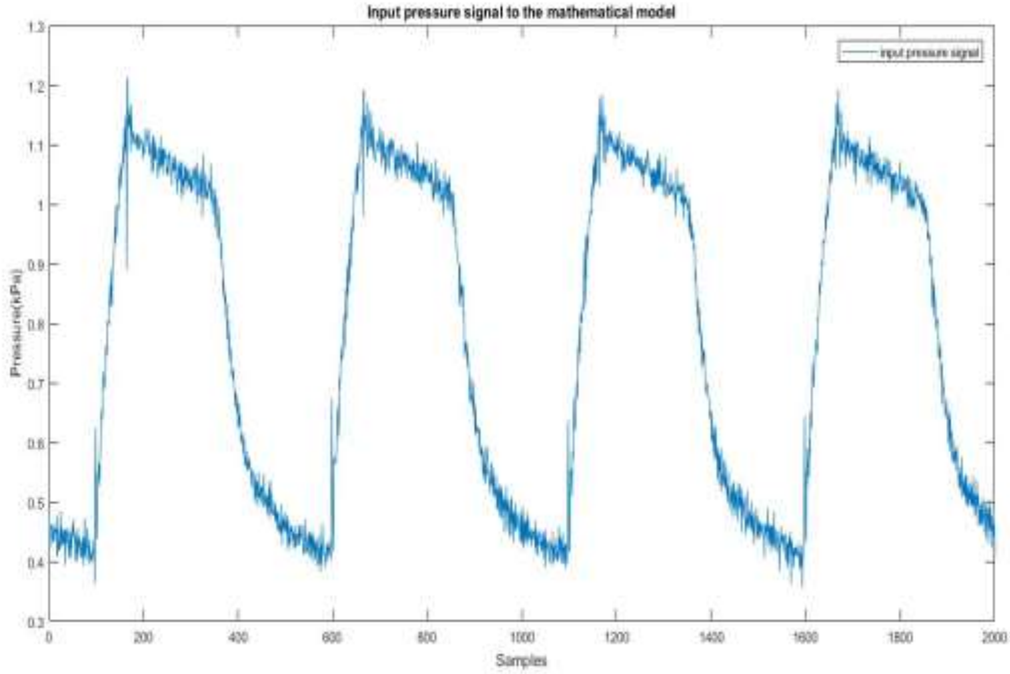


Figure 14: Input pressure to the model

The above displayed pressure values were used as an input to the five compartment model described in figure (3).

4.2.2 Output comparison with physical model

For the comparison of the output signal from the mathematical model and the physical model, we will consider the lobe 'LD'

Compartment	Abbreviation	Volume
Left Lower	LD	35l

Table 18: Compartments of the model, Compartment 'LD'

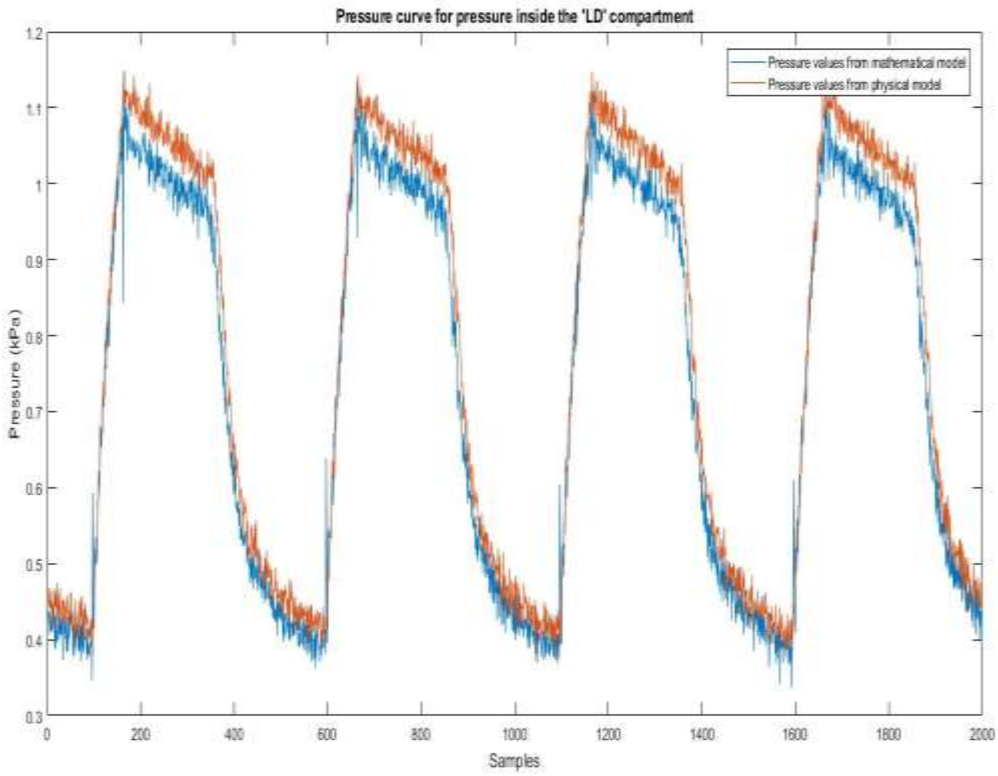


Figure 15: Pressure peak comparison of physical model vs mathematical model for 'LD' compartment

The blue curve depicts the pressure inside the Left Lower compartment simulated from the mathematical model and the red curve depicts the pressure inside the Left Lower compartment obtained from the experimental results on the physical model.

The output comparison between the physical and mathematical model shows a very close similarity in the pressure peaks and can hence satisfy the fact that the mathematical model is very close to representing the performance of the results from the physical model.

4.3 Verification of the model for a different lobe

To verify the working of model, we can use a different regime and a different lobe to prove that the output results are similar to the lobe 'LD' under regiment 'A'.

Now for consideration, the regime 'F' i.e. 500 ml volume at 12 BPM at I:E ratio 1:3 and PEEP set at 8cm H_2O.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
F	500	12	1:3	8

Table 19: Regimes of the Ventilator, Regime 'F'

We will use the lobe 'PS' for the consideration.

Compartment	Abbreviation	Volume
Right Middle	PS	10l

Table 20: Compartments of the model, Compartment 'PS'

The input pressure signal used for this looks like as below:

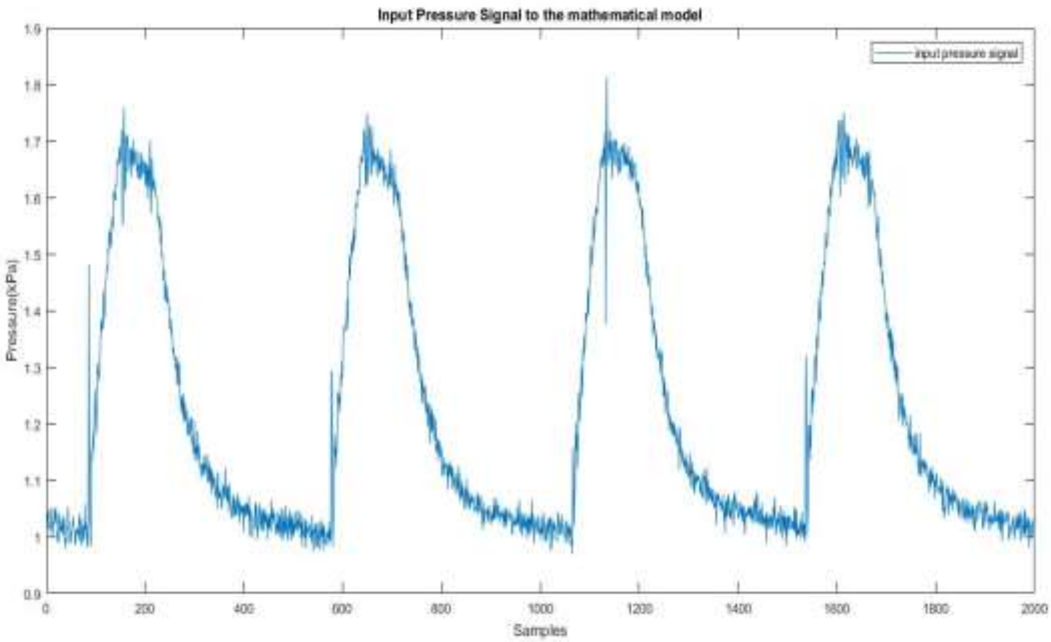


Figure 16: Input Pressure, Regime 'F' to the model

4.4 Pressure Comparison with Physical model for'PS' Compartment

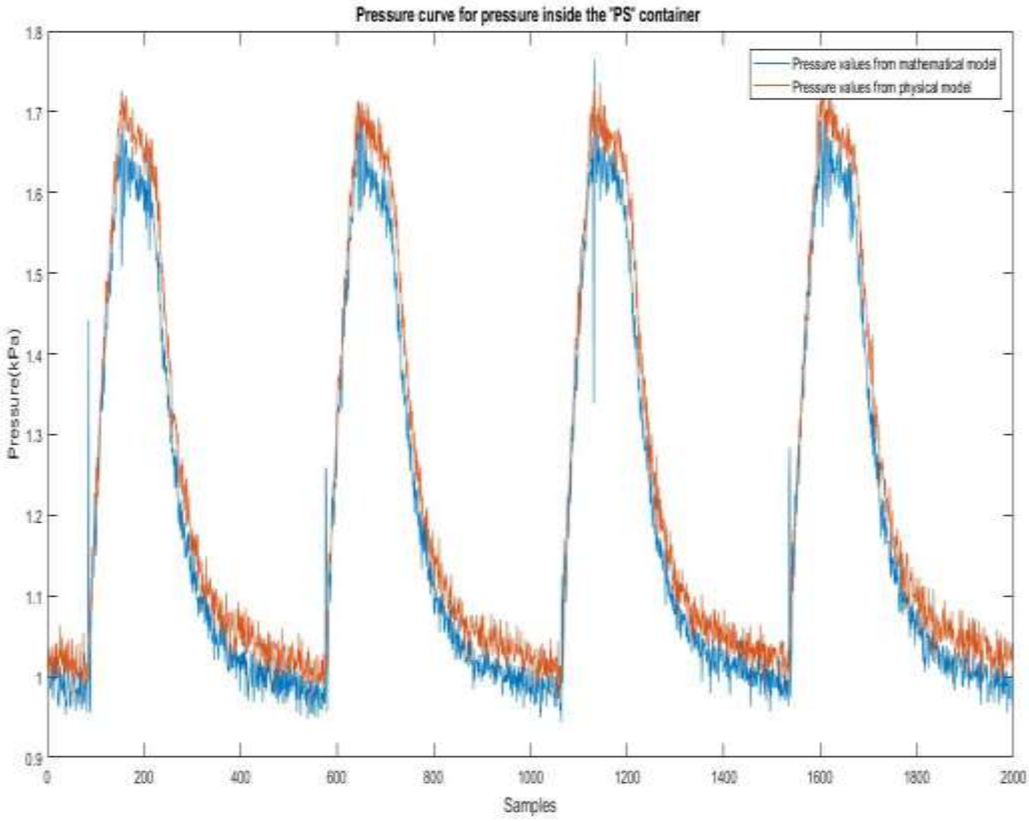


Figure 17: Pressure peak comparison of Mathematical model vs Physical model for 'PS' compartment

