The Prediction of the Joint Stiffness in Riveted Steel Bridges

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DIPLOMA THESIS ASSIGNMENT FORM

I. PERSONAL AND STUDY DATA

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II. DIPLOMA THESIS DATA

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- to create the nonlinear material models of riveted joints for representative bridges,
- to perform the load/stifness analysis,
- to evaluate the results,
- to use the existing and this research to establish the predictive formula.

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Name of Diploma Thesis Supervisor: doc. Ing. Pavel Ryjáček, Ph.D.

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III. ASSIGNMENT RECEIPT

I declare that I am obliged to write the Diploma Thesis on my own, without anyone's assistance, except for provided consultations. The list of references, other sources and consultants' names must be stated in the Diploma Thesis and in referencing I must abide by the CTU methodological manual "How to Write University Final Theses" and the CTU methodological instruction "On the Observation of Ethical Principles in the Preparation of University Final Theses".

Assignment receipt date Student's name

AUTHOR´S DECLARATION

I hereby declare that, This Master Thesis is my own work and investigation. The sources of information published by others authors has been quoted according methodical guide for ethical development of University final thesis.

Prague, 07 January, 2018

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Marcos Bryan Flores Pazmiño

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A mi amada familia: Marco, Sonia, Eli, Johana por estar presentes en esta búsqueda de la felicidad.

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ABSTRACT

Many riveted steel bridges were built around one hundred years ago. The problem lay down on many of those bridges have not been designed for their current life cycle or work services. The present report considers the modeling of eleven connections in the program IDEA StatiCa of five different types of old riveted steel bridges in the Czech Republic and focuses on the prediction of two types of formulas regarding the rotational stiffness depending on its inertia: the first one is based on those sections with low inertia and the second one in those sections with higher inertias. It has taken two types of connections: beam-cross sections and cross beam-stringers, in bridges that are mostly trussed.

It has to be consider, on the past there were not technological advances which allowed a correct prediction of the structural behavior, consequently, the connections were considered pinned or fixed, this calculation procedure delivers with uncertain internal forces in the element. For the analyzed connections (riveted) it is difficult to predict their behavior and their initial rotational stiffness, due to the number of elements that make up such as: plates, rivets, angles.

Nowadays, exist better tools which improve the general analysis of structures, and allows us to have a better idea of the structural behavior, the computational model resembles reality. As a result, the two formulas were compared independently which every bridge and its characterization (Type of bridge, type of connection, height of the element, etc.). Also, the average percentage for each formula presented on previous studies were compared to the one obtained in this study, Therefore, it is recommended these formulas to save calculation time in riveted bridges with adequate safety coefficients, taking into account the error values previously mentioned.

KEYWORDS

Steel Bridges, Riveted Connections, IDEA StatiCa, Modelling, Initial Rotational Stiffness, Moment of inertia, Formula.

RESUMEN

Muchos puentes de acero remachados fueron construidos hace cien años. El problema radica en que muchos de esos puentes no han sido diseñados para su ciclo de vida actual o sus servicios de trabajo.

El presente informe consiste en el modelado de once conexiones en el programa IDEA StatiCa de cinco tipos diferentes de puentes de acero remachados en la República Checa y se centra en la predicción de dos tipos de fórmulas sobre la rigidez rotacional en función de su inercia: la primera fórmula se basa en aquellas secciones con baja inercia y la segunda en aquellas secciones con mayor inercia. Ha tomado dos tipos de conexiones: secciones transversales de vigas y travesaños transversales, en puentes que en su mayoría son cerchas.

Hay que tener en cuenta que en el pasado no hubo avances tecnológicos que permitieran una correcta predicción del comportamiento estructural, por lo tanto, se puede inferir que en los elementos se calcularon con incertidumbre con respecto a las fuerzas internas, por ejemplo en el tipo de conexiones: Fijado, Semi-rígido o conexión fija. Para las conexiones analizadas (remachadas) es difícil predecir su comportamiento y su rigidez rotación inicial, debido a la cantidad de elementos que lo componen, como: placas, remaches, ángulos.

Hoy en día, existen mejores herramientas que mejoran el análisis general de las estructuras, y nos permite tener una mejor idea del comportamiento estructural, el modelo computacional se asemeja a la realidad.

Como resultado, las dos fórmulas se compararon independientemente de cada puente y su caracterización (tipo de puente, tipo de conexión, altura del elemento, etc.). Además, el porcentaje promedio para cada fórmula presentada en estudios previos se comparó con el obtenido en este estudio. Por lo tanto, se recomienda que estas fórmulas ahorren tiempo de cálculo en puentes remachados con coeficientes de seguridad adecuados, teniendo en cuenta los valores de error mencionados anteriormente.

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1. INTRODUCTION

Many riveted steel bridges were built around one hundred years ago. The problem lay down on many of those bridges have not been designed for their current life cycle or work services. Today, many of them have been partially repaired or prepare for its new services required, see (1). Despite of the time has elapsed, there is no apparent damage to the structure due to deterioration or fatigue, but a technical study is necessary to verify the real state of the structure and, if necessary, propose reinforcement alternatives.

The present report considers old riveted steel bridges in the Czech Republic and focuses on the prediction of a formula that relates the initial rotational stiffness of riveted connections with the inertia of the element. It has taken two types of connections: beam-Cross sections and cross beam-stringers, in bridges that are mostly Trussed, as well as, it is based on master's thesis of on Óscar Minor "The Impact of the Connection Stiffness on the behavior of a Historical Steel Railway Bridge", see (2), where it can be observed, that there is a directly proportional relationship between the numbers of rivets with the initial rotational stiffness, as well as, the inertia of the element analyzed with the initial rotation rotational Stiffness. Thus, the intention of this work is to deepen the knowledge of the latest one. It has been found that there is a relationship between rigidity in the connections and lateral deformation of structures. see (3).

It has to be consider, in the past there were not technological advances, such as computational programs, which allowed a correct prediction of the structural behavior, consequently, the connections were considered pinned or fixed , this calculation procedure delivers with uncertain internal forces in the element, for instance: Pinned connections induces only axial force on the elements, and at the middle of the element there is a greater bending moment, in the other hand at the extremes of the element the moment is zero. Semi-rigid and Fixed connection produces bending moment and shear force, in short, internal stresses on the elements and the bending moment are distributed between the extremes and at the middle of it, see [3].

For the analyzed connections (riveted) it is difficult to predict their behavior and their initial Rotational Stiffness, due to the number of elements that make up such as: plates, rivets, angles.

Currently, exist better tools which improve the general analysis of structures, and allows us to have a better idea of the structural behavior, the computational model resembles reality, so resources and calculation time can be optimized. The purpose of this study is to obtain a formula to facilitate old riveted bridge calculations and save valuable time, reduce uncertainties regarding the behavior of the elements.

2. STATE OF THE ART

To begin this study, the different types of bridges are listed, which are classified by their main structural system, many bridges depend on span, carriageway width and types of traffic, see [4].

Girder bridges

Bending moment on the middle of the span is main structural action. Girder bridges may be either solid web girders, truss girders or box girders, for example: plate girder bridges for less than 50 m and box girders for continuous spans up to 250 m, see (4).

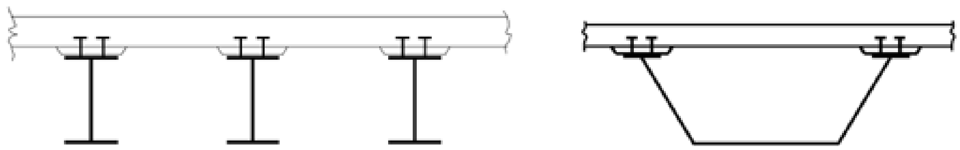


Figure 1. Typical type of girder bridges. Ref. [4]

Trussed structure

Members are subjected to axial forces, the loads are applied on the nodes and the members have pinned connections that do not transfer any shear forces or flexural moments. They are simply supported at the ends. Truss bridges are suitable for the span range of 30 m to 375 m, see [4].

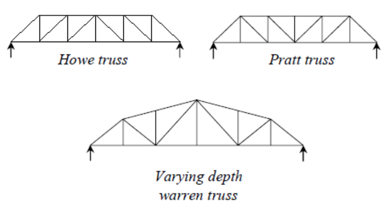


Figure 2. Typical type of trussed structure. Ref. [4]

Rigid frame bridges

In this type of structure, the main acting forces are flexure with some axial force, this bridges are suitable in the span range of *25 m* to *200 m*, see [4].

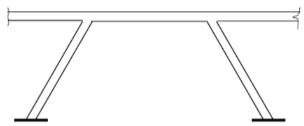


Figure 3. Rigid frame bridge. Ref. [4]

Arch bridges

The arch is the main structural element. The main force is axial compression in arch rib, combined with some bending. Typically loads are transferred to the foundations due to the arches. The span suitable for this structure is between *100m* to *500 m*, see [4].

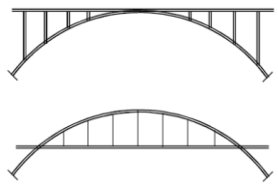


Figure 4. Arch bridges. Ref. [4]

Cable stayed bridges

The structural system is based on vertical cables which support the main longitudinal girders. The span suitable for this structure is between *150 m* to *700 m*, see [4].

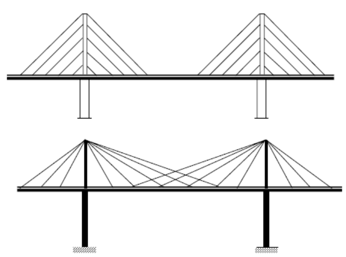


Figure 5. Cable stayed bridges. Ref. [4]

Suspension bridges

The bridge deck is suspended from cables, anchored to the ground at two ends and passing over towers erected near the two edges of the gap. This is the best solution for long span bridges between 500 m and over 2000 m, see [4].

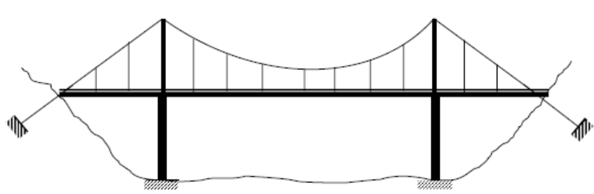


Figure 6. Suspension bridges. Ref. [4]

Classification based on the position of carriageway

There are three different of bridges depending of the position of the carriageway: 1. Deck type, 2. Through type and 3. Semi-through type.

Deck Type Bridge: The carriageway is on the top of the main load carrying members. In the case of deck type plate girder bridge, the railway is located on the top flanges and in the case of deck type truss girder bridge, the railway is located at the top chord level, see [4].

Through Type Bridge: The carriageway is at the bottom level of the main load carrying members. In the case of through type plate girder bridge, the railway is placed at the level of bottom flanges and in the case of the through type truss girder bridge, the railway is placed at the bottom chord level. The bracing of the top flange or lateral support of the top chord under compression is also required, see [4].

Semi through Type Bridge: The deck is in between the top and the bottom of the main load carrying members. The bracing of the top flange or top chord under compression is not done, the lateral restraint in the system is obtained usually by the U-frame action of the verticals and cross beam acting together, see [4].

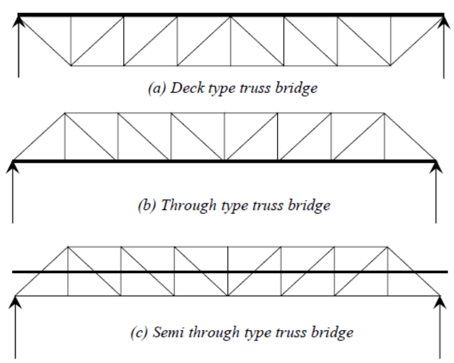


Figure 7. Classification based on the position of the carriageway. Ref. [4]

2.1. Classification of connections

According to EN 1993-1-8, see (5), the classification if the connections is based on the effects of behavior of the join, there are three simplified joint models as follows:

- Simple or pinned: The joint may be assumed not to transmit bending moments;
- Continuous or rigid: The behavior of the joint may be assumed to have no effect on the analysis;

- Semi-continuous or semi rigid: The behavior of the joint needs to be considered in the analysis.

Method of global analysis	Classification of joint		
Elastic	Nominally pinned	Rigid	Semi-rigid
Rigid-Plastic	Nominally pinned	Full-strength	Partial-strength
Elastic-Plastic	Nominally pinned	Rigid and full-strength	Semi-rigid and partial-strength Semi-rigid and full-strength Rigid and partial-strength
Type of joint model	Simple	Continuous	Semi-continuous

Table 1. Type of joint model. Ref. [5]

The design moment-rotation characteristic of a joint used in the analysis may be simplified by adopting any appropriate curve, including a linearized approximation, provided that the approximate curve lies wholly below the design moment-rotation characteristic, see [5]. Joints may be classified by their stiffness and by their strength.

Classification by stiffness

According Eurocode EN 1993-1-8, see [5], a joint may be classified as rigid, nominally pinned or semi-rigid according to its rotational stiffness, by comparing its initial rotational stiffness $S_{j,ini}$ with the classification boundaries as it is showed in the figure (8).

Nominally pinned joints

A nominally pinned joint should can transmit the internal forces, without developing significant moments which might adversely affect the members, or the structure and the joint should be able of accepting the resulting rotations under the design loads, see [5].

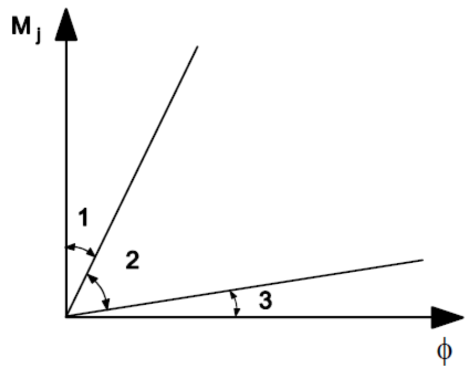
Rigid joints

Joints classified as rigid may be assumed to have sufficient rotational stiffness to justify analysis based on full continuity, see [5].

Semi-rigid joints

A joint which does not meet the criteria for a rigid joint or a nominally pinned joint should be classified as a semi-rigid joint. Semi-rigid joints provide a predictable degree of interaction between members, based on the design moment-rotation characteristics of the joints. Those joints should be capable of transmitting the internal forces and moments, see [5].

Classification boundaries



Zone 1: rigid, if $S_{j,ini} \geq k_b EI_b / L_b$

where:

$k_b = 8$ for frames where the bracing system reduces the horizontal displacement by at least 80 %

$k_b = 25$ for other frames, provided that in every storey $K_b/K_c \geq 0,1$ *)

Zone 2: semi-rigid

All joints in zone 2 should be classified as semi-rigid. Joints in zones 1 or 3 may optionally also be treated as semi-rigid.

Zone 3: nominally pinned, if $S_{j,ini} \leq 0,5\, EI_b / L_b$

*) For frames where $K_b/K_c < 0,1$ the joints should be classified as semi-rigid.

Key:

K_b is the mean value of I_b/L_b for all the beams at the top of that storey;
K_c is the mean value of I_c/L_c for all the columns in that storey;
I_b is the second moment of area of a beam;
I_c is the second moment of area of a column;
L_b is the span of a beam (centre-to-centre of columns);
L_c is the storey height of a column.

Figure 8. Classification boundaries. Ref. [5]

Classification by strength

A joint may be classified as full-strength, nominally pinned or partial strength by comparing its design moment resistance $M_{j,Rd}$ with the design moment resistances of the members that it connects. When classifying joints, the design resistance of a member should be taken as that member adjacent to the joint, see [5].

2.2. Type of connections

According Euro code 1993- 3, see [5], there are different types of connections depending of the configuration: Bolts, Rivets, Welded, hydride, etc...

Bolts, nuts and washers

Categories of bolted connections

Shear connections

- Category A: Bearing type
- Category B: Slip-resistant at serviceability limit state
- Category C: Slip-resistant at ultimate limit state

Tension connections

- Category D: non-preloaded
- Category E: preloaded, see [5].

Welded connections

The provisions in this section apply to weldable structural steels conforming to EN 1993-1-8 and to material thicknesses of 4 mm and over. The provisions also apply to joints in which the mechanical properties of the weld metal are compatible with those of the parent metal, should be also checked the welds subject to fatigue, see [5].

Rivets

The material properties, dimensions and tolerances of steel rivets should comply with the requirements given on the national standards from every European country, see [5]. Historic steel bridges dating to before 1970 were built with rivets. Rivets were nearly always used to fasten together built-up structural steel on bridges. Rivets were also frequently used for the connections on steel bridges. Today, rivets are not used anymore, instead of them, welds and high strength bolts provide the functions, see (6).

A rivet consists of a first rivet head – called manufactured head or shop head – formed by crushing the end of the cut segment of a cylindrical bar iron or steel called rivet shank. It connects Iron and steel plates and sections. The rivets were heated and then driven this process is called hot riveting. At that time the hot-riveting technique allowed to introduce advances in fabrication of iron and steel construction. The advantages of structural riveting e.g., reliability, affordability, design possibilities–permitted the development of new girder and column shapes, construction but also truss work. These innovations helped for the widespread construction of short and large span steel structures, see [4] and see (7). Additionally, riveted connections present a considerable amount of rigidity, but there are several uncertainties to account for this in the design of a joint, see [2].

Structural applications of hot rivets

Hot rivets have two principal applications the first one is the fabrication of built-up sections like columns and beams and the second application is assembling of structures, skeleton frames or portal frames. Typically, built-up sections are mainly made of flat plates, angles shapes, L-sections, T-sections or U-sections connected by rivets. First, large girders in bending were fabricated and then columns in wrought iron and shortly later with steel. Solidwedded sections required rivet to ensure continuity in both longitudinal and transverse directions. In the transverse directions the constituent plates and sections were connected by rivets to effectively fabricate the built-up actions. In addition, web and chord member had to be extended in the longitudinal direction of the built-up section for large span lengths, see [7].

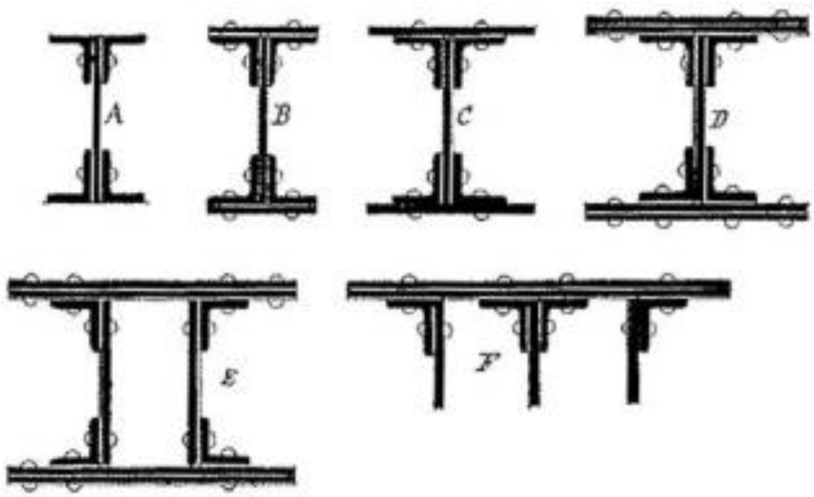


Figure 9. Structural application of rivets. Ref. [7]

The assemble of structures, skeleton frames, portal frames or truss are other applications of the rivets. The rivet shank complete fills the rivet hole after driving, for designing propose, the contribution of the frictional strength is neglected, the riveted connections behave in pure shear/bearing. The applied loads are uniformly distributed within the rivets of a given joint, see [7].

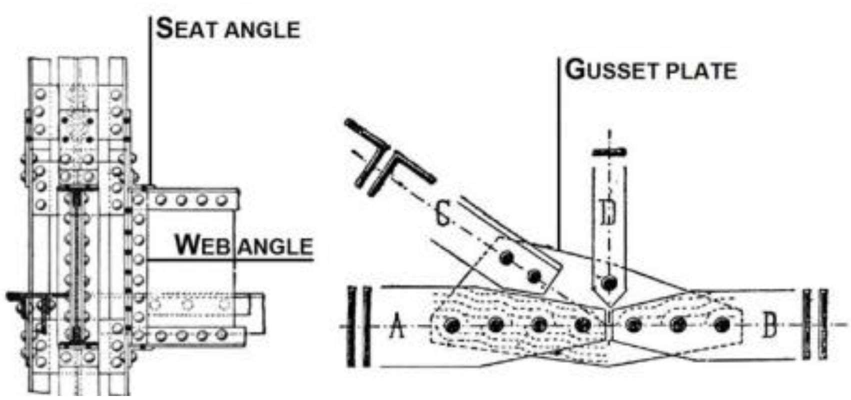


Figure 10. Connections of rivets. Ref. [7]

2.3. Differences between rivets and bolts

Today, numerous barriers have to be overcome when dealing with the repair or strengthening of existing riveted connections. Being the predominant joining technique on the late 19th century and the beginning of 20th century, but nowadays the know – how has being forgotten due to it cost, time consuming and modern technologies such bolts. That is the main reason of the lack of information on the design, in addition, it is difficult to accurately predict the actual strength and stiffness of riveted connections, see figure (11), as the quality of riveting is variable. Currently rivets are usually replaced with high strength bolts, or proprietary fasteners such a hock bolts, or tension control bolts, see [2] and see [7].

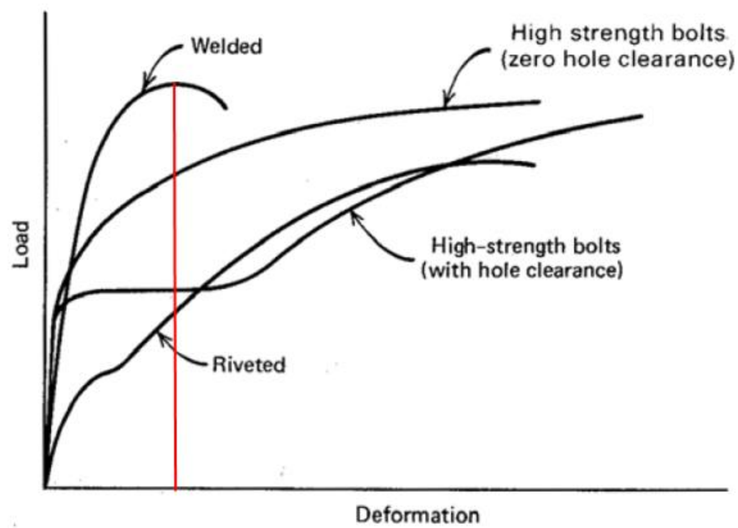


Figure 11. Comparison between riveted and bolted connections. Ref. [8]

The figure (11), it is easy to appreciate the differences between rivets, bolts with zero clearance and bolts with clearance, see (8).and how they are working due to the load applied.

Bolts are easy to replace if it is necessary, in the other hand, rivets are considered as a permeated as welding. The material of the rivet used to be cheaper but not the labor cost, it also may improve the stiffness of the connection and they can compensate hole misalignments, see [7]. Depending of the bolt they work in pure shear/bearing and tension. Riveted connections behave in pure shear/bearing, see [5] and see [7]. For both cases the applied loads are uniformly distributed within the rivets of a given joint, see [7].

2.4. The rotational stiffness

A connection transmits the forces from one member to another. These forces can be axial force, shear, bending moment, torsion or a combination of them as it is usually the real case. Forces such as axial, shear, bending moment and torsion or a combination of all of them together which produce deformations of the structure, but the largest deformation is the rotation caused by the bending moment. As it was explained before on this document, in the past century, joints have been analyzed and designed considering that they are either pinned or fully rigid. but they represent two extreme conditions of the real behavior and it does exclude the third alternative of semi rigid. It is common to express the rotational deformation as a function of the bending moment applied in the connection, obtaining a moment-rotation curve, there are some different examples on the figure (11) with examples of curves for different types of connections. A fully pinned is represented on horizontal axis and the rigid behavior in the vertical axis; but in real structures the behavior falls always somewhere in between, see (9) and see [2].

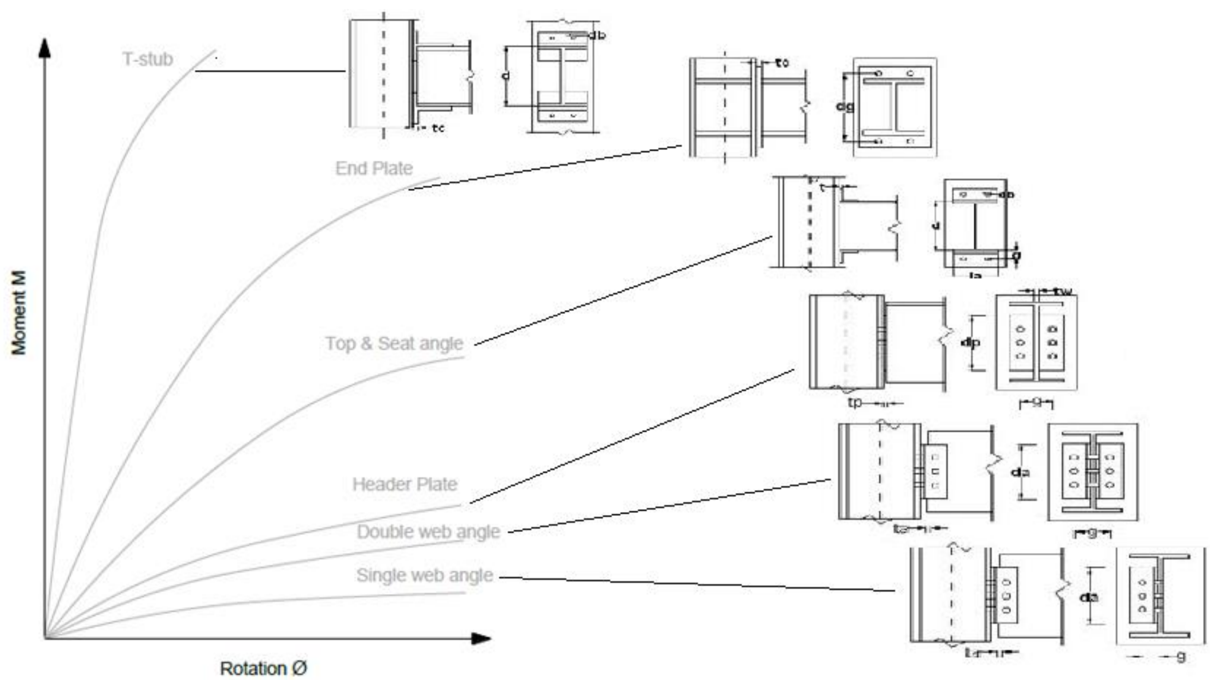


Figure 12. Moment- rotation curves for different connections. Ref. [9]

As it is showed the T- stub connection is more rigid as it can resist high levels of moment with small rotation. On the other hand, single web angle tent to the pinned connection allowing large rotation with minimal transmission of bending moment, see [9].

Moment-curvature diagrams are usually obtained from physical tests, but for the analysis and design, it is possible to idealize the behavior of the connection with a linear relation between moment and rotation, followed by a state in which the rotation increases without resisting any more bending moment. This linear relationship is defined as the rotational stiffness S_j, see [9] and see [2].

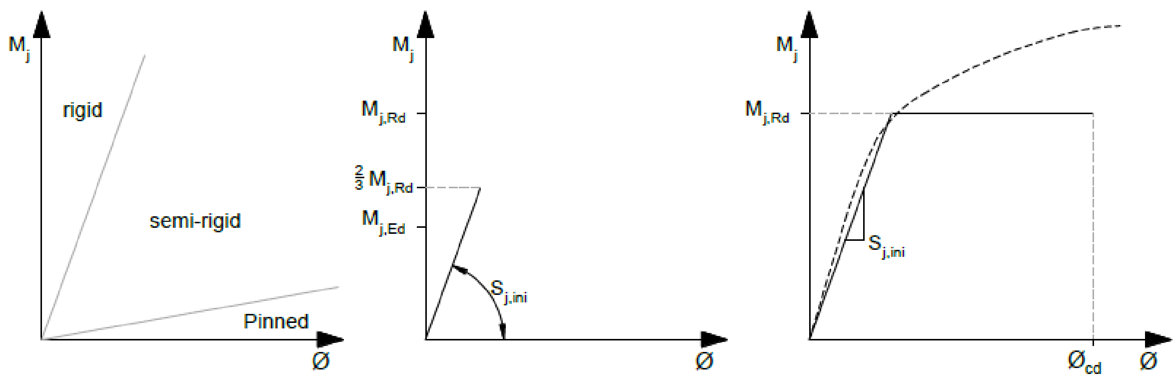


Figure 13. Classification of connections according to the stiffness. Ref. [2]

The moment required to produce unit rotation in a joint. EN 1993-1-8. The rotational stiffness of a joint should be determined from the flexibilities of its basic components, each represented by an elastic stiffness coefficient, see [5].

