Aerospace Department

Centrifuge for small experiments
A CubeSat-class satellites

Master’s Thesis

Suren Ali-Ogly

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Abstract:
This diploma work deals with construction of centrifuge machine for hypergravity simulation for samples of CubeSAT sizes. First, a small overview of centrifuges is presented, afterwards will be performed calculation of needed performance according to requirements and given conditions. Last, design of the centrifuge machine will be performed using known methods and technics, like analytical hands calculations, analytical calculations with the help of MATLAB® software and Finite Element Method (FEM).
Declaration:

I declare that I wrote my master (diploma) thesis independently and I used only references that are mentioned in attached list.

Prohlášení:

Prohlašuji, že jsem svou bakalářskou práci vypracoval samostatně a použil jsem pouze podklady uvedené v přiloženém seznamu.

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2. DVD

CAD files
Drawings is PDF
Excel file
FEM files
Matlab files
Word and PDF document of thesis

List of used software

Siemens NX 11.0
Microsoft Excel/Word 2013
Beta CAE Ansa 16.0
Siemens FEMAP v11.3
Matlab R2014b
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Nomenclature, Abbreviations, Acronyms

1B – one-beam centrifuge’s head variant/ simple one-beam centrifuge variant
1B-AWR – one-beam centrifuge with all possible windows reinforced variant
1B-R – one-beam centrifuge with one reinforced frame window variant
2B – AWR – two-beam centrifuge with all possible windows reinforced variant
2B – two-beam centrifuge’s head variant/ simple two-beam centrifuge variant
2B-R – two-beam centrifuge with one reinforced frame window variant
A – area
CHP – centrifuge head’s plate
\( C_c \) – centrifugal force of a capsule
\( C_D, c_D \) – drag coefficient [-]
CG – centre of gravity
\( D_p \) – pitch diameter [mm]
e – eccentricity
E – Young’s modulus [MPa]
\( F_C \) – Capsule force, for (3.85) p. 47
\( F_C \) – centrifugal force [N]
\( F_D, F_d \) – drag force [N]
\( G_p \) – the weight load of 45x45 profile
\( g \) – gravitational acceleration, \( g = 9.81 \ [\text{m.s}^{-2}] \)
G – shear modulus [MPa]
Gc – the weight loading of a capsule
I – mass moment of inertia [kg.m²]
j – reserve factor [-]
J – second polar moment of area [m²], [mm⁴]
k – stiffness [N/m], [N/mm]
\( k_{bru} \) – shear – bearing efficiency factor
\( k_t \) – net tension efficiency factor
\( k_{try} \) – efficiency factor for transverse yield load
M, M(x) – moment, Bending moment [N.m], [N.mm]
MBP – motor base plate
\( M_p, M_d \) – moment caused by drag force/ Drag moment [N.mm], [N.mm]
m – mass [kg]
\( m_c \) – mass of a capsule [kg]
\( m_p \) – mass of a 45x45 profile [kg]
n – number of revolutions
O – centrifugal force on a rotating shaft
p – contact pressure (feather key)
\( P_Y \) – yield transverse load
\( P_{bru} \) – ultimate load for shear – tear out and bearing failure
\( P_{bru} \) – bushing yield bearing load
\( P_{tu} \) – ultimate load for tension failure
q – distributed force [N.m⁻¹], [N.mm⁻¹]
r – radius/ rotational distance [m], [mm]
Re – Reynolds number [-]
RF – reserve factor [-]
\( R p_{0.2} \) – yield strength [MPa]
rpm – number of revolutions per minute
rps – number of revolutions per second
t – time [s]
T – torque [N.m], [N.mm]
TB – taper bush
v – circumferential velocity/ translation velocity [m/s]
V, V(x) – shearing force [N]
w – distributed force [N.m^1], [N.mm^1]
W – section modulus [m^3], [cm^3], [mm^3]
λ – fitting factor [-]
Ω – eigen frequency/ critical angular velocity
L – angular momentum [kg.m^2.s^-1]
f – safety factor [-]
ρ – density [kg.m^3]
σ – stress [MPa]
σ_y – yield stress [MPa]
σ_{cy} – compression yield stress of bushing material
σ_{red} – reduced stress [MPa]
σ_{tux} – ultimate tensile stress in x – direction of the material
σ_{tuy} – tensile yield stress of lug material in y – direction
τ – shear stress [MPa]
v - deflection [mm]
ω – angular velocity [rad/s]
1. Introduction

A centrifuge is a machine, respectively device, which uses a rotation of an object around fixed axis to produce a centripetal force, and due to reaction – centrifugal force.

There are variety of the applications of a centrifuge. It is widely used in pharmaceutical, food and general chemistry industry (using sedimentation principle). Centrifuge that can provide very high rotational speed can separate particles to an extremely low scale, down to molecules for example. Washing machines, pumps also uses centrifuge for their work. It can be used for geotechnical purposes. For example geotechnical centrifuge C60 (Figure 1-1), which is designed by Actidyn, can simulate behavior of structures on foundation and of soil mechanics, with the help of reduced scale model.

![Figure 1-1 C60, geotechnical centrifuge [1]](image1)

Another industries, where centrifuge is used are aviation and space technologies. Using centrifuge machines we can simulate flight phases, loading factors from maneuvers and so on. This can be very helpful for pilots who just have begun their professional path. For example, using a Phoenix centrifuge (Figure 1-2) of NASTAR Center we can simulate human performance under high accelerations, space training, high risk maneuvers training. [2]

![Figure 1-2 Phoenix centrifuge [2]](image2)

Purpose of this work is to design a centrifuge that can carry a payload of 15 kg and can accelerate up to 10g. Normally to simulate a space launch for the small centrifuges, for example CubeSat, required acceleration of 5g, however it is advantageous to use a centrifuge for a larger scope of work, that’s why was made a decision to design a centrifuge for 10g.
2. Preliminary Performance Calculation

The assignment of this part is to find the optimum performance characteristic as well as the optimum arm length of the arm-centrifuge machine.

The demand is to obtain the gravity acceleration of the value of 10 g, by using an advantage of the centrifugal force. We have given preliminary design parameters of the capsule, where will be hold the experimental unit. The capsule has the shape of the cylinder with the height of \( H = 0.65 \text{ m} \), diameter \( D = 0.34 \text{ m} \) and mass \( m = 15 \text{ kg} \).

2.1. The Centrifugal and Drag Force

*Figure 2-1 Centrifugal force [3]*

According to Newton's Law of Inertia, an object in motion tends to follow a straight line. Applying a sideways force on the object can overcome the inertia and cause the object to take a curved path. That force is called a centripetal force.

Newton's Third Law or Action-Reaction Law states that for every applied force, there is an equal and opposite force. In other words, when you apply a force on a rope in swinging an object around you, you will feel an equal and opposite force pulling the object away from you. This force is the centrifugal force. [3]

*Figure 2-2*

The centrifugal force is:

\[
F_c = m\omega^2 r = \frac{mv^2}{r}
\]  

The required gravitational acceleration is 10 g. Then:

\[
m \times 10g = m\omega^2 r = \frac{mv^2}{r}
\]  

(2.2)
Where \( m \) is the mass of the capsule with experimental unit.

We can see that the mass is located on the boss sides of the equation, so we can using advantage of the equation calculation and cut them.

\[
10g = \omega^2 r = (2\pi n)^2 r \quad \text{or} \quad 10g = \frac{v^2}{r}
\]  

(2.3)

The task is to find the most useful and the most advantageous angular velocity (rotations) and arm radius.

Limitation parameters are the electric motor performances, where the one of the most important parameter is the torque of a motor. The values of torque are represented as the dependency diagram of Torque [N.m] versus speed [rps].

The given electric motor is ES-MH 342200 3-phase steepen motor. Torque-speed characteristic are shown on the Figure 2-3.

![Figure 2-3 Torque vs speed char, of the ES-MH 34xx series [4]](image)

Very important thing that have to be considered during calculations of optimum values of speed and arm length is the drag force that will act opposite direction of the circumferential velocity of the structure, other words, the drag force will cause a resistance force and moment to the working structure. It will directly effect on the ability of structure to reach the maximum angular speed. If the moment caused by drag force will be lower than a torque of a motor, the centrifugal machine will have ability to increase the revolution speed. The machine cannot overcome the specific value of angular velocity, because \( t=\)otherwise the drag moment will be higher than motor torque. This specific value is the maximum speed that is reached when the drag moment is equal to the torque of the given motor. Note that the torque is different for every different rotation speed.

The equation of the drag force is:

\[
F_D = \frac{1}{2} \rho v^2 A_C D
\]

Where \( \rho \) is the density of the working fluid, which is the air in our case. For the first shoot of the calculation will be used the value of 1.225 kg/m\(^3\), that represents the air density on the sea level at 15 °C. More correct value will be obtained from ambient condition of the laboratory.

\( C_D \) is the drag coefficient. Its value depends on the working fluid, shape of streamlining body (cylinder) and Reynolds number. See Figure 2-4.
$A$ is the normal cross-section area of the streamlined body. Specifically in our case:

$$A = H \times D = 0.65 \times 0.34 = 0.221 [m^2]$$  \hspace{1cm} (2.5)

![Figure 2-4 Dependency of the Drag Coefficient on the Re [5]]

### 2.2. Preliminary calculation of the performance and arm length

As it was mentioned in the beginning, the required load should reach $10 \text{ g} = g_{req}$. As the first step we will try to guess the arm length. Let’s say it will be $r = 0.5 \text{ m}$. Then using this value of radius we will find the appropriate angular velocity.

$$\omega_{req} = \sqrt{\frac{g_{req}}{r}} = \sqrt{\frac{9.81}{0.5}} = 14.00714 \left[ \text{rad} \text{s}^{-1} \right]$$  \hspace{1cm} (2.6)

Transform it to the revolutions speed:

$$rps = \frac{\omega}{2\pi} = \frac{14}{2\pi} = 2.2293 \left[ \text{rps} \right]$$

Or

$$rpm = 60 \times \frac{\omega}{2\pi} = 60 \times \frac{14}{2\pi} = 133.7583 \left[ \text{rpm} \right]$$  \hspace{1cm} (2.7)

The circumferential speed on the tip of the arm is:

$$v = \omega r = 14 \times 0.5 = 7 \left[ \text{m} \text{s}^{-1} \right]$$  \hspace{1cm} (2.8)

Respective Reynolds number for obtained speed and cylindrical speed is:

$$Re = \frac{v D}{\nu} = \frac{7 \times 0.34}{1.46 \times 10^{-5}} = 163097 \left[ \right]$$  \hspace{1cm} (2.9)

The suitable drag coefficient for this Reynolds number is $c_D = 1.1 \left[-\right]$. The drag force acted on the capsule is then:

$$F_{DC} = \frac{1}{2} \rho v^2 A c_D = \frac{1}{2} \times 1.225 \times 7^2 \times 0.221 \times 1.1 = 7.3035 \left[ N \right]$$  \hspace{1cm} (2.10)
Moment caused by drag force of capsule. Note that this moment caused only by one loaded arm.

\[ M_{Dc} = F_{Dc} \times r = 7.3035 \times 0.5 = 3.6517 \, [N.m] \]  
(2.11)

Applying same method for other radiiues, we will construct tables.