The blue curve depicts the pressure inside the Right Middle compartment simulated from the mathematical model and the red curve depicts the pressure inside the Right Middle compartment obtained from the experimental results on the physical model.

The output comparison between the physical and mathematical model shows a very close similarity in the pressure peaks and can hence satisfy the fact that the mathematical model is very close to representing the performance of the results from the physical model.

4.5 Other Results to test the working of Mathematical Model

4.5.1 Verification of the mathematical model using ’PH‘ compartment

It is necessary to prove that the mathematical model works under all the regimes and for all the compartments, hence using a different combination of regimes and compartments, the functionality of the mathematical model and its ability to provide pressure results close to that expected from physical experiments shall be proved.

To verify the working of model, we can use a different regime and a different lobe to prove that the simulation results are similar to physical experiments.

Now for consideration, the regime ‘C’ i.e. 500 ml volume at 12 BPM at I:E ratio 1:2 and PEEP set at 4cm H_2O.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
C	500	12	1:2	4

Table 21: Regimes of the ventilator, Regime 'C'

We will use the lobe ‘PH’ for the consideration.

Compartment	Abbreviation	Volume
Right Upper	PH	25l

Table 22: Compartments of the model, Compartment 'PH'

4.5.1.1 Pressure comparison with physical model for ‘PH‘ compartment

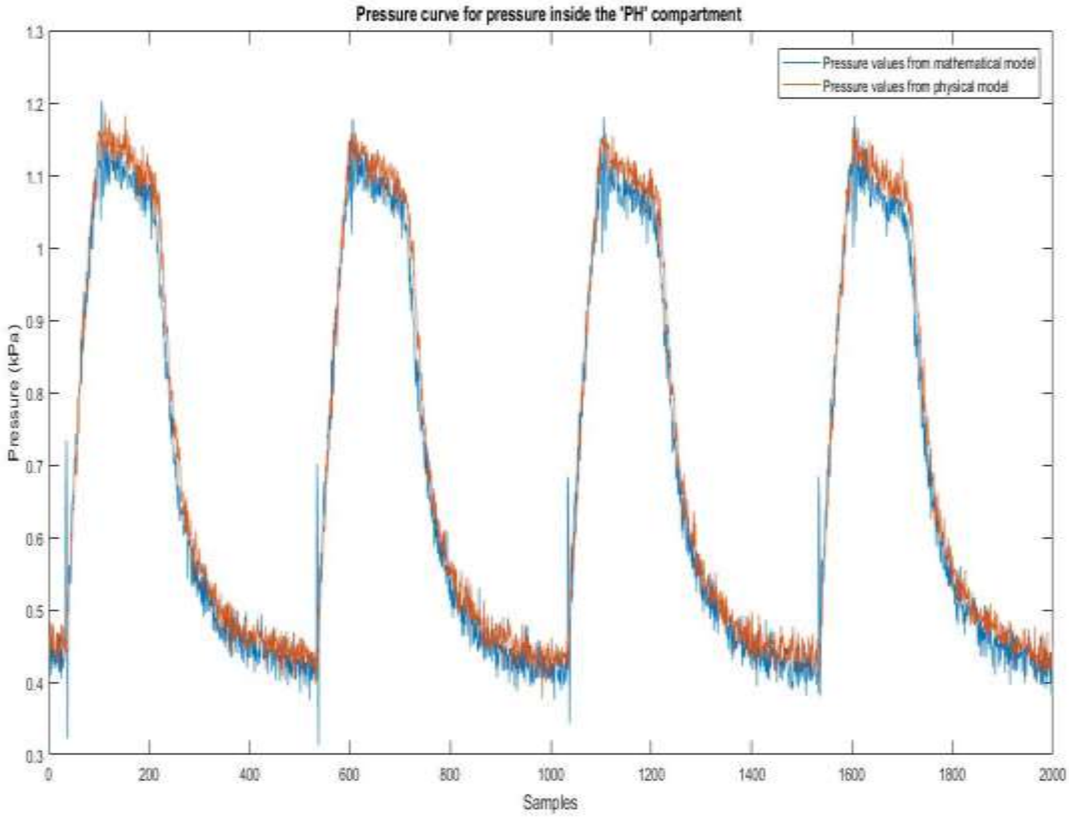


Figure 18:Pressure peak comparison of Mathematical model vs Physical model for ’PH‘ compartment

The blue curve depicts the pressure inside the Right Upper compartment simulated from the mathematical model and the red curve depicts the pressure inside the Right Upper compartment obtained from the experimental results on the physical model.

The output comparison between the physical and mathematical model shows a very close similarity in the pressure peaks and can hence satisfy the fact that the mathematical model is very close to representing the performance of the results from the physical model.

4.5.2 Verification of the mathematical model using ’PD‘ compartment

This time we use the ‘PD’ compartment to check the appropriate working of the mathematical model.

Now for consideration, the regime ‘E’ i.e. 500 ml volume at 12 BPM at I:E ratio 1:3 and PEEP set at 4cm H_2O.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
E	500	12	1:3	4

Table 23: Regimes of the ventilator, Regime 'E'

For the compartment ‘PD’,

Compartment	Abbreviation	Volume
Right Lower	PD	25l

Table 24:Compartments of the model, Compartment 'PD'

4.5.2.1 Pressure comparison with physical model for ‘PD’ compartment

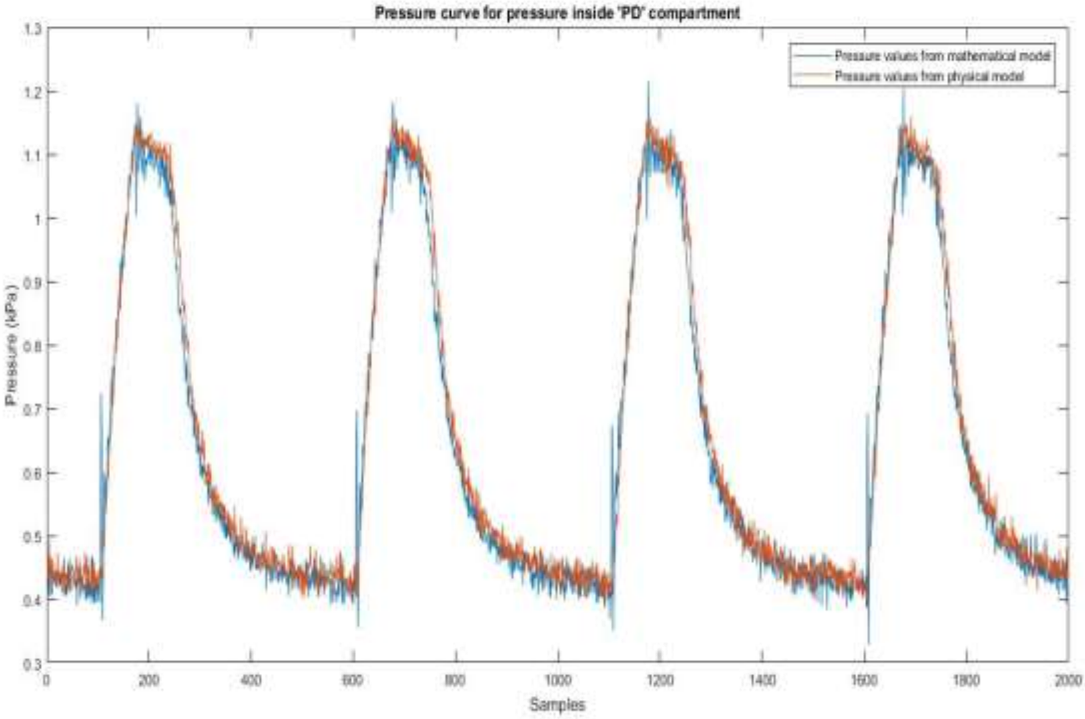


Figure 19:Pressure peak comparison of Mathematical model vs Physical model for ’PD‘ compartment

The blue curve depicts the pressure inside the Right Lower compartment simulated from the mathematical model and the red curve depicts the pressure inside the Right Lower compartment obtained from the experimental results on the physical model.

The output comparison between the physical and mathematical model shows a very close similarity in the pressure peaks and can hence satisfy the fact that the mathematical model is very close to representing the performance of the results from the physical model.

4.5.3 Use of Mathematical model to analyse the change in I:E ratio

Using the mathematical model it is possible to analyse the changes in various parameters of ventilatory regimes such as PEEP and I:E ratio. Two different regimes can be analysed to observe the changes in I:E ratio in a compartment of the model.