2.5. Calculation of joints according to Eurocode

According to Eurocode EN 1993-1-8, see [5], and as it was already mention before in this document, the joints could be classified by the stiffness in rigid, normally pinned or semi-rigid. A rigid connection must satisfy the following condition:

$$S_{j,ini} \geq \frac{k_b \, E \, I_b}{L_b} \qquad (1)$$

Where:

k_b = 8, for frames where bracing systems reducing horizontal displacements at least 80%

k_b = 25, for other frames, in which in every story Kb/Kc > 0.1

K_b is the mean value of I_b/L_b for all the beams at the top of 'that storey' and K_c is the mean value of I_c/L_c for all the columns in 'that storey'. A pinned connection must satisfy the following condition:

$$S_{j,ini} \leq \frac{0.5\ E\ I_b}{L_b} \tag{2}$$

Where:

E Is the elastic modulus,

I_b The second moment of area of the beam,

L_b The span center to center of the beam,

From the parameter k_b, this classification was taught for joints in structural frames. A truss connection is a case of a frame system, especially similar to a braced frame, see [2].

According to Eurocode EN 1993-1-8, see [5], the rotational stiffness should be determined from the flexibilities of its basic components, each represented by an elastic stiffness coefficient k_i. These elastic stiffness coefficients are for general application

The rotational stiffness S_j is computed as:

$$S_j = \frac{E z^2}{\mu \sum_i \frac{1}{k_i}} \tag{3}$$

Where:

z I_s the lever arm,

μ I_s the stiffness ratio $S_{j,ini}/S_j$

The Eurocode EN 1993-1-8 provides formulas to compute the stiffness of basic joint components in the table 6.111 of Eurocode EN 1993-1-8, see [5].

2.6. The Component Method (CM)

The component method (CM) is the main philosophy for the determination of the bearing capacity and the stiffness of the joint included in the Eurocode EN 1993-1-8, see [5]. This method applies to any type of steel or composite joints, whatever the geometrical configuration, the type of loading (axial force and/or bending moment) and the type of member sections. This method considers any joint as a set of individual basic components. For the particular joint shown in Figure 1 (steel joint configuration with an extended endplate connection subjected to hogging bending moments), the relevant components are given. (10), see [10].

This method considers the joint as a system of interconnected components, and consists on:

1. Identification of the active components in the joint being considered;
2. Characterization or evaluation of the stiffness and/or resistance characteristics for each individual basic component (specific characteristics - initial stiffness, design resistance, ... - or the whole load-deformation curve); the distribution of forces needs to be done satisfying the equilibrium in the joint.
3. Assembly of all the constituent components and evaluation of the stiffness and/or resistance characteristics of the whole joint (specific characteristics - initial stiffness $S_{j,ini}$, design resistance M_j,Rd, ... - or the whole moment-rotation curve),see [10].

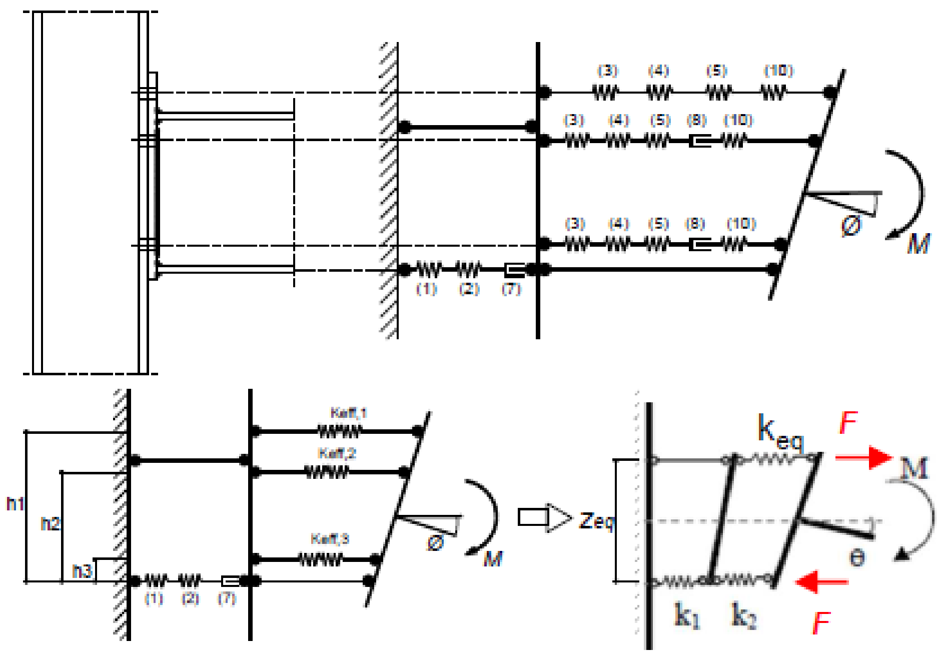


Figure 14. Schematical representation of the component method. Ref. [10]

On the figure (14), it is describing a steel joint an extended end-plate connection subjected to hogging moments identification of the active component- spring model, see [10].

The formulas proposed on the present report are specified for H and I sections, and they are not valid for hollowed sections. In the bridges analyzed the elements are a mix of angles and plates assembled together which configure H and I sections.

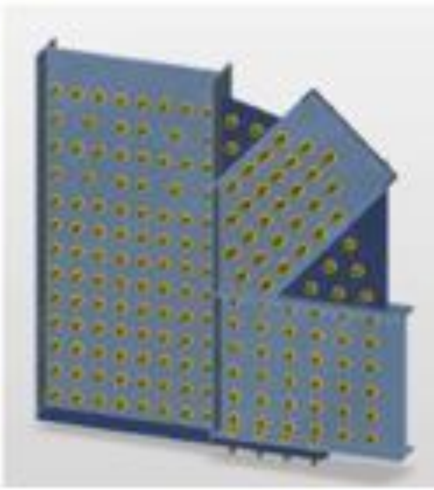
2.7. Previous investigation on joint stiffness

The bases of the present report are focused on two previous studies:

- A technical report studying made by SUDOP about the axial and rotational stiffness in the connections of a steel railway bridge, Tábor-Písek bridge, over the Vlatava River in the km 41,791, see (11). From now it will be referred as Tábor-Písek bridge.

Figure 15. Tábor- Písek Bridge. ref. Wikipedia

The bridge was built in 1886 and consist of three lattice trusses with a span of 84.4 m each. The sections were built with angles and plates and riveted connections and plates. The characterization and modeling of this bridges was made on the software IDEA Statica version 5, see [2].



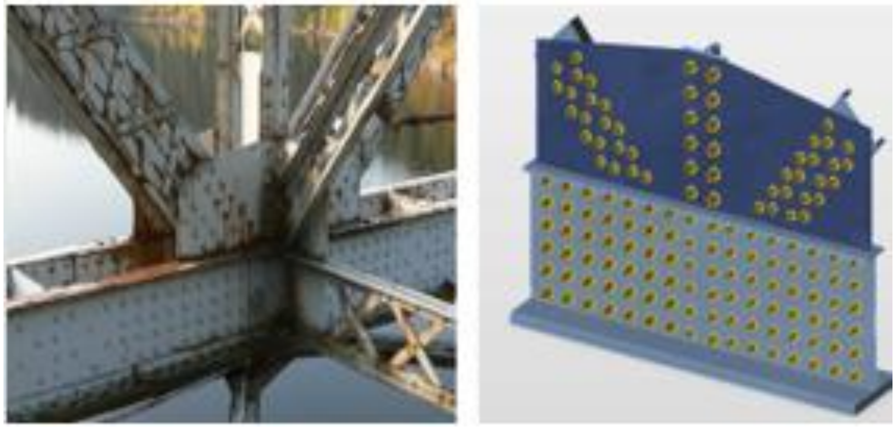


Figure 16. Types of connections analyzed in the report on the Tábor – Písek bridge ref. [11]

- The second one is a Master Thesis "The Impact of the Connection Stiffness on the behavior of a Historical Steel Railway Bridge" made in the Czech Technical University in Prague made by Oscar Minor, see [2], this report is the principal reference and the starting point for the present study, in which briefly analyzes the interaction between inertia moment and rotational stiffness This report includes the characterization of the connections and the modelling of the different joints on the software IDEA StatiCa version 8.0.15.43212. and the final analysis with CSI Bridge of the steel railway bridge that connects Vyšehrad and Smichov, over the Vltava River in the center of Prague. It is usually referred to as *most pod Vyšehradem*. The bridge has a total length of 218 m and is divided in three sections, each one formed by polygonal arched trusses supported on masonry pillars over the river bed. In the Vyšehrad end, the bridge is continued by 4 spans of 19 m each made of steel girders; while in the Smichov side, it is followed by an embankment. The structure allocates two railways between the trusses, and pedestrian ways in cantilever at both sides. Since its construction in 1901. The connections are analyzed with models created in the software IDEA StatiCa, see [2]. From now it will be referred as Vyšehradem bridge

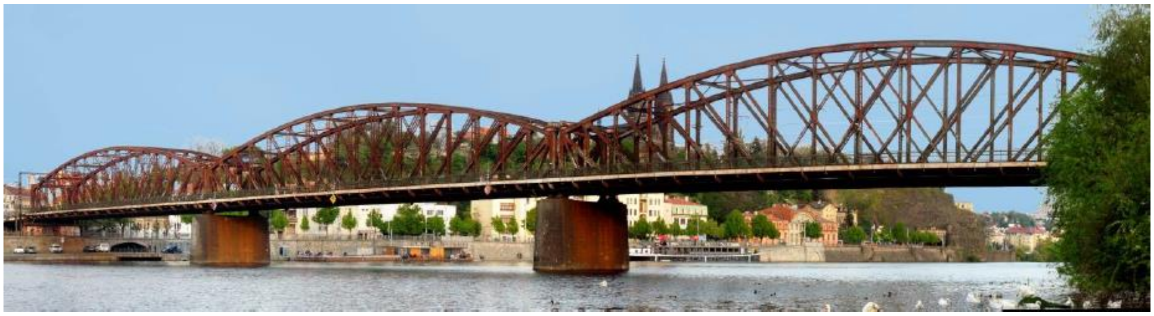


Figure 17. Steel Bridge near Vyšehrad. Ref. [2]

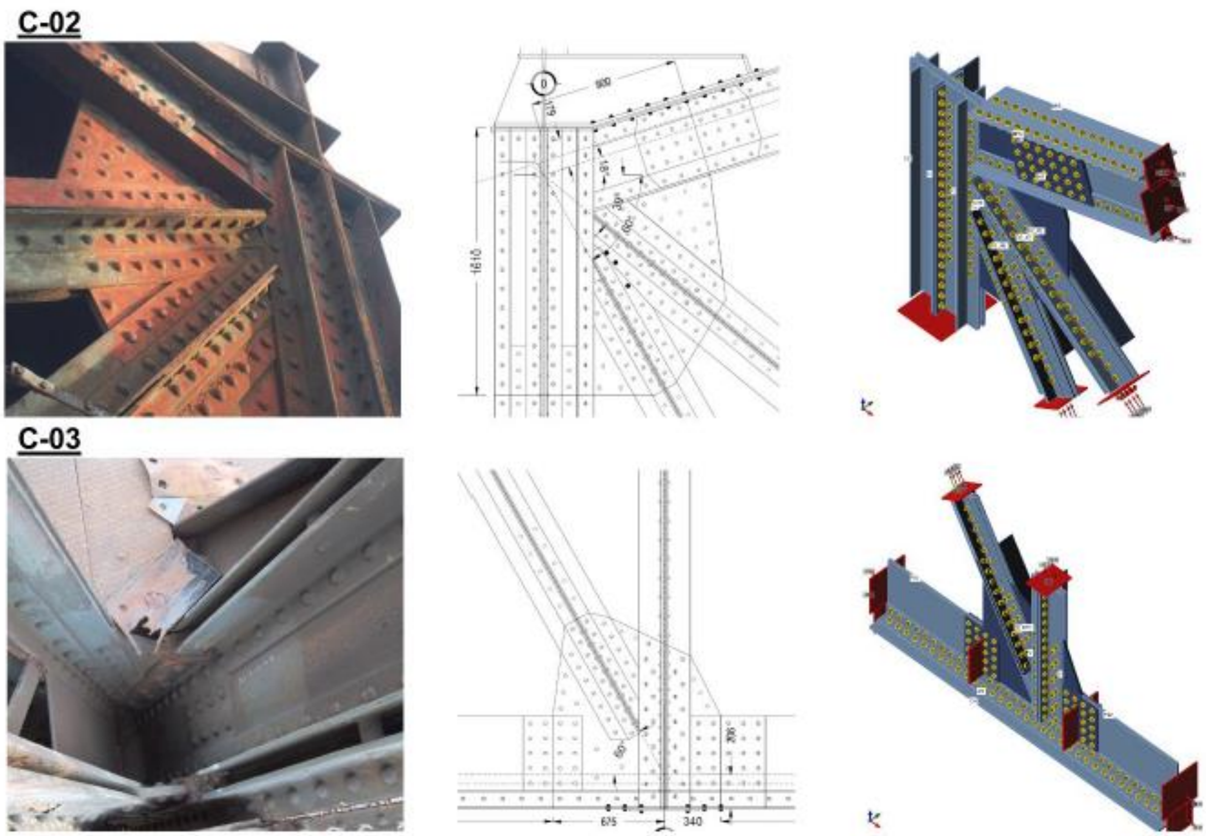


Figure 18. Type of connections of Steel Bridge near Vyšehrad. Ref. [2]

3. JOINTS STIFFNESS ANALYSIS METHODS

The railway transportation was introduced in the Czech territory during the Austro-Hungarian Empire at the beginning of the nineteenth century with a horse-drawn railway between Linz (Austria) and Ceske Budejovice. The following decades other lines were constructed joining major cities. The first formal train between Vienna and Prague was opened on 1845, and the route from Prague to Dresden was completed in 1851, see (12) and see (13). At the end of nineteenth century in Europe, several number of bridges built around the same age during the boom of railways construction, therefore, many of them share characteristics of geometry, materials and type of connections, see [13], it will be taken into account in order to analyze if there are any kind of relationship between the moment of inertia of the joints and its initial stiffness.

3.1. Component Method

As it was explained in the section 2.6, this method applies to any type of steel or composite joints, whatever the geometrical configuration, this method considers any joint as a set of individual basic components.

This method considers the joint as a system of interconnected components, and consists on:

1. Identification of the active components in the joint being considered.
2. Characterization or evaluation of the stiffness.
3. Assembly of all the constituent components, see [10].

3.2. Final Element Method (FEM)

The structural stress-analysis problem, the engineer seeks to determine displacements and stresses throughout the structure, which is in equilibrium and is subjected to applied loads. For many structures, it is difficult to determine the distribution of deformation using conventional methods, and thus the finite element method is necessarily used, see (14).

There are two general direct approaches traditionally associated with the finite element method as applied to structural mechanics problems. One approach, called the force, or flexibility, method, uses internal forces as the unknowns of the problem. To obtain the governing equations, first the equilibrium equations are used. Then necessary additional equations are found by introducing compatibility equations. The result is a set of algebraic equations for determining the redundant or unknown forces, see [14]

The second approach, called the displacement, or stiffness method, assumes the displacements of the nodes as the unknowns of the problem. For instance, compatibility conditions requiring that elements connected at a common node, along a common edge, or on a common surface before loading remain connected at that node, edge, or surface after deformation takes place are initially satisfied. Then the governing equations are expressed in terms of nodal displacements using the equations of equilibrium and an applicable law relating forces to displacements, see [14].

3.3. CBFEM

According to IDEA StatiCa, the weak point of standard Component method is in analyzing of internal forces and stress in a joint. CBFEM replaces specific analysis of internal forces in joint with general FEM, see [15].

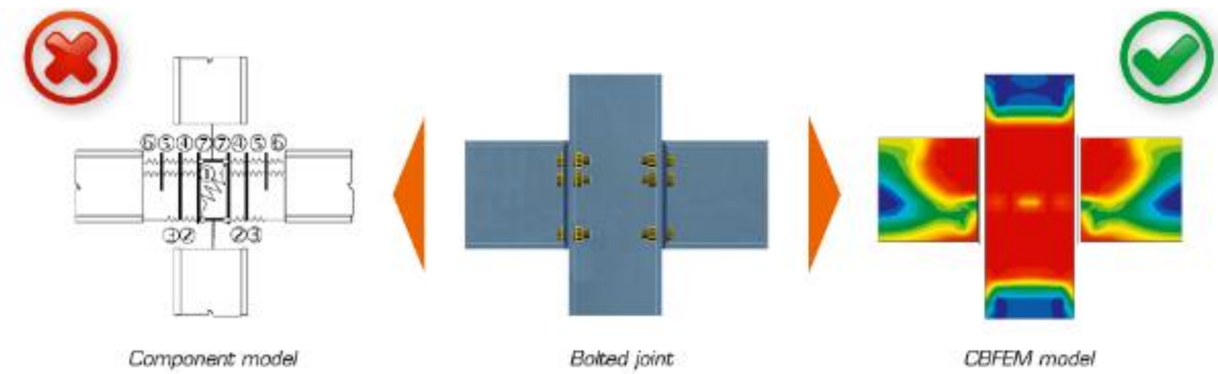


Figure 19. CBFEM versus Component method. Ref. [15]

Check methods of specific components like bolts or welds are done according to standard Component method. For the fasteners – bolts and welds – special FEM components had to be developed to model the welds and bolts behavior in joint. All parts of 1D members and all additional plates are modelled as plate/walls. These elements are made of steel (metal in general) and the behavior of this material is significantly nonlinear. The real stress-strain diagram of steel is replaced by the ideal plastic material for design purposes in building practice. The advantage of ideal plastic material is, that only yield strength and modulus of elasticity must be known to describe the material curve. The granted ductility of construction steel is 15 %. The real usable value of limit plastic strain is 5% for ordinary design (1993-1-5 appendix C Paragraph C.8 note 1). The stress in steel cannot exceed the yield strength when using the ideal elastic-plastic stress-strain diagram. Internally, plates are modeled as shell elements with 6 degrees of freedom in each node: 3 translations (ux, uy uz) and 3 rotations (φx, φy, φz), see [15],

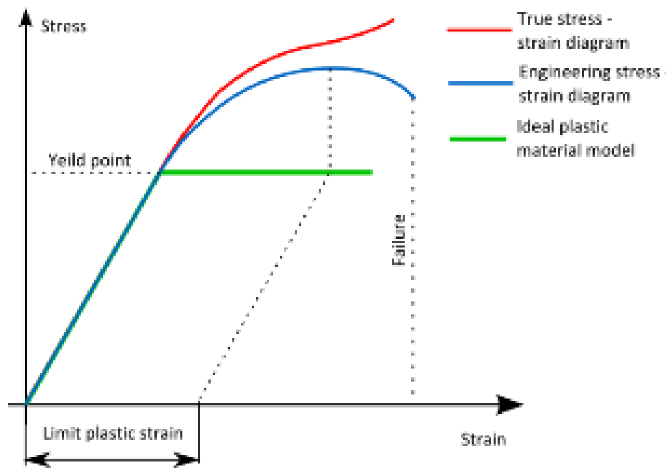


Figure 20. Real tension curve and the Ideal elastic- plastic diagram of material. Ref. [15]

CBFEM method tries to create to model the real state precisely. The analysis plate/walls are not interconnected, no intersections are generated between them, unlike it is used to when modelling structures and buildings. Mesh of finite elements is generated on each individual plate independently on mesh of other plates. Welds are modelled as special massless force interpolation constraints, which ensure the connection between the edge of one plate and the surface or edge of the other plate. Using plastic distribution, solid elements with elastic-plastic material diagram with respect to weld throat thickness, position and orientation are inserted between interpolation links. Yielding of welds allows for redistribution of peak stresses along the weld length. This unique calculation model provides very good results – both for the point of view of precision and of the analysis speed. The method is protected by patent, see [15].

Two approaches of modelling welds are implemented.

The first option of weld model between plates is direct merge of meshes of welded plates. The load is transmitted through a force-deformation constrains to opposite plate. This model does not respect the stiffness of the weld and the stress distribution is conservative.

Stress peaks, which appear at the end of plate edges, in corners and rounding, govern the resistance along the whole length of the weld. To eliminate the effect of stress peaks three methods for evaluation of the weld can be chosen, see [15]:

- Maximal stress (conservative)
- Average stress on weld
- Linear interpolation along weld, see [15].

The second approach uses an improved weld model. A special elastoplastic element is added between the plates. The element respects the weld throat thickness, position and orientation. Ideal plastic model is used and the plasticity state is controlled by stresses in the weld throat section. The stress peaks are redistributed along the longer part of the weld length, see [15].

Bolted connection consists of two or more clasped plates and one or more bolts. Plates are placed loosely on each other. A contact element is inserted between plates in the analysis model, which acts only in compression. No forces are carried in tension, see [15].

Shear force is taken by bearing. Special model for its transferring in the force direction only is implemented. IDEA StatiCa Connection can check bolts for interaction of shear and tension. The bolt behavior is implemented according following picture, see [15].

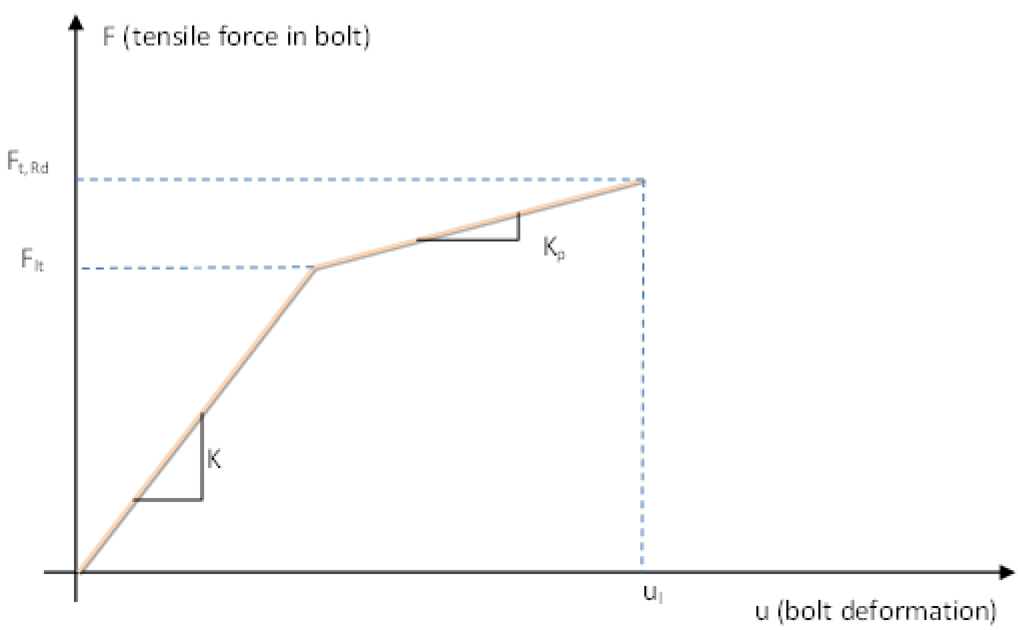


Figure 21. Bolts for interaction of shear and tension. Ref. [15]

Where

K linear stiffness of bolt,

K_p stiffness of bolt at plastic branch,

F_{lt} limit force for linear behavior of bolt,

$F_{t,Rd}$ limit bolt resistance,

u_l limit deformation of bolt.

The contact between plates is treated according to the penalty method, which is basically the application of a penalty stiffness added between the node and the opposite plate when penetration of a node into an opposite surface is detected, see [15] and see [2].

The tensile force is transmitted to the plates by interpolation between the bolt shank and the nodes in the plate. For compression, the force is transmitted from the bolt shank to the plate in the bolt hole by interpolation links between the shank and hole edges nodes. Finally, the interaction between the axial and the shear forces is considered. The higher the tensile forces the less shear force is resisted by the bolt, see [15] and see [2].

Types of results obtained with IDEA StatiCa. Model of connection, automatic generated mesh, equivalent stress with deformation, and stresses in each plate.
Another of the different results that can be obtained with the software is the shear force that is acting in each fastener, is it is verified in Figure (22). For the example shown, only axial force is applied to the elements so the resultant shear force is all in the same direction for all the fasteners. The force on the double angles is 10 KN and so is the sum of the shear forces of bolt in that section. The plate is loaded with other 10 KN, so the sum of the shear forces in the bolts for that plane is the total load of 20 KN. It can be noted that the load is distributed regularly in the bolts. This regularity is lost when bending force is acting on the member, which is the real case in many structures, and to account for this is complicated without the correct tools, see [2] and see [15].

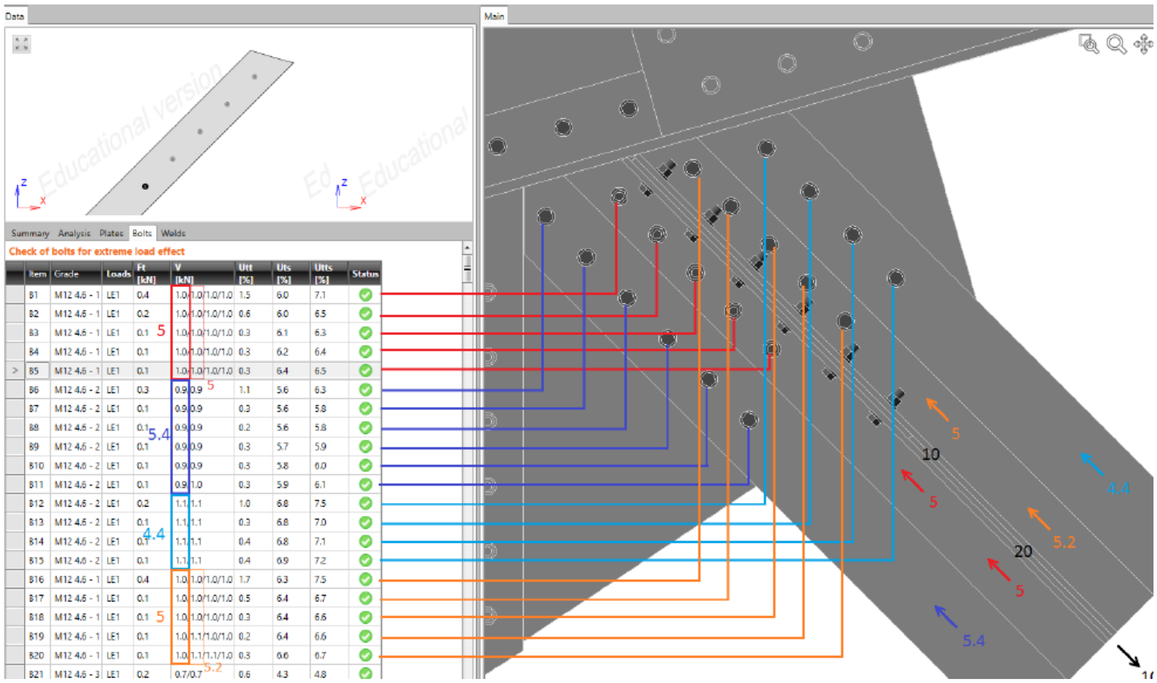


Figure 22. Shear force in the bolts. Ref. [2]

3.3.1. IDEA StatiCa – CBFEM method

The work will consist of advanced numerical modelling of old riveted joints that will be done by the CBFEM method with the software IDEA Connections. Some old railway bridges will be selected according of its difficulty to determinate the classification of the connection, in-situ inspected, their joints evaluated, and select specific joints which will be modelled to obtain joint stiffness. Identify the correct value of stiffness in the joint is essential for bridge modelling, where joints are semi-rigid, and it has a significant impact on the internal forces of each element and in the global behavior of the structure.