<table>
<thead>
<tr>
<th>Arm [m]</th>
<th>0.5</th>
<th>0.6</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>rps</td>
<td>2.229</td>
<td>2.035</td>
<td>1.8202205</td>
<td>1.576</td>
<td>1.409937</td>
<td>1.28709</td>
<td>1.192</td>
</tr>
<tr>
<td>rpm</td>
<td>133.758</td>
<td>122.1041</td>
<td>109.21323</td>
<td>94.581</td>
<td>84.5962</td>
<td>77.22542</td>
<td>71.497</td>
</tr>
<tr>
<td>Angle [(^\circ)]</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
</tr>
<tr>
<td>( v_{circum} ) [m/s]</td>
<td>7.004</td>
<td>7.672</td>
<td>8.577</td>
<td>9.904</td>
<td>11.073</td>
<td>12.131</td>
<td>13.102</td>
</tr>
<tr>
<td>( F_{Dc} ) [N]</td>
<td>7.303</td>
<td>8.76</td>
<td>10.955</td>
<td>14.61</td>
<td>18.25</td>
<td>21.91</td>
<td>25.56</td>
</tr>
<tr>
<td>( M_D ) [N.mm]</td>
<td>3.6517</td>
<td>5.258</td>
<td>8.216</td>
<td>14.61</td>
<td>22.823</td>
<td>32.866</td>
<td>44.733</td>
</tr>
<tr>
<td>Re [-]</td>
<td>163096</td>
<td>178663</td>
<td>199752</td>
<td>230653</td>
<td>257878</td>
<td>282492</td>
<td>305126</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arm [m]</th>
<th>2</th>
<th>2.25</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) [rad/s]</td>
<td>7.003</td>
<td>6.603</td>
<td>6.264</td>
</tr>
<tr>
<td>rps</td>
<td>1.115</td>
<td>1.051</td>
<td>0.997</td>
</tr>
<tr>
<td>rpm</td>
<td>66.88</td>
<td>63.05</td>
<td>59.82</td>
</tr>
<tr>
<td>Angle [(^\circ)]</td>
<td>84.29</td>
<td>84.29</td>
<td>84.29</td>
</tr>
<tr>
<td>( v_{circum} ) [m/s]</td>
<td>14.007</td>
<td>14.856</td>
<td>15.66</td>
</tr>
<tr>
<td>( F_{Dc} ) [N]</td>
<td>29.21393</td>
<td>32.86568</td>
<td>36.517</td>
</tr>
<tr>
<td>( M_D ) [N.mm]</td>
<td>58.43</td>
<td>73.95</td>
<td>91.29</td>
</tr>
<tr>
<td>Re [-]</td>
<td>326193</td>
<td>345980</td>
<td>364695</td>
</tr>
</tbody>
</table>

**Table 1 Finding optimum values**

Angle is the angle of inclination that is denoted as \( \alpha \) on a figure below.

![Figure 2-5 Inclination angle](image)

For easier comparison of obtained values, we will construct dependency diagrams.
Figure 2-6 Velocity versus arm length $r$

Figure 2-7 Drag moment and revolution with the changing arm length $r$

The less the moment caused by drag force, the better. The smallest one is convenient for smallest radius, but for smallest radius, we have to have the higher speed. As my personal view, the optimum solution is the arm length of 1m.

<table>
<thead>
<tr>
<th>Arm [m]</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$ [rad/s]</td>
<td>9.904</td>
</tr>
<tr>
<td>$\text{rps}$</td>
<td>1.576</td>
</tr>
<tr>
<td>$\text{rpm}$</td>
<td>94.581</td>
</tr>
<tr>
<td>Angle [$^\circ$]</td>
<td>84.29</td>
</tr>
<tr>
<td>$v_{\text{circum}}$ [m/s]</td>
<td>9.904</td>
</tr>
<tr>
<td>$F_{DC}$ [N]</td>
<td>14.61</td>
</tr>
<tr>
<td>$M_{D}$ [N.mm]</td>
<td>14.61</td>
</tr>
<tr>
<td>$Re$ [-]</td>
<td>230653</td>
</tr>
</tbody>
</table>

Table 2 Selected parameters

For these chosen values will be continued further calculations.
The essential difference between a centrifuge with a swinging capsule and a centrifuge with a fixed one is the way how the centrifugal force applies on the capsule. The bottom of the capsule is constantly perpendicular to the sum of the earth gravitation vector and of the centrifuge acceleration vector and the distance between centre of gravity of the capsule and center of rotation increases and this causes the increment of acceleration of a particular point, like centre of gravity. In order to find a gradient of the acceleration of this point, including the effect of an angle increasing (increasing a distance between point and rotation centre) we will use an assumption that arm length is 1m and the distance between arm’s tip and centre of gravity of the tested capsule is 425 mm, see Figure 3-6, p.17. Dependency between centrifugal acceleration of a point distanced from rotation centre and rotational speed can be found using equation (2.1), where radius r is:

\[
r = R + a = R + 0.425 \times \sin \alpha
\]  

(2.12)

Where R = 1m, is the chosen arm length, a is an additional distance due to capsule displacement, \( \alpha \) is a deflection angle and determined by the equation (2.13).

\[
\tan \alpha = \frac{g_r}{g} = \frac{x \times g}{g} = x \\
\alpha = \text{atan} x
\]  

(2.13)

Where x is a coefficient describing a centrifugal acceleration. For example, if we are talking about 10g acceleration, then x = 10.

Using that logic we can state that:

\[
x \times g = \omega^2 \times (R + a \times \sin \alpha) \\
x \times g = \omega^2 \times (R + a \times \sin (\text{atan} x))
\]  

(2.14)

The expression shown in (2.14) can be solved iteratively.

1) \( x_i \times g = \omega_i^2 \times (R + a \times \sin \alpha_i) \)
2) \( a_{i+1} = \text{atan}(x_i) \)
3) \( x_{i+1} \times g = \omega_{i+1}^2 \times (R + a \times \sin (\alpha_{i+1})) \)
4) \( x_i = x_{i+1}; \quad \alpha_i = \alpha_{i+1}; \quad \omega_i = \omega_{i+1} \)  

(2.15)

Initial conditions are:

\[
\alpha(\omega_0) = 0, \quad \omega_0 = 0.001 \left[ \frac{\text{rad}}{s} \right]
\]  

(2.16)

Then the gradient of centrifugal acceleration can be constructed as a graph, that is shown on the Figure 2-8. From this figure, we can see that in fact we will reach 10g acceleration in CG of a capsule when arms will rotate at 1.33 rotations per second, 1.2 rps for the bottom and 1.5 rps for the top. Even though we found a rotation speed for which preliminary CG of a tested cylindrical capsule of 650x350 mm dimension reaches 10g, and this value is less than the one mention in Table 2, we will use for further calculations the speed of 1.567 rps. The reason is that the centre gravity point is not necessarily located in middle of a capsule or tested sample might be located in different point than the capsule geometrical centre, but rotation speed 1.567 rps corresponds to the 10g acceleration at 1m distance far from rotation centre.
2.3. Acceleration and time when the required speed will be reached.

Preliminary

To find the angular acceleration of the structure as the first step we have to find the angular momentum.

\[ L = I \cdot \omega \left( \frac{kg}{s} \right) \left( m^2 \right)^2 \]  

(2.17)

Where the \( I \) is the moment of inertia and \( \omega \) is the angular speed.

To find the moment of inertia there was created the very first preliminary design of arms together with the capsules on the both sides of it. Using properties function of CAD software Inventor 2016® we have found the preliminary moment of inertias. I have created two variants: 1st when it is on static and when we have reached our required 10g.

\[ I_{\text{max}} = 36127435.887 \text{[kg.mm}^2] = I_{yy} \approx 40 \text{[kg.m}^2]\]
I_{\text{max}} = 60548333.883 \, [\text{kg.mm}^2] \approx 60 \, [\text{kg.m}^2].