For this observation Regime 'B' i.e. Volume of 500 ml at 12 BPM with PEEP 8cm H_2O and I:E ratio 1:1 is compared to Regime 'D' i.e. Volume of 500 ml at 12 BPM with PEEP 8cm H_2O and I:E ratio 1:2. Here, the I:E ratio is the only different parameter and the mathematical model can be used to graphically observe this difference in the compartment.

Under these ventilatory conditions, the mathematical model can be used to simulate the pressure distribution in the 'LH' compartment.

Compartment	Abbreviation	Volume
Left upper	LH	25l

Table 25: Compartments of the model, 'LH' compartment

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
B	500	12	1:1	8
D	500	12	1:2	8

Table 26: Regimes of the ventilator, Regime 'B' vs Regime 'D'

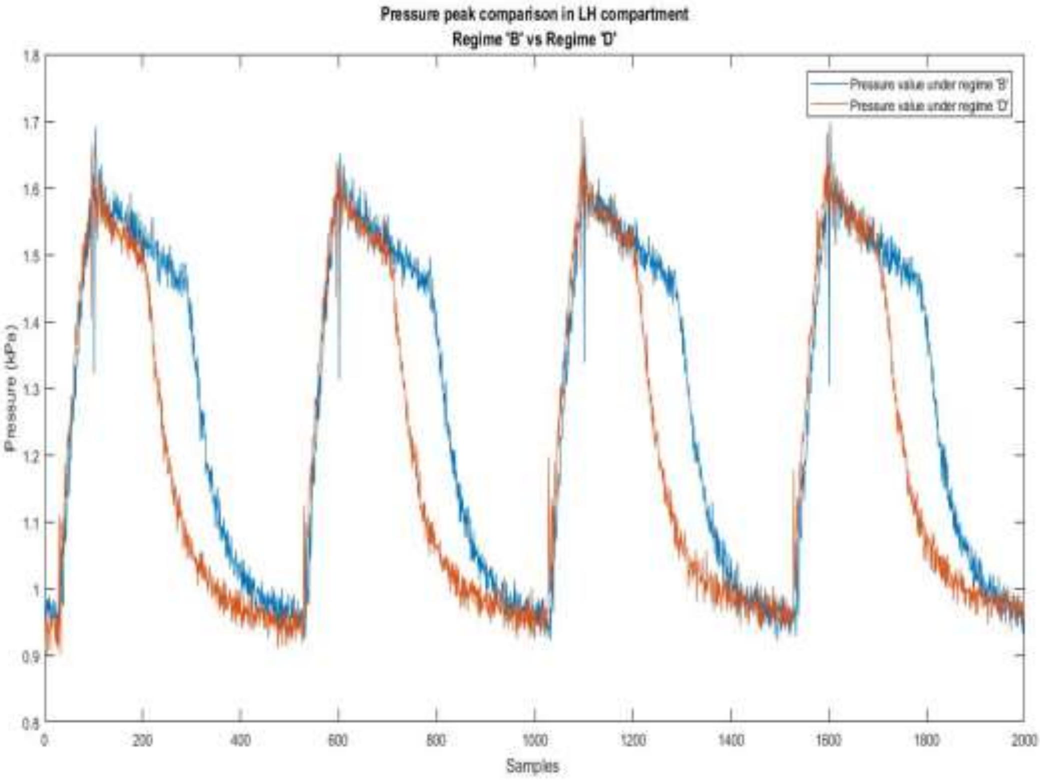


Figure 20: Regime 'B'vs 'D' comparison for pressure curve

The blue curve indicates the pressure distribution in the left upper 'LH' compartment of the respiratory model during regime 'B', the I:E ratio under regime 'B' is 1:1 and hence it can be observed that area under the curve for inspiratory and expiratory phase is the same. Since Breaths per minute are set to 12, then each breath (i.e. one inspiratory and expiratory phase) is 5 seconds long. With I:E ratio at 1:1, the inspiratory and expiratory phase are both 2.5 seconds long.

The red curve indicates the pressure distribution in the left upper 'LH' compartment of the respiratory model during regime 'D', the I:E ratio under regime 'D' is 1:2 and hence it can be observed that area under the curve for expiratory phase is longer than the area under the curve for inspiratory phase. Since Breaths per minute are set to 12, then each breath (i.e. one inspiratory and expiratory phase) is 5 seconds long. With I:E ratio at 1:2, the inspiratory phase is 1.66 seconds and expiratory phase is 3.33 seconds long.

4.5.4 Use of Mathematical model to analyse the change in PEEP

Using the mathematical model it is possible to analyse the changes in various parameters of ventilatory regimes such as PEEP and I: E ratio. Two different regimes can be analysed to observe the changes in PEEP in a compartment of the model.

For this observation Regime ‘A’ i.e. Volume of 500 ml at 12 BPM with PEEP 4cm H_2O and I: E ratio 1:1 is compared to Regime ‘B’ i.e. Volume of 500 ml at 12 BPM with PEEP 8cm H_2O and I: E ratio 1:1. Here, the PEEP is different in both the regimes and the mathematical model can be used to graphically observe this difference in a compartment.

Under these ventilatory conditions, the mathematical model can be used to simulate the pressure distribution in the ‘LD’ compartment.

Compartment	Abbreviation	Volume
Left Lower	LD	35l

Table 27: Compartments of the model, Compartment 'LD'

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4
B	500	12	1:1	8

Table 28: Regimes of the ventilator, Regime 'A' vs Regime 'B'

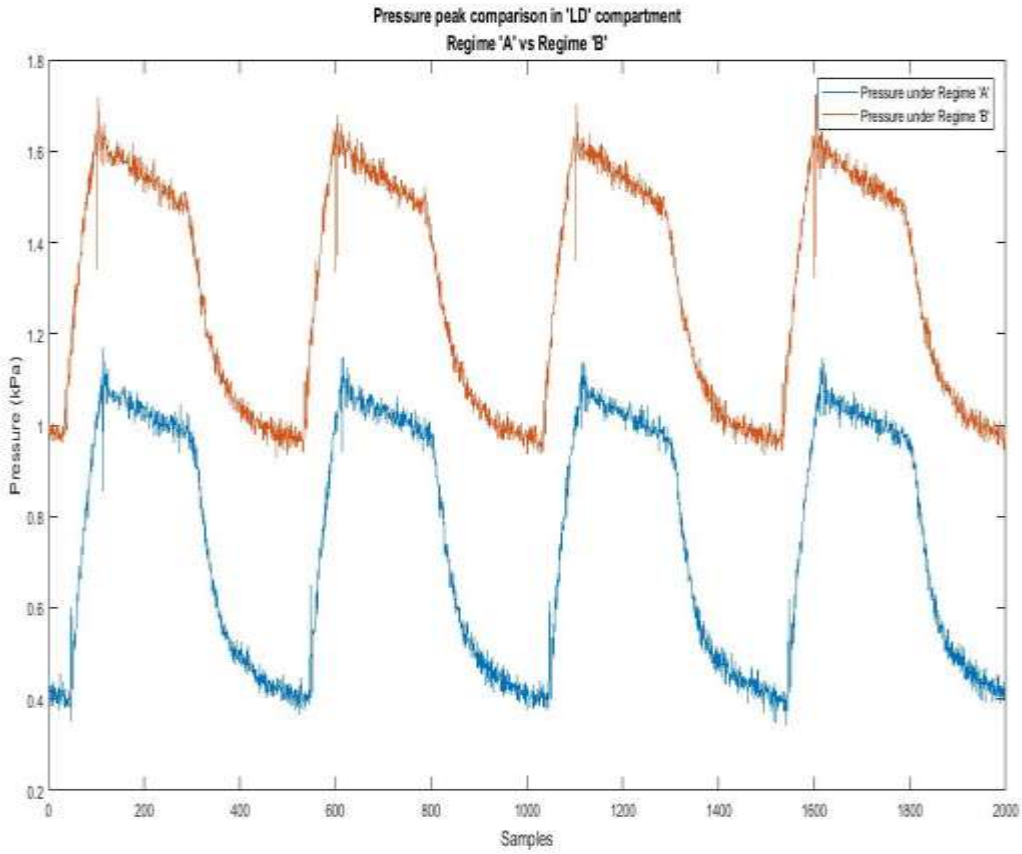


Figure 21: Pressure peak comparison, Regime 'A' vs 'B'

The blue curve indicates the pressure distribution in the left lower ‘LD’ compartment of the respiratory model during regime ‘A’, the PEEP under regime ‘A’ is 4cm H_2O and hence it can be observed that curve originates at approximately 0.4 kPa.

The red curve indicates the pressure distribution in the left lower ‘LD’ compartment of the respiratory model during regime ‘B’, the PEEP under regime ‘B’ is 8cm H_2O and hence it can be observed that curve originates at approximately 1kPa.

Using such a comparative experiment, it is possible to analyse the performance of the ventilator under any regime and it is possible to observe the pressure distribution in every compartment.

4.5.5 Use of Mathematical model to analyse the change in PEEP & I:E ratio

Using the mathematical model it is possible to analyse the changes in various parameters of ventilatory regimes such as PEEP and I: E ratio. Two different regimes can be compared to observe the changes in PEEP as well as I: E ratio in a compartment of the model.

For this observation Regime ‘A’ i.e. Volume of 500 ml at 12 BPM with PEEP 4cm H_2O and I: E ratio 1:1 is compared to Regime ‘F’ i.e. Volume of 500 ml at 12 BPM with PEEP 8cm H_2O and I: E ratio 1:3. Here, the PEEP and I:E ratio, both are different in both the regimes and the mathematical model can be used to graphically observe this difference in a compartment.

Under these ventilatory conditions, the mathematical model can be used to simulate the pressure distribution in the ‘PS’ compartment.