The connections were made with a specialized software. As it was described before in the section 2.6, the component method is the analysis of the stiffness on separated components that conform a normal connection, but this method doesn't work property for riveted connections due to its complexity which involves hundreds of rivets and different plates. An alternative is to model the connection in finite element software like IDEA StatiCa, see (15), which allows modelling any type of connection, and the results obtained are either the capacity member in the connection, or the analysis of stiffness of the elements.

The IDEA StatiCa version 8.0.15.4637 allows to model in 3D steel joints and get the values of stress, deformations, fasteners and rigidity of the connection. It could perform four type of analysis; these are:

- **Stress/strain**, response of the joint to applied design load
- **Stiffness analysis**, stiffness of connection of selected member of the joint
- **Member capacity design**, Joint is designed not on design load, but on maximal capacity of connected member,
- **Joint design resistance**, ratio between design load and maximal load is determined for the whole joint, see [15].

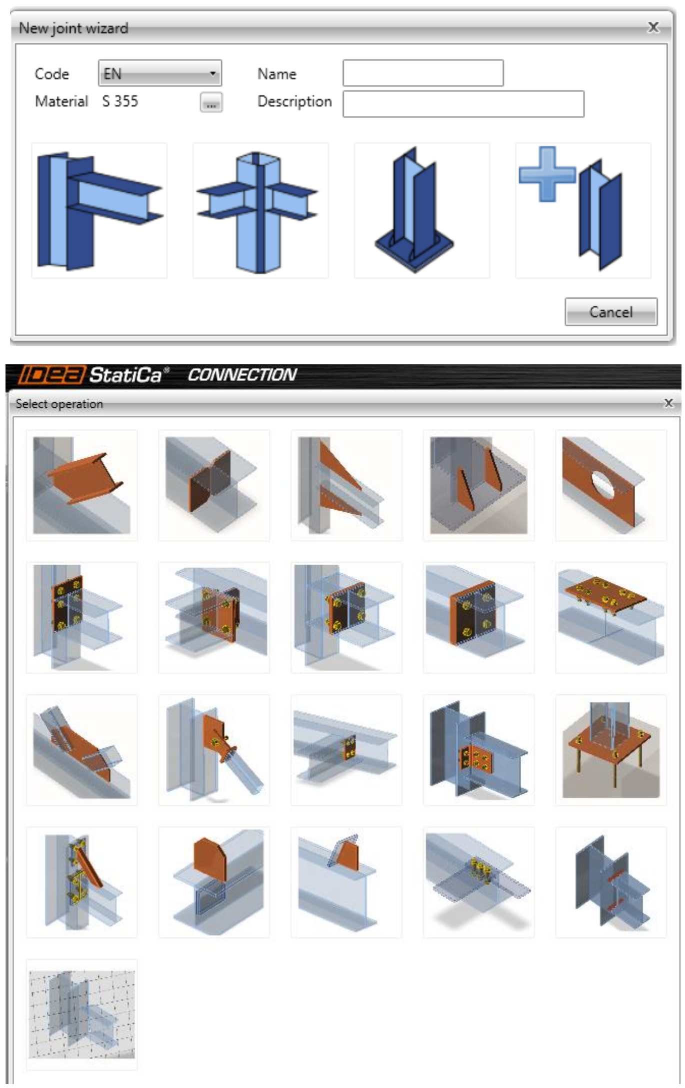
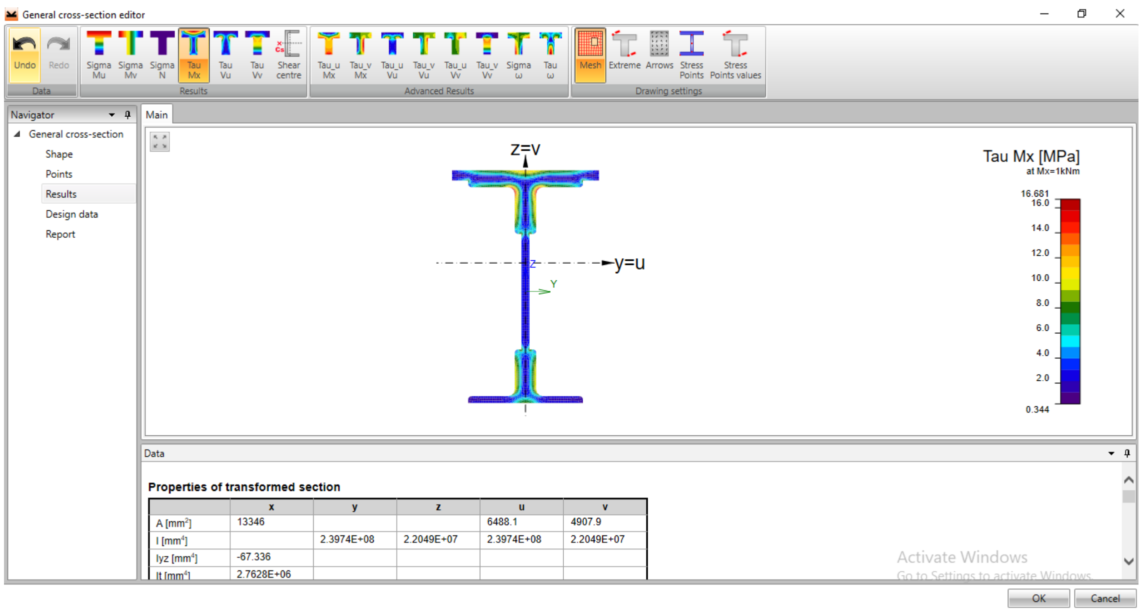


Figure 23.Initial windows in IDEA StatiCa.. Ref. [15]

As It is showed on the figure 23 we have a range of possibilities for different types of connections and its tools allow modelling close to a real case. IDEA StatiCa used the CBFEM method with is based on the component method, see [15].

There is a possibility to use a user defined section, and some experiments where made using this to model the complete section as one member. It helps on the computational time since there were not fund any inconsistences on the construction of every model and the results will be the correct ones.



	x	y	z	u	v
A [mm²]	13346			6488.1	4907.9
I [mm⁴]		2.3974E+08	2.2049E+07	2.3974E+08	2.2049E+07
Iyz [mm⁴]	-67.336				
It [mm⁴]	2.7628E+06				

Figure 24.Define General Section in IDEA StatiCa. Ref. [15]

All the bridges on this present report are connected by rivets but there is not option for modeling them in IDEA StatiCa. This deficiency has been resolved replacing rivets with screws of user-defined dimensions corresponding to rivet parameters, see [11]. The figures (25) and (26) correspond to the curves of the shear loads vs deformation for rivets and bolts. User definition in IDEA StatiCa allows to change the characteristics of the bolt, so if it is changed the gross cross section area and the tensile stress area of the bolt; it will behave as a rivet. In the figure (26) the behavior of failure plate number 1 is closer to the behavior of the rivets showed in figure (25). The rivets yield around 60 Kips with approximately 0.18 inches and the in failure plate ,1 bolts yield around 60 Kips with approximately 0.15 inches. In order to get more accurate data, it was decided to modelling as the diameter of the rivet-bolt is equal to the hole in which it is placed.

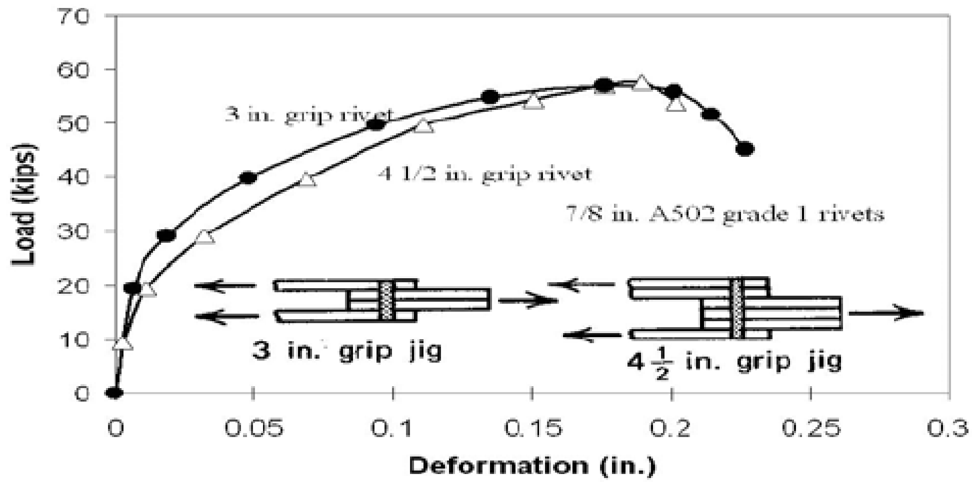


Figure 25. Shear versus deformation curves for A 502 grade 1 rivets. Ref [8].

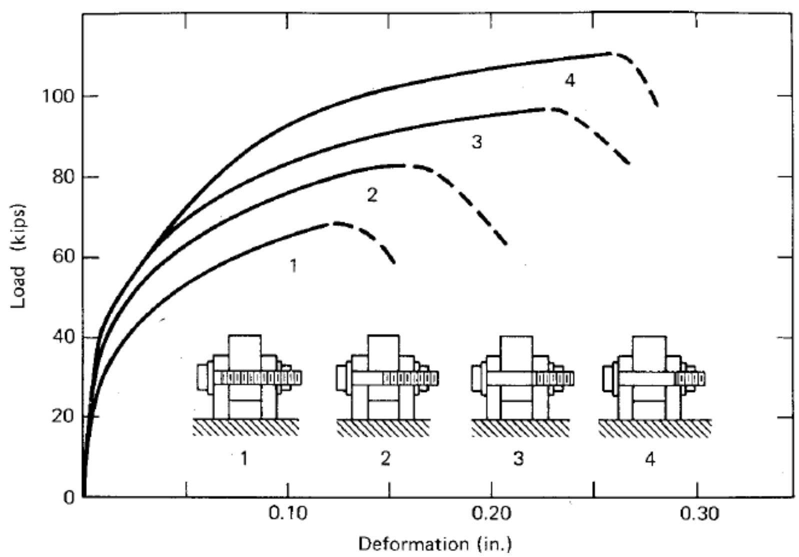


Figure 26. Shear load versus deformation curves for different failure plates. Ref [8].

4. THE GOALS OF THE THESIS

The purpose of the presented thesis is to analyze relationships between the stiffness and the moment of inertia for different steel railway bridges and evaluate the prediction formula of initial rotational stiffness (S_j) for the riveted connections in old steel bridges. The purpose of the prediction formula is for the creation of numerical models of riveted bridges, without difficult modeling of the connections.

The necessary steps to achieve the goals are:

- to select the representative types of steel bridges to be analyzed,
- to select typical group of connections in the selected steel bridged,
- to evaluate the connections geometry in order to model them,
- to create detail 3D CBFEM models in IDEA StatiCa and obtain the initial rotational stiffness of the detail models,
- to analyze the results, compare with existing results, evaluate the simple relationship and define the prediction formula,
- to compare the prediction formula with the CBFEM models to estimate the average percent error.

5. SELECTIONS OF BRIDGES DEPENDING OF THE DETALING

The project “Methods of expert assessment of railway bridges and determination of prediction procedures” is contracted by the Czech Railway Infrastructure Administration, SZDC by its initials in Czech, and executed by the Czech Technical University in Prague. The project is about 9 different types of bridges, each of them has a different and specific characteristic. They are the most representative bridges in the railway system in the Czech Republic, for instance: truss bridges, girder bridges, arch bridges, deck type, semi through, through bridges. In this section are described 5 bridges of different kind.

5.1. Libocany -TU 502

The official name of the bridges is Libocany this technical report will be referred as TU 502. The bridge is located over the river Ohře through Libočan on the kilometer 200,916, with the following GPS coordinates 50 ° 19'52.678"N 13 ° 31'6.762"E, this bridge connects Mladotice (mimo) – Žatec (mimo) (vč. Žatec západ); The bridge data are: length 129.40 m, width 8.10 m, height of the bridge: 13.20 m. (16), see [16]. This is a semi through type truss bridge.

Figure 27.TU 502 bridge. Ref. [16]

5.1.1. Geometry

The bridge has a total length of 119.40 m and is divided in two sections 58.50 m formed by trusses supported on masonry pillars over the river bed, the year od manufacture was on 1907 and repair and coat at the year 1982. All the original structure is made of angles and plates joined with rivets. The structure is made of steel, beam truss, plain, rivet joints, ending of the truss perpendicular, the diagonals are made of angles.

- Dimensions: span - 58,00 m, length - 58,50 m, width - 5,30 m
- Main beams: truss, riveted, composite system diagonal and vertical (angles + flat materials), height max. - 6,34 m, width of the upper belt of the main beam - 320 mm, length - 58,50 m, axial distance of the main beams - 4.80 m.
- Transverse bracing between ribbed trusses, riveted
- Longitudinal bracing between the longitudinal straight angles (L profiles) unidirectional

Year of manufacture: 1907 and Year of repair: 1982, from the front of the main beams at the beginning and end to the right - Coat IX. 1982

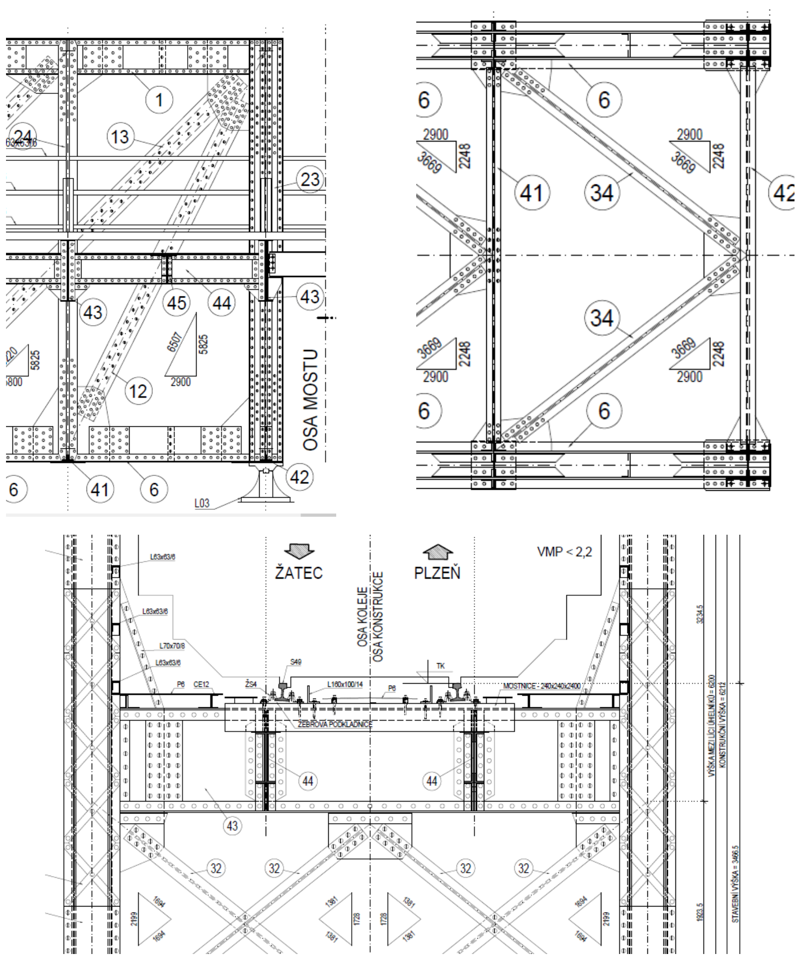


Figure 28. Geometry of the TU 502 bridge. Ref. [16]

5.1.2. Group of connections

On this present report it will be analyzed only the connections which are considered complicated to determinate its initial rotational stiffness. The characterization of joints helps on the analysis and data processing of the connections mas simplify the time of computing. In this work focuses on the connections in the deck which are divided in two groups: The connection between the main girder (or truss) with the cross beam; and the connections between cross beam with the stringer. On this bridge, it was important to define the initial stiffness on the connection between the truss and the cross beam at it is showed on the graphic below.

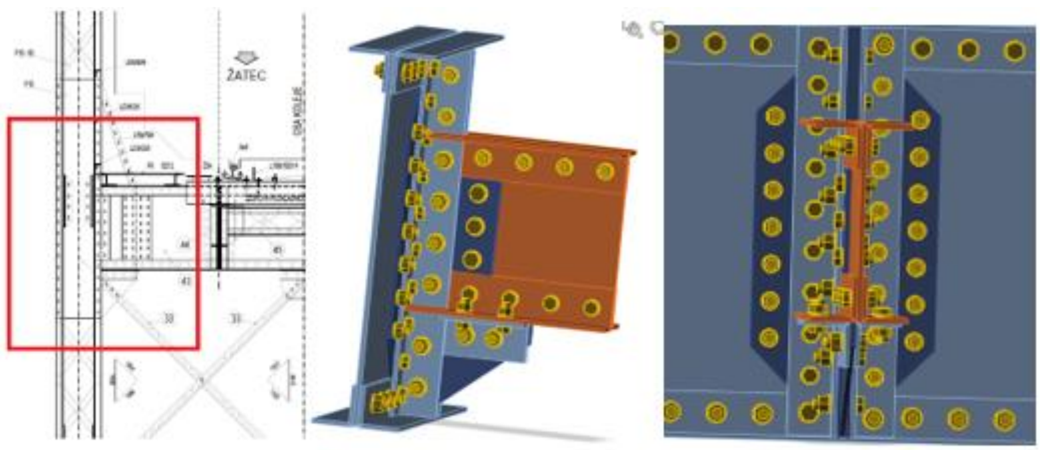


Figure 29. Joint of interest on TU 502. Ref. [16]

On the modelling uses the profiles on the figure (30)

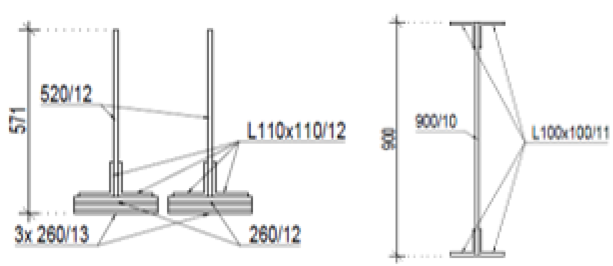


Figure 30. Profiles. Ref. [16]

5.2. Postoloprty – Vrbka- TU 581

The official name of the bridges is or Postoloprty - Vrbka and for this technical report will be referred as TU 581. The bridge is located over the river Chomutovka on the kilometer 215.615 m, with the following GPS coordinates 50 ° 21'50.653"N, 13 ° 41'54.458"E, this bridge connects Žatec (mimo) – České Zlatníky (mimo) (vč. Obrnice); The bridge data are: length 66.10 m, width 7.35 m, height of the bridge: 19.95 m, see [16]. This is a semi through type girder bridge.

Figure 31. TU 581 bridge. Ref. [16]

5.2.1. Geometry

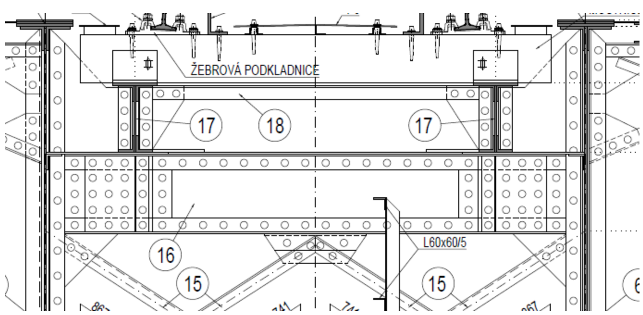
The bridge has a total length of 66.1 m and is divided in two sections 20.92 m formed by Girder, supported on masonry pillars over the river bed, the year od manufacture was on 1872 and repair and at the year 1911 and on 1972. All the original structure is made of angles and plates joined with rivets. The structure is made of steel, beamed, welded joints and rivets, recessed bridge, end of perpendicular.

- Dimensions: span - 20,52 m (MES), length - 20,92 m, width - 7,35 m
- Main beams: Fully riveted, length 20,92 m, flange width 280 mm, height 2020 mm, axle distance 2.70 m. Rectangles: riveted, height - 350 mm, flange width - 170 mm, axial distance - 1.80 m, placed on cross members with transverse intermediate

reinforcement from double angles. Rectangles: 10 pcs: top panel, height - 430 mm, bottom truss, height 930 mm, axle distance - 2,30 m.

- Longitudinal bracing lower (composite), single "L" profiles + horizontal stacking sheets.
- Longitudinal bracing top intermediate (composite), single "L" profiles + horizontal joint plates.
- Laying the bearing structure: on the bearings.
- Type and layout of bearings: fixed - steel stool on the support O 01. movable - steel three-roller on pillar P 01.
- The distance between the longitudinal members of the constructions K 01 and the construction K 02 on the pillar P 01: approx. 120 mm.
- Distance of main beams K 01 and construction K 02 on pillar P 01 left 530 mm, right 530mm.

Year of manufacture: 1872, the manufacturer's label is not on the construction, year of the first repair 1911 and last repair in 1972, designation of the company carrying out PKO and year of execution placed on the wall of the main beam at the beginning to the right of natural – mo- louny 1972.



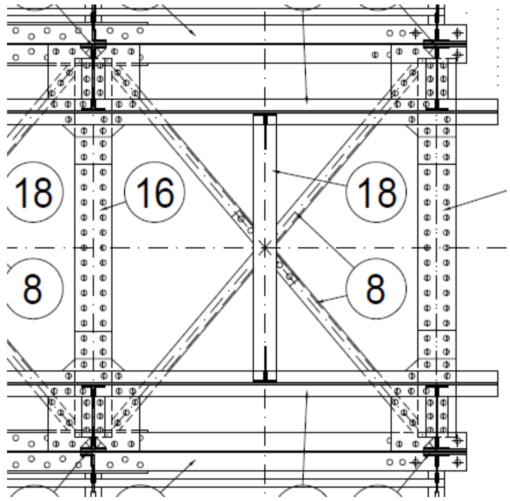


Figure 32. Geometry of the TU 581 bridge. Ref. [16]

5.2.2. Group of connections

On this present report it will be analyzed only the connections which are considered complicated to determine its initial rotational stiffness. The characterization of joints helps on the analysis and data processing of the connections mas simplify the time of computing. In this work focuses on the connections in the deck which are divided in two groups: The connection between the main girder (or truss) with the cross beam; and the connections between cross beam with the stringer. On this bridge, it was important to define the initial stiffness on the connection between the main girder with the cross beam and the connections between cross beam with the stringer as it is showed on the graphic below.

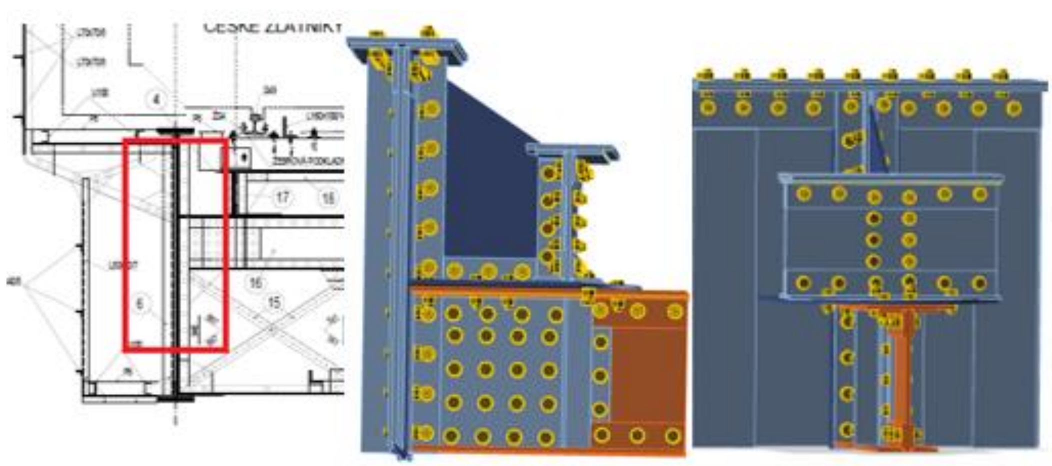


Figure 33. Joint of interest on TU 581, main girder with the cross beam. Ref. [16]

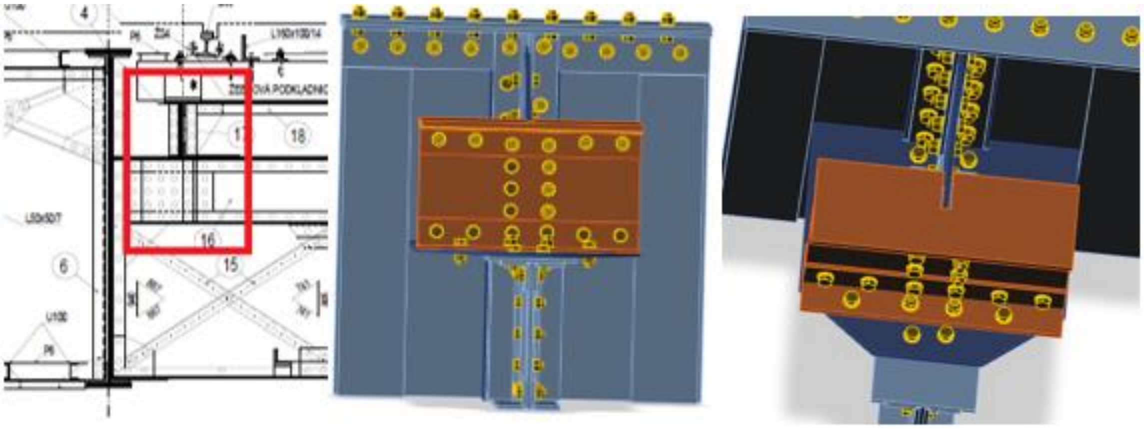


Figure 34.Joint of interest on TU 581. Cross beam with the stringer. Ref [16]

On the modelling uses the profiles on the figure (35)

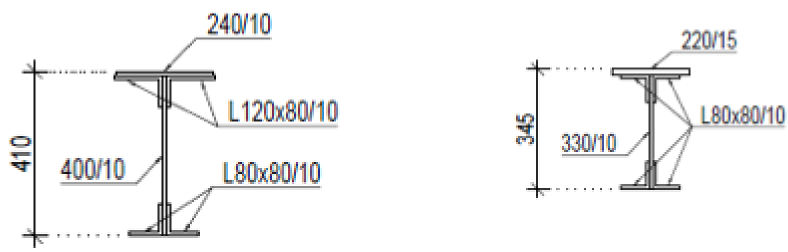


Figure 35. Profiles. Ref. [16]

5.3. Kojetín- TU 2101

The official name of the bridges is Kojetín and for this technical report will be referred as TU 2101. The bridge is located over the river Morava on the kilometer 74.798, with the following GPS coordinates 49 ° 21'1.7 "N, 17 ° 19'18.7" E, this bridge connects Brno hl.n. (mimo) - Přerov (mimo) (přes Chrlice); The bridge data are: length 130 m, width 5.5 m, height of the bridge: 5.40 m, see [16]. This is a through type truss bridge.

Figure 36. TU 2101 Bridge. Ref. [16]

5.3.1. Geometry

The bridge has a total length of 130 m and is divided in three sections: two equal sections of 36.50 m and the middle one of 48.7 m formed by trusses supported on masonry pillars over the river bed, the year od manufacture was on 1907 and repair and coat at the year 1982. All the original structure is made of angles and plates joined with rivets. The structure is made of steel.

First and third structures

- Steel, bridge construction. Construction perpendicular. Element bottom element.
- Length of construction 36,50 m (MES), span 35,68 m (MES), width 5,50 m (MES).
- Main steel beams, riveted, trusses - base system with vertices, height up to 4250 mm, width 280 mm, axial distance 5000 mm. The lower longitudinal stiffening of the hl. of beams from double steel profiles L 120x120x14 mm, rivet connections.
- Cross-bars, steel, ribbed, riveted I profiles, height 910 mm, lower flange width 275 mm, axial distance 3560 mm, connections to the main beams rivets.