The next calculation will be provided for I = 60 \, [\text{kg.m}^2]. Using (2.17) we find L = 594.2726 \, [\text{kg.m}^2/s]

If torque is given from the motor characteristics, using formula (2.18) we can find a required time until the machine reach required angular speed.

\[
T = \frac{dL}{dt}
\]

\[
T \, dt = dL
\]

\[
\int T \, dt = \int dL
\]

(2.18)

If we assume that torque is constant from 0 to 2 rps, it value will be 20 \, [\text{N.m}].

\[
20 \int dt = L
\]

\[
\int dt = L \frac{594.2726}{20} = 29.7136
\]

\[
\int_{0}^{t} dt = t = 29.7136 \approx 30 \, [s]
\]

(2.19)

Calculation within drag force resistance will be performed with assumption that drag force will be a constant value.

\[
T' = T - M_{Dc}
\]

\[
T' = 20 - 14.6069 = 5.3930 \, [\text{N.m}]
\]

(2.20)

Using (2.19) we can recalculate the time.

\[
t = \frac{L}{T''} = \frac{594.2726}{5.3930} = 110.1927 \, [s] \approx 115 \, [s]
\]

(2.21)

For results that are more accurate, we will no longer assume the torque of a motor as a constant value and drag moment too. Next table will help to create it numerically.
<table>
<thead>
<tr>
<th>T [N.m]</th>
<th>ω [rad/s]</th>
<th>v [m/s]</th>
<th>Re [-]</th>
<th>$M_T$ [N.m]</th>
<th>$\alpha$ [rad/s^2]</th>
<th>ΔT [N.m]</th>
<th>$\alpha'$ [rad/s^2]</th>
<th>t' [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
<td>20</td>
<td>0.33</td>
<td>0</td>
</tr>
<tr>
<td>20.5</td>
<td>2.325</td>
<td>2.325</td>
<td>54139</td>
<td>0.805</td>
<td>0.342</td>
<td>19.695</td>
<td>0.328</td>
<td>7.082</td>
</tr>
<tr>
<td>21.2</td>
<td>5.97</td>
<td>5.97</td>
<td>139004</td>
<td>5.305</td>
<td>0.353</td>
<td>15.895</td>
<td>0.265</td>
<td>22.532</td>
</tr>
<tr>
<td>20.44</td>
<td>9.425</td>
<td>9.425</td>
<td>219481</td>
<td>13.587</td>
<td>0.341</td>
<td>6.853</td>
<td>0.114</td>
<td>82.515</td>
</tr>
<tr>
<td>20.35</td>
<td>9.905</td>
<td>9.905</td>
<td>230653</td>
<td>15.005</td>
<td>0.34</td>
<td>5.345</td>
<td>0.09</td>
<td>111.19</td>
</tr>
<tr>
<td>20.05</td>
<td>12.566</td>
<td>12.566</td>
<td>292641</td>
<td>23.513</td>
<td>0.334</td>
<td>-3.463</td>
<td>-0.058</td>
<td>-217.72</td>
</tr>
</tbody>
</table>

Table 3 Time calculation

Figure 2-10 Moment equilibrium diagram (For only one arm loaded)

Figure 2-11 Time versus rps

If we will remain the arm length as 1m, but there will be additionally added some more materials it will increase the mass of the structure, which follows into increasing of inertia moment. This will have consequences like time increasing and increasing loading.
<table>
<thead>
<tr>
<th>I [kg.m²]</th>
<th>𝛼' [rad/s²]</th>
<th>t[s]</th>
<th>t[min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.267</td>
<td>37.063</td>
<td>0.618</td>
</tr>
<tr>
<td>30</td>
<td>0.1782</td>
<td>55.595</td>
<td>0.9266</td>
</tr>
<tr>
<td>50</td>
<td>0.107</td>
<td>92.658</td>
<td>1.544</td>
</tr>
<tr>
<td>80</td>
<td>0.067</td>
<td>148.25</td>
<td>2.471</td>
</tr>
<tr>
<td>100</td>
<td>0.053</td>
<td>185.32</td>
<td>3.1</td>
</tr>
<tr>
<td>120</td>
<td>0.0445</td>
<td>222.38</td>
<td>3.71</td>
</tr>
<tr>
<td>150</td>
<td>0.036</td>
<td>277.975</td>
<td>4.633</td>
</tr>
<tr>
<td>200</td>
<td>0.027</td>
<td>370.633</td>
<td>6.18</td>
</tr>
<tr>
<td>250</td>
<td>0.0214</td>
<td>463.291</td>
<td>7.72</td>
</tr>
<tr>
<td>300</td>
<td>0.0178</td>
<td>555.95</td>
<td>9.266</td>
</tr>
<tr>
<td>400</td>
<td>0.0134</td>
<td>741.266</td>
<td>12.35</td>
</tr>
</tbody>
</table>

*Table 4 Inertia vs Time*

*Figure 2-12 Time increasing with moment of inertia*
3. Design

This chapter is dedicated for the general design steps of specific centrifuge’s parts, such as: Centrifuge arms; Transmission design; Shaft design; Motor mounting; Lug and Pin design; Centrifuge Head and Frame.

3.1. Centrifuge arms strength calculation

Centrifuge arms are the part of the centrifuge machine that carry two significant loadings, which are mass load of the tasted sample (cylindrical capsule) and centrifugal force appeared on this sample due to rotation around fixed axis. Generally, we can divide loadings for two separate cases that arms will carry, those are Static case (when no rotation of the centrifuge happens) and Motion case (with rotation).

3.1.1. Static case

Static case it is the case when the structure of the centrifugal machine is loaded only by weight of capsule and own weight.

\[
\begin{align*}
\text{Figure 3-1 Static case loading}
\end{align*}
\]

Where Gc represents the weight loading of a capsule. \( G_p \) represents the weight load of 45x45 profile and can be represented as distributed load \( w \). \( R_B \) and \( M_B \) are reaction at the fixed connection, which is the center of rotation. Their values are:

\[
\begin{align*}
 w &= m_p \frac{g}{L} = \frac{G_p}{L} = 4 \times \frac{9.81}{1} \approx 40 \left[ \frac{N}{m} \right] \\
 G_c &= m_c g = 15 \times 9.81 \approx 150 \left[ N \right] \\
 R_B &= G_c + G_p = 150 + 40 = 190 \left[ N \right] \\
 M_B &= G_c L + w L \frac{L}{2} = L \left( G_c + \frac{1}{2} G_p \right) = 1 \times \left( 150 + \frac{1}{2} \times 40 \right) = 170 \left[ N.m \right]
\end{align*}
\]

Calculation

\[
\begin{align*}
 V(x) &= -R_B + w x \\
 M(x) &= -M_B + R_B x - w \frac{x^2}{2}
\end{align*}
\]

Where \( x \) is from 0 to \( L \) (=1m).
Stress calculation

\[ \sigma_{\text{max}} = \frac{f \cdot M(x)_{\text{max}}}{I} \times \tau = \frac{f \cdot M(x)_{\text{max}}}{W} \]

Where \( W \) is the section modulus, \( f \) --is the safety factor. The maximum stress can be said as the yielding stress of the material. Material is AlMgSi0.5F25 and its strength parameters are:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{p02} )</td>
<td>200 [MPa]</td>
</tr>
<tr>
<td>( E )</td>
<td>70000 [MPa]</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.33 [-]</td>
</tr>
<tr>
<td>( G )</td>
<td>27000 [MPa]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2700 [kg/m(^3)]</td>
</tr>
</tbody>
</table>

Table 5 Material property

In order to find the needed amount of profiles to be assembled to handle the loading of the structure, we will find the section modulus \( W \) and compare it with the \( W \) of the given profile, which is 45 x 45 Alutec k&k.

\[ W = f \cdot \frac{M(x)_{\text{max}}}{R_{p02}} \]

\[ W = 2 \cdot \frac{170}{200 \cdot 10^6} = 1.7 \cdot 10^{-6} [m^3] = 1.7 [cm^3] \]

Compare it with the profile \( W \): \( \frac{7.72}{1.7} = 4.5 \) times it is bigger than we are required. It means that one profile is enough to handle the loading stress.

Deformation calculation:

We will use Mohr’s integral for that.
\[ v_t = \int_0^L \frac{M(x)}{EI} \cdot m(x) \, dx = 0.004520 \, [m] = 4.52 \, [mm] \]

where \( m(x) = x - 1 \) \hspace{1cm} (3.7)

Which is acceptable.

**Reserve factor**

Reserve factor “\( j \)” for chosen one profile will follow from next calculations:

\[ \sigma_{\text{max}} = f \cdot \frac{M(x)_{\text{max}}}{W} = 2 \cdot \frac{170}{7.72 \cdot 10^{-6}} = 22020725 \, [Pa] = 22.0207 \, [MPa] \]  \hspace{1cm} (3.8)

**Shear stress:**

\[ \tau_{\text{max}} = f \cdot \frac{V(x)_{\text{max}}}{A_p} = 2 \cdot \frac{190}{9.096 \cdot 10^{-4}} = 417786.3 \, [Pa] = 0.4178 \, [MPa] \]  \hspace{1cm} (3.9)

Where \( A_p \) is the cross-section area of the 45 x 45 profile.

Using HMH (von Mises) stress criterion we will find a reduced stress, also known as equivalent stress.

\[ \sigma_{\text{red}} = \sqrt{\sigma^2 + (\sqrt{3} \cdot \tau)} \]  \hspace{1cm} (3.10)

In our case it will be:

\[ \sigma_{\text{red}} = \sqrt{22.0207^2 + 3 \cdot 0.4178^2} = 22.0326 \, [MPa] \]  \hspace{1cm} (3.11)

Comparing this value with Yielding stress of the material we will find out what is the reserve factor for this kind of load. Comparison is providing by dividing yielding stress to the maximum applicable stress on the structure, and the value of the reserve factor must be bigger than one, \( j(=RF)>1 \).

\[ j = \frac{R_{\text{P0z}}}{\sigma_{\text{red}}} = \frac{200}{22.0326} \approx 9 \, [-] > 1 \]  \hspace{1cm} (3.12)

**3.1.2. Motion case**

In this case will be calculated strength criterion for the load that will be applied on the structure during the required motion, which is 10g load on the capsule and 94.6 rpm.

Now the applied forces have been changed relative to the previous loading case. Besides \( G_C \) and \( G_P \) (w) will appear:

Centrifugal force on capsule: \( C_C = m_c \omega^2 r = 15 \cdot 9.9045 \cdot 1 = 1471.5 \, [N] \)

Centrifugal force on profile (calculated by dividing arm for 10 parts, calculating for each part the C force, and then summarize them, since they are acting on the same axis on the same direction. We will neglect the force couple (moment) from the place where centrifugal force acts to the central axis of profile, since the deformation distance is very small) using formula (2.1) \( C_P = 135 \, [N] \).

We have to also consider the effect of the drag force appeared by the air resistance during motion.
Drag force on applied in the capsule have already been calculated in Table 2 and its value is: \( F_{Dc} = 14.6070 \, [N] \)

The calculation of the drag force on profile will be more complicated since circumferential velocity depends on radius \( (v = \omega r) \) and drag force according to (2.10) is function of area and square of circumferential velocity. This makes some complication the calculation procedure.