Compartment	Abbreviation	Volume
Right Middle	PS	10l

Table 29: Compartment of the model, Compartment 'PS'

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4
F	500	12	1:3	8

Table 30: Regimes of the ventilator, Regime 'A' vs Regime 'F'

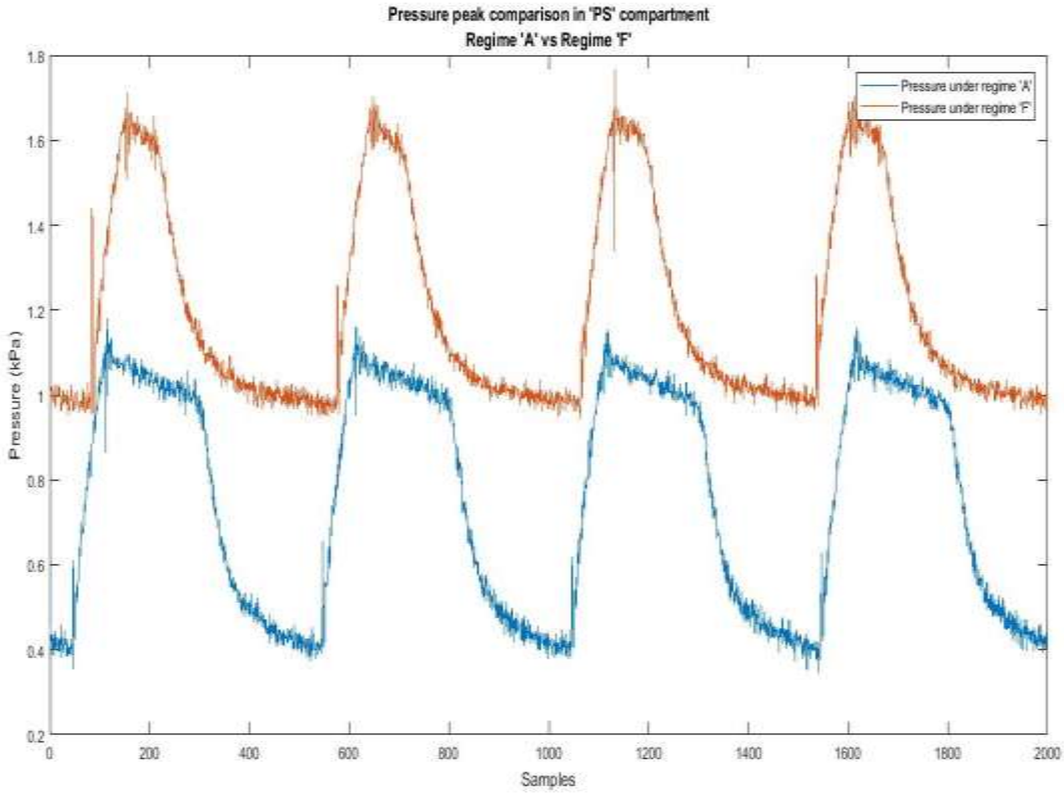


Figure 22: Pressure peak comparison, Regime 'A' vs 'F'

The blue curve indicates the pressure distribution in the Right Middle ‘PS’ compartment of the respiratory model during regime ‘A’, the PEEP under regime ‘A’ is 4cm H_2O and hence it can be observed that curve originates at approximately 0.4 kPa. Also, it can be observed that the I: E ratio under regime ‘A’ is 1:1 and hence it can be observed that area under the curve for inspiratory and expiratory phase is the same. Since Breaths per minute are set to 12, then each breath (i.e. one inspiratory and expiratory phase) is 5 seconds long. With I: E ratio at 1:1, the inspiratory and expiratory phase are both 2.5 seconds long.

The red curve indicates the pressure distribution in the left lower ‘LD’ compartment of the respiratory model during regime ‘B’, the PEEP under regime ‘B’ is 8cm H_2O and hence it can be observed that curve originates at approximately 1kPa. The I: E ratio under regime ‘F’ is 1:3 and hence it can be observed that area under the curve for expiratory phase is longer than the area under the curve for inspiratory phase. Since Breaths per minute are set to 12, then each breath (i.e. one inspiratory and expiratory phase) is 5 seconds long. With I: E ratio at 1:3, the inspiratory phase is 1.25 seconds and expiratory phase is 3.75 seconds long

Using such a comparative experiment, it is possible to analyse the performance of the ventilator under any regime and it is possible to observe the pressure distribution in every compartment.

5. Dead Space analysis in the five compartment model

Dead space, in physiology can be defined as the volume of air which is inhaled but does not participate in the exchange of gases; it could be because of either of the two reasons.

i) Air remains in the conducting airways.

ii) Air reaches the alveoli that are not perfused or poorly perfused.

In other words, not all the air that is breathed in during inspiration is available for exchange of gases. Mammals breathe in and out of their lungs and waste the part of the inspired air that remains in the conducting pathways where the gas exchange does not occur. In humans, about 1/3rd of every resting breath has no change in Oxygen and Carbon dioxide levels. In adults, it is approximately 150 ml.

When a patient is ventilated, the breathing pattern, the rate of breathing and the tidal volume of the air delivered to the patient is dictated by the ventilator. Because of the dead space, taking deep breaths at a low frequency (for example. 500 ml at 12 BPM = 6l/min) is more efficient than shallow breaths at high frequency (for example 250 ml at 24 BPM =6l/min) although the minute volume is the same (6l/min), a significant amount of shallow breaths is dead space and does not allow oxygen to get into blood.

Dead space can be increased by breathing through a tube with smaller diameter and with a larger resistance. The smaller diameter of the tube traps the air and delays the supply of air during each inspiration as it does not have sufficient space to deliver the required volume of breath during each inspiration hence a person inhales a significant quantity of air that has remained in the tube from the previous exhalation cycle.

To monitor the pressure trends in a compartment of the lungs under additional dead space, the diameter of the pathway providing flow to the demijohn is reduced and the pressure curve is compared with pressure curve for the compartment with a pathway of regular size and a larger size and the results are analysed in this chapter.

5.1 Pressure comparison for ’PH‘ compartment in physical model

In the five-compartment model of the respiratory system, the silicon tube connecting the central airways to the demijohn for the Right Upper ‘PH’ compartment was replaced by another silicon tube of a much smaller diameter to increase the dead-space. This could result in a time delay in distribution of pressure inside the compartment and this can be analysed graphically.

The pressure distribution in the Right Upper ‘PH’ compartment is compared with pressure distribution in Left Upper ‘LH’ compartment. Compartment ‘LH’ still has the regular silicon tube connecting the demijohn with central airways. This comparison was made for Regime ‘A’ of the Ventilator.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4

Table 31: Regimes of the Ventilator, Regime 'A'

Compartment	Abbreviation	Volume
Left upper	LH	25l
Right Upper	PH	25l

Table 32: Compartments of the Ventilator, Compartment 'PH' and 'LH'

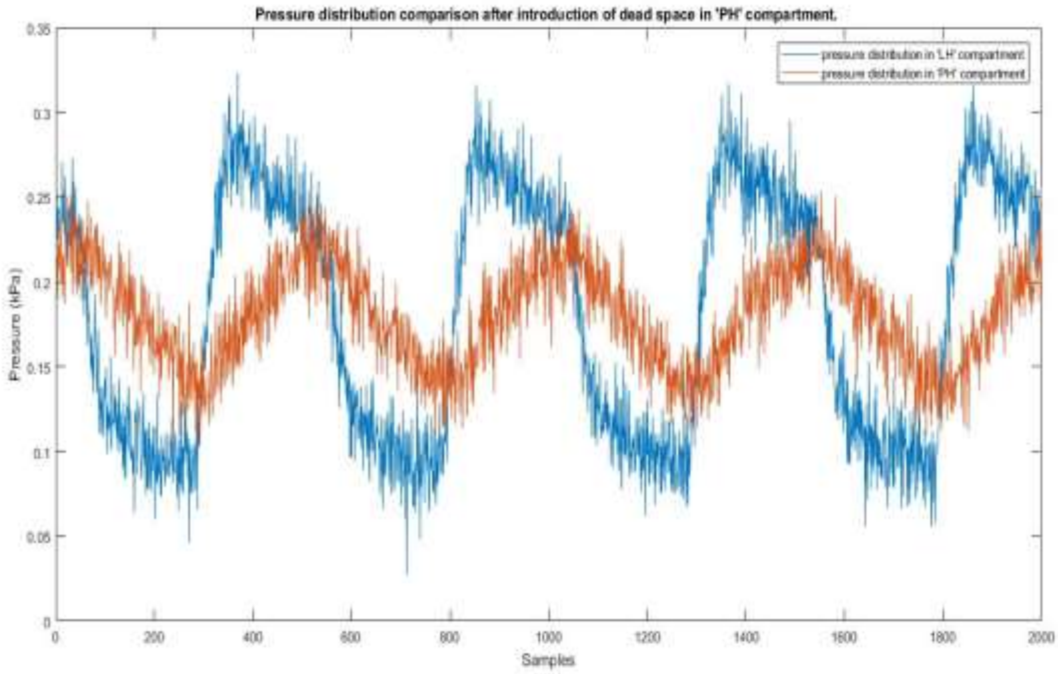


Figure 23: Pressure distribution comparison after dead space introduction in a compartment

The blue curve indicates the pressure distribution in ‘LH’ compartment, the silicon tube connecting the respiratory pathway to the demijohn is of regular size and hence there is no additional dead-space in the compartment to delay the pressure curve.

The red curve depicts the pressure distribution in ‘PH’ compartment; a silicon tube with very small diameter causes an additional dead-space which not only causes a time delay in pressure distribution inside the demijohn but also leads to a change in pressure amplitude. The pressure inside the ‘PH’ compartment is significantly lesser than the pressure inside the ‘LH’ compartment.

This can also be concluded by another example in which a different regime of ventilator is used to compare the effects of introduction of dead-space in ‘PH’ compartment.

5.2 Pressure comparison for ’PH‘ compartment using regime ’F‘

In the five-compartment model of the respiratory system, the silicon tube connecting the central airways to the demijohn for the Right Upper ‘PH’ compartment was replaced by another silicon tube of a much smaller diameter to increase the dead-space. This could result in a time delay in distribution of pressure inside the compartment and this can be analysed graphically.