- Steel, full length, riveted I profiles, height 460 mm, lower flange width 210 mm, axial distance 1800 mm, connections to ribbed cross members. Transversal reinforcement of steel plates profiles U 160x65 mm, rivet connections. Longitudinal bracing of steel rail sections L 80x80x9 mm, rivet connections.
- Load bearing support - bearing: steel bearings - O 01 fixed, P 01 movable two-roller.

Second Structure

- Steel, bridge construction. Construction perpendicular. Element bottom element.
- Length of construction 48,70 m (MES), span 47,60 m (MES), width 5,50 m (MES).
- Main steel beams, riveted, lattice - basic system with vertices, height up to 6000 mm, belt width 300 mm, axial distance 5000 mm. The upper belt of the truss beams is shaped L 130x130x14 mm, rivet connections. Upper transverse bracing of main beams made of steel, riveted, full-width I profiles, height approx. 250 mm, riveted connections.
- Crossbars, steel, flat, riveted I profiles, height 900 mm, width of lower flanges 330 mm, axial distance 5200 mm, connections to the main beams rivets.
- Steel, full length, riveted I profiles, height 660 mm, width of lower flanges 280 mm, axial distance 1800 mm, connections to ribbed cross members. Transversal reinforcement of steel plates profiles U 160x65 mm, rivet connections. Longitudinal bracing of steel rail sections L 80x80x9 mm, rivet connections.
- Load bearing support - bearing: Steel Pulley Bearings - P 01 fixed stationary, P 02 movable two-roller.
- Production and construction year 1953 and repair 1974.

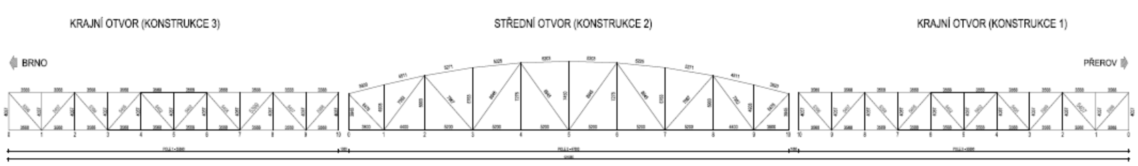


Figure 37.Over all structure TU 2101. Ref. [16]

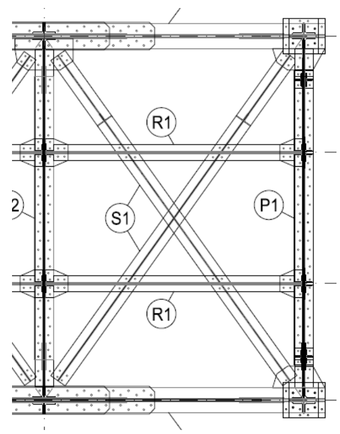


Figure 38.First and third structure TU 2101. Ref. [16]

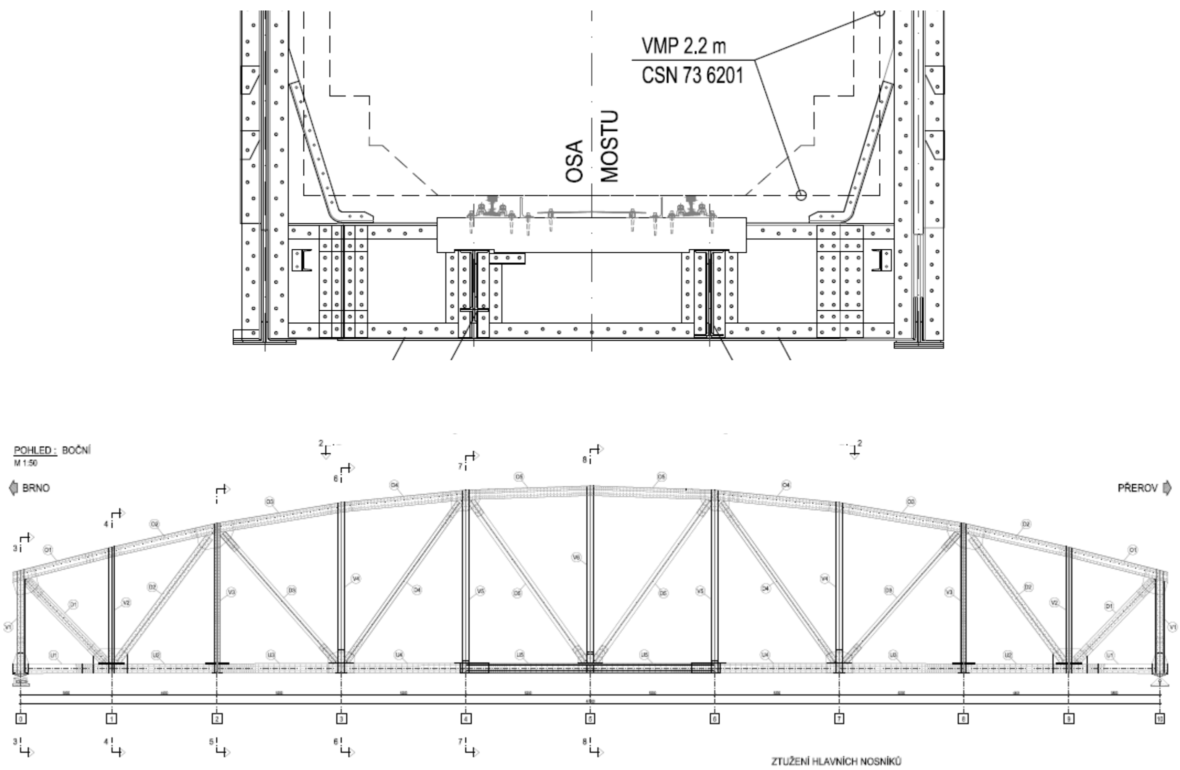


Figure 39.Central structure TU 2101. Ref. [16]

5.3.2. Group of connections

On this present report it will be analyzed only the connections which are considered complicated to determinate its initial rotational stiffness. The characterization of joints helps on the analysis and data processing of the connections mas simplify the time of computing. In this work focuses on the connections in the deck which are divided in two groups: The connection between the main girder (or truss) with the cross beam; and the connections between cross beam with the stringer. On this bridge, it was important to define the initial stiffness on the connection between the main girder with the cross beam and the connections between cross beam with the stringer as it is showed on the graphic below.

First and third structure

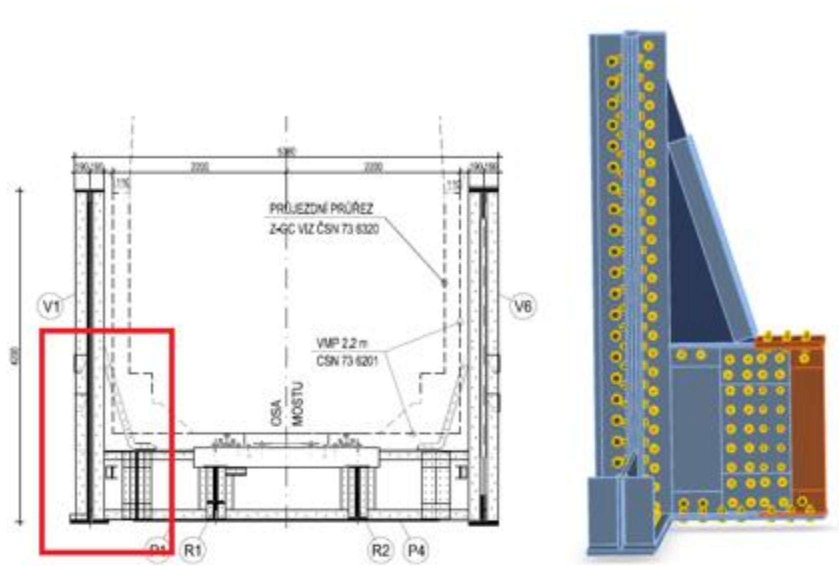


Figure 40. Joint of interest on TU 2101. Main girder with the cross beam. Ref. [16]

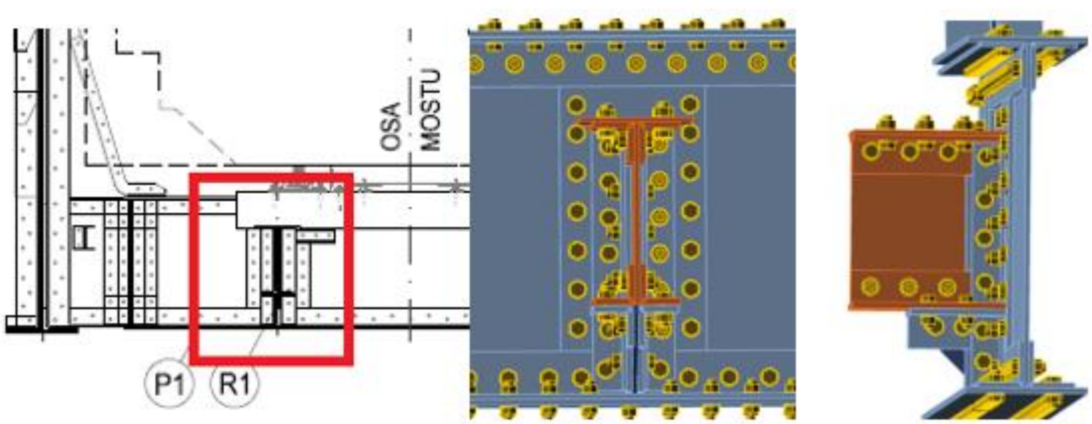


Figure 41. Joint of interest on TU 2101. Cross beam with the stringer. Ref. [16]

On the modelling uses the profiles on the figure (42)

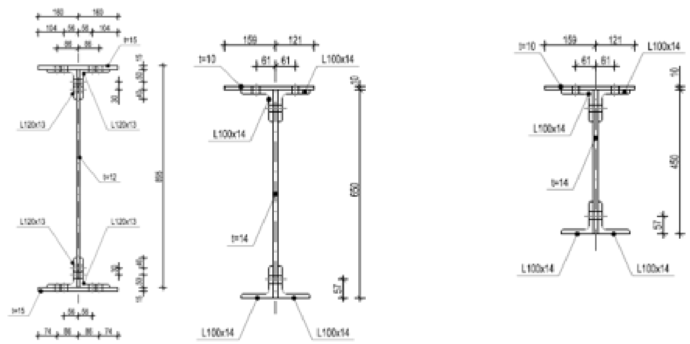


Figure 42. Profiles. Ref. [16]

Middle Structure

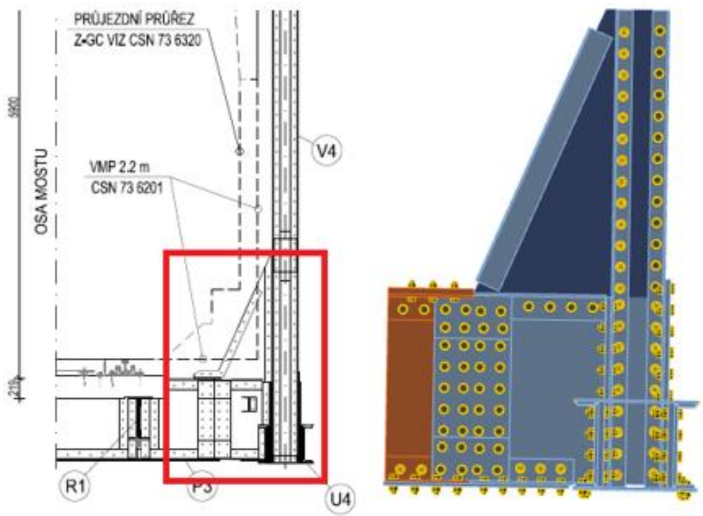


Figure 43. Joint of interest on TU 2101. Main girder with the cross beam. Ref. [16]

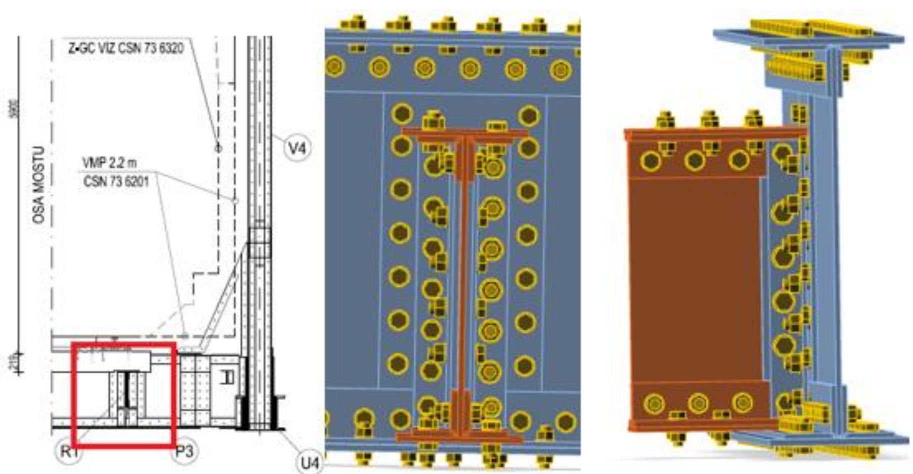


Figure 44. Joint of interest on TU 2101.Cross beam with the stringer. Ref. [16]

5.4. Domašov nad Bystřicí- TU 2191

The official name of the bridges is Domašov nad Bystřicí and for this technical report will be referred as TU 2191. The bridge is located over the river Hrubá on the kilometer 28.162 m, with the following GPS coordinates 49 ° 43'52.074 "N, 17 ° 26'29.108" E, this bridge connects Olomouc hl. n. (m) (O. hl. n. Bělidla vč) - Krnov; The bridge data are: length 31.55 m, width 5.37 m, height of the bridge: 4.82 m, Bridging Length: 18.55 m, see [16]. This is a deck type girder bridge.

Figure 45.TU 2191 Bridge. Ref. [16]

5.4.1. Geometry

The bridge has a total length of 20.10 m formed by steel beams supported on masonry pillars over the river bed, the year od manufacture was on 1899. All the original structure is made of angles and plates joined with rivets. The structure is made of steel, beam, truss, the diagonals are made of angles.

- Steel bridge construction. Construction perpendicular. The bridge head is recessed.
- Length of construction 20,10 m (MES), span 19,50 m (MES), width 4,55 m (MES). Main steel beams, solid, riveted "I" profiles 1855 mm high, 275 mm flange widths and axial distance of the main beams 2500 mm.
- Longitudinal bracing of main beams made of steel "L" profiles top 70x70x8.
- Cross reinforcement of main steel beams, height 1180 mm, axial distance 1940 mm.
- Cross bars, lattice, "L" profiles, axial distance 1940 mm.

- Steel, solid, welded "I" profile height 335 mm, flange width 150 mm, axial distance 1800 mm.
- longitudinal bracing of steel "L" profiles 80x80x9
- transverse bracing of steel profiles U 140x60.
- The manufacturer's table or PKO painting label is not available.
- Structural bearing - bearing: steel plate - O 01 fixed, O 02 movable.

Production and construction year 1899 (MES), 1962 repair and PKO 1980.

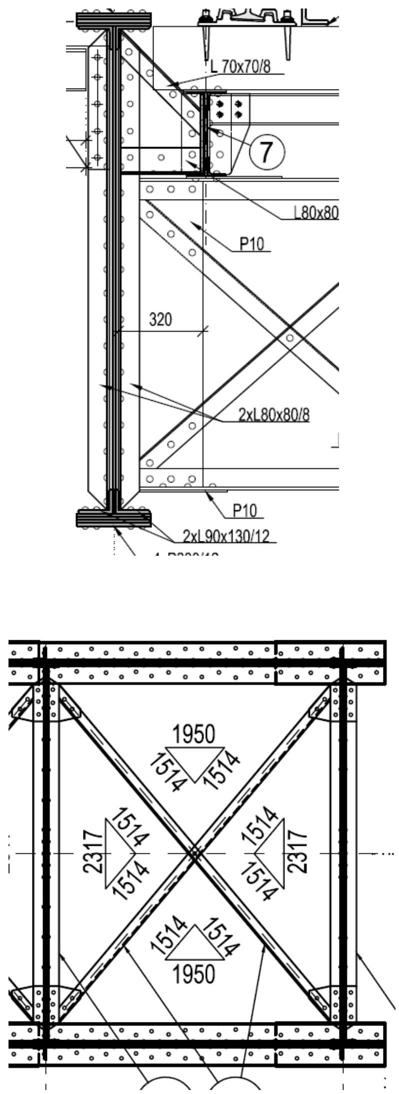


Figure 46.Geometry of the TU 2191 bridge. Ref. [16]

5.4.2. Group of connections

On this present report it will be analyzed only the connections which are considered complicated to determinate its initial rotational stiffness. The characterization of joints helps on the analysis and data processing of the connections mas simplify the time of computing. In this work focuses on the connections in the deck which are divided in two groups: The connection between the main girder (or truss) with the cross beam; and the connections between cross beam with the stringer. On this bridge, it was important to define the initial stiffness on the connection between the main girder with the cross beam as it is showed on the graphic below.

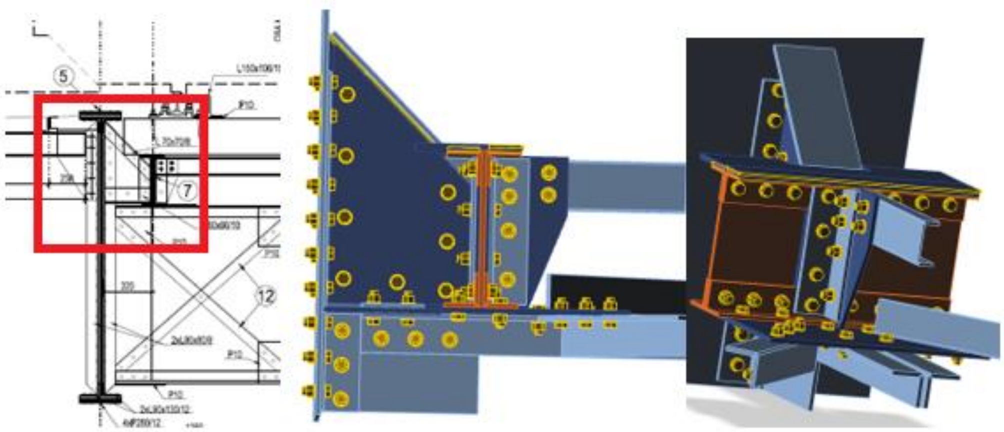


Figure 47. Joint of interest on TU 2191. Ref. [16]

5.5. Luzna - TU 2362

The official name of the bridges is Luzna and for this technical report will be referred as TU 2362. The bridge is located over the river Luženka on the kilometer 25.938, with the following GPS coordinates 49 ° 14'28.2 "N, 18 ° 1'29.3" E, this bridge connects Horní Lideč (včetně) -Vsetín (včetně); The bridge data are: length 91.90 m, width 15.12 m, height of the bridge: 20.20 m, Bridging Length: 75.20 m, see [16]. This is a deck type truss bridge.

Figure 48. TU 2362 Bridge. Ref. [16]

5.5.1. Geometry

The bridge has a total length of 75.20 m and is divided in two sections 38.70 m formed by trusses supported on masonry pillars over the river bed, the year od manufacture was on 1937 and repair and coat at the year 1992. All the original structure is made of angles and plates joined with rivets. The structure is made of steel, truss, rivet joints, ending of the truss perpendicular, the diagonals are made of angles.

- Steel, bridge construction. Construction perpendicular. Elevation elemental, upper.
- Construction length 38,70 m (MES), span 38,00 m (MES), width 5,15 m (MES 3,36 m).
- Main steel beams, riveted, trusses - basic system with vertices; height 4700 mm, lower belt widths up to 300 mm, axial distance 3000 mm. Longitudinal reinforcement of the main beams profiles L, bottom L 90x90x10 mm, upper L 100x100x12 mm.
- Steel, full length, riveted I profiles, height 500 mm, belt width 250 mm, axial distance 1800 mm. The longitudinal girders are mounted on the crossbars and fixed by bolts.
- Longitudinal bracing steel profile sections T 200x100 mm. Transversal reinforcement of steel profiles U 220x80 mm. Rivet bindings.
- Cross bars, ribbed, riveted I profiles, height 620 mm, width 250 mm, axial distance 3800 mm, connections to hl. rivet beams. Transverse bracing of hl. beams from doubles steel profiles L 100x100x12 mm, rivet joints. Total height up to 4680 mm.
- Structural bearing - bearing: Bearing Steel Bearings - initially movable three-roller, at the end solid fixed.

Production and construction year 1937 and repair 1992. Producer table on the object - Vitkovice Ironworks. PKO - 1982.

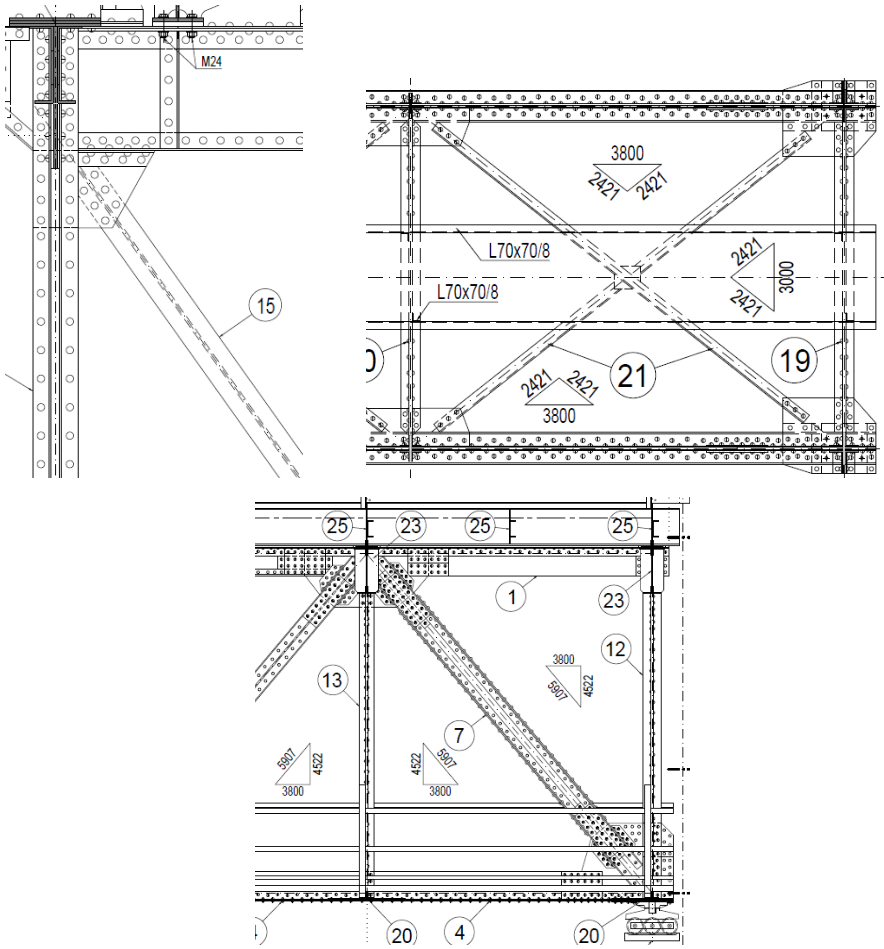


Figure 49. Geometry of the TU 2362 bridge. Ref. [16]

5.5.2. Group of connections

On this present report it will be analyzed only the connections which are considered complicated to determinate its initial rotational stiffness. The characterization of joints helps on the analysis and data processing of the connections mas simplify the time of computing. In this work focuses on the connections in the deck which are divided in two groups: The connection between the main girder (or truss) with the cross beam; and the connections between cross beam with the stringer. On this bridge, it was important to define the initial stiffness on the connection between the main girder with the cross beam as it is showed on the graphic below.

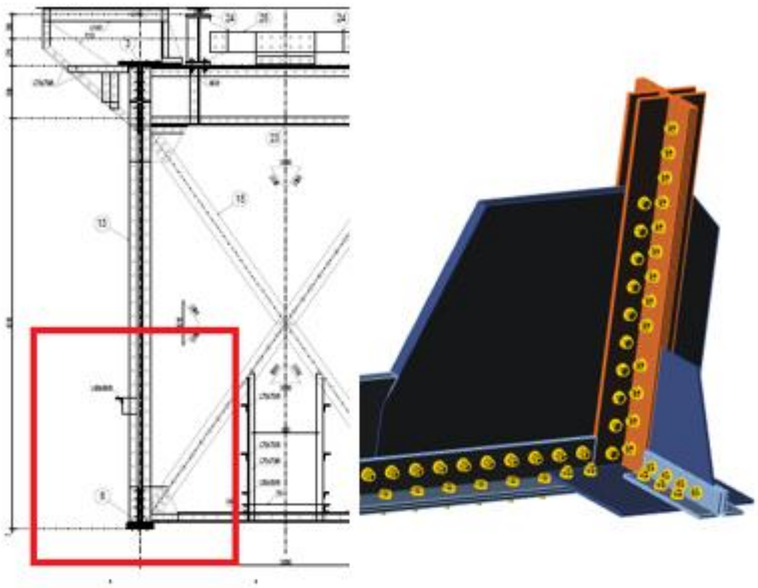


Figure 50. Joint of interest on TU 2362. Bottom section of the truss on the supports. Ref. [16]

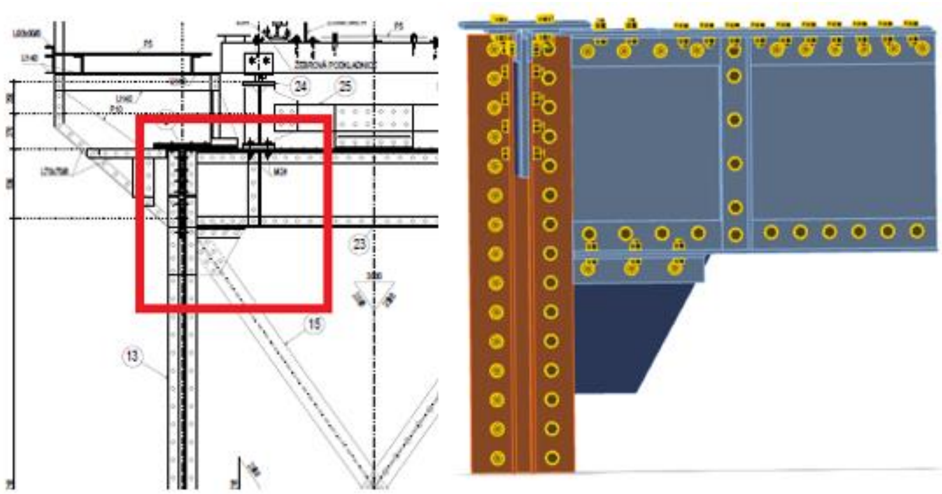


Figure 51. Joint of interest on TU 2362. Upper section of the truss on the supports. Ref. [16]

6. CBFEM MODELING

The modelling of the CBFEM models for all the bridges mentioned in the section 5 are made with steel plates, angles and bolts/riveted connections. The characterization and modeling of this bridges was made on the software IDEA StatiCa described in the section 3.3.1. The table (2) is a resume of the models with its characteristics.