For easier and simplify, but relatively accurate calculation, I have decided to divide the arm for 10 equal parts, which is \( \Delta r = 1/10 = 0.1 \, m \) each part and I assume that velocity is constant for each part, which is the velocity appeared in the center of its part. For better understanding see Figure 3-4. Velocity is calculated by:

\[
v_i = \omega \left( (i - 1) + \frac{1}{2} \right) \Delta r \quad \ldots i = 1 \ldots 10
\]

\[
(3.13)
\]

Since \( \Delta r \) is constant, the cross-section area required for drag force calculation will be also constant.

\[
\Delta A = \Delta r \ast (45 \ast 10^{-3}) = 0.0045 \, [m^2]
\]

(3.14)

The drag force will be calculated by equation (2.10):

\[
F_{DPi} = \frac{1}{2} \rho v_i^2 \Delta A \ast c_{DP}
\]

(3.15)

Where \( c_{DP} = 2.5 \) is the drag coefficient for rectangular shapes, since there is no information provided about aerodynamic characteristics of this profile.

Provided calculation were tabulated, see Table 6.
Let us assume this drag force as the distributed force as function of distance r, →q(r). We need to find this function q(r). The easiest way to find analytical function of this force is to make a trend line with analytical function by special function of Microsoft Office Excel® software. This function is:

\[ y = 0.676x^2 \]

If we try to find analytically by ourselves we will obtain:

\[
k = \frac{1}{2} \cdot \rho \cdot (45 \cdot 10^{-3}) \cdot \omega^2 \cdot c_{DP} \cdot \frac{\Delta r}{L} = \frac{1}{2} \cdot 1.225 \cdot (45 \cdot 10^{-3}) \cdot 9.9045^2 \cdot 2.5 \cdot \frac{0.1}{1} = 0.67597 \left[ \frac{N}{m^3} \right]
\]

(3.17)

\[
q(r) = k \cdot r^2 = 0.67597 \cdot r^2
\]

(3.18)

In order to provide strength calculation we should know external applied forces and geometrical values. Geometrical values for this load case were taken from Figure 3-6.
If we assume that forces, that were mentioned above, acts in the center of gravity (centroid) and inclination angle is $84^\circ$, we can analytically calculate required distances $a$ and $b$.

$$a = (0.325 + 0.1) \cdot \sin 84^\circ \approx 0.4 \text{ [m]}$$
$$b = (0.325 + 0.1) \cdot \cos 84^\circ \approx 0.04 \text{ [m]}$$

(3.19)

**Calculation of strength**

**X-Y plane**

**Drag force:**

$$F_{DP_{resultant}} = \int_0^L q(r)dr = \int_0^L 0.676 \cdot r^2 dr = 0.676 \cdot \frac{L^3}{3} = 0.2253 \text{ [N]}$$

(3.20)

Place where drag force (3.20) is applied:

$$x_C = \frac{\int_0^L q(x) \cdot x \, dx}{\int_0^L q(x) \, dx} = \frac{0.676 \cdot \frac{L^4}{4}}{0.676 \cdot \frac{L^3}{3}} = \frac{3}{4}L$$

(3.21)

Using relative force transformation theory from analytical static mechanics the centrifugal arm will be loaded by next load:

Moment caused by capsule’s drag force:
\[ m_{FDc} = F_{DC} \alpha = 14.607 \times 0.4 = 5.843 \text{ [N.m]} \]  

(3.22)

The transformed drag force itself \( F_{DC} = 14.607 \text{ [N]} \).

**Reactions:**

\[
M_b^I = F_{DC} L + m_{FDc} + F_{DP} x_c \\
M_b^I = 14.607 \times 1 + 5.843 + 0.2253 \times \frac{3}{4} = 20.6 \text{ [N.m]} 
\]

(3.23)

\[
R_b^I = F_{DC} + F_{DP} = 14.607 + 0.2253 = 14.832 \text{ [N]} 
\]

(3.24)

**Shearing force:**

\[
V(x) = -R_b^I + k \frac{x^3}{3} = -14.832 + 0.676 \frac{x^3}{3}, \quad ...x = 0 \text{ to } L 
\]

(3.25)

**Bending moment:**

\[
M(x) = R_b^I x - k \frac{x^3}{3} - \frac{x^4}{4} - M_b^I = 14.832 \times x - \frac{0.676x^4}{12} - 20.6 \quad ...x = 0 \text{ to } L 
\]

(3.26)

![Graph of Shearing force and Bending moment](image)

*Figure 3-8 Distribution of Shear force and Bending moment*

Maximum Shearing force is \( V(x)_{\text{max}} = 14.8323 \text{ [N]} \)

Maximum Bending moment \( M(x)_{\text{max}} = 20.6000 \text{ [N.m]} \)

**Shear stress:**

\[
\tau_{\text{max}} = f \frac{V(x)_{\text{max}}}{A} = 2 \times \frac{14.8323}{9.096 \times 10^{-4}} = 32614.3543 \text{[Pa]} = 0.0326 \text{[MPa]} 
\]

(3.27)

**Bending stress:**

\[
\sigma_{z\text{max}} = f \frac{M(x)_{\text{max}}}{W} = 2 \times \frac{-20.6}{7.72 \times 10^{-6}} = -5341644.275 \text{ [Pa]} = -5.34166 \text{ [MPa]} 
\]

(3.28)
Z-Y Plane

Figure 3-9 Z-Y View. Torque loading

\[ T = F_D c \times b = 14.607 \times 0.04 = 0.5813 [N.m] \]  (3.29)

Shear stress from torque:

\[ \tau_T = f \times T \times \frac{t_{\text{max}}}{J} \]  (3.30)

Where \( t_{\text{max}} \) is the maximum thickness = 4 mm, J is polar moment of inertia

\[ J = I_X + I_Y = 1.738 \times 10^{-7} + 1.738 \times 10^{-7} = 3.476 \times 10^{-7} [m^4] \]  (3.31)

\[ \tau_T = 2 \times 0.5813 \times \frac{0.004}{3.476 \times 10^{-7}} = 13380.41 [Pa] = 0.013 [MPa] \]  (3.32)

X-Z Plane

External loads: \( m_{GC} = G_c \times a = 60 [N.m] \); \( m_{CC} = C_c \times b = 58.56789 [N.m] \)

Reactions:

\[ R_X = C_c + C_p = 1471.5 + 135 = 1606.5 [N] \]  (3.33)

\[ M_B^I = G_c \times L + G_p \times \frac{L}{2} - m_{CC} + m_{GC} = 171.4321 [N.m] \]

\[ R_B^I = M_B^I + G_p \times \frac{L}{2} - m_{GC} + m_{CC} = 192.8642 [N] \]  (3.34)

Shearing force:

\[ V(x) = -R_B^I + wx = 192.8642 + 40 \times x \ ... x = 0 to L \]  (3.35)

Bending Moment:

\[ M(x) = R_B^I \times x - M_B^I - w \times \frac{x^2}{2} \ ... x = 0 to L \]  (3.36)

Normal internal force (neglecting temperature expansion)

\[ N(x) = C = C_c + C_p = 1606.5 \]  (3.37)
Shearing force and Bending moment distribution

**Tensile stress:**

\[
\sigma_x = \frac{N(x)}{A} = \frac{f \times 1606.5}{9.096 \times 10^{-4}} = 4098483.2160 \text{ [Pa]} = 4.0984 \text{ [MPa]}
\]  

(3.38)

**Bending Stress:**

\[
\sigma_y = f \times \frac{M(x)_{\text{max}}}{W} = 2 \times \frac{-171.432}{7.72 \times 10^{-7}} = -44412463.99 \text{ [Pa]} = -44.4124 \text{ [MPa]}
\]  

(3.39)

**Shear stress:**

\[
\tau = f \times \frac{V(x)_{\text{max}}}{A} = 2 \times \frac{-192.8642}{9.096 \times 10^{-4}} = -42.40843268 \text{ [Pa]} = 0.4241 \text{ [MPa]}
\]  

(3.40)

**HMH stress criterion and reserve factor**

Von Misses (HMH) stress criterion reduce all stress components to one total normal stress.

\[
\sigma_{\text{red}} = \sqrt{\frac{1}{2} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6 \times (\tau_x^2 + \tau_y^2 + \tau_z^2) \right]} 
\]  

(3.41)

Specifically in our load case:

\[
\sigma_{\text{red}} = \sqrt{\frac{1}{2} \left[ (4.1 - (-44.41))^2 + (-44.41 - 5.34)^2 + (-5.34 - 4.1)^2 + 6 \times (0.42^2 + 0.013^2 + 0.033^2) \right]} 
\]  

\[
\sigma_{\text{red}} = 49.59 \approx 50 \text{ [MPa]}
\]  

(3.42)

Using relation (3.12) for finding reserve factor:

\[
j = \frac{R_{P02}}{\sigma_{\text{red}}} = \frac{200}{50} = 4 \text{ [-]} > 1
\]  

(3.43)
3.1.3. Summary

Calculations that were provided shows us that it is enough to use one 45 x 45 profile of 1m length, because it satisfies stress requirements for static case load (when centrifuge is not rotating) and motion case load (for maximum rotational speed). Even though using one 45x45 profile is enough for centrifugal arm, the machine, specifically the rotating head (=centrifuge head) will be designed in order to have an option to use 2-beam of 45x45 profile for one arm, preliminary design if this variant was shown on the Figure 2-9. Since one “centrifuge head” will support both variants, it is good to create an identifications for each variant of a centrifuge. One-beam variant will have a designation 1B, two-beam variant will have a designation 2B.
3.2. Belt transmission

Since the centrifugal machines will be driven by electric motor, it is sufficient and reliable to design a machine the way, that electric motor will transfer its torque and revolutions by belt transmission. It will help to avoid problems with rotation axis misalignment that can negatively affect work of the motor by damaging a motor bearing and decreasing efficient of the motor performance. For this purpose a timing belt (teeth belt) transmission was chosen, because this kind of a belt has better transmission characteristics due to lack of slipping.