The pressure distribution in the Right Upper ‘PH’ compartment is compared with pressure distribution in Left Upper ‘LH’ compartment. Compartment ‘LH’ still has the regular silicon tube connecting the demijohn with central airways. This comparison was made for Regime ‘F’ of the Ventilator.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
F	500	12	1:3	8

Table 33: Regimes of the ventilator, Regime 'F'

Compartment	Abbreviation	Volume
Left upper	LH	25l
Right Upper	PH	25l

Table 34: Compartments of the model, Compartment 'LH' and 'PH'

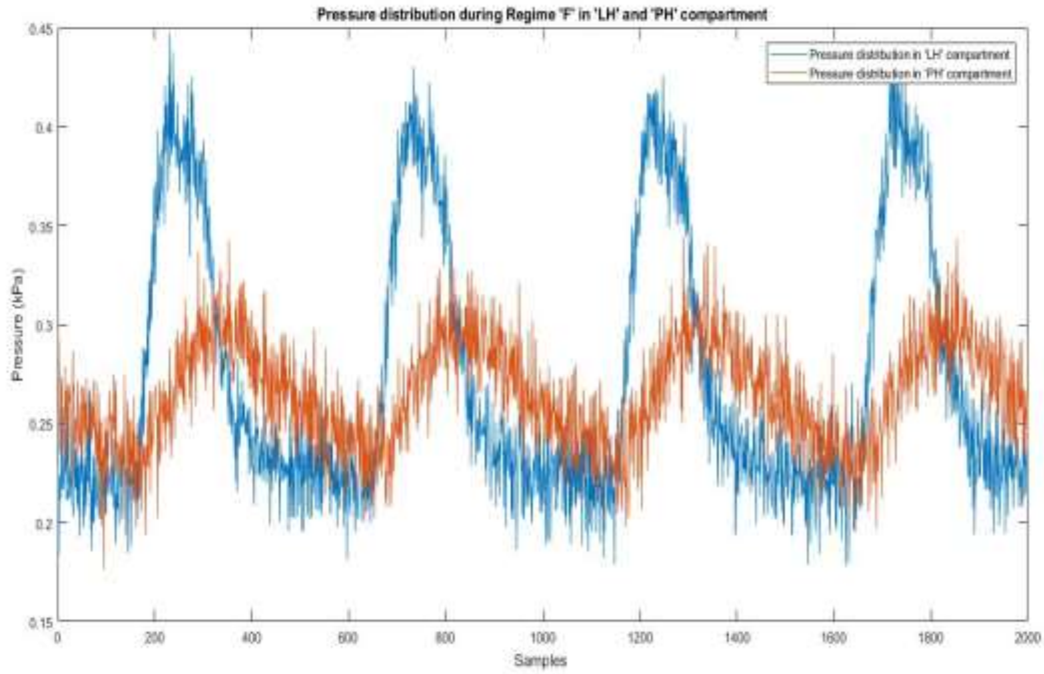


Figure 24: Pressure distribution in 'LH' and 'PH' compartment after adding dead-space

The blue curve indicates the pressure distribution in ‘LH’ compartment, the silicon tube connecting the respiratory pathway to the demijohn is of regular size and hence there is no additional dead-space in the compartment to delay the pressure curve.

The red curve depicts the pressure distribution in ‘PH’ compartment. The pressure inside the ‘PH’ compartment is significantly lesser than the pressure inside the ‘LH’ compartment.

From the pressure curves above, an additional observation can be made, that the pressure distribution in the ‘PH’ compartment is not affected by the change in I:E ratio. In ‘LH’ compartment, where on changing the regime, the I:E ratio change provides a significant difference in the time period for inspiration and expiration phase such is not the case in ‘PH’ compartment.

This fact, that the addition of dead-space leads to a state in which the change in I:E ratio does not create any significant change in inspiratory and expiratory phase can be verified by another example wherein the pressure distribution in ‘PH’ compartment can be observed under regime ‘A’ and regime ‘F’.

5.3 Pressure comparison for ’PH‘ compartment using regime ’F‘ and regime ’A‘

As seen from the pressure curves in figure 21, upon introduction of dead-space in the ‘PH’ compartment, the inspiratory and expiratory phases tend to become the same. i.e. the time required to inspire the fixed volume of air into the demijohn and the time required to expire the same volume of air from the demijohn becomes the same irrespective of the set I:E ratio, this can be further explained by comparing the pressure curves during regime ‘A’ and regime ‘F’ delivered to ‘PH’ compartment.

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4
F	500	12	1:3	8

Table 35 : Regimes of the ventilator, Regime 'A' and 'F'

Compartment	Abbreviation	Volume
Right Upper	PH	25l

Table 36: compartment of the model, compartment 'PH'

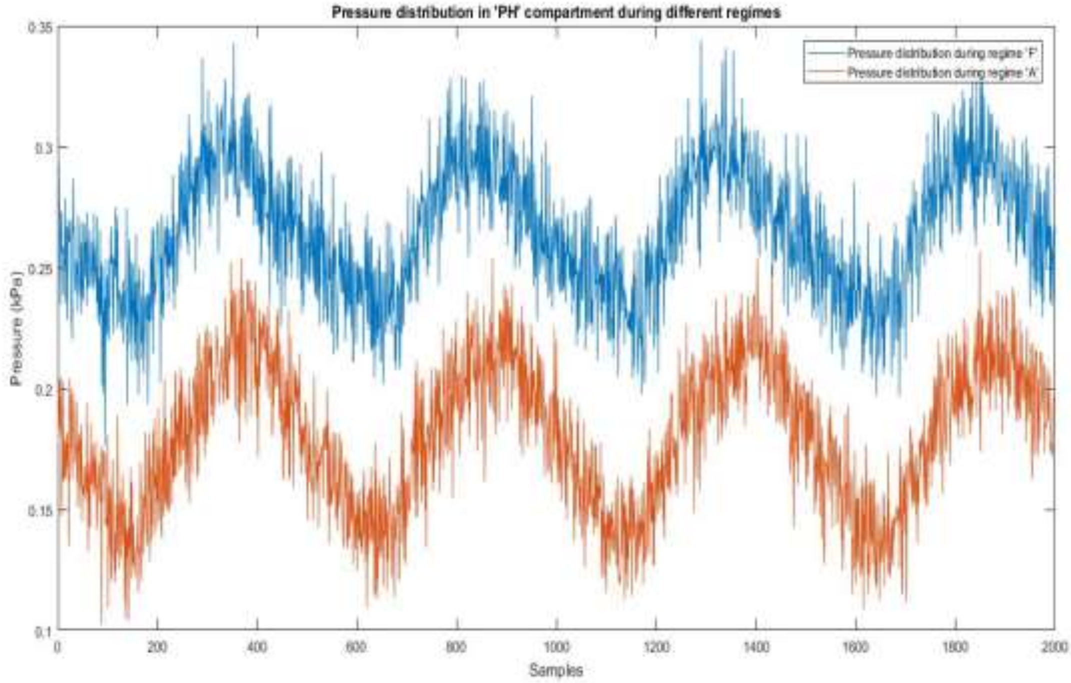


Figure 25: Pressure distribution in 'PH' compartment during Regimes 'A' and 'F'

The blue curve indicates the pressure inside the ‘PH’ compartment during regime ‘F’ and the red curve indicates the pressure during regime ‘A’.

During regime ‘F’ the I:E ratio is set to 1:3 and hence the expiratory phase is expected to be longer than the inspiratory phase whereas during regime ‘A’ the I:E ratio is set to 1:1 which means the inspiratory and expiratory phase are expected to be of the same size.

However, from the pressure curves in figure 22 it is quite evident that there is no significant difference in length of the inspiratory and expiratory phase during regime ‘F’, hence we can assume that when the silicon tube of a smaller diameter is used for connecting the respiratory pathway to the demijohn, the space required for gas exchange gets reduced and a lot of air during the expiratory phase gets retained in the pathway and is re-inspired during the inspiratory phase, hence there isn’t sufficient time for the expiration as there is another breath on its way before the expiratory phase is complete.

5.4 Mathematical Model for Dead Space Analysis in ’PH‘ compartment.

The mathematical model described in figure 9 and figure 10 can be used to compare the results from the physical model for dead space analysis in the ‘PH’ compartment. Using the geometric dimensions of the ‘PH’ compartment and acoustic elements for the ‘PH’ compartment, it is possible to analyse the pressure distribution in the demijohn after addition of deadspace.

Demijohn	L - length of tube (cm)	S - cross section area (cm^2)	V- volume (l)	R_t – Radius of tube (cm)
PH	30	0.0314	25	0.10

Table 37 : Geometric Dimensions of 'PH' compartment

PH (Right Upper)	
r_a	1.404 x 10^7 $N.s.m^{-5}$
c_a	16.67 x 10^{-8} $m^4.s^2.kg^{-1}$
m_a	1.2 x 10^5 $kg.m^{-4}$

Table 38: Acoustic elements of Right Upper, 'PH' demijohn

Using the mathematical model below, it is possible to observe the pressure distribution in ‘PH’ compartment after introduction of deadspace.