Name		Characterization	Type of connection	Height of the member analyzed (mm)	Section of reference
Libocany	TU 502	semi through type truss bridge	Cross Beam-stringer	430	5.1
Postoloprty	TU 581	semi through type girder bridge.	Main girder-cross beam	410	5.2
Postoloprty	TU 581	semi through type girder bridge.	Cross beam-Stringer	348	5.2
Kojetín	TU 2101	through type truss bridge	Main girder-cross beam	910	5.3
Kojetín	TU 2101	through type truss bridge	Main girder-cross beam	910	5.3
Kojetín	TU 2101	through type truss bridge	Cross Beam - stringer	460	5.3
Kojetín	TU 2101	through type truss bridge	Cross Beam - stringer	683	5.3
Domašov nad Bystřicí	TU 2191	deck type girder bridge	Cross Beam - stringer	320	5.4
Luzna	TU 2362	deck type truss bridge.	Cross Beam - stringer	603	5.5
Luzna	TU 2362	deck type truss bridge.	Corner Upper	250	5.5
Luzna	TU 2362	deck type truss bridge.	Corner lower	250	5.5

Table 2. Resume of the CBFEM models

6.1. Input data into the program

Proprieties of material

For Properties of materials, the Eurocode, see [5], recommends to apply the national codes for calculations purposes. In Czech applies Appendix A of the SŽDC SR5. (17), see [17]. For those bridges described before, taken the consideration the year of manufacture was used cast steel as a material.

As it was described on the section 3.3, the real stress-strain diagram of steel is replaced by the ideal plastic material for design purposes in building practice the advantage of ideal plastic material is, that only yield strength and modulus of elasticity must be known to describe the material curve, see [15]. The figure (20) describes the real tension curve and the ideal elastic-plastic diagram of material used in this thesis.

Year of manufacture	Material	Allowable stresses σ_{adm} [MPa]	Guaranteed yield strength f_y [MPa]	Strength limit f_u [MPa]	γ_{M0}	γ_{M1}	γ_{M2}	Standard
until 1894	Wrought Iron (Svářkové železo)	130	210	340	1.1	1.2	1.3	
1895-1904	Wrought Iron (Svářkové železo)	130	210	340	1.1	1.2	1.3	Nařízení 97/1904
	Cast steel (plávková ocel)	140	230	360	1.1	1.2	1.3	

Table 3. Properties of Material. Ref. [17]

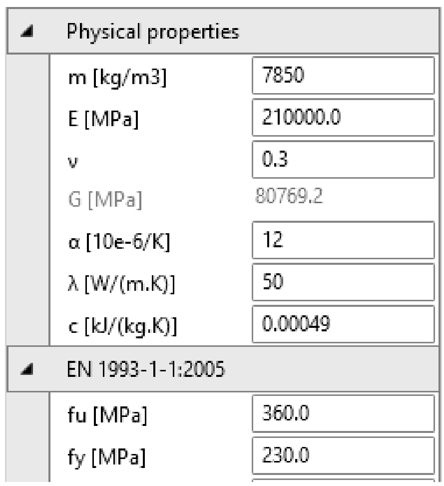


Figure 52. Physical properties of the material. Ref. [15]

Similarly, the rivet strength was considered in accordance with the Appendix A of the SŽDC SR5, see [16].

Strength characteristics	Rivets		Precision Bolts	
	In constructions made of material with a yield strength			
	fy ≤ 300 MPa	fy > 300 MPa	fy ≤ 300 MPa	fy > 300 MPa
f_y [MPa]	200	245	300	
f_u [MPa]	310	440	500	

Table 4. Properties of Material of the Rivets. Ref. [17]

As it was described on the section 3.3, the bolt behavior is implemented according the figure (21) which describes bolts for interaction of shear and tension.

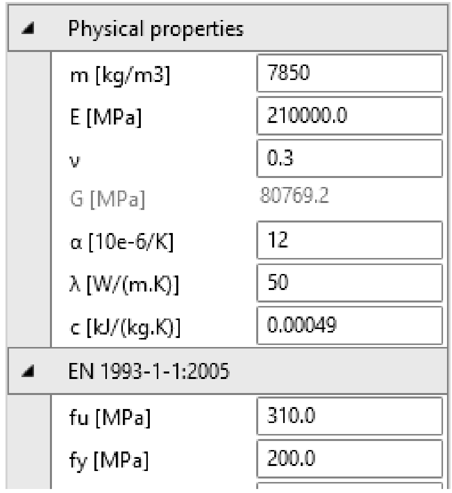


Figure 53. Physical properties of the bolts. Ref [15].

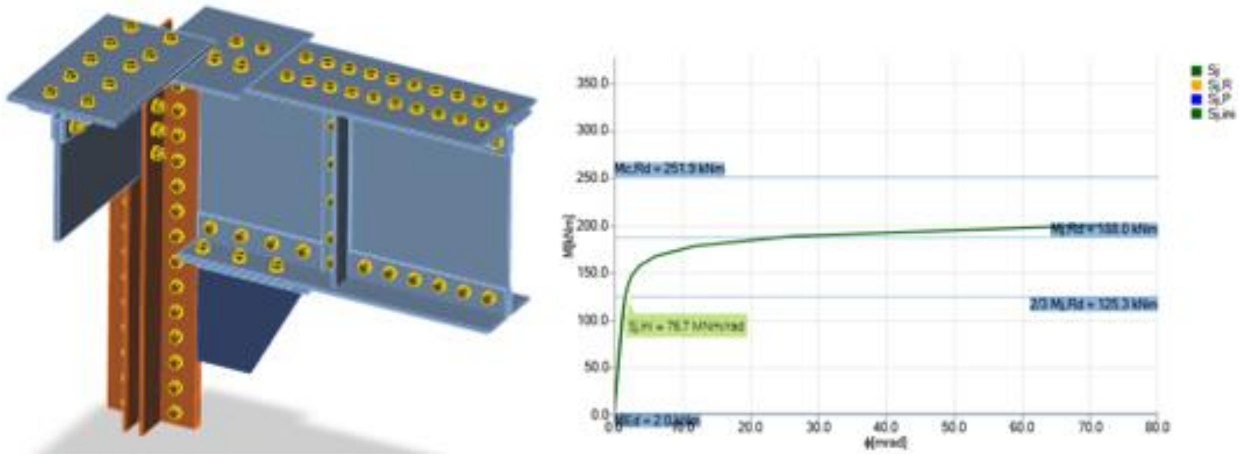
Loads applied

The stiffness, according to the formulas on the Eurocode EN 1993-1-8, see [5], is a property of the material (elasticity modulus), the flexibility of each component (*K* coefficients) and the geometry (Inertia); and loading should not be a determinant condition. The loads for each connection were low, to ensure the behavior of the material is within the elastic range. For all the models the axial force was 2 kN, for M_y and M_z were 1 kNm.

6.2. Output data from the program

For the stiffness analysis, the programs provides the moment-rotation diagram, including the initial stiffness $S_{j,ini}$ in light green, and the boundaries: pinned limit in blue, and rigid limit in yellow.

This work is about to predicting a formula of the initial stiffness S_j based on the Inertia of the element. The bonders $S_{j,R}$ and $S_{j,P}$ are according to EN 1993 1-8 with equations (1) and (2).



Rotational stiffness

Name	Comp.	Loads	Mj,Rd [kNm]	Sj,ini [MNm/rad]	Φc [mrad]	L [m]	Sj,R [MNm/rad]	Sj,P [MNm/rad]	Class.
1	My	LE1	188.0	76.7	71.5	4.70	66.3	1.3	Rigid

Secant rotational stiffness

Name	Comp.	Loads	M [kNm]	Sjs [MNm/rad]	Φ [mrad]
1	My	LE1	2.0	91.0	0.0

Figure 54. Rotational Stiffness provided from IDEA StatiCa. Ref. [15]

Symbol explanation

Symbol	Symbol explanation
$M_{j,Rd}$	Bending resistance
$S_{j,ini}$	Initial rotational stiffness
$S_{j,s}$	Secant rotational stiffness
Φ	Rotational deformation
Φc	Rotational capacity
$S_{j,R}$	Limit value - rigid joint
$S_{j,P}$	Limit value - nominally pinned joint

Table 5. Description table of IDEA StatiCa. Ref. [15]

6.3. Verification

For the verification of this work, several alternatives models were taken into account to understand the operation of the program.

There are three alternatives: the first models are created with the elements in its real dimensions and the elements don't have any supported element which it means, they have 6 degrees of freedom in the node: x, y, z, M_x, M_y, M_z.

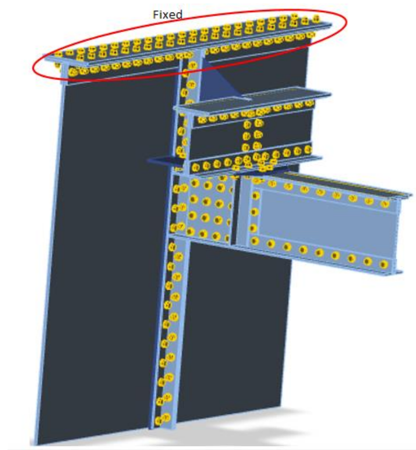


Figure 55.TU 581 no fixed – long elements

The second models are created with the short elements up to 40 cm and the elements don't have any supported element which it means, they have 6 degrees of freedom in the node: x, y, z, M_x, M_y, M_z.

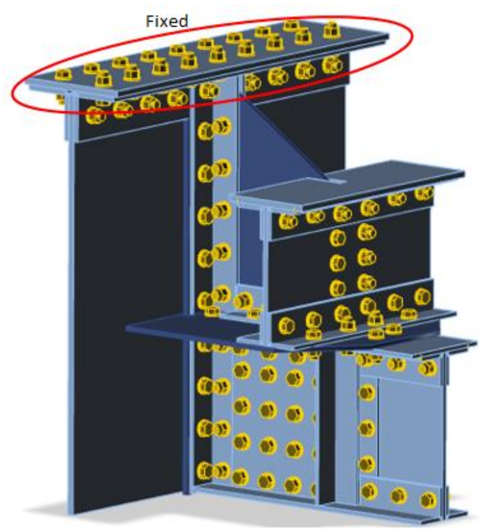


Figure 56. TU 581 no fixed short

The last models are created with the short up to 40 cm element and the elements on the extreme of the joint are fixed, it means, they have 0 degrees of freedom on de node: x, y, z, M_x, M_y, M_z.

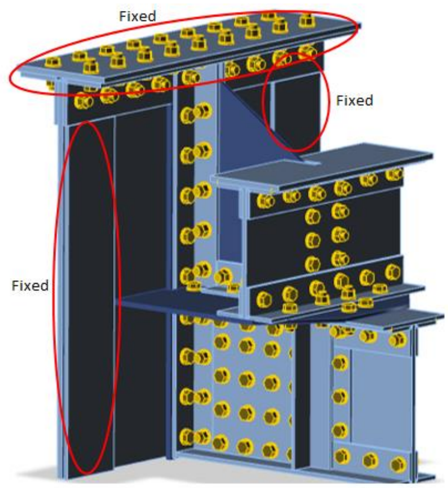


Figure 57.TU 581 fixed short

The last alternative is used in the thesis; because the previous models were made only to validate the veracity of the program; and finally the previous studies are carried out in this way.

TU 502, Cross Beam - stringer, Stiffness		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	0	18.5	764.3	15.3	18.5	40%
	Long Connection	247.7	30.8	764.3	15.3	33.8	
Mz	Short connection	35.6	2	47.6	1	2.2	-18%
	Long Connection	33.5	1.7	47.6	1	1.8	
TU 581, Main girder-Cross beam, Stiffness, Long members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Fixed connection	562.5	35.4	468.3	9.4	35.7	0%
	No Fixed Connection	590.9	35.4	468.6	9.4	36.8	
Mz	Fixed connection	111.1	3.6	28	0.6	3.8	0%
	No Fixed Connection	111.1	3.6	28	0.6	3.8	
TU 581, Main girder-Cross beam, Stiffness, fixed members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	594.5	53.1	468.6	9.4	56.7	-50%
	Long Connection	562.5	35.4	468.3	9.4	35.7	
Mz	Short connection	110.5	8.8	28	0.6	9.6	-144%
	Long Connection	111.1	3.6	28	0.6	3.8	

TU 581,Cross beam-Stringer, Stiffness, fixed members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	154	3.9	331.2	6.6	0	Semi-Rigid
Mz	Short connection	68.2	4.3	30.5	0.6	6.2	Semi-Rigid

TU 2101, Main girder-Cross beam, Stiffness, Long members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Fixed connection	3022.2	2014000000	3412.4	68.2	137700000	0%
	No Fixed Connection	3022.2	2014830000	3412.4	68.2	137766000	
Mz	Fixed connection	188.1	448.6	100.6	2	18409930	8%
	No Fixed Connection	186.1	488.6	100.6	2	18409930.2	

TU 2101, Main girder-Cross beam, Stiffness, Fixed members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	680.2	260.1	3412.4	68.2	266.5	100%
	Long Connection	3022.2	2014000000	3412.4	68.2	137700000	
Mz	Short connection	168	7.3	100.6	2	9.6	98%
	Long Connection	188.1	448.6	100.6	2	18409930	

TU 2101, Main girder-Cross beam2, Stiffness, Long members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Fixed connection	826.6	734.4	3412.4	68.2	920	0%
	No Fixed Connection	826.6	734.4	3412.4	68.2	920	
Mz	Fixed connection	234.7	156484402	100.6	2	18409930.2	0%
	No Fixed Connection	234.7	156484402	100.6	2	18409930	

TU 2101, Main girder-Cross beam2, Stiffness, Fixed members		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	559.7	210.5	3412.4	68.2	223.7	71%
	Long Connection	826.6	734.4	3412.4	68.2	920	
Mz	Short connection	188.7	26.9	100.6	2	27.4	100%
	Long Connection	234.7	156484402	100.6	2	18409930.2	

TU 2101, Cross Beam - stringer, Stiffness		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	212.7	18.3	575.6	11.5	29.8	-34%
	Long Connection	212.7	13.7	575.6	11.5	19.6	
Mz	Short connection	115.3	4	37.9	0.8	5.1	-122%
	Long Connection	123.6	1.8	37.9	0.8	2	

TU 2101, Cross Beam - stringer2, Stiffness		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	250.4	4.4	1718.5	34.4	15	68%
	Long Connection	109.9	13.8	1718.5	34.4	13.8	
Mz	Short connection	125.1	3	53.9	1.1	3.3	-58%
	Long Connection	119.4	1.9	53.9	1.1	2	
TU 2191, Cross Beam - stringer, Stiffness		**Rotational Stiffness**					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	100.3	2.5	395.4	7.9	13.9	43%
	Long Connection	92.9	4.4	395.4	7.9	9783015	
Mz	Short connection	39.4	26256963.2	8.2	0.2	2000000	-4%
	Long Connection	37.9	25285509	8.2	0.2	2000000	
TU 2362, Cross Beam - stringer, Stiffness		**Rotational Stiffness**					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	198.5	14.8	1101.5	22	220.6	3%
	Long Connection	197.1	15.2	1101.5	22	252.1	
Mz	Short connection	78.8	3.3	89	1.8	37.9	-22%
	Long Connection	78.5	2.7	89	1.8	27.2	

TU 2362 Corner Upper		Rotational Stiffness					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	188	76.7	66.3	1.3	91	Rigid
Mz	Short connection	130.9	40.8	29.3	0.6	225.6	Rigid
TU 2362 Corner lower		**Rotational Stiffness**					
		Mj. Rd	Sj. Ini	Sj,R	Sj, P	Sjs	Difference in %
		kNm	MNm/rad	MNm/rad	MNm/rad	MNm/rad	
MY	Short connection	69.9	2.3	66.3	1.3	2.7	Semi-Rigid
Mz	Short connection	118.1	78739181	29.3	0.6	5368580.6	Rigid

Table 6. Verification of the models

After the analysis it is remarkable two data: the first one, the rotational Stiffness does not vary if an element is fixed at the end, nor some elements in the connection are fixed or not; because at the moment of analyzing rotational stiffness the program fixes the other elements and only leaves free the element of interest. The second and most important, is due to the length of the element analyzed, the difference varies by up to 100% and this is due to the program includes the rigidity of the element; adding the rigidity introduced, consequently it uses the rigidity in two occasions which causes a significant variation in the results. It is recommended to use short connections because the rigidity of the connection is taken into account once.

7. RESULTS

Since, more data available enrich the database in order to obtain better results. The last two previous studies: O. Minor's master thesis (The Impact of the Connection Stiffness on the behavior of a Historical Steel Railway Bridge), see [2], and the SUDOP Study (The axial and rotational stiffness in the connections of a steel railway bridge, Tábor-Písek bridge), see [11], which serve as a starting point and reference for this present document, were considered into the analysis.

7.1 Results of the CBFEM models

The data from the CBFEM models are showed in the figure (60):

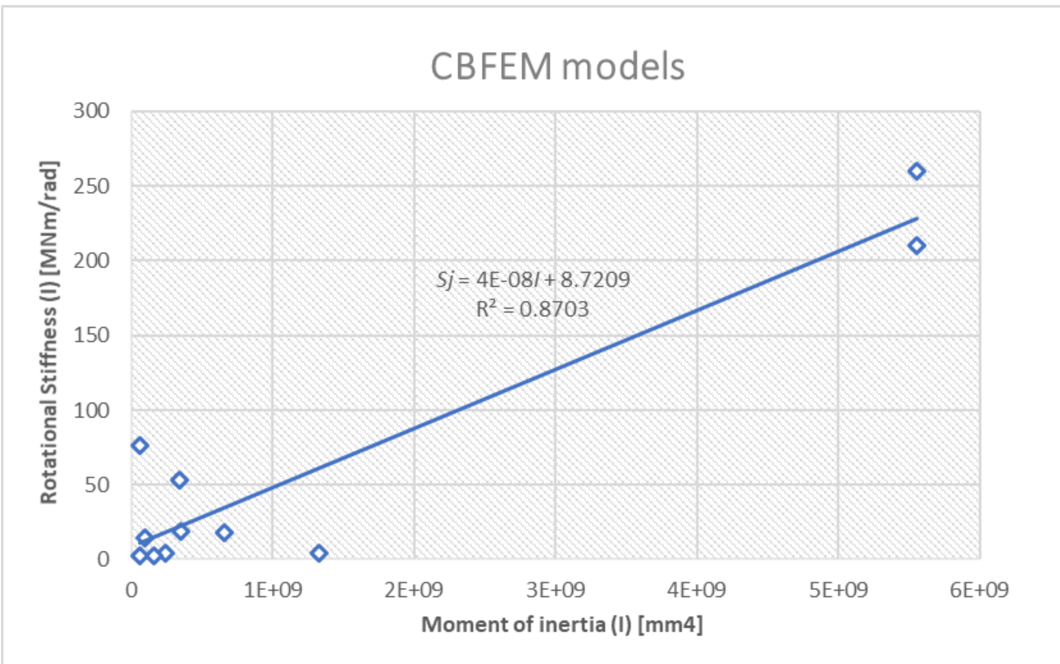


Figure 58. Formula from CBFEM models.

$$S_j = 4x10^{-8}I - 8.7209 \quad [MNm/rad] \tag{4}$$

7.2 Results of the previous studies

The Tábor-Písek bridge, truss bridge, presents the following data and provided the following formula, see [11]:

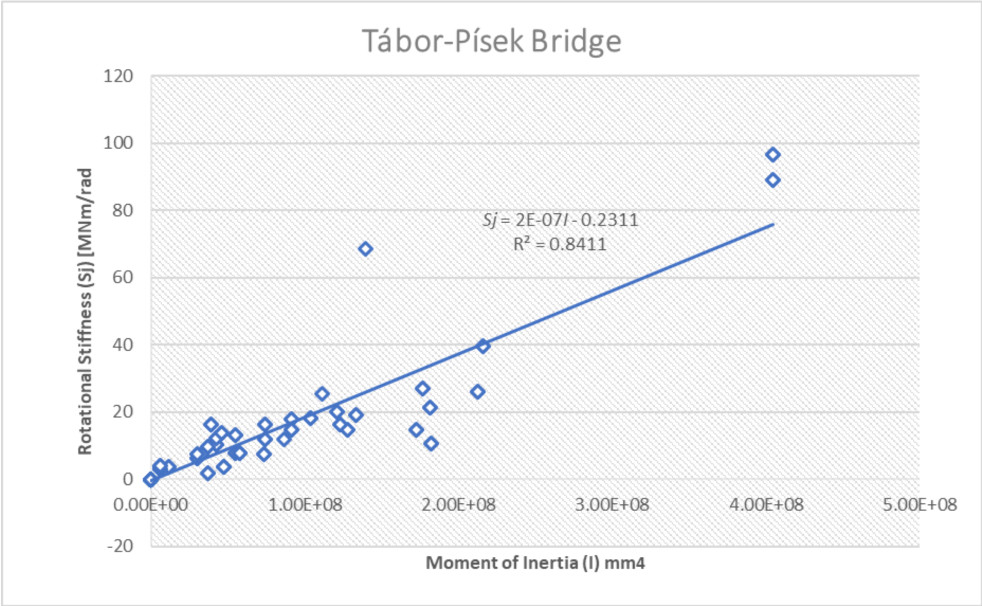


Figure 59. Formula from Tábor-Písek bridge. Ref. [11]

$$S_j = 2x10^{-7}I - 0.2311 \qquad [MNm/rad] \tag{5}$$

The bridge Pod Vyšehradem, truss bridge, presents the following data and provided the following Formula, see [2]:

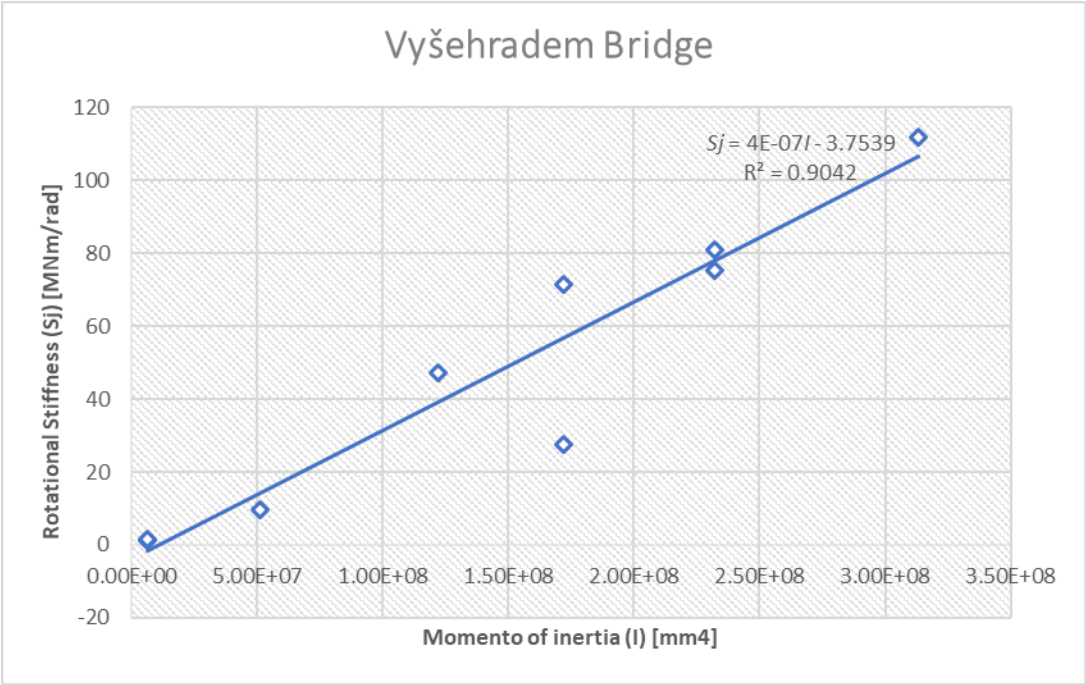


Figure 60.Formula from Vyšehradem bridge. Ref. [2]

$$S_j = 4x10^{-7}I - 3.7539 \qquad [MNm/rad] \tag{6}$$

7.3 Comparison between studies

The figure (61) represents all data compared between them, it can be intuited that the relationship changes as the inertia increases.

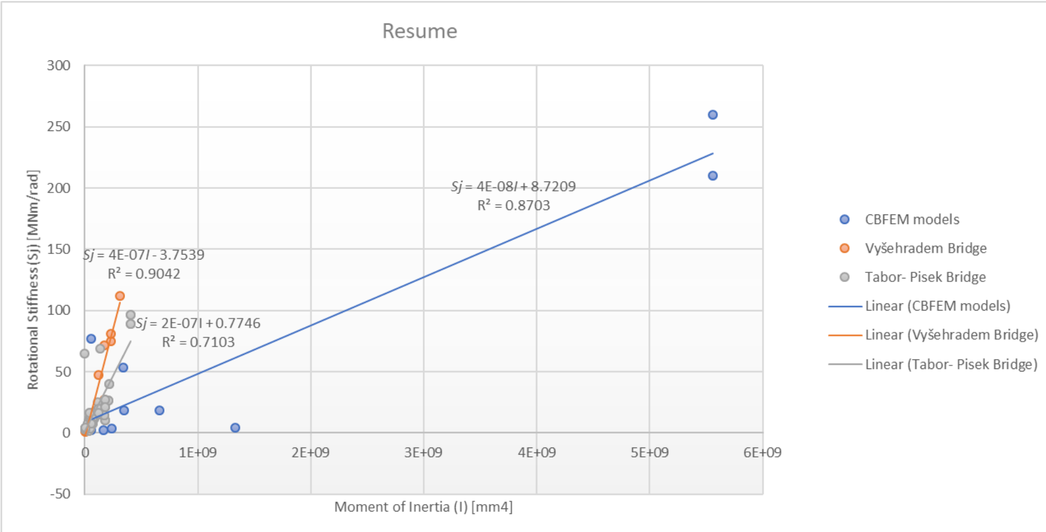


Figure 61. Compilation of the formulas

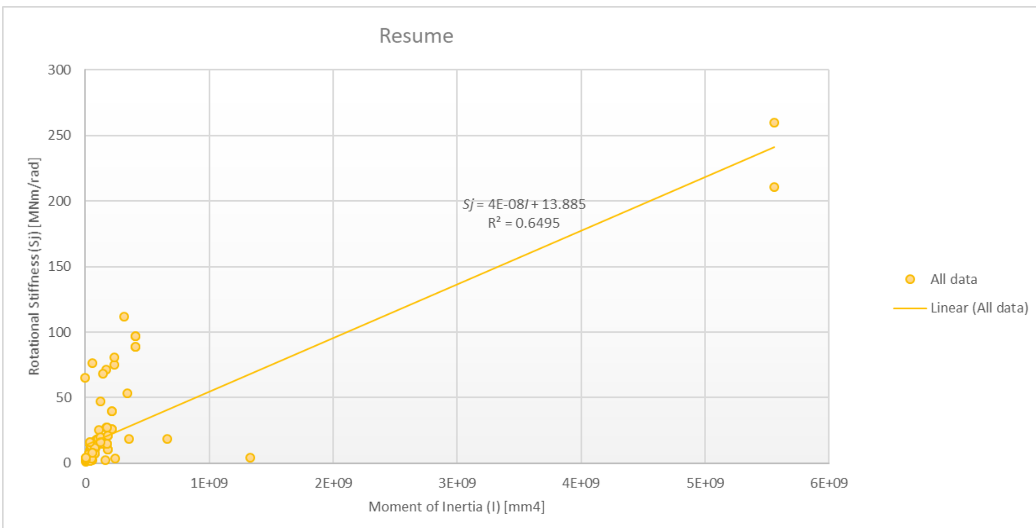


Figure 62. General compilation of the formulas

$$S_j = 4x10^{-8}I - 0.6495 \ \ [MNm/rad] \qquad (4)$$

The figure (62) shows how the relationship changes when the inertia increases, leaning slightly to the right. Through statistical analysis you can understand how inertia affects the rotational stiffness, therefore, the relationship is changing.

8. PREDICTION FORMULA EVALUATION

In order to improve the data analysis due to their volume. It was used the curve of the gauss bell or a data's normal distribution which is a graphic representation of the normal distribution in a group of data. These are distributed in low, medium and high values, creating a graph of bell shaped and symmetric with respect to a certain parameter. Obtaining this function is based on the least squares method, and tries to find the most likely values for X parameters based on N observations, taking into account the uncertainty introduced by the errors in the observations, see (18).

After applying normal distribution analysis in all the data available. It was found 2 big concentrations of data, therefor, two types of predictions regarding the Rotational Stiffness: the first are those sections with low rotational stiffness which in this case are: members in truss bridges and small profiles (marked in a red square on the figure (63)); and the second those sections with higher rotational stiffness which in this case are: big profiles, rigid connections between girder and cross beams, rigid connections between stringers and cross beams (marked in a green square on the figure (63)).