Traditionally belt transmission consists of sprockets and belt. Considering that manufacturing abilities and budget are limited, the best solution here will be to choose desirable sprockets and belts that are already manufactured and can be easily found in the market.

The supplier of the belt transmission components is chosen Pikron s.r.o.

Sprockets

As the point to start from, we will use information about shaft diameter of the motor. Diameter of the electric motor ES-MH342200 [7] is 19 [mm], feather key length is \( s = 30 \) [mm], feather key width is \( b = 6 \) [mm]. Motor torque will be assumes constant in the level of 20 [Nm].

Sprocket for timing belt requires a special taper bush. From the web page of the supplier of a belt transmission, we can barely choose from:

<table>
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<tr>
<th>TB</th>
<th>1210</th>
<th>1610</th>
<th>2012</th>
</tr>
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<td>Dmax [mm]</td>
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<td>57</td>
<td>70</td>
</tr>
<tr>
<td>L [mm]</td>
<td>25.4</td>
<td>25.4</td>
<td>31.8</td>
</tr>
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<td>Clamping moment [Nm]</td>
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<td>20</td>
<td>31</td>
</tr>
<tr>
<td>Prize [CZK]</td>
<td>100</td>
<td>105</td>
<td>140</td>
</tr>
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*Table 7 Taper bushes parameters*

For the taper bushes that are mentioned in the Table 7 matches pulleys from the Table 8.
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<th>Belt width [mm]</th>
<th>Type</th>
<th>D_pitch [mm]</th>
<th>Pitch [mm]</th>
<th>Prize [CZK]</th>
<th>F_Te [N]</th>
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Table 8 Pulleys

Where $F_{Te}$ is the effective tension force which is calculated by:

$$F_{Te} = 2 \cdot \frac{T}{D_p} = 2 \cdot \frac{20}{D_p} [N]$$  \hspace{1cm} (3.44)

Gear ratio for the toothed pulleys can be calculated by next equation:

$$i = \frac{D_{p1}}{D_{p2}}$$  \hspace{1cm} (3.45)

For example, I marked in the Table 8 by blue colour pulleys for motor shaft (driver) and centrifuge shaft (driven), with the gear ratio of 1.33, by green line I marked fitted pulleys with the gear ratio 1.25.

Different gear ratios needs for different torque requirements. For example, for fixed driving torque of 20 N.m the bigger gear ratio will make bigger torque for driven pulley, with relation shown in equation (3.50)

Ratios that we can reach using items from the chosen supplier are 1.25, 1.375, 1.4, 1.6, 1.75, and 2. In order to compare them and to choose the optimum one, we will construct equilibrium diagram, where we will compare drag moment from rotation and the torque.
\[ T \ 1:i = T_{\text{output}} = T_{\text{input}}(= T \ 1:1) \ast i \ast \eta \]

where \( \eta = 0.95 \).

\[ (3.46) \]

\[ L = I \ast \omega; \ T = \frac{dL}{dt} \rightarrow dt = \frac{d(I\omega)}{dT} \rightarrow \Delta t = \frac{\Delta L}{\Delta T} \]

Equation (3.47) states, that the more excess of the torque we have the less time we need, as well as the less angular momentum makes less time. \( \Delta T \) is shown on the Figure 3-14. This figure shows, that to rotate shaft by transmitting force using a timing belt with efficiency \( \eta = 0.95 \) with the angular speed \( \omega_{\text{out}} = 1.58 \) rad/s, we need to reach the speed of the electric motor rotor \( \omega_{\text{in}} = 2.3645 \) rad/s, where the torque transmitted by the shaft to the centrifuge’s arms is approximately 28.4 N.m. The difference between this torque and resistive drag moment relative to the \( \omega_{\text{out}} \) angular speed will be our excess capacity of the torque that will influence the time to reach desired angular velocity.

Previously we stated that the required rotational speed is 1.58 [rps = 1/s] (see paragraph §2.2), with this speed our shaft has to rotate in order to reach desired acceleration of 10g. In this case rotor of an electric motor has to rotate with rotational speed of 1.58 * i. See table below.

<table>
<thead>
<tr>
<th>Ratio [-]</th>
<th>1.25</th>
<th>1.375</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor speed [1/s]</td>
<td>1.9704465</td>
<td>2.167491153</td>
<td>2.20690008</td>
<td>2.3645358</td>
<td>2.522172</td>
<td>2.7586251</td>
</tr>
</tbody>
</table>

Table 9 Table of desired speed

We have to reference these values of the rps when working with Figure 3-13. For gearing ratio \( i = 1.25 \) rotor speed should be 1.97 [1/s] and according to the Figure 3-13, maximum rotational speed of centrifuge arms that we can reach is about 1.437 [1/s], which does not satisfy desired task. From the diagram also we can read that maximum reachable speed is approximately the same as needed rotational speed, that is 1.58 [1/s] for ratio \( i = 1.5 \).

For example if we choose \( i = 1.5 \), then we can find what excess capacity of a torque, that will help us to decrease time to reach needed rotational speed.

Figure 3-13 Equilibrium of moments for various gear ratios, for two loaded arms
As the final decision, there were chosen next items, which combination will make gearing ratio $i = 1.6$:

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<thead>
<tr>
<th></th>
<th>Driving</th>
<th>Driven</th>
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</thead>
<tbody>
<tr>
<td>Taper bush</td>
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<tr>
<td>Sprocket</td>
<td>HTD/GT 40 – 8M – 20</td>
<td>HTD/GT 64 – 8M – 20</td>
</tr>
</tbody>
</table>

*Table 10 Transmission combination, $i = 1.6$*
3.3. Rotating Shaft

A shaft is an important part of a centrifuge machine that transfers rotation from a motor (or any other driver) to the centrifuge’s head and connects centrifuge’s head to a frame. Hence, a shaft should be designed not only to carry static load from the centrifuge’s head (arms, tested sample under the 10g load for symmetric and asymmetric case etc.), but also rotates at desired speed with required mass attached safely. That is why this chapter is divided for two parts: Static Calculation and Calculation of a Critical Speed.

3.3.1. Shaft. Static

For shaft, design there was draw schematic diagram of it with relatable loadings on it on the Figure 3-15. Preliminary shaft design is shown on the Figure 3-16, it just shows tendency of how is shaft needed to be design acceding to the constructions requirements, diameters can be changed.

Support reactions are:

\[ B \cdot a + M_B = 0 \]

\[ B = \frac{-M_B}{a} = -\frac{-600000}{322} = 1863.45 \, [N] = -A \]  \hspace{1cm} (3.48)
Bending moment distribution is shown on the Figure 3-17, where the maximum absolute value of the bending moment is equal to 600000 N.mm.

![Figure 3-17 Bending moment of the shaft](image)

If we apply for all section (1,2,3 and 4) this maximum bending moment, normal stresses on these sections are (using equation $\sigma = \frac{M}{Z}$, where $Z = \frac{l}{r} = \frac{\pi d^3}{32}$):

<table>
<thead>
<tr>
<th>i [-]</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ [MPa]</td>
<td>391.14</td>
<td>186.51</td>
<td>278.41</td>
<td>573.96</td>
</tr>
</tbody>
</table>

*Table 12 Normal stresses from bending moment in each section*

If we use the same material to produce the shaft as material that are Alutek profiles are made from, than its properties are:

| Rp02=$\sigma_y$ | 200 [MPa] |
| E             | 70000 [MPa] |
| $\mu$         | 0.33 [-]   |
| G             | 27000 [MPa] |
| $\rho$        | 2700 [kg/m3] |

*Table 13 AlMgSi0.5F25 general properties*

Then, from known material properties we will find allowable stress and comparing it with the values from Table 12, we can say if this design is safe or not.

$$\sigma_{all} = \frac{\sigma_y}{f} = \frac{200}{2} = 100 \text{ [MPa]}$$

(3.49)

It is clearly seen, that no values from the Table 12 are smaller than 100 MPa, i.e. that we have to redesign shaft diameters.

We also have to take into account the fact of the presence of shearing stress from the torque $T$, which is located between sections 1 and 2. To consider the affection of the shearing stress we will use HMH hypothesis to reduce normal and shearing stresses into one equivalent stress, and according to designing philosophy, this value has to be less than allowable stress $\sigma_{all}$. The value of the torque for the gear ratio $i = 1.6$ is 32 224 N.mm.

$$\sigma_{red} = \sqrt{\sigma^2 + 3 \cdot \tau^2} \leq \sigma_{all}$$

(3.50)

$$\sigma_{all}^2 \geq \sigma^2 + 3 \cdot \tau^2 = \frac{M_B^2}{Z^2} + \frac{T^2}{W^2}, \text{ where } Z = \frac{\pi d^3}{32}; \ W = \frac{\pi d^3}{16},$$

(3.51)
\[ d \geq \sqrt[6]{\frac{(32M_B)^2 + (16T)^2}{(\pi\sigma_{all})^2}} \]

\[ d \geq 39.39 \text{[mm]}, \text{and we choose } d = 45 \text{[mm]} \]

This diameter is minimum required one, so it is referred to the \( d_4 \), but according to the chosen design, it is minimum dimeter of the shaft, so we will reference this value as the minimum one.

Since, we have change the diameter, we have to recalculate bearing for the shaft at section 4, tapper bush and sprocket for shaft’s section 3 and for both sections recalculate and re-choose feather keys.

**Bearings**

Using very useful tool for bearing life calculation from bearing producer itself (SKF Engineering tool: http://www.skf.com/group/knowledge-centre/engineering-tools/skfbearingcalculator.html), we can calculate, if specific bearing satisfies given conditions. We choose bearing SKF with designation 16009.
Figure 3-18 Bearing life calculation

Analogical bearing to SKF 16009 is CSN 024630 SKF. To be sure if provided bearing calculation using SKF bearing calculator tool are correct, we will provide calculation of the bearing life using Autodesk Inventor® 2016 Professional, special bearing designer tool. Results are shown in Table 14. Results are very similar, so we can surely state that the CSN 024640 SKF bearing with inner diameter of 45 mm and width 10 mm is sufficient to fulfill needed requirements.