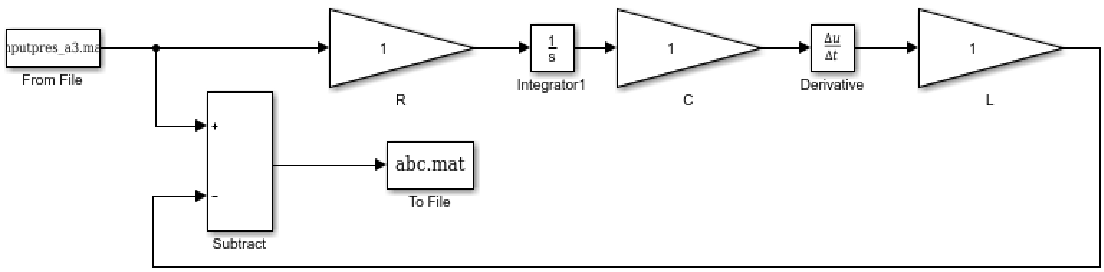


Figure 26: Mathematical model for a single compartment

For this we will compare the results from the mathematical model with the pressure inside the ‘PH’ compartment during the physical experiments, during regime ‘A’

Regimes	Volume(ml)	BPM	I:E ratio	PEEP(cmH_2O)
A	500	12	1:1	4

Table 39: Regimes of the Ventiltaor, Regime 'A'

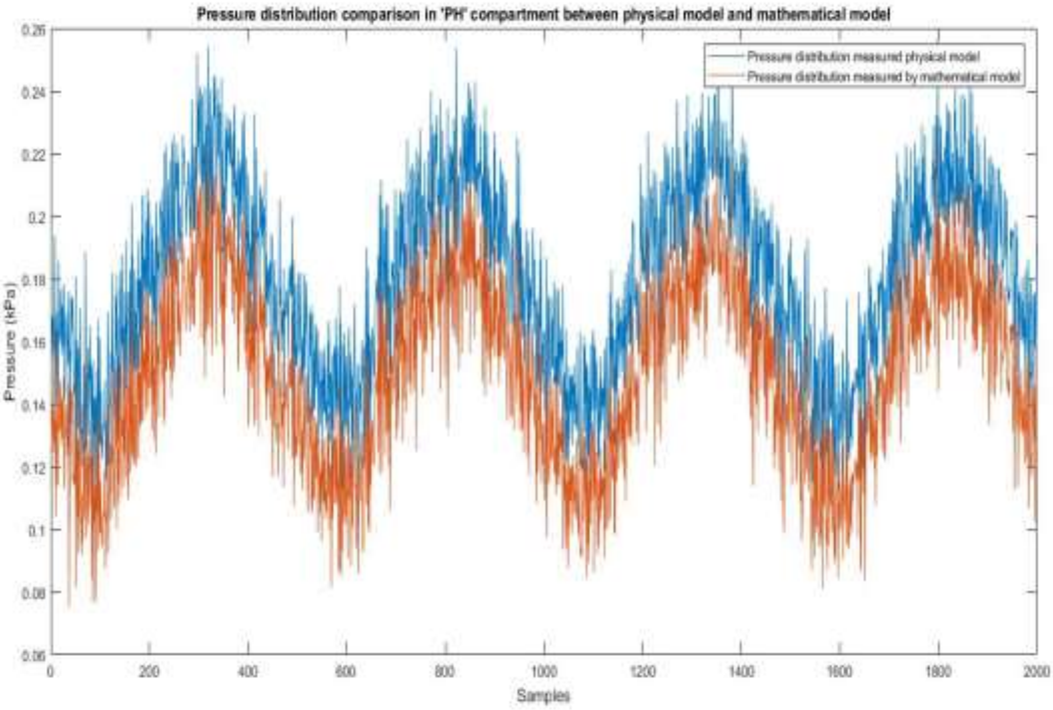


Figure 27: Dead space analysis in physical model and mathematical model

The blue curve depicts the pressure inside the 'PH' compartment measured experimentally on the physical model, whereas the red curve depicts the pressure inside the compartment, simulated by the mathematical model.

5.5 Conclusions.

From the observations from the dead space analysis in the models of the respiratory system, it can be concluded that on reducing the diameter of the silicon tube connecting the respiratory pathway to the demijohn, additional dead space is added in the compartment, which in turn affects the pressure distribution inside the ‘PH’ demijohn.

Due to the added deadspace the change in I:E ratio in the ventilator settings, does not affect the pressure distribution inside the demijohn. The inspiratory phase and expiratory phase are of the same time length which is due to retention of air in the tubes and during inspiration, the air inside the tube which is retained is re-inspired into the compartment.

In physiology, the dead space in a breathing apparatus is the space in which the breathing gas has to flow in both the directions. Due to additional dead space, the respiratory effort required for getting the same amount of usable air increases and there is also a risk of accumulation of carbon dioxide.

Dead space in breathing circuit can be reduced by using separate passages for intake and exhaust of air and by using one way valves.

6. Pressure Distribution in the model under High Frequency Oscillatory Ventilation.

High Frequency ventilators have been designed to eliminate many problems that the conventional ventilators create when they try to mimic the normal breathing. During normal breathing, a negative pressure is created which allows the gas to be drawn into the lungs by creating a negative pressure with our diaphragm. However, the conventional ventilators available today rely on positive pressure for assisting in breathing. The gas is pushed into the lungs rather than being pulled. This has greatly simplified the patient's care. But, lungs, especially premature lungs are not designed to tolerate a high amount of positive pressure.

The Lungs of infants are not fully developed to be able to tolerate a high level of positive pressure. Applying the pressure from outside as in conventional ventilators causes problems, the airways get distended and the alveoli get ruptured.

High Frequency Ventilators have been developed to reduce the chances of problems associated with conventional ventilators; these HFVs do not replicate the normal breathing but they assist in ventilation by using a much smaller tidal volume being delivered to the patient at high frequency rate. Animals and clinical studies have proven that smaller tidal volumes cause less lung injury. [14]-[15]

High frequency ventilators are different than other mechanical ventilators because they don't imitate normal breathing but help in gas exchange in a manner that is similar to panting in animals.

The two basic factors driving High frequency ventilation are smaller than normal tidal volumes and higher than normal frequencies of breathing. We can fairly assume that ventilation or CO_2 elimination is proportional to frequency x tidal volume.

$$\dot{V}_{min} = f \times V_T \text{-----} (3)$$

Where,

$\dot{V}_{min}$ = Minute Volume;

f = Ventilator Frequency;

V_T = Tidal Volume

Thus, if practically considered, there has to be a lower limit to the tidal volume that can affectively provide sufficient alveolar ventilation. This lower limit is related to the effective or physiologic dead-space by the following equation.

$$\dot{V}_A = f \times (V_T - V_D) \text{-----} (4)$$

Where,

$\dot{V}_A$ = Alveolar ventilation;

V_D = Physiologic Deadspace

Thus, as the tidal volume of the ventilator approaches the physiologic deadspace, the Alveolar ventilation becomes zero.

Hence it is necessary to examine the pressure distribution in the different compartments of the model under High Frequency Ventilation to establish the minimum pressure and frequency required to provide sufficient ventilation in the alveoli.

Figure 28: SensorMedics 3100B High Frequency Oscillatory Ventilator

For the analysis of pressure distribution in the different compartments of the model, a SensorMedics 3100B High frequency Oscillatory Ventilator has been used at a frequency of 5.0 Hz at an pressure of 20 cm H_2O and I:E ratio of 1:1.

6.1 Pressure comparison inside the compartments and at the input to the respiratory pathways.

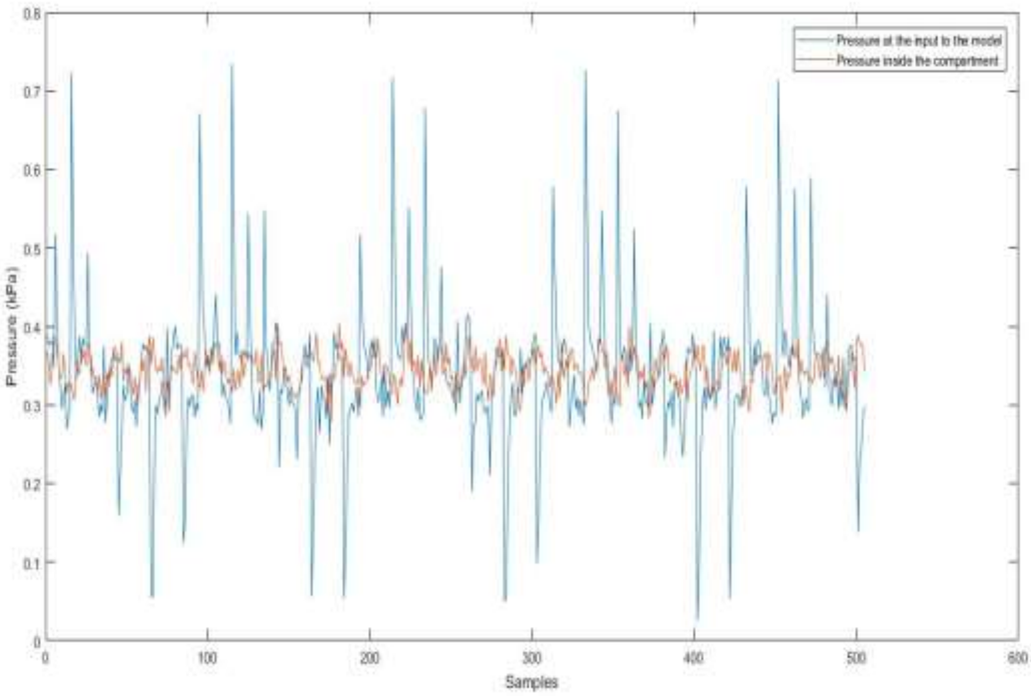


Figure 29: Pressure at the input to the pathway vs pressure inside the 'LH' compartment

The blue curve indicates the pressure measured at the input to the respiratory pathway, the red curve indicates the pressure inside the ‘LH’ compartment. From the figure it is quite evident that the pressure inside the compartment is much lower than the pressure at the input to the respiratory pathway, this could be due to the fact that the shallow breaths delivered at a very low tidal volume may not be able to create a pressure difference inside the compartment.

Also, much of the air gets retained inside the pathways before it enters the demijohn.

To verify these results, we consider another observation wherein the pressure inside various compartments is visualised.

6.2 Pressure comparison inside different compartments

The pressure distribution inside different demijohns is compared to observe the effects of High Frequency Ventilation. For this, the pressure inside the compartments, Left Lower ‘LD’, Right Upper ‘PH’ and Right Middle ‘PS’ are compared.