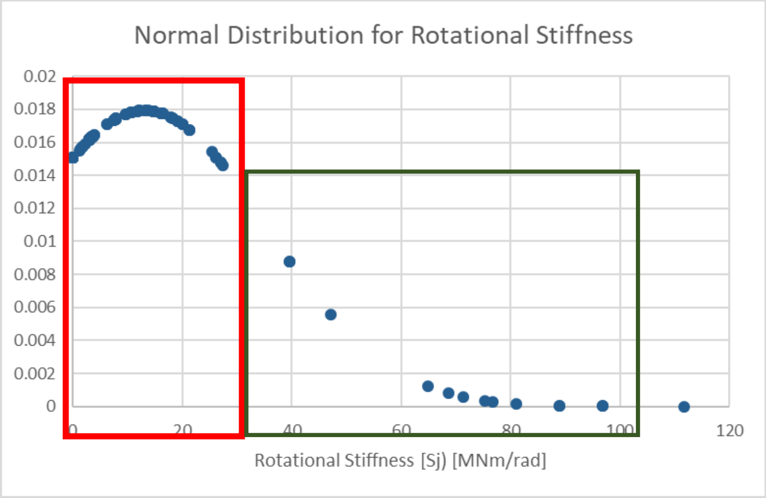


Figure 63. Normal Distribution of all the data.

After analyzing all the available data. It can be examined that 5 out of the 7 bridges, correspond to truss bridges, therefore, there are a predominance of connections in a truss bridge, so in the first group, the formula will suit better to a bridge with this characteristic. In the other hand, the second group will suit better with another formula. Both formulas will be checked with the CBFEM models presented before in this thesis, in order to understand the behavior of them. The two groups are:

1. Low rotational stiffness: Truss bridges and small profiles
2. Higher rotational stiffness: big profiles, girder and cross beams cross beam.

8.1. Low rotational stiffness: Truss bridges and small profiles

The highest concentration of data is found for values less than 30 MNm/rad on the Rotational Stiffness value of the curve of the distribution, the values greater than this number are not representative on truss, therefore, they are excluded from the prediction but the values of the cases of study of this document for girder bridges are preserved to measure their behavior, in order to compared them.

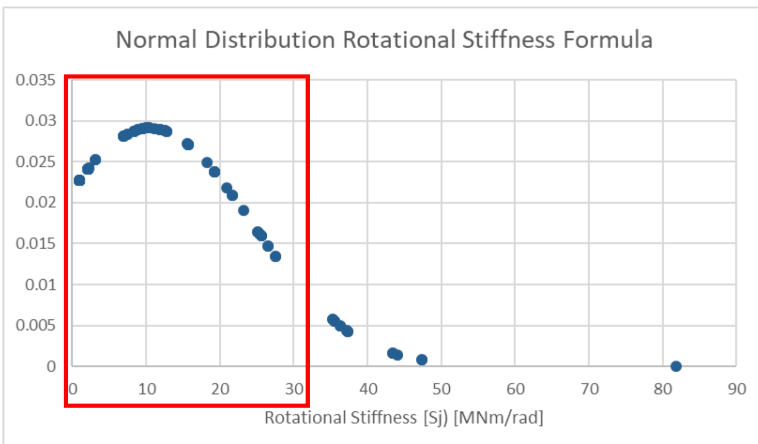


Figure 64. Normal Distribution of the data of the elements with low Inertia.

It can be found the first formula of interest from the linear relationship of the rotational stiffness (S_j) and the value of inertia (I) of the analyzed section.

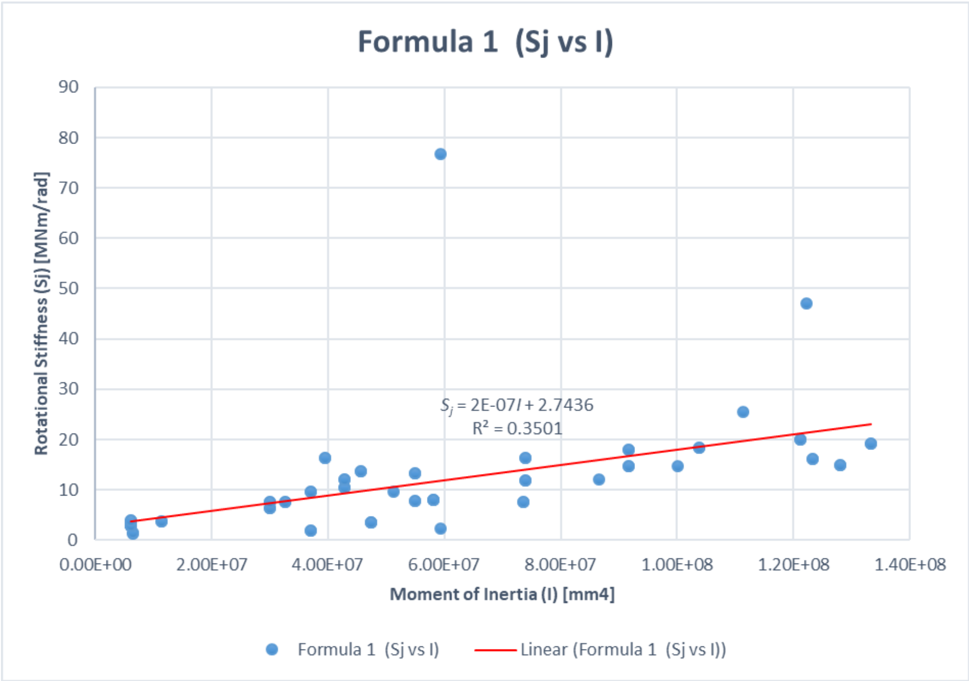


Figure 65. Prediction of the first formula with small rotation stiffness

$$S_j = 2x10^{-7} I - 2.7436\ [\mathrm{MNm/rad}] \qquad (5)$$

$$R^2 = 0.3501$$

Where;

S_j Rotational Stiffness in MNm/rad

I Inertia in $mm^4 \leq 30\ [MNm/rad]$

R^2 The dependence of the variable S_j over the value I

The average percentage error between the computational value of the rotational Stiffness and the value obtained by the formula, varies by 62%. On this first formula of interest (8). The value of R^2 indicates a positive correlation between the two variables. It means, when one value depends of the other one, so, when the Inertia Increases the Rotational Stiffness also increases.

8.2. Higher rotational stiffness: big profiles, girder and cross beams cross beam.

A different analysis was done for higher rotational stiffness, where the small inertia values were eliminated. It brings a new normal distribution curve, figure (66).

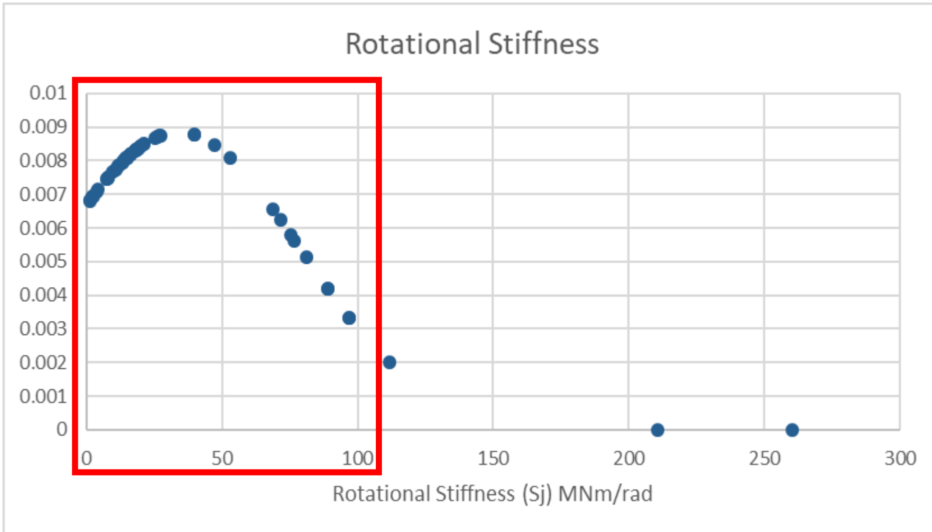


Figure 66. Normal Distribution for higher S_j

It can be appreciated that the highest concentration of data is found for values less than 100 MNm/rad on the Rotational Stiffness value. Hence, in this curve there are almost all values within it.

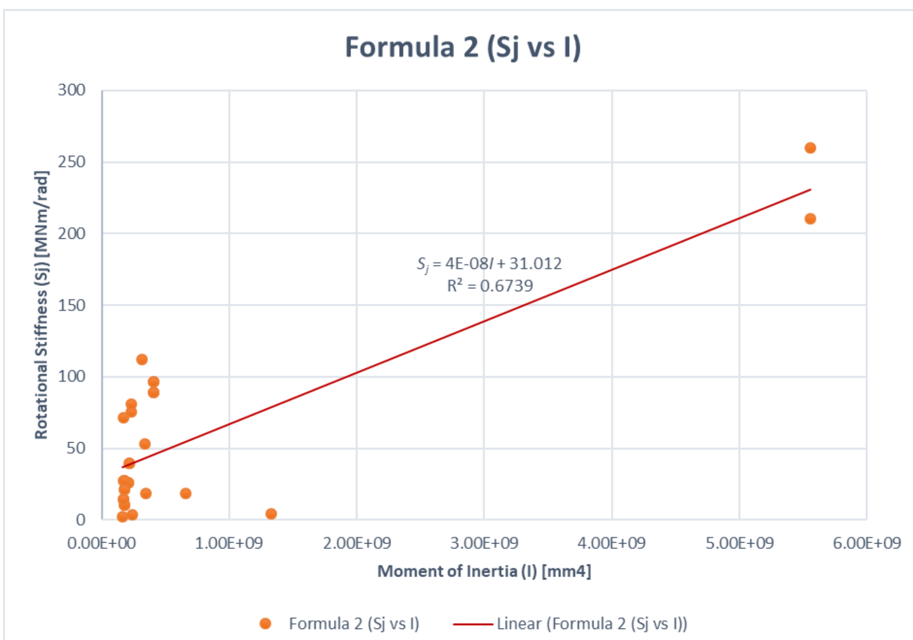


Figure 67. Prediction formula with high rotation stiffness

$$S_j = 4x10^{-8}I - 31.012 \ \ [MNm/rad] \qquad (9)$$

$$R^2 = 0.6739$$

Where;

S_j Rotational Stiffness in MNm/rad

I Inertia in mm^4$\geq 30\ [MNm/rad]$

R^2 The dependence of the variable S_j over the value I

The average percentage error between the computational value of the rotational Stiffness and the value obtained by the formula, varies by 213%. The second formula of interest (9) is similar to the one obtained in the beam bridges, therefore, it fits well with the analysis carried out in this document.

The value of R^2 indicates a positive correlation between the two variables. It means, when one value depends of the other one, so, when the Inertia Increases the Rotational Stiffness also increases.

Although the value is not as accurate at It would be expected but it provides a better understanding of the connection. As it was mention before in this thesis, for the designing of riveted connections it was assumed that the connections were pinned (Hinge) or fixed (rigid). These formulas bring a new possibility, semi rigid connection, in order to save time doing analysis to obtain a value of the rotational stiffness.

8.3. Boundaries of the formulas

As it was mention before, every formula should be used for a different type of riveted connection depending of the rotational stiffness. Since, the formulas of the rotational stiffness depend of the inertia and at the same time the inertia depends of the profile´s height, it can be determinate a steel profile with a specific cross section which could be used as a boundary between the two formulas.

This value is obtained by using the inertia parallel axes theorem, equation (10). Replacing the value of 30 MNm/rad as rotational stiffness (S_j) on the first formula (8), it results in a determined inertial value ($\mathrm{I} = 151371800$ mm^4). In order to calculate the cross section of the profile the flanges area is assumed and calculated from elements of 100 mm base x 10 mm high (these flange measurements were the most usual in this connections).

$$\frac{\mathrm{bh}^3}{12} + 2\mathrm{Ad} \leq \mathrm{I} \qquad (10)$$

$$I \leq 151371800 \text{ mm}^4$$

Since, it is an estimate value.

$$2d \approx h$$

Replacing d by h.

$$\frac{bh^3}{12} + 4Ah \leq 151371800 \qquad (11)$$

Solving the equation.

$$h \leq 566.33\ mm$$

For the first formula the range of the cross section's height goes from 0 mm to 550 mm and for the second formula is until 900 mm because it was the maximum cross section analyzed in this thesis.

8.4. Final formulas

The final formula riveted connections in truss bridges and profiles less than 550 mm height is:

$$S_j = 2x10^{-7}I - 2.7436\ [MNm/rad] \qquad (8)$$

Where;

S_j Rotational Stiffness in MNm/rad

I Inertia in mm^4

R^2 The dependence of the variable S_j over the value I

The final formula riveted connections in girder bridges and profiles more than 550 mm until 900 mm height is:

$$S_j = 4x10^{-8}I - 31.012\ [MNm/rad] \qquad (96)$$

Where;

S_j Rotational Stiffness in MNm/rad

I Inertia in mm^4

R^2 The dependence of the variable S_j over the value I

In the figure (68) it can be appreciating the different slopes of the formulas and it shows how the relationship changes when the inertia increases, leaning slightly to the right. For both cases it is a liner relationship with a positive slop between the Inertia and the Rotational stiffness.

The boundaries for both formulas are:

$$X = I = 151371800 \text{ mm}^4$$

$$Y = S_j = 30 \text{ MNm/rad}$$

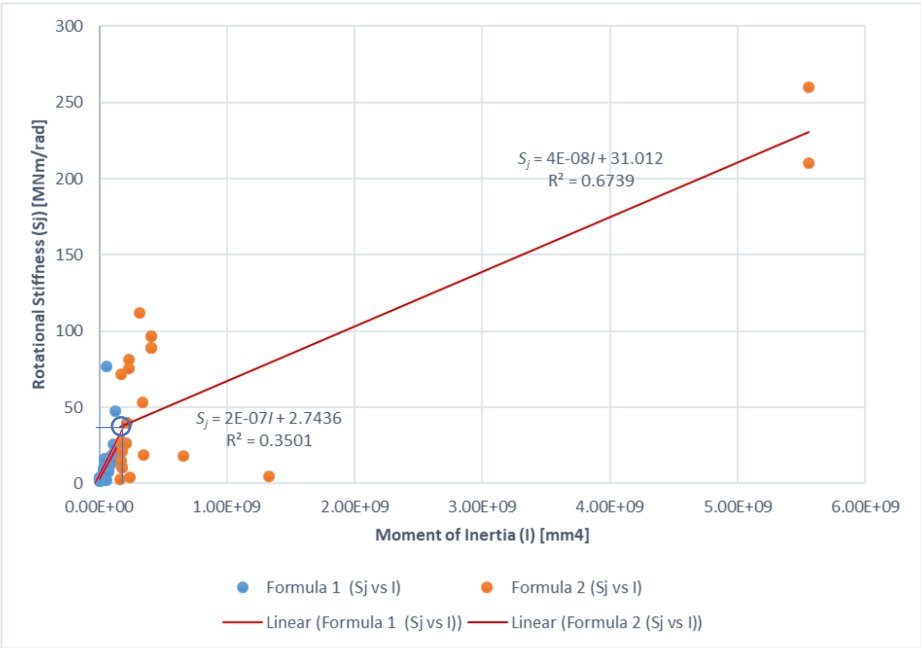


Figure 68. Boundaries of the final formulas

8.5. Comparison the two formulas with the CBFEM models

It is interesting to analyze the two formulas with the computational value of the S_j in the bridges in this study, the values change significantly depending of the bridge characterization.

The table (7) and (8) are organized as follows:

On the first chart: Type of bridge, notation assigned, type of connection and height of the element analyzed. Subsequently the inertia of the analyzed section, the Rotational Stiffness obtained with IDEA StatiCa, the Rotational Stiffness obtained with the formula and finally the percentage of error between the theoretical and the computational value.

8.5.1. Case 1: First formula

It is described on the following table:

Comparison of the first formula with the cases of study		#	*I* mm^4	S_j MNm/rad	S_j *(formula)* MNm/rad	Error %
Diferent Bridges	Beam-TU 502, Cross Beam - stringer, h=430mm	1	3.49E+08	18.5	72.6196	293%
	beam-TU 581, Main girder-Cross beam, h=410mm	2	3.39E+08	53.1	70.5396	33%
	up- TU 581,Cross beam-Stringer, h=348mm	3	2.4E+08	3.9	50.6836	1200%
	big-TU 2101, Main girder-Cross beam, h=910mm	4	5.56E+09	260.1	1114.0736	328%
	big-TU 2101, Main girder-Cross beam2, h=910mm	5	5.56E+09	210.5	1114.0736	429%
	within-TU 2101, Cross Beam - stringer, h=460mm	6	6.58E+08	18.3	134.3156	634%
	within- TU 2101, Cross Beam - stringer2, h=683mm	7	1.33E+09	4.4	268.5436	6003%
	up-TU 2191, Cross Beam - stringer, h=320mm	8	1.64E+08	2.5	35.5476	1322%
	Truss- TU 2362, Cross Beam - stringer, h=603mm	9	1E+08	14.8	22.7696	54%
	Truss- TU 2362 Corner Upper, configuration 238,250	10	59388000	76.7	14.6212	81%
	truss- TU 2362 Corner lower, configuration 238,250	11	59388000	2.3	14.6212	536%

Table 7. Comparison of the first formula with cases of study

First impressions are: as expected, profiles with a high web do not fit on the prediction.

Second, the connections Cross beam - Stringer where the Stringer is over the Cross beam does not correspond with the prediction. Figure (69).

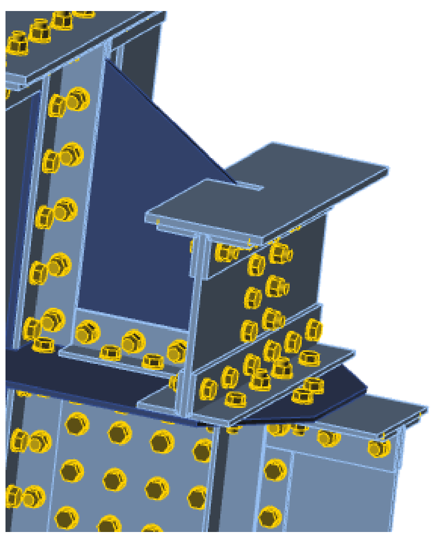


Figure 69. Example on the TU 581 bridge.

Third, the connections Cross beam - Stringer where the Stringer is within the Cross beam does not correspond with the prediction. For this case can be analyzed the rotational Stiffness of the connection depending on the number of rivets. Figure (70).

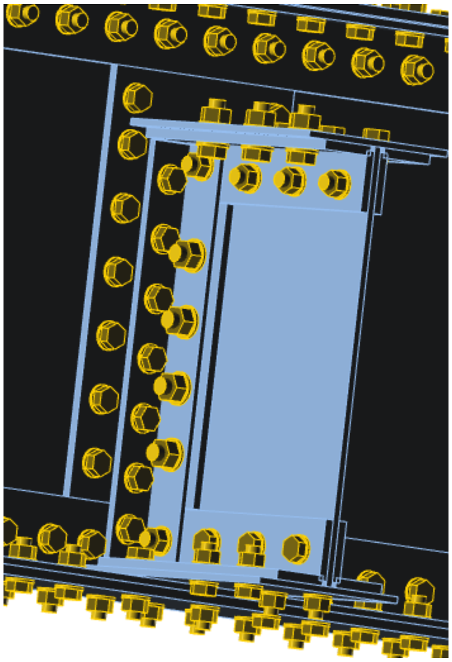


Figure 70. Example on the TU 2101 bridge.

And the last two connections have a different configuration of an IPE profile, so they do not fit on the prediction. Figure (71).

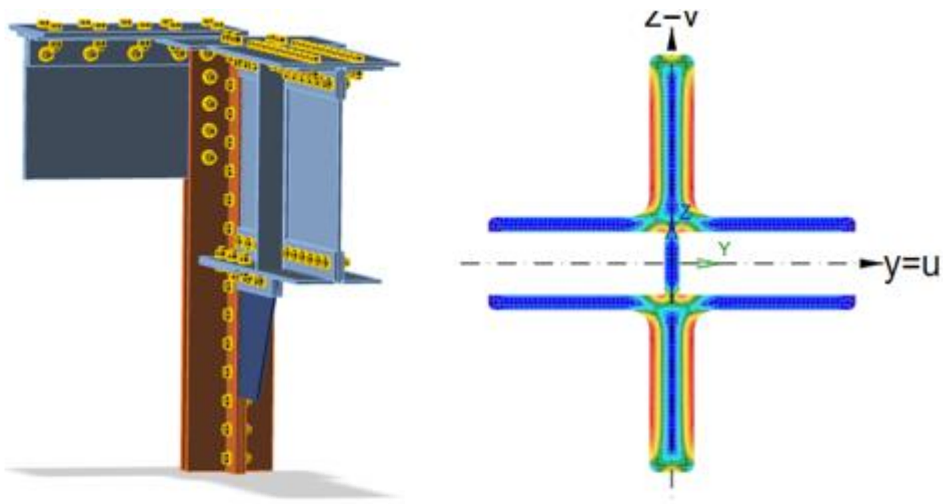


Figure 71. Non IPE profiles.

In general, on the analyzed bridges those profiles have a higher height on their web so they do not fit very well to this formula.

8.5.2. Case 2: Second formula

It is described on the following table:

Comparison of the second formula with the cases of study		#	I	S_j	S_j (formula)	Error %
			mm^4	MNm/rad	MNm/rad	
Diferent Bridges	Beam-TU 502, Cross Beam - stringer, h=430mm	1	3.49E+08	18.5	44.9872	143%
	beam-TU 581, Main girder-Cross beam, h=410mm	2	3.39E+08	53.1	44.5712	16%
	up- TU 581,Cross beam-Stringer, h=348mm	3	2.4E+08	3.9	40.6	941%
	big-TU 2101, Main girder-Cross beam, h=910mm	4	5.56E+09	260.1	253.278	3%
	big-TU 2101, Main girder-Cross beam2, h=910mm	5	5.56E+09	210.5	253.278	20%
	within-TU 2101, Cross Beam - stringer, h=460mm	6	6.58E+08	18.3	57.3264	213%
	within- TU 2101, Cross Beam - stringer2, h=683mm	7	1.33E+09	4.4	84.172	1813%
	up-TU 2191, Cross Beam - stringer, h=320mm	8	1.64E+08	2.5	37.5728	1403%
	Truss- TU 2362, Cross Beam - stringer, h=603mm	9	1E+08	14.8	35.0172	137%
	Truss- TU 2362 Corner Upper, configuration 238,250	10	59388000	76.7	33.38752	56%
	truss- TU 2362 Corner lower, configuration 238,250	11	59388000	2.3	33.38752	1352%

Table 8. Comparison of the second formula with cases of study

The first impressions are: as expected, the profiles with a low web do not fit on the prediction.

Second, for both cases the connections Cross beam- Stringer where the Stringer is over the Cross beam does not correspond with the prediction. Figure (72).

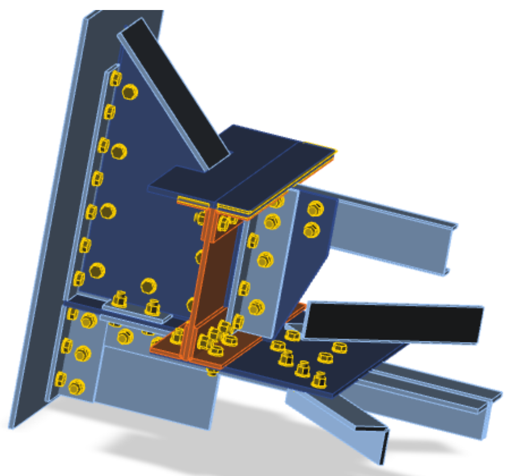


Figure 72. Example on the TU 2191 Bridge.

In general, this formula is coupled much greater range of bridges regardless of their characterization except for the cases listed above.

8.5.3. Comparison with other formulas

In this study, emphasis has been made on comparing with the previous formulas proposed by similar studies, in order to provide a better understanding.

8.5.3.1. Comparison with First formula

Comparting the average percentage error with the first formula. Annex 13

Formula	% Error
Tabok - Pisek	54%
Minor´s Thesis	134%
Present Study	62%

Table 9. Comparison the average percentage error with the first formula

Also, the first and the third formula are very similar, this is obvious when most of the data come from the study of Tabok-Pisek bridges, see [13], and it adapts well to connections in truss bridges with lower moment of inertia or less than 550 mm high.

8.5.3.2. Comparison with Second formula

The formulas of the two previous studies do not fit in connections with biggest moment of inertia. Which makes sense, the elements which conform Truss bridges are relative small, therefore, the proposed formula is better suited to connections with profiles greater than 550 mm and even other types of bridges such as Girder Bridges. Annex (14).

Formula	% Error
Tabok - Pisek	392%
Minor´s Thesis	826%
Present Study	213%

Table 10. Comparison the average percentage error with the second formula

9. CONCLUSIONS

The aim of the thesis was to evaluate the relationship between the rotational stiffness and the moment of inertia in connections on different riveted steel railway bridges.

The results of 3D CBFEM models were used for the evaluation of the predictive formula. Two different formulas were created, first for the low rotational stiffness and second for the higher rotational stiffness. Together they form the bilinear curve with the internal boundary at the rotation stiffness of 30 MNm/rad and the second group with the rotational stiffness value between 30 MNm/rad and 100 MNm/rad.

The average percentage error of the two formulas described in Oscar Minor's thesis. see [2], compared with the formulas obtained in this present thesis have a higher average percentage error for both cases. This fact, inclines to think that this formula is a little more accurate and delivers more precise values. However, it is important to remember that the average percentage error for the first formula is 62% for profiles less than 550 mm in height and 213% for profiles greater than 550 mm according to the obtained formula, with a value of R^2 of 0.3501 and 0.6739 respectively, which indicates the correlation between the two variables.

Therefore, it is recommended to use these formulas to save calculation time for semi-rigid connections in riveted bridges with adequate safety coefficients, taking into account the error values previously mentioned. However, compared to the existing design praxis, where the designer use hinge or rigid connection only, this predictive formula significantly improves the precision of the global numerical models.