<table>
<thead>
<tr>
<th>Results</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic rating life</td>
<td>$L_{10}$</td>
</tr>
<tr>
<td>Adjusted rating life</td>
<td>$L_{10a}$</td>
</tr>
<tr>
<td>Calculated static safety factor</td>
<td>$S_{ok}$</td>
</tr>
<tr>
<td>Power lost by friction</td>
<td>$P_z$</td>
</tr>
<tr>
<td>Necessary minimum load</td>
<td>$F_{min}$</td>
</tr>
<tr>
<td>Static equivalent load</td>
<td>$P_0$</td>
</tr>
<tr>
<td>Dynamic equivalent load</td>
<td>$P$</td>
</tr>
<tr>
<td>Over-revolving factor</td>
<td>$k_e$</td>
</tr>
<tr>
<td>Equivalent speed</td>
<td>$n_e$</td>
</tr>
<tr>
<td>Strength Check</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Table 14 Autodesk Inventor Bearing calculation for CSN 024630 SKF
Feather Key

![Diagram of shaft and hub with feather key](image)

*Figure 3-19 Loads on the shaft and hub with feather key. [9, p. 38]*

Contact pressure is

\[ p = \frac{4M_k}{d \cdot h \cdot l_a} \]  

(3.52)

Where \( M_k \) is the torque applied to a shaft, which is in our case is \( T_2 \). \( l_a = l - b \) is the effective feather key length.

*Centrifuge head*

In the section 4, for the shaft diameter 45 mm, according to standards [10] we will choose feather key CSN 02 2562 A 14 x 9 x l.

Suitable, \( l \) can be determined using equation (3.52), if we assume \( p \) as allowable pressure \( p_{alt} \approx \tau_{alt} = 70 \text{ [MPa]} \). Then \( l_{a_{min}} = 15.6 \text{ [mm]} \). To be suitable to the standard we call \( l_a = 26 \text{ [mm]} \), so that \( l = 40 \text{ [mm]} \). Therefore, needed feather key is CSN 02 2562A 14 x 9 x 40.

Contact pressure is:

\[ p = \frac{4T}{d \cdot h \cdot l_a} = \frac{4 \cdot 32224}{45 \cdot 9 \cdot (40 - 14)} = 12.2 \text{ [MPa]} \]  

(3.53)

Value of contact pressure of 12.2 MPa is several times less, than allowable one. Required safety \( j > 5 \).

In the section 4, for the shaft diameter 45 mm, according to standards [10] we will choose feather key CSN 02 2562 A 14 x 9 x 40.
**Tupper bush**

The taper bush is manufactured for usage with a feather key and the shaft diameter calculation should take into account this fact.

For the chosen diameter of the shaft of 28 mm, desirable taper bush TB2012, according to manufacturer, has a pocket with the width of $b = 14$mm, see Figure 3-20.

From the Czech standard tables, suitable feather key can be **CSN 02 2562A 14 x 9 x 40**.

Contact pressure is:

$$p = \frac{4T}{d \cdot h \cdot l_a} = \frac{4 \cdot 32224}{50 \cdot 9 \cdot (40 - 14)} = 11.016 \text{ [MPa]}$$

(3.54)

Value of contact pressure of 11.016 MPa is several times less, than allowable one. Required safety $j > 4$.

**Conclusion of the shaft static design**

Section 2 of the shaft will have diameter $d_2 = 55$ mm, so it will refer as the pulley support. Shaft diameter of the section $I$, will be 45 mm, which was calculated for maximum bending moment of absolute value 600 000 N.mm. For this diameter is suitable thrust bearing and its housing – UCF209 by GISS.

Final table of the shaft dimensions is:

<table>
<thead>
<tr>
<th>Section</th>
<th>Length</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>45</td>
</tr>
</tbody>
</table>

*Table 15 Modified shaft dimensions*
3.3.2. Critical speed of the shaft

Critical angular velocity of a rotating shaft is the angular velocity that will reach the value of Eigen frequency (Eigen angular velocity) of the shaft with all masses attached. If the angular velocity of the shaft will be the same value as the Eigen velocity of the working shaft will appear the phenomena called Resonant, which means that the deformation at some point of the shaft will be infinity big value that, obviously, will cause a failure of a part.

In order to find Eigen frequencies of the shaft, we are going to solve this problem with consideration that the shaft is massless and ideal, i.e. has no eccentricity \( e = 0 \). The shaft has two masses attached to it, first one is the pulley from the belt transmission, second is the combination of several parts (arms, working capsule or another test examples, fasteners, bearings, lugs etc.), assumed as one solid. For the more precise calculation, we will respect non-homogeneity of shaft’s cross-section (resp. second polar moment of area) and its character of the support (resp. boundary conditions). See Figure 3-21.

![Figure 3-21 Simplified diagram of the shaft for Eigen frequency calculation. I II III and IV represents each section. Black rectangles represents mass wheels](image)

### 3.3.2.1. Massless shaft, zero eccentricity and one mass attached

If there is no eccentricity \( e = 0 \) mm, then the location of the centrifugal force \( O_{I,II} \) from the attached mass exists on the shaft’s axis of rotation.

First, we will find Eigen frequency for the first mass mounted to the shaft (mass on the section I). If we assume that the material of the shaft will have linear behavior according and will behave like a spring, then the next equation will be valid

\[
F = k \cdot y
\]  \hspace{1cm} (3.55)

Where \( k \) is the stiffness of the shaft at certain point, \( y \) is the deflection and \( F \) is the force needed to provide on the shaft the deflection \( y \) at certain point.

If we apply these assumptions and knowing equation of the centrifugal force, then we will obtain next:

\[
m_i \cdot y \cdot \omega^2 = k \cdot y \]  \hspace{1cm} [11] \hspace{1cm} (3.56)

The deflection \( y \) as a function of angular velocity is then:

\[
y = \frac{0}{m_i \omega^2 - k} = \frac{0}{\omega^2 - \frac{k}{m_i}}
\]  \hspace{1cm} (3.57)
From the equation (3.57), we can say that the deflection \( y \) will reach any value (infinity) as soon as the value of angular will tend to the value of \( \frac{k}{m} \). From here, we can state, that \( y(\Omega) \to \pm \infty \), where \( \Omega \) is:

\[
\Omega^2 = \frac{k}{m_l} \rightarrow \Omega = \sqrt{\frac{k}{m_l}} \quad (3.58)
\]

Same method is valid for the mass attached on the section II.

**Figure 3-22 Deflection \( y \) as a function of angular velocity, \( e=0 \)**

### 3.3.2.2. Massless shaft, with non-zero eccentricity and one mass attached

For the case of non-zero eccentricity \( (e \neq 0) \), the equation (3.56) transforms onto the next form:

\[
m_l (y + e) \omega^2 = k \cdot y \quad (3.59)
\]

The deflection as function of angular velocity, now, has this form

\[
m_l y \omega^2 + me \omega^2 - ky = 0
\]

\[
y(m \omega^2 - k) = -me \omega^2
\]

\[
y = \frac{-me \omega^2}{m \omega^2 - k} = \frac{me \omega^2}{k - m \omega^2} = \frac{e \omega^2}{\frac{k}{m} - \omega^2} \quad (3.60)
\]

Again, as we mentioned for the equation (3.57), the deflection \( y \) will tend to be infinity when angular velocity \( \omega \) will tend to the value of \( \frac{k}{m} \), but now significant difference is in numerator. In case of (3.57), we had 0, which means, that the deflection will be zero at any angular velocity except, when it reaches the value of \( \Omega \). For the non-zero eccentricity numerator of the last equation in (3.60) is non-zero value, but the function of angular velocity and eccentricity. The presence of eccentricity represents more realistic rotor behavior.

If we provide the limit of the equation (3.60), where angular velocity tends to infinity we will find that the deflection will be the same as eccentricity

\[
\lim_{\omega \to \infty} y(\omega) = \frac{e \omega^2}{\frac{k}{m} - \omega^2} = \frac{e}{\frac{k}{m} - \omega^2} = \frac{e}{0 - 1} = -e \quad (3.61)
\]
Same method is valid for the mass attached on the section II.

\[
\begin{align*}
\sum_{i=1}^{2} \frac{m_i}{k_{ii}} y_i + \sum_{i=1}^{2} \frac{m_i}{k_{ij}} y_j + \sum_{i=1}^{2} \frac{m_i}{k_{ji}} y_i + \sum_{i=1}^{2} \frac{m_i}{k_{jj}} y_j &= O_1 + O_2 \\
\end{align*}
\]

Transform to another form

\[
\begin{align*}
y_1 &= \frac{O_1}{k_{11}} + \frac{O_2}{k_{12}}; \quad y_2 = \frac{O_1}{k_{21}} + \frac{O_2}{k_{22}} \\
\end{align*}
\]

Since centrifugal forces \(O_1\) and \(O_2\) are also functions of the deflection \(y_1\) and \(y_2\) respectively, the dependency of the deflection \(y\) from angular velocity \(\omega\) can be determined by following equations:

\[
\begin{align*}
y_1 &= \frac{m_1 (y_1 + e_1) \omega^2}{k_{11}} + \frac{m_2 (y_2 + e_2) \omega^2}{k_{12}} \\
y_2 &= \frac{m_1 (y_1 + e_1) \omega^2}{k_{21}} + \frac{m_2 (y_2 + e_2) \omega^2}{k_{22}} \\
\end{align*}
\]

\[
\begin{align*}
y_1 &= \frac{m_1 y_1 \omega^2}{k_{11}} - \frac{m_2 y_2 \omega^2}{k_{12}} = \frac{m_1 e_1 \omega^2}{k_{11}} + \frac{m_2 e_2 \omega^2}{k_{12}} \\
y_2 &= \frac{m_1 y_1 \omega^2}{k_{21}} - \frac{m_2 y_2 \omega^2}{k_{22}} = \frac{m_1 e_1 \omega^2}{k_{21}} + \frac{m_2 e_2 \omega^2}{k_{22}} \\
\end{align*}
\]