Compartment	Abbreviation	Volume
Left Lower	LD	35l
Right Upper	PH	25l
Right Middle	PS	10l

Figure 30: Compartments of the model, Compartment 'LD', 'PH' and 'PS'

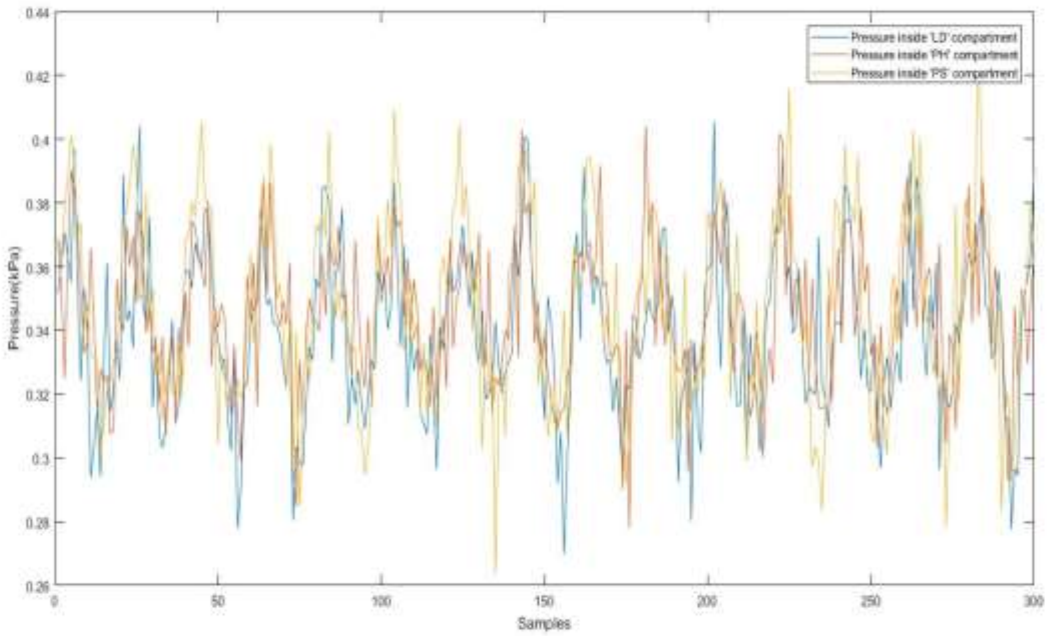


Figure 31: Pressure inside 'LD', 'PH' and 'PS' compartment under HFOV

The blue curve indicates the pressure in ‘LD’ compartment, red curve indicates the pressure in ‘PH’ compartment and yellow curve indicate pressure in ‘PS’ compartment. The pressure in all three compartments looks almost similar in magnitude.

6.3 Conclusions and discussions

From the analysis of pressure distribution under High Frequency Oscillatory Ventilation, it can be concluded that the pressure inside the compartments is much lower than the input pressure to the model. This could be due to the fact that during HFOV, the air gets retained in the respiratory pathways (giving rise to additional dead space) or due to shallow breaths at low tidal volumes, the pressure generated is not sufficiently dissipated into the demijohn during each breath.

There is not much difference in pressure distribution in the different compartments of demijohn; hence the pressure inside the demijohns is pretty much stabilized for all compartments of the model.

From such observations we can conclude that combining the relationships of gas velocity, dead space, breathing frequency and lung mechanics it is possible to achieve appropriate minute ventilation and compensate for very small tidal volumes.

7. PCO_2 and gas pressure analysis for a single compartment.

PCO_2 is the partial pressure of carbon dioxide, a measure of the relative concentration of the gas in air or in a fluid, such as plasma. PCO_2 values are sensitive indication of efficiency or level of ventilation in the lungs; it is expressed quantitatively in millimetres of mercury (mm Hg). Alveolar PCO_2 directly reflects pulmonary gas exchange in relation to blood flow: alveolar PCO_2 usually decreases as the respiration rate increases. Normal values for arterial and alveolar PCO_2 are between 35 and 45 mm Hg. Higher levels occur in conditions of slow blood flow and respiration. Below-normal values are caused by hyperventilation and lead to respiratory alkalosis. The goal of this experiment is to try and determine how the PCO_2 changes on change in Breath per minute (BPM) and Tidal Volume (V_T) in a fixed volume plastic compartment of 28litres. It is hypothesized that the PCO_2 value depends on the Tidal volume and the Minute Volume introduced into the compartment irrespective of the frequency of the breath and also the gas pressure inside the compartment is dependent only on the Tidal Volume being delivered inside the compartment. These hypotheses shall be proven by experimental results and graphical interpretation.

The purpose of this study is to analyse the changes in PCO_2 levels in a fixed volume compartment under different ventilatory regimes using a volume controlled mode of ventilation to understand the dependency of PCO_2 levels on various parameters such as Breath Per Minute (BPM), Tidal Volume (V_T) and Minute Volume ($\dot{V}$). At the same time the study also analyses the changes in gas pressure inside the compartment to understand its dependency on the above mentioned ventilator parameters as well.

7.1 Methods and Materials

The experimental setup for this study includes a 28 litre plastic compartment in which gas is introduced, a Siemens servo ventilator 300, controlled in a volume controlled mode, two piezoelectric pressure sensors along with Siemens SIRECUST 404-1 multi-parameter monitoring device for monitoring the pressure inside the compartment and on the input to the compartment and Radiometer ABL 800 FLEX Blood gas analyser for measuring the PCO_2 inside the compartment and at the input to the compartment.