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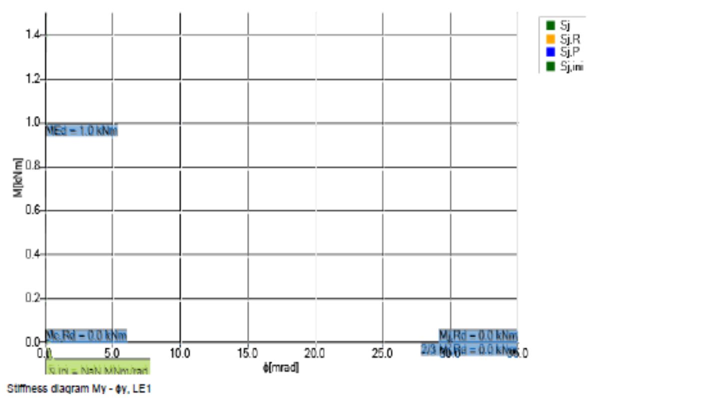
ANNEXES

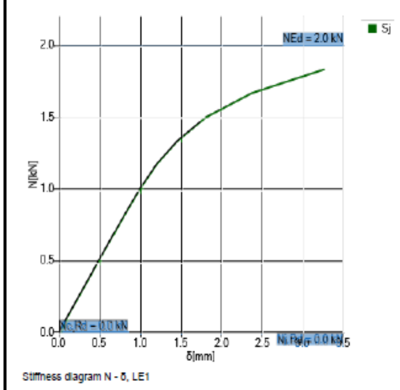
Annex 1: Results provided for the program IDEA StatiCa of the connection: TU502. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness

Name:	TU 502	Load effects						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	349380000	0	349380000
comment:	Short connection	2	0	0	0	1	0	Iz	16131000	0	16131000

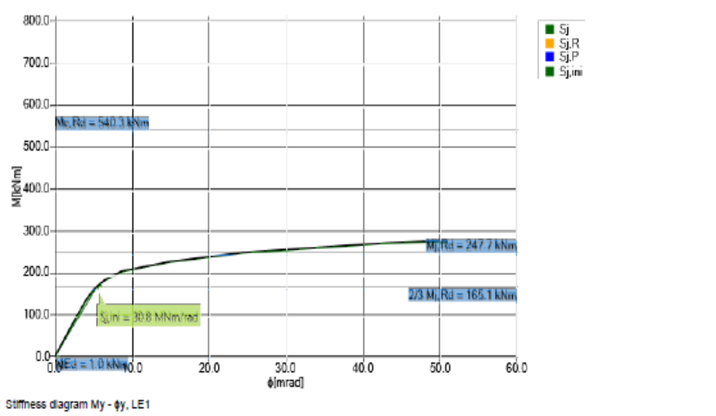
Cross beam - stringer

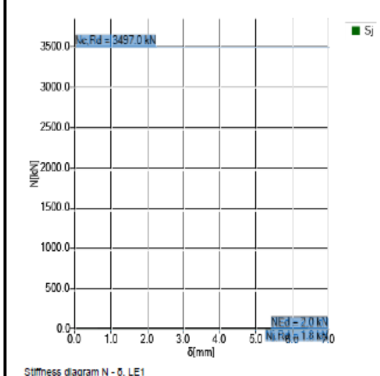
Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
0	18.5	0.1	30.2	2.1	764.3	15.3	18.5	semi-rigid	N	2	0	0	126





Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
247.7	30.8	0	50.9	2.4	764.3	15.3	33.8	semi-rigid	N	2	1.8	0	2000000



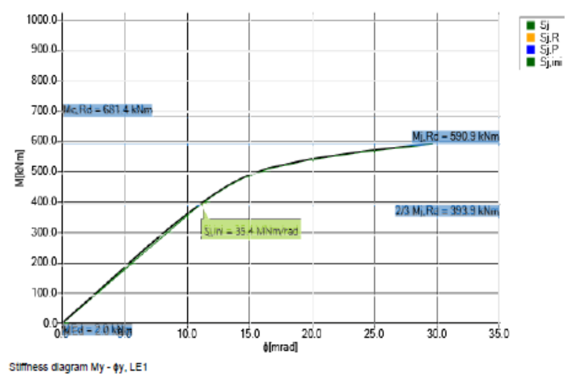


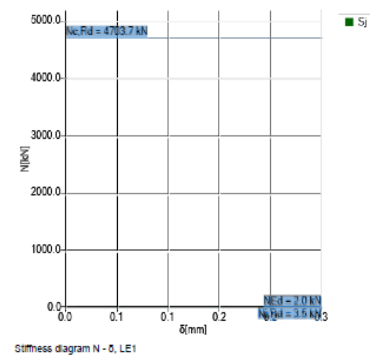
Annex 2: Results provided for the program IDEA StatiCa of the connection: TU581. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU 581	**Load effects**						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	338980000	0	3.39E+08
comment:	Long connection	2	0	0	0	2	0	Iz	20269000	0	20269000

Main Girder- Cross beam no fixed

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
590.9	35.4	0.1	29.6	3.6	468.6	9.4	36.8	semi-rigid	N	2	3.5	0	1230

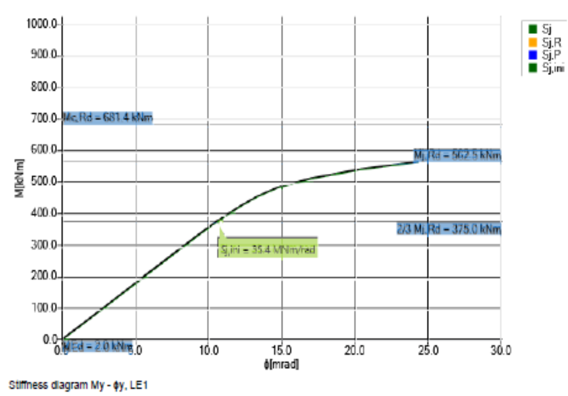


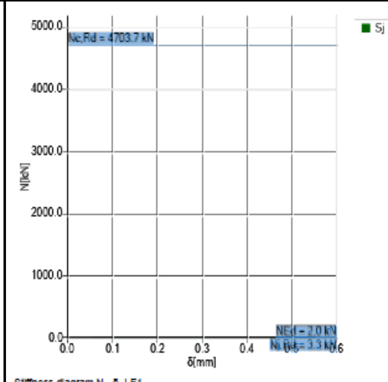


Name:	TU 581	**Load effects**						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	Long connection	2	0	0	0	-2	0	

Main Girder- Cross beam fixed

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
562.5	35.4	0.1	24.4	3.8	468.3	9.4	35.7	semi-rigid	N	2	3.3	0	960

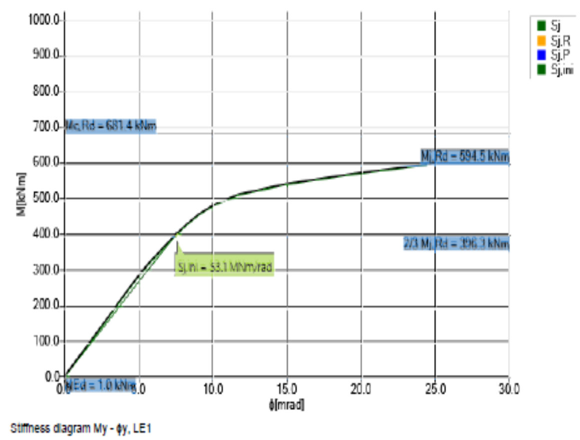


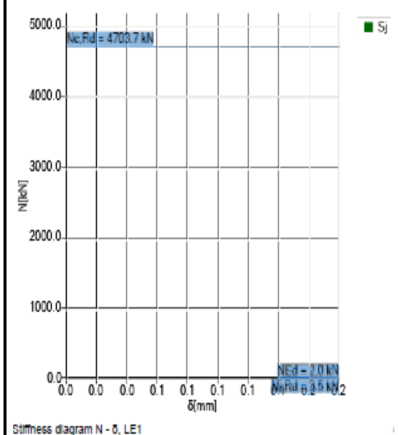


Name:	TU 581	Load effects						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	Short connection	2	0	0	0	2	0	

Main Girder- Cross beam no fixed

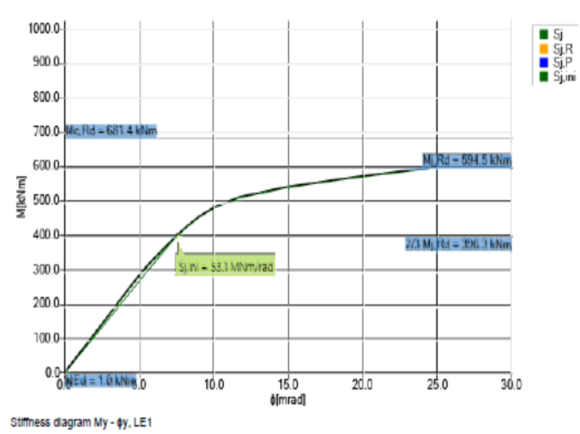
Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
594.5	53.1	0	24.5	3.8	468.6	9.4	56.7	semi-rigid	N	2	3.5	0	1824

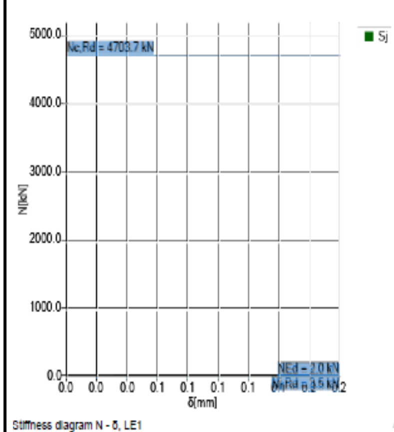




Main girder- cross beam. Fixed connection.

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
594.5	53.1	0	24.5	3.8	468.6	9.4	56.7	semi-rigid	N	2	3.5	0	2031



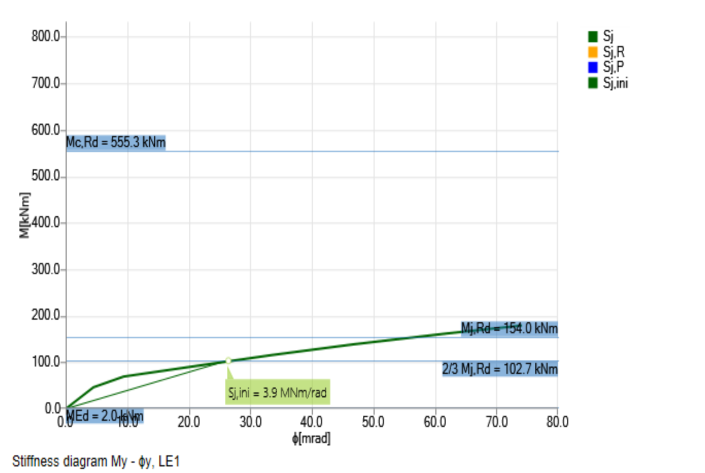


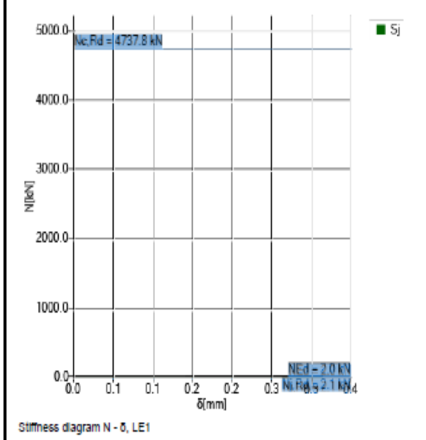
Annex 3: Results provided for the program IDEA StatiCa of the connection: TU581. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU 581	Load effects						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	239700000	0	2.4E+08
comment:	short connection	1	0	0	0	2	0	Iz	22049000	0	22049000

Cross beam - Stringer fixed

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
154	3.9	0.2	74.2	3.8	331.2	6.6	0	Pinned	N	1	0.6	0	53



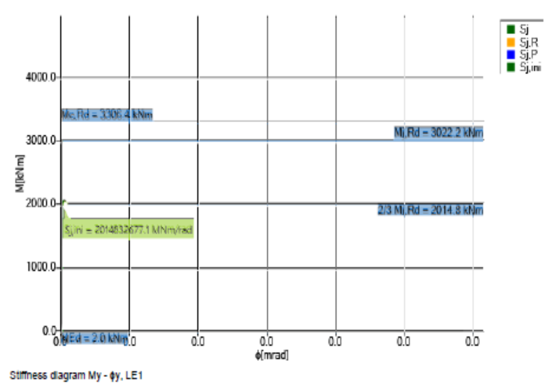


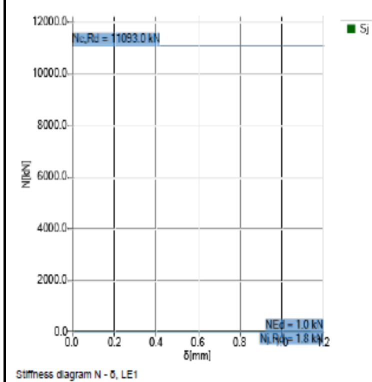
Annex 4: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	**Load effects**						Moment of inertia		
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4		
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy 3899900000	1656750000	5556650000
comment:	Long connection	1	0	0	0	2	0	Iz 114920000	30720000	145640000

Main Girder- Cross beam no fixed

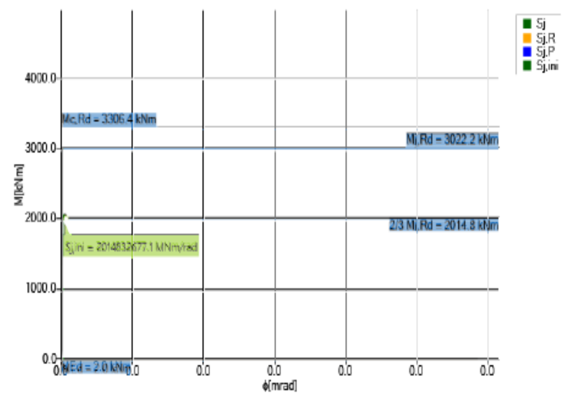
Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
3022.2	2014830000	0	0	6	3412.4	68.2	137766000	Rigid	N	1	1.8	0	1000000



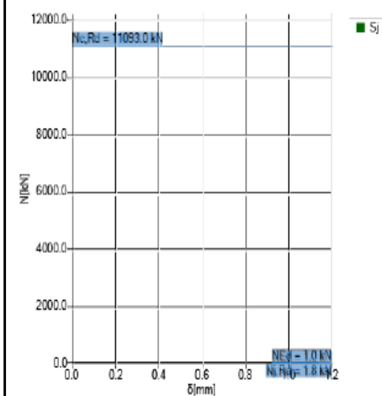


Main Girder- Cross beam fixed

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
3022.2	2014000000	0	0	6	3412.4	68.2	137700000	Rigid	N	1	1.8	0	1000000



Stiffness diagram My - ϕy, LE1

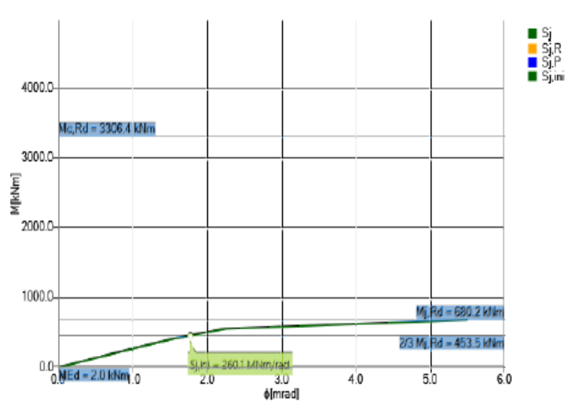


Stiffness diagram N - δ, LE1

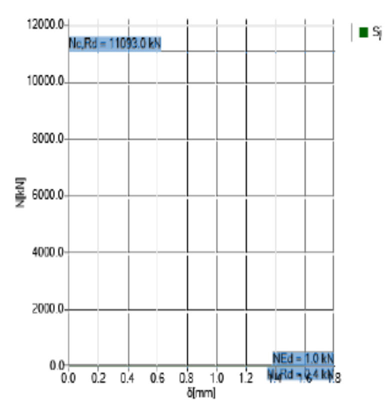
Short connection.

Main Girder- Cross beam fixed

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
680.2	260.1	0	5.5	6	3412.4	68.2	266.5	semi-rigid	N	1	0.4	0	1000000



Stiffness diagram My - ϕy, LE1



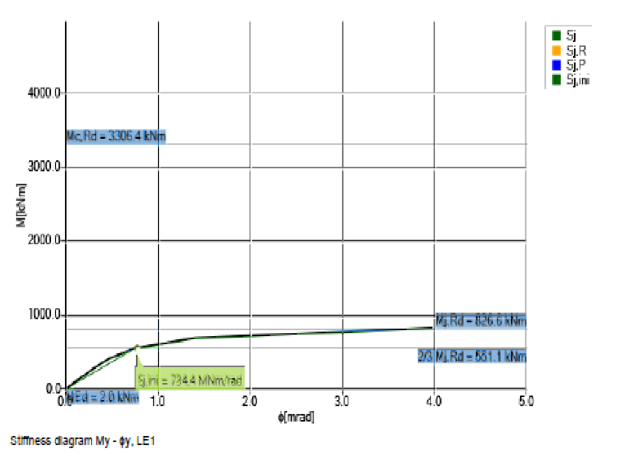
Stiffness diagram N - δ, LE1

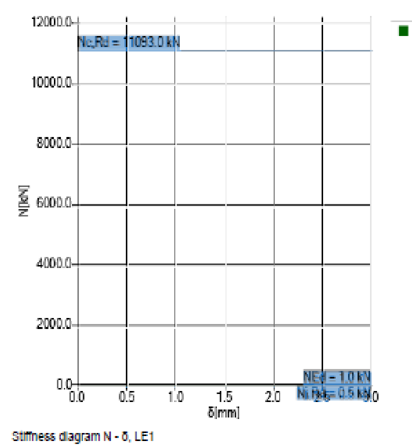
Annex 5: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	**Load effects**						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	3.9E+09	1.66E+09	5556650000
comment:	Long connection	1	0	0	0	2	0	Iz	1.15E+08	30720000	145640000

Main Girder- Cross beam no fixed

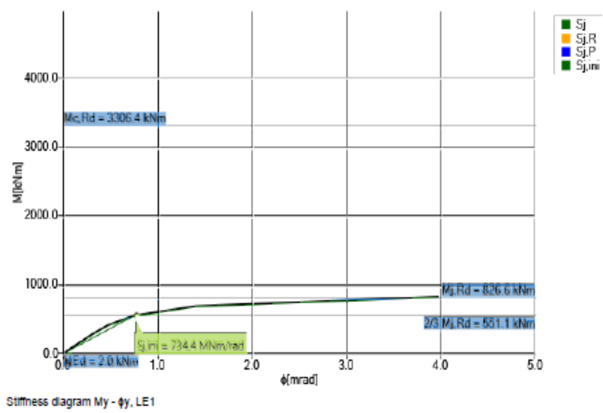
Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
826.6	734.4	0	4	6	3412.4	68.2	920	Semi-Rigid	N	1	0.5	0	1000000

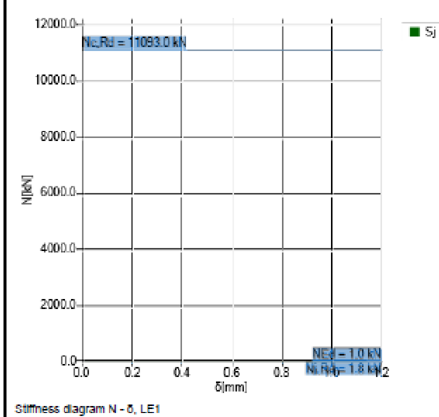




Main Girder- Cross beam fixed

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
826.6	734.4	0	4	6	3412.4	68.2	920	Semi-Rigid	N	1	0.5	0	1000000

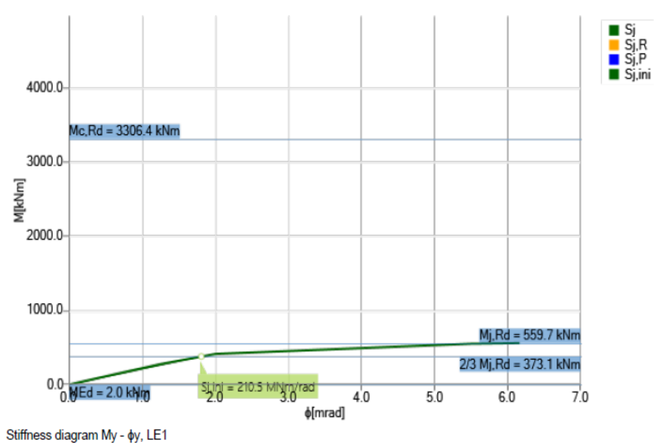


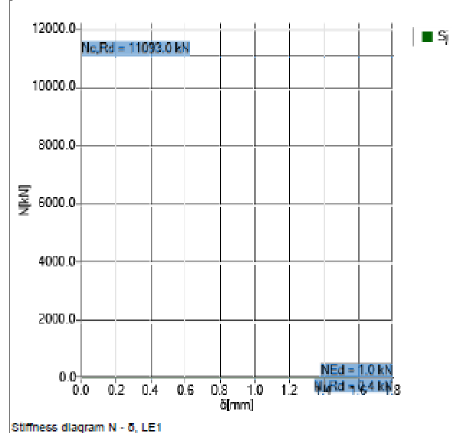


Short Connection

Main Girder- Cross beam fixed

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
559.7	210.5	0	6.2	6	3412.4	68.2	223.7	semi-rigid	N	1	0.3	0	1000000



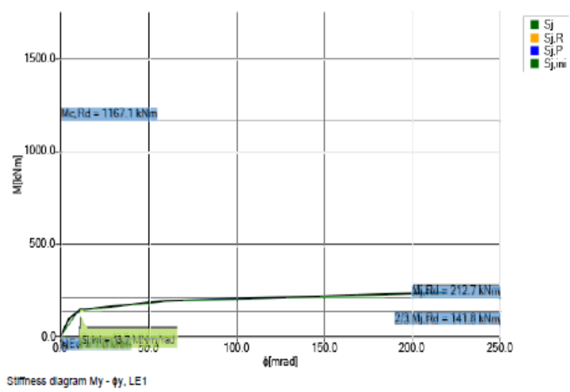


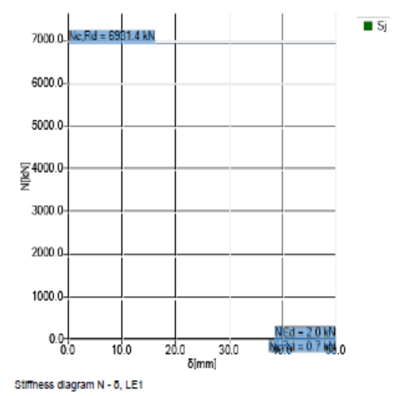
Annex 6: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	**Load effects**						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	657860000	0	6.58E+08
comment:	Long connection	2	0	0	0	1	0	Iz	43328000	0	43328000

Cross beam- Stringer

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
212.7	13.7	0.1	219.3	6	575.6	11.5	19.6	Semi-rigid	N	2	0.7	0	228

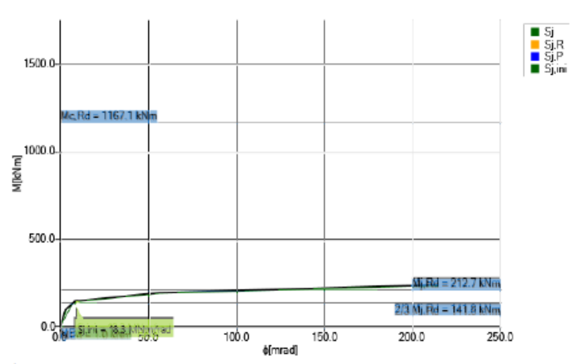


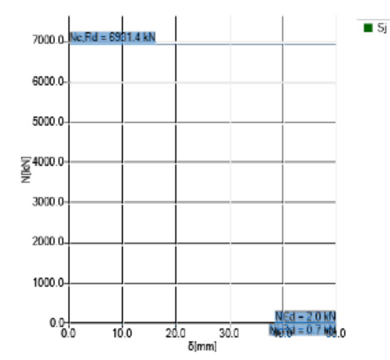


Name:	TU2101	**Load effects**						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	short connection	2	0	0	0	1	0	

Cross beam - Stringer

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
212.7	18.3	0	214.9	6	575.6	11.5	29.8	Semi-Rigid	N	2	0.7	0	271



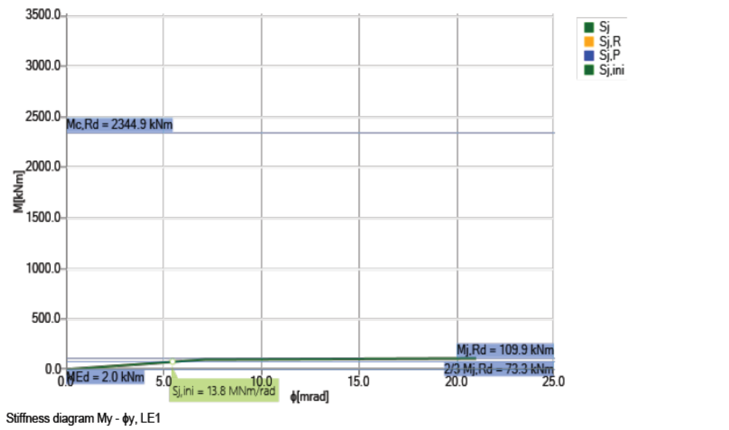


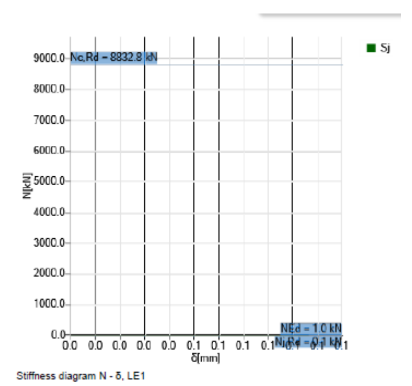
Annex 7: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	**Load effects**						Moment of inertia		
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4		
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	1329000000	0 1.33E+09
comment:	Long connection	1	0	0	0	2	0	Iz	260650000	0 2.61E+08

Cross beam- Stringer

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
109.9	13.8	0.1	21	6	1718.5	34.4	13.8	Pinned	N	1	0.1	0	385

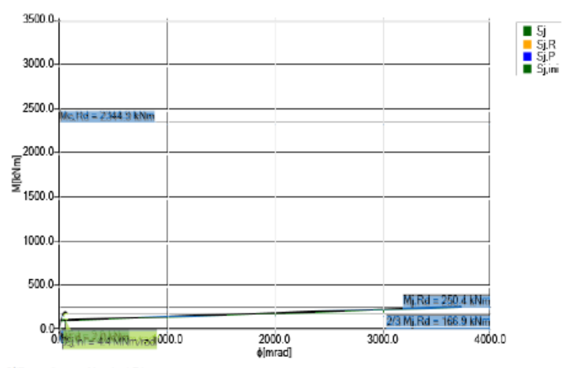


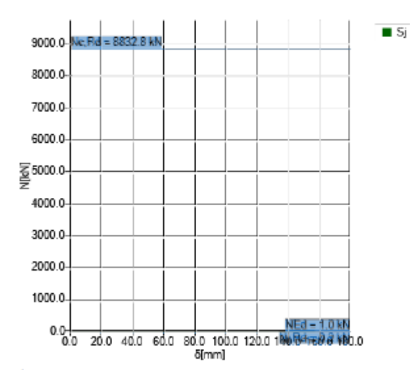


Name:	TU2101	**Load effects**						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	short connection	1	0	0	0	2	0	

Cross beam - Stringer

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
250.4	4.4	0.1	3734.8	6	1718.5	34.4	15	Pinned	N	1	0.2	0	496



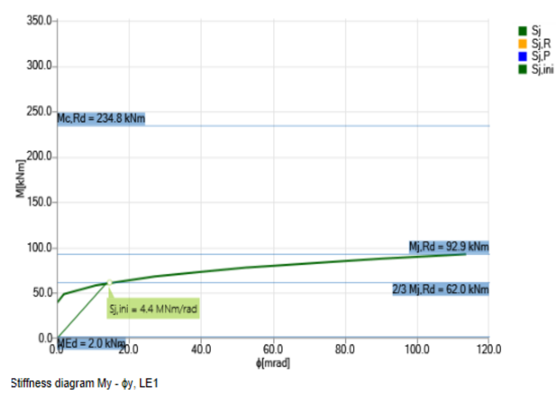


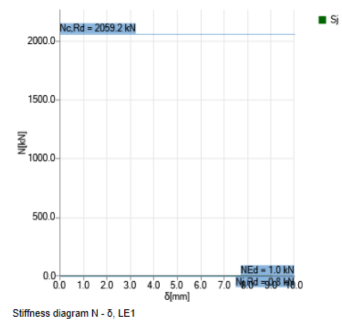
Annex 8: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	Load effects						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	164020000	0	164020000
comment:	Long connection	1	0	0	0	2	0	Iz	14727000	0	14727000

Cross beam- Stringer

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
92.9	4.4	0	113.6	1.5	395.4	7.9	9783015	Pinned	N	1	0.8	0	1000000