Let’s say that \(\frac{m_1 e_1 \omega^2}{k_{11}} + \frac{m_2 e_2 \omega^2}{k_{12}} = S_1\) and \(\frac{m_1 e_1 \omega^2}{k_{21}} + \frac{m_2 e_2 \omega^2}{k_{22}} = S_2\)

\[
(3.63)
\]
\[
y_1 \left(1 - \frac{m_1 \omega^2}{k_{11}}\right) - y_2 \cdot \frac{m_2 \omega^2}{k_{12}} = S_1
\]
\[
y_2 \left(1 - \frac{m_2 \omega^2}{k_{22}}\right) - y_1 \cdot \frac{m_1 \omega^2}{k_{21}} = S_2
\]

Let’s say that \(1 - \frac{m_1 \omega^2}{k_{11}} = a_{11}; \frac{m_2 \omega^2}{k_{12}} = a_{12}; \frac{m_1 \omega^2}{k_{21}} = a_{21}; \left(1 - \frac{m_2 \omega^2}{k_{22}}\right) = a_{22}\)

\[
y_1 a_{11} - y_2 a_{12} = S_1
\]
\[
y_1 a_{21} + y_2 a_{22} = S_2 \rightarrow -/ - \ast (-1)
\]

\[
y_1 a_{11} - y_2 a_{12} = S_1
\]
\[
y_1 a_{21} - y_2 a_{22} = -S_2
\]

We can create now system of linear algebraic evacuation in matrix form:

\[
\begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
S_1 \\
S_2
\end{bmatrix}
\]
If \[
\begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix} = A,
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = y \text{ and } \begin{bmatrix}
S_1 \\
S_2
\end{bmatrix} = S, \text{ then}
\]
\[
y = A^{-1} \ast S
\] (3.64)

In order to find Eigen frequencies, we can assume that the matrix S, where eccentricities are, is equal to 0.
Then the equation (3.64) has the form:

\[
\begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (3.65)

This equation has two solutions, first it is when deflections are zero, which is not interesting for us, because it gives us nothing, and when system is linearly dependent and deflections y can be anything. Which means matrix A is singular:

\[
\det \begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix} = 0
\]
\[
a_{11} \ast (-a_{22}) + a_{12} \ast a_{21} = 0
\] (3.66)

Values for the angular velocity that we will obtain from the equation (3.66) are Eigen values, respectively, Eigen frequencies, \(\Omega_1\) and \(\Omega_2\).
3.3.2.4. Appyling calculation

In order to find Eigen frequency $\Omega_{I}$ or $\Omega_{II}$ it is significantly important to calculate shaft’s bending stiffness in the place of centrifugal force from the attached mass.

![Figure 3-24 Shaft, calculation model](image)

**Section I – Mass I – massless shaft**

To find deflection at the place where centrifugal force $O_1$ is applied, we have to find reactions in supports (bearings) and solve Mohr’s integral. However, the way that shaft is supported is so called *statically indeterminate* (3 unknowns for 2 equilibrium equations), so we have to apply deformation conditions in order to find missing equation. The needed deformation condition is that the vertical deflection $y_B$, at the position of the support B is equal to zero, $y(l_1) \approx y_B = 0$ mm. Knowing that boundary condition we can state, that:

$$y(l_1) = 0 = \int_0^{l_3} \frac{M(x)}{EI_x} m_i(x) dx$$  \hspace{1cm} (3.67)

Where $m_i(x)$ is the bending from the unit load applied at $l_1$.

Since our shaft does not have constant cross-section along its length and not distributes within x by some specific function, we can rewrite Mohr’s integral (3.67) as the sum of integrals of each section.

$$0 = \int_0^{l_1} \frac{M(x)}{EI_1} m_i(x) dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_i(x) dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_i(x) dx$$  \hspace{1cm} (3.68)

Where, if we say $O_1 = F_1$ and using the method of section we can write:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$[0, l_1]$</th>
<th>$[l_1, l_2]$</th>
<th>$[l_2, l_3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(x)$</td>
<td>$-F_1 \times x$</td>
<td>$-F_1 \times x + R_{B_1} \times (x - l_1)$</td>
<td></td>
</tr>
<tr>
<td>$m_i(x)$</td>
<td>0</td>
<td>$-(x - l_1)$</td>
<td></td>
</tr>
</tbody>
</table>

$\pi d_i^4 / 64$

|  | $\pi d_1^4 / 64$ | $\pi d_2^4 / 64$ | $\pi d_3^4 / 64$ |

*Table 16 Mohr’s integral values for the shaft at loading $O_1$, for unit load at support B*

By solving equation (3.68) we find the reaction force $R_{B_1}$. Using software MATLAB® we can fast and easy find the needed solution. The script with the solution can be found in the attachments.
We assumed that the centrifugal force $O_1$ approaches to the free end of the shaft’s section I, in order to be on the safety side of the problem. So the stiffness will be at this place will be $k_{11} = \frac{o_{1}}{y_{11}} = \frac{F_{1}}{y_{11}}$, where the first index shows the place and second index shows the force applied, so the stiffness $k_{11}$ is saying that this stiffness is the stiffness of the shaft at the place 1 (where the first force is applied) from the force $O_1$. So the $k_{12}$ will be the stiffness at the place 1, that will be caused by the force $O_2$.

To find stiffness $k_{11}$ again, we have to use Mohr’s integral to find the deflection $y_{11}$, with applying unit load at the same place as centrifugal force $O_1$. Again using an advantage of the MATLAB®, we can solve the integral equation (3.70), where all needed values are written in the

$$y_{11} = \int_{0}^{l_1} \frac{M(x)}{EI_1} m_1(x) dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_1(x) dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_1(x) dx$$

(3.70)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$[0, l_1]$</th>
<th>$[l_1, l_2]$</th>
<th>$[l_2, l_3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(x)$</td>
<td>$-F_1 \times x$</td>
<td>$-F_1 \times x + R_{B1} \times (x - l_1)$</td>
<td></td>
</tr>
<tr>
<td>$m_1(x)$</td>
<td>$-1 \times x$</td>
<td>$-1 \times x + r_{B1} \times (x - l_1)$</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$\pi d_1^4$</td>
<td>$\pi d_2^4$</td>
<td>$\pi d_3^4$</td>
</tr>
<tr>
<td></td>
<td>$\frac{64}{64}$</td>
<td>$\frac{64}{64}$</td>
<td>$\frac{64}{64}$</td>
</tr>
</tbody>
</table>

Table 17 Mohr’s integral values for the shaft at loading $O_1$, for unit load at the place of $O_1$

Where $r_{B1}$ is the reaction at the support B from the unit load at the place of force $O_1 = F_1$. Symbolic result for $y_{11}$ you can find in the attachments.

From the equation (3.55) we can find the stiffness $k_{11}$, which was solved by MATLAB® with the using values for our shaft. Symbolic result, again, in the attachments.

$$k_{11} = \frac{F_{1}}{y_{11}} = 1.9479329E + 04 = 19\,479.329 \left[ \frac{N}{mm} \right]$$

(3.71)

Preliminary mass was read from the CAD file, assuming geometry and density for all parts involved at the place 1 according to the Figure 3-24. For “One Beam” (1B) modification the mass is $m_{10} = 40$ kg and we will take a reserve 10 kg for future sensors and for additional masses if appears, so the 1B mass is $m_1 = 50$ kg. “Two Beam” (2B) modification has mass of $M_{10} = 50$ kg and with extra mass 2B is $M_1 = 60$ kg. Using equation (3.58) we obtain:
<table>
<thead>
<tr>
<th>Modification</th>
<th>1B</th>
<th>2B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigen angular velocity, ( \Omega_{01} ) [rad/s]</td>
<td>19.73</td>
<td>18.018</td>
</tr>
<tr>
<td>Eigen frequency, ( f_{01} ) [1/s]</td>
<td>3.14</td>
<td>2.86</td>
</tr>
<tr>
<td>Eigen angular velocity, ( n_{01} ) [rpm]</td>
<td>188.40</td>
<td>171.98</td>
</tr>
</tbody>
</table>

*Table 18 Eigen values for case 1*

To reach 10g acceleration for the chosen 1m arm requires 9.905 [rad/s] or 94.582 [rpm], or 1.577 [rps=1/s]. In order to be conservative let us assume that maximum required speed is 2 [1/s] and the related Eigen value should not be lower than this value. Results from Table 18 shows that we are okay.

If we add the effect of eccentricity, then with the help of equation (3.60) it is possible to construct the graph \( y(\omega) \) for eccentricities \( e \in [0.1; 1] \)

![Graph](image)

*Figure 3-25 y(\omega) diagram for \( M_1 \)*

The maximum allowable deflection can be calculated from the allowable stress (bending stress).

\[
\sigma_{\text{alt}} = \frac{M_{\text{max}} (F_{\text{alt}})}{I} \Rightarrow F_{\text{alt}} = f(\sigma_{\text{alt}}) \Rightarrow y_{11,\text{alt}} = \frac{F_{\text{alt}}}{k_{11}}
\] (3.72)

In order to clearly know where is the maximum bending moment along the shaft, let’s say that the \( F = 1 \) N so we can construct the diagram for the bending moment distribution \( M(x) \). Using information from the Table 16 and equation (3.69), we can construct next diagram.
From the Figure 3-26 it is seen that the maximum absolute value of the bending moment is situated when $x = 100 \text{ mm}$ or $x = l_1$. In that case the maximum bending moment is $M_{\text{max \ all}} = F_{\text{all}} \cdot l_1$. Then follows:

$$\sigma_{\text{all}} = \frac{F_{\text{all}} \cdot l_1}{d_1 \cdot l_1} \Rightarrow F_{\text{all}} = \sigma_{\text{all}} \cdot \frac{l_1}{d_1} \cdot \frac{1}{l_1} = 100 \cdot \frac{\pi \cdot 45^3}{32} \cdot \frac{1}{100} = 8946.176 [N]$$

(3.73)

$$y_{11\text{all}} = \frac{F_{\text{all}}}{k_{11}} = \frac{8946.176}{19479.329} = 0.459 [mm]$$

(3.74)

Therefore, the Figure 3-25 will finally look as follows.