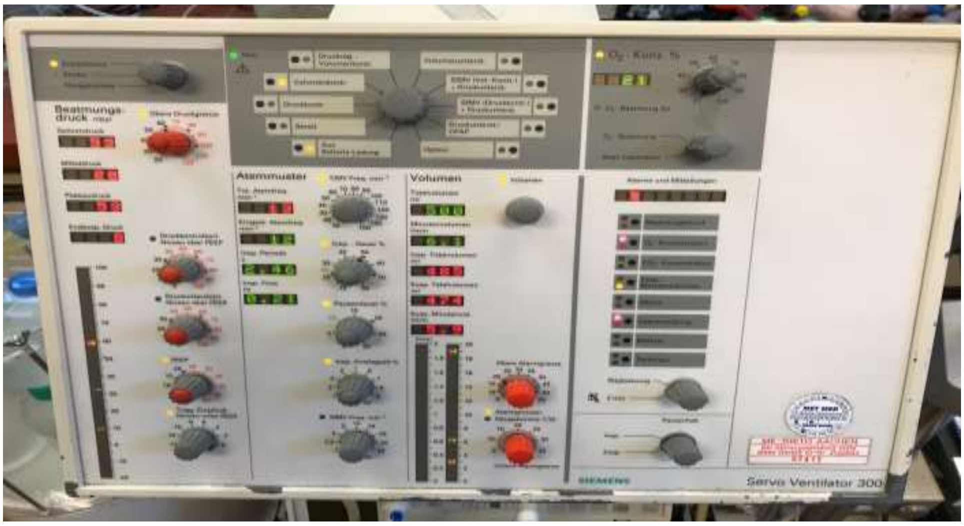


Figure 32:Siemens Servo Ventilator 300

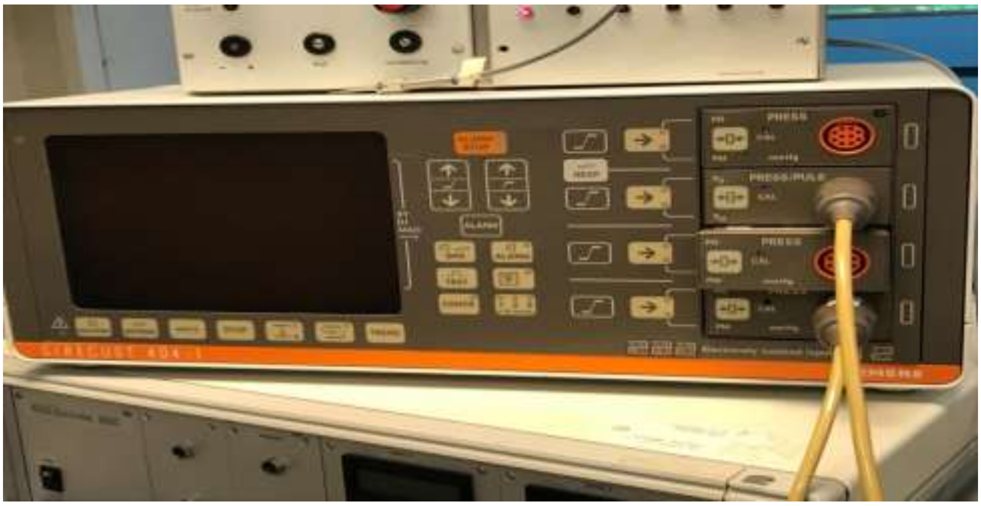


Figure 33: Siemens Sirecust 404-1 multi-parameter monitoring device

Figure 34: Radiometer ABL 800Flex Blood-Gas Analyser

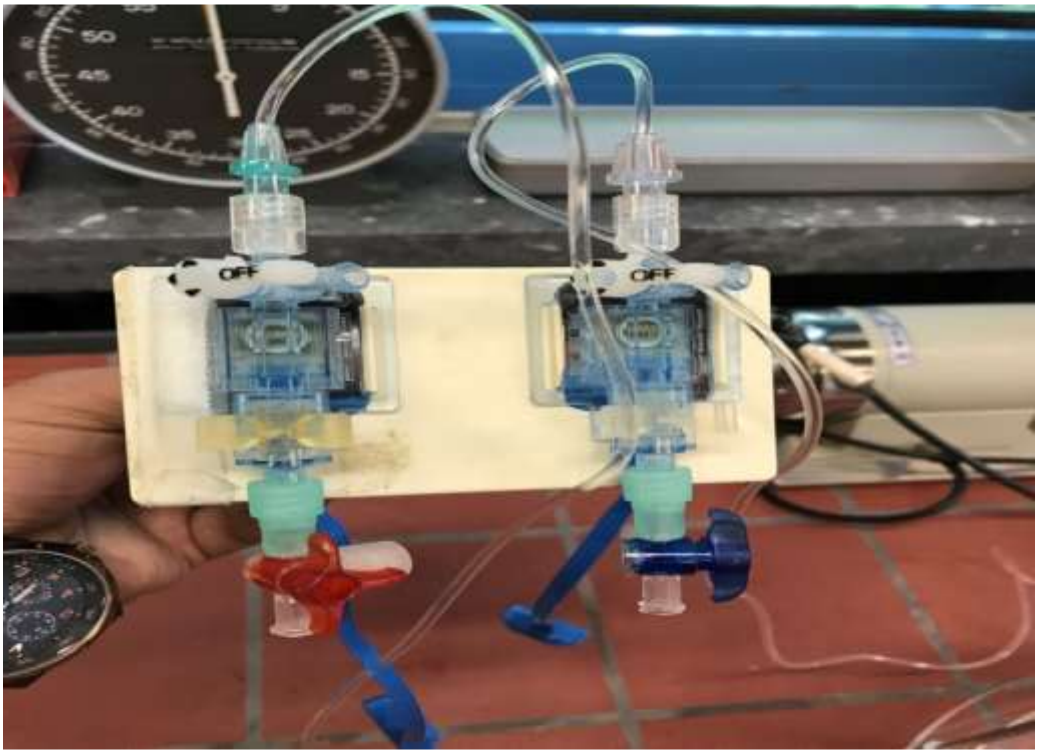


Figure 35: Piezoelectric pressure sensors

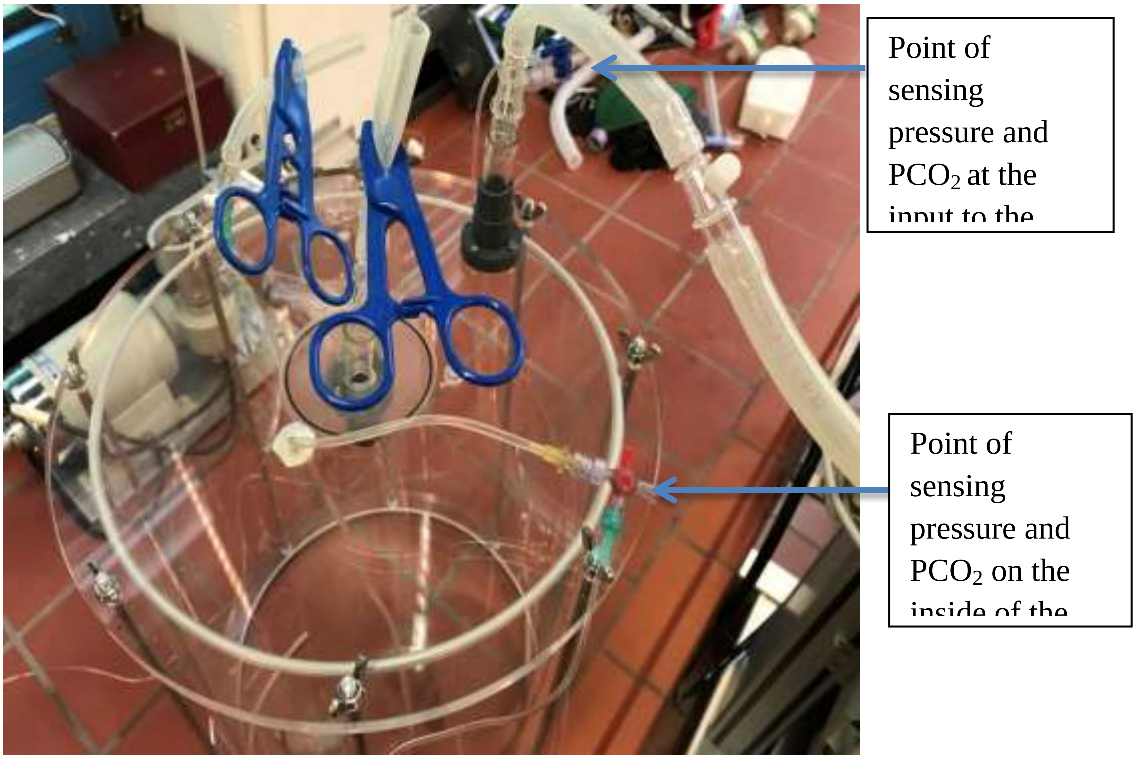


Figure 36: 28L compartment

Figure 37: Experimental Setup

For the study, a mixture of gas was introduced into the compartment with help of the ventilator. The gas introduced is mixture of air and carbon dioxide with a fixed proportion (200 ml) of CO_2 per breath. The Siemens servo ventilator 300 was operated in volume controlled continuous mandatory ventilation mode. Three different regimes were used to obtain experimental results.

Regime	BPM	Tidal Volume (litres) V_T	Minute Volume $\dot{V}$ (litres)
X	12	0.5	6
Y	12	1	12
Z	24	0.5	12

Table 40: Regimes of the Ventilator

In the experiment, the PCO_2 values were observed at two different points in the setup.

1. At the input of the gas into the compartment.
2. Inside the compartment.

Alongside the PCO_2 observation, the gas pressure was also observed at the two points in the setup as stated above.

7.2 Results and Discussions

From the experimental observations, the following results were obtained.

Breaths per minute (BPM)	Tidal Volume V_T (Litre)	Minute Volume $\dot{V}$ (Litre)	PCO_2 (input) mmHg	PCO_2 (inside the compartment) mmHg	Pressure (inside the compartment) mmHg
12	0.5	6	69.6	69.6	5
12	1	12	34.5	35.2	12.5
24	0.5	12	35.2	37.5	5.5

Table 41: Observational Results

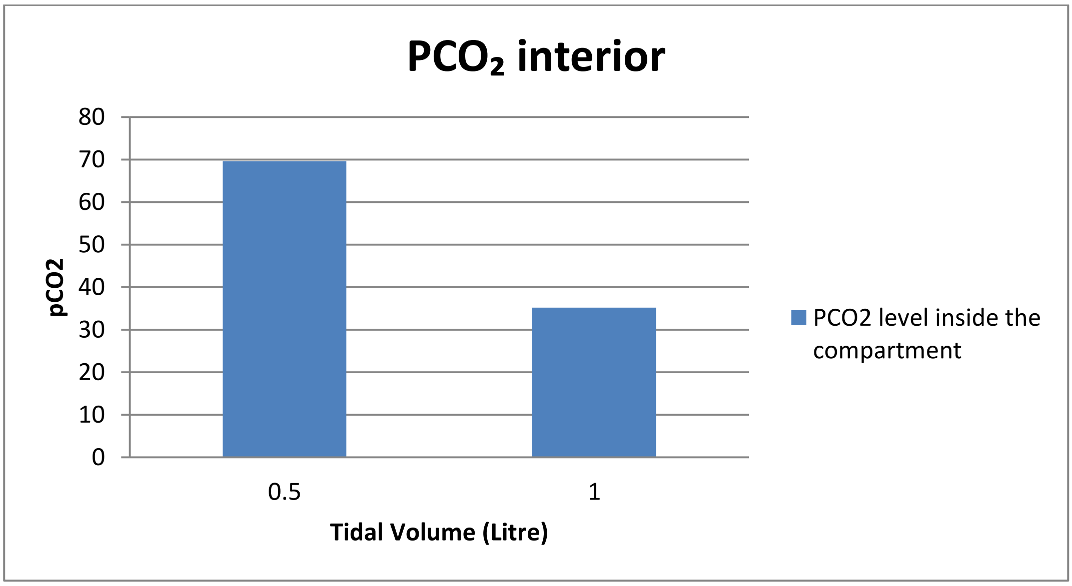


Figure 38:PCO_2 dependency on Tidal Volume at 12 BPM

By observing the figure above, it can be concluded that the PCO2 value inside the compartment are a function of Tidal Volume irrespective of the Breath per Minute. It decreases with increase in Tidal Volume. In fact, it can also be said that PCO2 values are liner with Minute Volume.

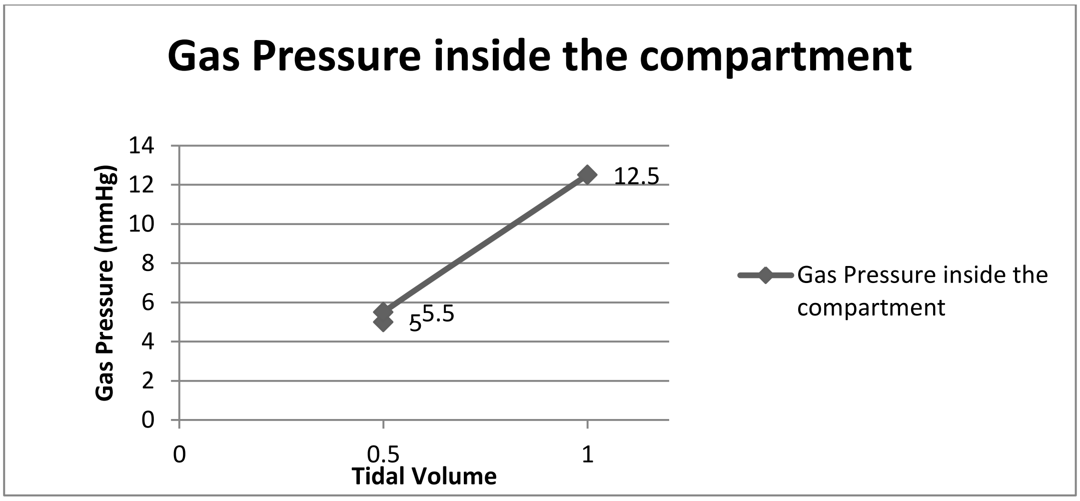


Figure 39:PCO$_2$ dependency on Tidal Volume at 12 BPM

From the chart above we can conclude that the gas pressure inside the compartment is also a function of the Tidal Volume, it increases with increase in Tidal Volume.

7.3 Conclusions

From the experimental results, three things can be notably concluded.

1. PCO_2 values are dependent on Tidal Volume irrespective of breaths per minute.
2. PCO_2 values are also a function of the Minute Volume and are relative to the change in Minute Volume.
3. The Pressure inside the compartment is dependent on the Tidal Volume and is also relative to increase in Tidal Volume.

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