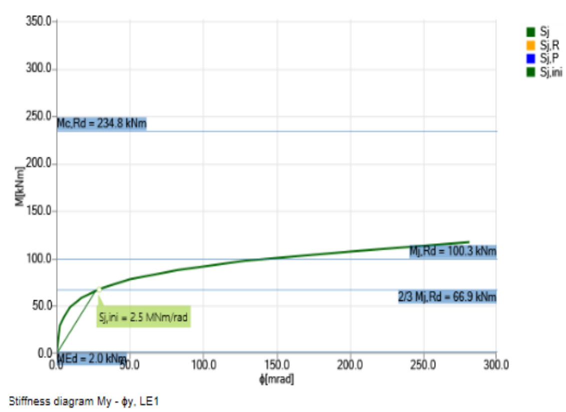


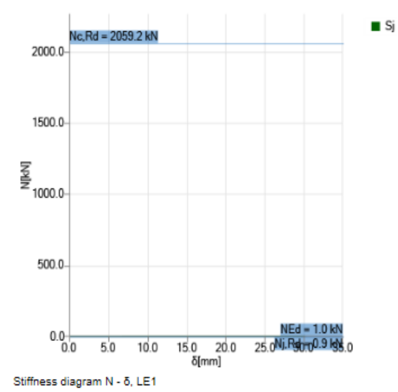


Name:	TU2101	Load effects						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	short connection	1	0	0	0	2	0	

Cross beam - Stringer

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
100.3	2.5	0.1	281.6	1.5	395.4	7.9	13.9	Pinned	N	1	0.9	0	1000000



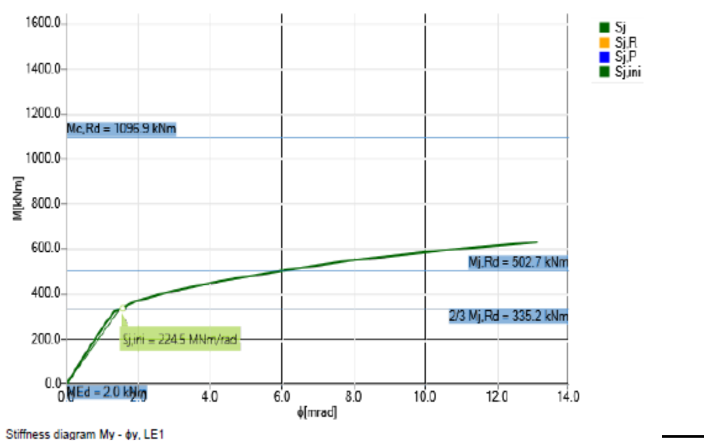


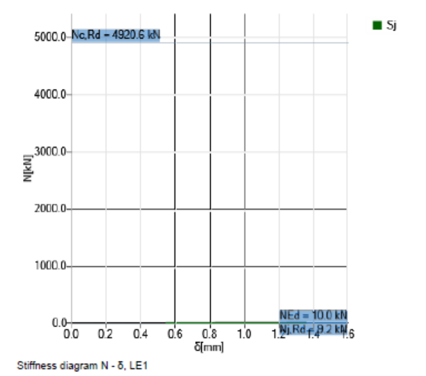
Annex 9: Results provided for the program IDEA StatiCa of the connection: TU2101. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2101	Load effects						Moment of inertia			
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	1001300000	0	1E+09
comment:	Long connection	10	0	0	0	2	0	Iz	43702000	0	43702000

Main Girder- Cross beam

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
197.1	15.2	0	13.1	3.8	1101.5	22	252.1	Semi-Rigid	N	10	9.2	0	1000000

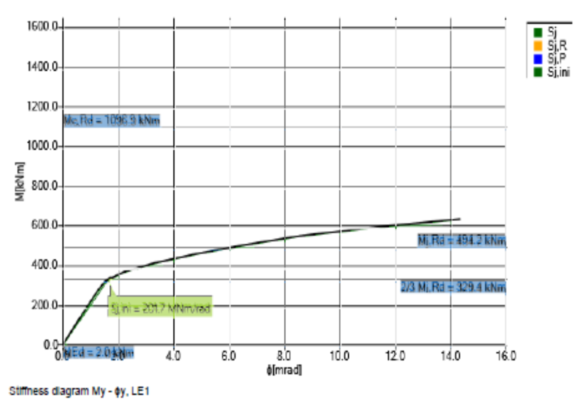


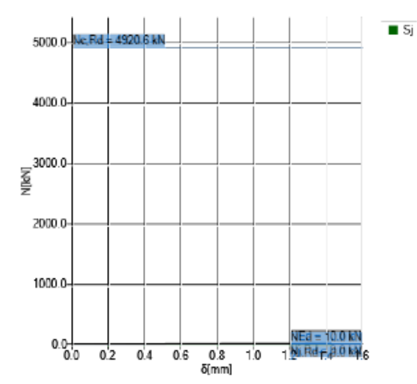


Name:	TU2101	Load effects						Moment of inertia
Type:	My	N	Vy	Vz	Mx	My	Mz	mm4
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	
comment:	short connection	10	0	0	0	2	0	

Main Girder- Cross beam

Rotational Stiffness									Axial stiffness				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
198.5	14.8	0	14.3	3.8	1101.5	22	220.6	Pinned	N	10	0.2	0	10000000



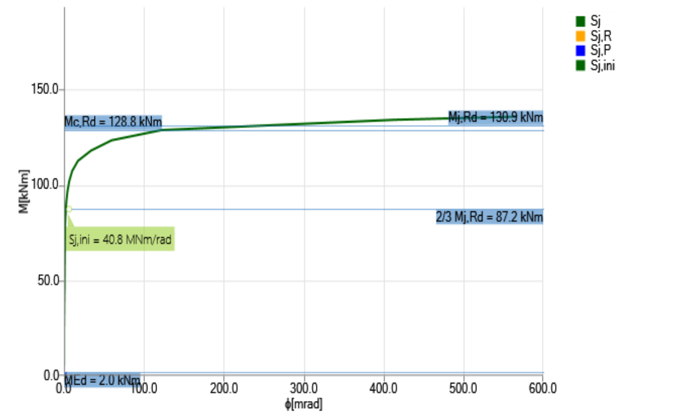


Annex 10: Results provided for the program IDEA StatiCa of the connection: TU2362. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

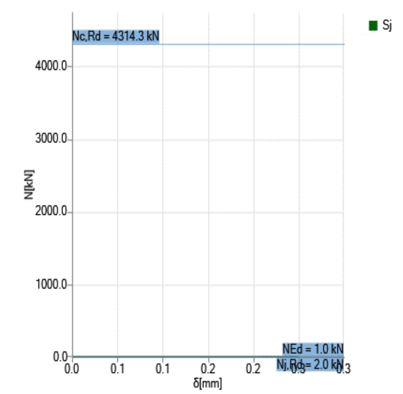
Name:	TU2362	**Load effects**						Moment of inertia			
Type:	Mz	N	Vy	Vz	Mx	My	Mz	mm4			
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	59388000	0	59388000
comment:	short connection	-1	0	0	0	0	2	Iz	26240000	0	26240000

Upper corner connection

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
130.9	40.8	0	567	4.7	29.3	0.6	225.6	Rigid	N	-1	-2	0	1000000



Stiffness diagram Mz - ϕz, LE1



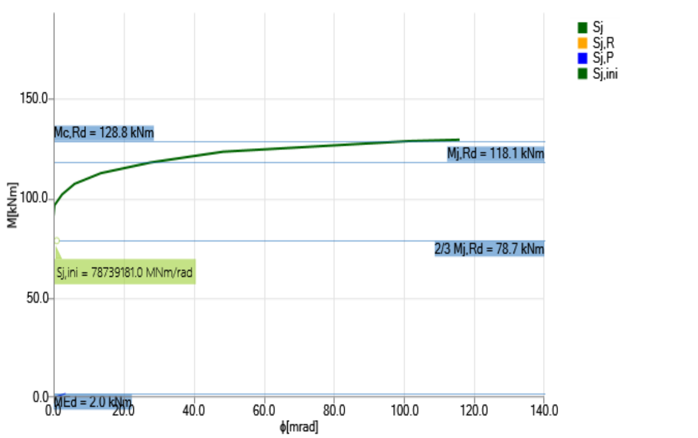
Stiffness diagram N - δ, LE1

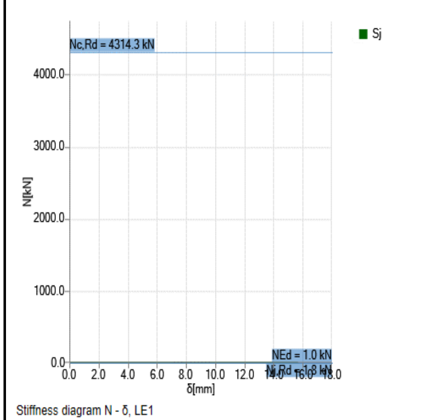
Annex 11. Here is described the load effects which are input on the program, the moment of inertia of the element analyzed, the initial rotational stiffness with its boundaries to rigid and pined connection, the initial rotation, the class of the connection: Pinned, Rigid, semi-rigid and the axial stiffness.

Name:	TU2362	**Load effects**						Moment of inertia		
Type:	Mz	N	Vy	Vz	Mx	My	Mz	mm4		
Analysis:	Stiffness	kN	kN	kN	kNm	kNm	kNm	Iy	59388000	0 59388000
comment:	short connection	-1	0	0	0	0	2	Iz	26240000	0 26240000

Lower corner connection

Rotational Stiffness									**Axial stiffness**				
Mj. Rd	Sj. Ini	ϕ	ϕc	L	Sj,R	Sj, P	Sjs			N	Nj, Rd	dx	St
kNm	MNm/rad	mrad	mrad	m	MNm/rad	MNm/rad	MNm/rad	Class	Comp.	kN	kN	mm	MN/m
118.1	78739181	0	115.8	4.7	29.3	0.6	5368580.6	Rigid	N	-1	-1.8	0	1000000





Annex 12: Comparison the average percentage error between formulas obtained on the different studies for all the profiles**.** General formula

All data		I	Sj	Sj tabor-Pisek		Sj Oscar		Sj Marcos	
		mm4	MNm/rad	Mnm/rad	%	Mnm/rad	%	Mnm/rad	%
CBFEM models	1	349380000	18.5	70.30	280%	136.00	635%	22.70	23%
	2	338980000	53.1	68.22	28%	131.84	148%	22.28	58%
	3	239700000	3.9	48.36	1140%	92.13	2262%	18.31	369%
	4	5556650000	260.1	1 111.75	327%	2 218.91	753%	230.99	11%
	5	5556650000	210.5	1 111.75	428%	2 218.91	954%	230.99	10%
	6	657860000	18.3	131.99	621%	259.39	1317%	35.04	91%
	7	1329000000	4.4	266.22	5950%	527.85	11897%	61.88	1306%
	8	164020000	2.5	33.22	1229%	61.85	2374%	15.28	511%
	9	100130000	14.8	20.45	38%	36.30	145%	12.73	14%
	10	59388000	76.7	12.30	84%	20.00	74%	11.10	86%
	11	59388000	2.3	12.30	435%	20.00	770%	11.10	382%
Vyšehrad em Bridge	1	232355208	75.256	46.89	38%	89.19	19%	18.02	76%
	2	313293874.7	111.8	63.08	44%	121.56	9%	21.25	81%
	3	232355208	81.028	46.89	42%	89.19	10%	18.02	78%
	4	171843208	71.464	34.79	51%	64.98	9%	15.59	78%
	5	171843208	27.471	34.79	27%	64.98	137%	15.59	43%
	6	122189141.3	47.112	24.86	47%	45.12	4%	13.61	71%
	7	51341866.67	9.724	10.69	10%	16.78	73%	10.77	11%
	8	6481237.333	1.192	1.72	44%	(1.16)	197%	8.98	653%
	9	6481237.333	1.486	1.72	15%	(1.16)	178%	8.98	504%
	1	1.40E+08	68.6	28.38	59%	52.16	24%	14.31	79%
	2	2.13E+08	26.2	42.95	64%	81.30	210%	17.23	34%
	4	1.82E+08	10.7	36.84	244%	69.08	546%	16.00	50%
	5	3.70E+07	1.8	7.82	335%	11.05	514%	10.20	467%
	7	8.66E+07	12	17.74	48%	30.89	157%	12.18	2%
	8	1.33E+08	19.1	27.08	42%	49.56	159%	14.05	26%
	9	6.15E+06	3.5	1.65	53%	(1.29)	137%	8.97	156%
	10	1.82E+08	10.7	36.84	244%	69.08	546%	16.00	50%
	11	3.70E+07	1.8	7.82	335%	11.05	514%	10.20	467%
	13	5.49E+07	13.2	11.40	14%	18.21	38%	10.92	17%
	14	9.17E+07	17.9	18.76	5%	32.92	84%	12.39	31%
	15	6.15E+06	3.5	1.65	53%	(1.29)	137%	8.97	156%
	17	4.28E+07	10.5	8.98	14%	13.37	27%	10.43	1%
	18	3.70E+07	9.7	7.82	19%	11.05	14%	10.20	5%
	19	6.15E+06	3.45	1.65	52%	(1.29)	137%	8.97	160%
	21	3.00E+07	6.3	6.42	2%	8.24	31%	9.92	57%
	22	3.00E+07	7.6	6.42	16%	8.24	8%	9.92	31%
	23	6.15E+06	3.45	1.65	52%	(1.29)	137%	8.97	160%
	25	3.27E+07	7.5	6.95	7%	9.31	24%	10.03	34%
	26	4.28E+07	12.1	8.98	26%	13.37	11%	10.43	14%
	27	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	29	9.17E+07	14.8	18.76	27%	32.92	122%	12.39	16%
	30	4.57E+07	13.7	9.56	30%	14.53	6%	10.55	23%
	31	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	33	1.28E+08	14.9	26.01	75%	47.43	218%	13.84	7%
	34	7.40E+07	16.4	15.22	7%	25.86	58%	11.68	29%
	35	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	37	1.73E+08	14.7	35.01	138%	65.43	345%	15.64	6%
	38	1.04E+08	18.3	21.17	16%	37.76	106%	12.87	30%
	39	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	41	2.16E+08	39.7	43.62	10%	82.65	108%	17.36	56%
	42	1.77E+08	27	35.79	33%	66.98	148%	15.79	42%
	43	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	45	4.04E+08	89	81.31	9%	158.03	78%	24.90	72%
	46	4.04E+08	96.8	81.31	16%	158.03	63%	24.90	74%
	50	1.21E+08	20	24.66	23%	44.72	124%	13.57	32%
	51	1.81E+08	21.3	36.70	72%	68.82	223%	15.98	25%
	52	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	54	7.40E+07	11.8	15.22	29%	25.86	119%	11.68	1%
	55	1.23E+08	16.2	25.09	55%	45.59	181%	13.66	16%
	56	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	58	5.49E+07	7.8	11.40	46%	18.21	133%	10.92	40%
	61	7.35E+07	7.6	15.13	99%	25.66	238%	11.66	53%
	62	1.14E+07	3.7	2.70	27%	0.81	78%	9.18	148%
	64	3.94E+07	16.3	8.31	49%	12.02	26%	10.30	37%
	65	5.81E+07	7.9	12.03	52%	19.47	146%	11.04	40%
	66	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
r- Pisek B	68	4.74E+07	3.6	9.90	175%	15.21	323%	10.62	195%
	69	4.74E+07	3.6	9.90	175%	15.21	323%	10.62	195%

	70	6.15E+06	4	1.65	59%	(1.29)	132%	8.97	124%
	74	1.11E+08	25.4	22.67	11%	40.76	60%	13.17	48%
	78	2.13E+08	26.2	42.95	64%	81.30	210%	17.23	34%
	79	5.49E+07	13.2	11.40	14%	18.21	38%	10.92	17%
	80	6.15E+06	3.5	1.65	53%	(1.29)	137%	8.97	156%
	84	1.33E+08	19.1	27.08	42%	49.56	159%	14.05	26%
	85	4.28E+07	10.5	8.98	14%	13.37	27%	10.43	1%
	86	6.15E+06	3.5	1.65	53%	(1.29)	137%	8.97	156%
	88	9.17E+07	17.9	18.76	5%	32.92	84%	12.39	31%
	89	3.00E+07	6.3	6.42	2%	8.24	31%	9.92	57%
	90	6.15E+06	3.45	1.65	52%	(1.29)	137%	8.97	160%
	92	3.70E+07	9.7	7.82	19%	11.05	14%	10.20	5%
	93	3.27E+07	7.5	6.95	7%	9.31	24%	10.03	34%
	94	6.15E+06	3.45	1.65	52%	(1.29)	137%	8.97	160%
	96	3.00E+07	7.6	6.42	16%	8.24	8%	9.92	31%
	97	9.17E+07	14.8	18.76	27%	32.92	122%	12.39	16%
	98	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	100	4.28E+07	12.1	8.98	26%	13.37	11%	10.43	14%
	101	1.28E+08	14.9	26.01	75%	47.43	218%	13.84	7%
	102	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	104	4.57E+07	13.7	9.56	30%	14.53	6%	10.55	23%
	105	1.73E+08	14.7	35.01	138%	65.43	345%	15.64	6%
	106	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	108	7.40E+07	16.4	15.22	7%	25.86	58%	11.68	29%
	109	2.16E+08	39.7	43.62	10%	82.65	108%	17.36	56%
	110	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	112	1.04E+08	18.3	21.17	16%	37.76	106%	12.87	30%
	113	4.04E+08	89	81.31	9%	158.03	78%	24.90	72%
	114	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	116	1.77E+08	27	35.79	33%	66.98	148%	15.79	42%
	117	1.21E+08	20	24.66	23%	44.72	124%	13.57	32%
	118	0.00E+00	64.9	0.42	99%	(3.75)	106%	8.72	87%
	122	4.04E+08	96.8	81.31	16%	158.03	63%	24.90	74%
	123	7.40E+07	11.8	15.22	29%	25.86	119%	11.68	1%
	124	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	126	1.81E+08	21.3	36.70	72%	68.82	223%	15.98	25%
	127	5.49E+07	7.8	11.40	46%	18.21	133%	10.92	40%
	128	6.15E+06	3.1	1.65	47%	(1.29)	142%	8.97	189%
	130	1.23E+08	16.2	25.09	55%	45.59	181%	13.66	16%
	133	3.94E+07	16.3	8.31	49%	12.02	26%	10.30	37%
	134	1.14E+07	3.7	2.70	27%	0.81	78%	9.18	148%
	136	7.35E+07	7.6	15.13	99%	25.66	238%	11.66	53%
	137	4.74E+07	3.6	9.90	175%	15.21	323%	10.62	195%
	138	6.15E+06	3	1.65	45%	(1.29)	143%	8.97	199%
	140	5.81E+07	7.9	12.03	52%	19.47	146%	11.04	40%
	141	5.81E+07	7.9	12.03	52%	19.47	146%	11.04	40%
	142	6.15E+06	4	1.65	59%	(1.29)	132%	8.97	124%
					139%		306%		112%

Annex 13: Comparison the average percentage error between formulas obtained on the different studies for all the profiles. Formula one.

Comparison of the First formula with the non-Truss bridges	#	I	Sj	Formula Sj tabor-Pisek	Error %	Formula Sj Minor's Thesis	Error %	Formula Sj	Error %
		mm4	MNm/rad	Mnm/rad		Mnm/rad		Mnm/rad	
Different Bridges	9	100130000	14.8	20.45	38%	36.30	145%	22.77	54%
	10	59388000	76.7	12.30	84%	20.00	74%	14.62	81%
	11	59388000	2.3	12.30	435%	20.00	770%	14.62	536%
	6	122189141	47.112	24.86	47%	45.12	4%	27.18	42%
	7	51341867	9.724	10.69	10%	16.78	73%	13.01	34%
	8	6481237.3	1.192	1.72	44%	-1.16	197%	4.04	239%
	9	6481237.3	1.486	1.72	15%	-1.16	178%	4.04	172%
	4	3.70E+07	1.8	7.82	335%	11.05	514%	10.15	464%
	6	8.66E+07	12	17.74	48%	30.89	157%	20.06	67%
	7	1.33E+08	19.1	27.08	42%	49.56	159%	29.40	54%
	8	6.15E+06	3.5	1.65	53%	-1.29	137%	3.97	14%
	10	3.70E+07	1.8	7.82	335%	11.05	514%	10.15	464%
	12	5.49E+07	13.2	11.40	14%	18.21	38%	13.73	4%
	13	9.17E+07	17.9	18.76	5%	32.92	84%	21.08	18%
	14	6.15E+06	3.5	1.65	53%	-1.29	137%	3.97	14%
	16	4.28E+07	10.5	8.98	14%	13.37	27%	11.31	8%
	17	3.70E+07	9.7	7.82	19%	11.05	14%	10.15	5%
	18	6.15E+06	3.45	1.65	52%	-1.29	137%	3.97	15%
	20	3.00E+07	6.3	6.42	2%	8.24	31%	8.74	39%
	21	3.00E+07	7.6	6.42	16%	8.24	8%	8.74	15%
	22	6.15E+06	3.45	1.65	52%	-1.29	137%	3.97	15%
	24	3.27E+07	7.5	6.95	7%	9.31	24%	9.28	24%
	25	4.28E+07	12.1	8.98	26%	13.37	11%	11.31	7%
	26	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	28	9.17E+07	14.8	18.76	27%	32.92	122%	21.08	42%
	29	4.57E+07	13.7	9.56	30%	14.53	6%	11.89	13%
	30	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	32	1.28E+08	14.9	26.01	75%	47.43	218%	28.34	90%
	33	7.40E+07	16.4	15.22	7%	25.86	58%	17.55	7%
	34	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	37	1.04E+08	18.3	21.17	16%	37.76	106%	23.50	28%
	38	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	41	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	46	1.21E+08	20	24.66	23%	44.72	124%	26.98	35%
	48	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	50	7.40E+07	11.8	15.22	29%	25.86	119%	17.55	49%
	51	1.23E+08	16.2	25.09	55%	45.59	181%	27.42	69%
	52	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	54	5.49E+07	7.8	11.40	46%	18.21	133%	13.73	76%
	57	7.35E+07	7.6	15.13	99%	25.66	238%	17.45	130%
	58	1.14E+07	3.7	2.70	27%	0.81	78%	5.02	36%
	60	3.94E+07	16.3	8.31	49%	12.02	26%	10.63	35%
	61	5.81E+07	7.9	12.03	52%	19.47	146%	14.36	82%
	62	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	64	4.74E+07	3.6	9.90	175%	15.21	323%	12.23	240%
	65	4.74E+07	3.6	9.90	175%	15.21	323%	12.23	240%
	66	6.15E+06	4	1.65	59%	-1.29	132%	3.97	1%
	70	1.11E+08	25.4	22.67	11%	40.76	60%	25.00	2%

	75	5.49E+07	13.2	11.40	14%	18.21	38%	13.73	4%
	76	6.15E+06	3.5	1.65	53%	-1.29	137%	3.97	14%
	80	1.33E+08	19.1	27.08	42%	49.56	159%	29.40	54%
	81	4.28E+07	10.5	8.98	14%	13.37	27%	11.31	8%
	82	6.15E+06	3.5	1.65	53%	-1.29	137%	3.97	14%
	84	9.17E+07	17.9	18.76	5%	32.92	84%	21.08	18%
	85	3.00E+07	6.3	6.42	2%	8.24	31%	8.74	39%
	86	6.15E+06	3.45	1.65	52%	-1.29	137%	3.97	15%
	88	3.70E+07	9.7	7.82	19%	11.05	14%	10.15	5%
	89	3.27E+07	7.5	6.95	7%	9.31	24%	9.28	24%
	90	6.15E+06	3.45	1.65	52%	-1.29	137%	3.97	15%
	92	3.00E+07	7.6	6.42	16%	8.24	8%	8.74	15%
	93	9.17E+07	14.8	18.76	27%	32.92	122%	21.08	42%
	94	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	96	4.28E+07	12.1	8.98	26%	13.37	11%	11.31	7%
	97	1.28E+08	14.9	26.01	75%	47.43	218%	28.34	90%
	98	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	100	4.57E+07	13.7	9.56	30%	14.53	6%	11.89	13%
	102	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	104	7.40E+07	16.4	15.22	7%	25.86	58%	17.55	7%
	106	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	108	1.04E+08	18.3	21.17	16%	37.76	106%	23.50	28%
	110	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	113	1.21E+08	20	24.66	23%	44.72	124%	26.98	35%
	117	7.40E+07	11.8	15.22	29%	25.86	119%	17.55	49%
	118	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	121	5.49E+07	7.8	11.40	46%	18.21	133%	13.73	76%
	122	6.15E+06	3.1	1.65	47%	-1.29	142%	3.97	28%
	124	1.23E+08	16.2	25.09	55%	45.59	181%	27.42	69%
	127	3.94E+07	16.3	8.31	49%	12.02	26%	10.63	35%
	128	1.14E+07	3.7	2.70	27%	0.81	78%	5.02	36%
	130	7.35E+07	7.6	15.13	99%	25.66	238%	17.45	130%
	131	4.74E+07	3.6	9.90	175%	15.21	323%	12.23	240%
	132	6.15E+06	3	1.65	45%	-1.29	143%	3.97	32%
	134	5.81E+07	7.9	12.03	52%	19.47	146%	14.36	82%
	135	5.81E+07	7.9	12.03	52%	19.47	146%	14.36	82%
	136	6.15E+06	4	1.65	59%	-1.29	132%	3.97	1%
					54%		134%		62%

Annex 14: Comparison average percentage error between formulas obtained on the different studies for big profiles. Formula 2.

Comparison of the second formula with the non- Truss bridges	#	I	Sj	Formula Sj tabor-Pisek	Error %	Formula Sj Minor's Thesis	Error %	Formula Sj	Error %
		mm4	MNm/rad	Mnm/rad		Mnm/rad		Mnm/rad	
Different Bridges	1	3.49E+08	18.5	70.30	280%	136.00	635%	44.99	143%
	2	3.39E+08	53.1	68.22	28%	131.84	148%	44.57	16%
	3	2.4E+08	3.9	48.36	1140%	92.13	2262%	40.60	941%
	4	5.56E+09	260.1	1111.75	327%	2218.91	753%	253.28	3%
	5	5.56E+09	210.5	1111.75	428%	2218.91	954%	253.28	20%
	6	6.58E+08	18.3	131.99	621%	259.39	1317%	57.33	213%
	7	1.33E+09	4.4	266.22	5950%	527.85	11897%	84.17	1813%
	8	1.64E+08	2.5	33.22	1229%	61.85	2374%	37.57	1403%
Steel Railway Bridge	1	2.32E+08	75.256	46.89	38%	89.19	19%	40.31	46%
	2	3.13E+08	111.8	63.08	44%	121.56	9%	43.54	61%
	3	2.32E+08	81.028	46.89	42%	89.19	10%	40.31	50%
	4	1.72E+08	71.464	34.79	51%	64.98	9%	37.89	47%
	5	1.72E+08	27.471	34.79	27%	64.98	137%	37.89	38%
	2	2.13E+08	26.2	42.95	64%	81.30	210%	39.52	51%
	3	1.82E+08	10.7	36.84	244%	69.08	546%	38.30	258%
	6	1.82E+08	10.7	36.84	244%	69.08	546%	38.30	258%
	12	1.73E+08	14.7	35.01	138%	65.43	345%	37.93	158%
	14	2.16E+08	39.7	43.62	10%	82.65	108%	39.65	0%
	15	1.77E+08	27	35.79	33%	66.98	148%	38.09	41%
	17	4.04E+08	89	81.31	9%	158.03	78%	47.19	47%
	18	4.04E+08	96.8	81.31	16%	158.03	63%	47.19	51%
	20	1.81E+08	21.3	36.70	72%	68.82	223%	38.27	80%
	27	2.13E+08	26.2	42.95	64%	81.30	210%	39.52	51%
	34	1.73E+08	14.7	35.01	138%	65.43	345%	37.93	158%
	36	2.16E+08	39.7	43.62	10%	82.65	108%	39.65	0%
	38	4.04E+08	89	81.31	9%	158.03	78%	47.19	47%
	39	1.77E+08	27	35.79	33%	66.98	148%	38.09	41%
	41	4.04E+08	96.8	81.31	16%	158.03	63%	47.19	51%
	43	1.81E+08	21.3	36.70	72%	68.82	223%	38.27	80%
					392%		826%		213%