**Section II – Mass 2 – massless shaft**

For the section II as well as for the first section we need to find reaction force in the bearing and then use it to find stiffness and after the Eigen frequency. Again, to find reaction force in the support B, we need to apply there a boundary condition that is saying that the vertical deflection at the support B is equal to zero. General equations to find the reaction at the support B for the section II will be same as for the section I, i.e. equation (3.67). Equation (3.68) will have another view, since the force $O_2$ is assumed to be from the sprocket that has $32 \text{ mm}$ width and assuming that the centrifugal force $O_2$ situates exactly in the center, hence its position is in $l_2 \cdot 32/2 \text{ mm} = l_{1.5}$. 
\[ y_b = 0 = \int_0^{l_1} \frac{M(x)}{EI_1} m_1(x) dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_2(x) dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_3(x) dx \]  

(3.75)

Values for the equation (3.75) are mentioned in the Table 19.

<table>
<thead>
<tr>
<th>x</th>
<th>[0; l_1]</th>
<th>[l_1; l_1.5]</th>
<th>[l_1.5; l_2]</th>
<th>[l_2; l_3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(x)</td>
<td>0</td>
<td>( R_{B_2} \cdot (x - l_1) )</td>
<td>( R_{B_2} \cdot (x - l_1) - F_2 \cdot (x - l_1.5) )</td>
<td></td>
</tr>
<tr>
<td>m(x)</td>
<td>0</td>
<td>(-1 \cdot (x - l_1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>( \frac{\pi d_1^4}{64} )</td>
<td>( \frac{\pi d_2^4}{64} )</td>
<td>( \frac{\pi d_3^4}{64} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 19 Integral values for the shaft at loading \( O_2 = F_2 \), for unit load at support B (apply for (3.75))

Using MATLAB we can solve this integral equation and find needed value \( R_{B_2} \) (the symbolic result for it you can find in attachments).

Finding a deflection at the place where force \( O_2 = F_2 \) appeared we can find a related stiffness \( k_{22} = \frac{F_2}{y_{22}} \).

\[ y_{22} = \int_0^{l_1} \frac{M(x)}{EI_1} m_1(x) dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_2(x) dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_3(x) dx \]  

(3.76)

Where all functions are:

<table>
<thead>
<tr>
<th>x</th>
<th>[0; l_1]</th>
<th>[l_1; l_1.5]</th>
<th>[l_1.5; l_2]</th>
<th>[l_2; l_3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(x)</td>
<td>0</td>
<td>( R_{B_2} \cdot (x - l_1) )</td>
<td>( R_{B_2} \cdot (x - l_1) - F_2 \cdot (x - l_1.5) )</td>
<td></td>
</tr>
<tr>
<td>m(x)</td>
<td>0</td>
<td>( r_{B_2} \cdot (x - l_1) )</td>
<td>( r_{B_2} \cdot (x - l_1) - 1 \cdot (x - l_1.5) )</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>( \frac{\pi d_1^4}{64} )</td>
<td>( \frac{\pi d_2^4}{64} )</td>
<td>( \frac{\pi d_3^4}{64} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 20 Integral values for equation (3.76)

Where \( r_{B_2} \) is the reaction at the support B from the unit load at the place of force \( F_2 \). Symbolic result for \( y_{22} \) you can find in the attachments. The stiffness is then:

\[ k_{22} = \frac{F_2}{y_{22}} = 2.82637293E + 05 = 282,637,293 \left[ \frac{N}{mm} \right] \]  

(3.77)

Mass of the complete sprocket is 3.45 kg, but to be on the conservative side let us assume that this mass will be \( M_2 = 4 \) kg. Using equation (3.58) we find Eigen values.

<table>
<thead>
<tr>
<th>Eigen angular velocity, ( \Omega_{02} ) [rad/s]</th>
<th>265.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigen frequency, ( f_{02} ) [1/s]</td>
<td>42.3</td>
</tr>
<tr>
<td>Eigen angular velocity, ( n_{02} ) [rpm]</td>
<td>2538.2</td>
</tr>
</tbody>
</table>

Table 21 Eigen values for mass \( M_2 \)
Results from the Table 21 shows that they are far beyond than required speed for 10g acceleration, which is I remind from conservative point of view is 120 [rpm] or 2 [1/s].

If we add the effect of eccentricity, then with the help of equation (3.60) it is possible to construct the graph $y(\omega)$ for eccentricities $e \in [0.1; 1]$. See Figure 3-28.

MATLAB® shows that deflection for the angular velocity $2 \times 2\pi = 12.566 \frac{[\text{rad}]}{s}$ is 2.04E-04, so for my point of view it is very small deflection, there is no point to solve the shaft for the allowable deflection for mass $M_2$.

Shaft with both mass attached – massless shaft

To find Eigen values for this case we have to calculate equation (3.66), which is determinant of matrix that contains masses, stiffness and angular velocities. This matrix was mentioned in (3.63) and (3.64). Stiffness $k_{11}$ and $k_{22}$ were found in previous parts of this paragraph. Our task now is to find stiffness $k_{12}$ and $k_{21}$ which for isotropic material should be the same values, $k_{12} = k_{21}$. To find stiffness $k_{12}$ we need to find deflection caused by force $F_2$ at the place of applied force $F_1$, which is $y_{12}$. $k_{12} = \frac{F_2}{y_{12}}$. To find deformation $y_{12}$, will be used general equation (3.76), but will contain another functions for bending moment from an applied force and bending moment from a unit load. Table for this integral equation is written below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$[0;l_1]$</th>
<th>$[l_1;l_{1,5}]$</th>
<th>$[l_{1,5};l_2]$</th>
<th>$[l_2;l_3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(x)$</td>
<td>0</td>
<td>$R_{B_1}(x - l_1)$</td>
<td>$R_{B_2}(x - l_1) - F_2 \ast (x - l_{1,5})$</td>
<td></td>
</tr>
<tr>
<td>$m(x)$</td>
<td>$-x$</td>
<td>$-x + \tau_{B_1}(x - l_1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$\frac{\pi d_1^4}{64}$</td>
<td>$\frac{\pi d_2^4}{64}$</td>
<td>$\frac{\pi d_3^4}{64}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 22 Integral values to solve (3.76) to find $y_{12}$

![Graph](image_url)
To find deflection at the place where \( F_2 \) applies but from the force \( F_1 \), \( y_{21} \) we need to use again general integral equation (3.76) and put there next values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>([0; l_1])</th>
<th>([l_1; l_{1.5}])</th>
<th>([l_{1.5}; l_2])</th>
<th>([l_2; l_3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M(x) )</td>
<td>(-F_1 \cdot x)</td>
<td>(-F_1 \cdot x + R_{B_1}(x - l_1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m(x) )</td>
<td>0</td>
<td>(a_{B_2}(x - l_1))</td>
<td>(a_{B_2}(x - l_1) - 1 \cdot (x - l_{1.5}))</td>
<td></td>
</tr>
<tr>
<td>( I )</td>
<td>(\frac{\pi d_1^4}{64})</td>
<td>(\frac{\pi d_2^4}{64})</td>
<td>(\frac{\pi d_3^4}{64})</td>
<td></td>
</tr>
</tbody>
</table>

*Table 23 Integral values to solve (3.76) to find \( y_{21} \)*

Dividing each force by related deflection, we can find all needed stiffness. Constructing something like a stiffness matrix specifically for a designed shaft, will look like:

\[
K = \begin{bmatrix}
 k_{11} & k_{12} \\
 k_{21} & k_{22}
\end{bmatrix} = \begin{bmatrix}
 19479.329 & -124295.99 \\
 -124295.99 & 282637.293
\end{bmatrix} \text{[N/mm]}
\] (3.78)

This matrix confirms the theory that \( k_{12} = k_{21} \).

Solving the equation (3.66) will give us next:

\[
\det \begin{bmatrix}
 a_{11} & -a_{12} \\
 a_{21} & -a_{22}
\end{bmatrix} = \det \begin{bmatrix}
 1 - m_2 \omega^2 & -m_2 \omega^2 \\
 m_2 \omega^2 & 1 - m_2 \omega^2
\end{bmatrix} = \det \begin{bmatrix}
 1 - \frac{\omega^2}{\Omega_0} & -\frac{m_2 \omega^2}{k_{12}} \\
 \frac{m_2 \omega^2}{k_{21}} & 1 - \frac{\omega^2}{\Omega_0}
\end{bmatrix} = 0
\]

\[
\Omega = \begin{bmatrix}
 \Omega_1 \\
 \Omega_2
\end{bmatrix} = \begin{bmatrix}
 18 \\
 331.6
\end{bmatrix} \text{[rad/s]} = \begin{bmatrix}
 171.8 \\
 3166.5
\end{bmatrix} \text{rpm}
\] (3.79)

Example in (3.79) was calculated for two beams modification (2B). Generally, we can exceed the rotational speed \( \Omega_1 \), if it will be done quickly, but specifically for the designed centrifugal machine we don’t need to reach angular speed more than 120 [rpm] = 12.556 [rad/s]. From this point of view we are satisfied with the designed shaft.

Adding eccentricity \( e \in [0.05; 1] \) to the shaft the function \( y(\omega) \) which is (3.64) the dependency for the deflection \( y_1 \) will be then as it shows Figure 3-29. For the \( y_2 \) see Figure 3-30.
To find allowable deflection, first we need to find a maximal bending moment on the shaft, preliminary to find a place of it and its value we can use several assumptions in order to simplify a work. We know that $M_1 = 60 \text{ kg}$ and $M_2 = 4 \text{ kg}$, which makes $M_2 = M_1 / 15$ and assuming that $y_2 = y_1$ then we can write next:

$$O_1 = M_1 \cdot y_1 \cdot \omega^2$$

$$O_2 = M_2 \cdot y_2 \cdot \omega^2 = \frac{M_1}{15} \cdot y_1 \cdot \omega^2 = \frac{O_1}{15}$$

(3.80)
If we say that centrifugal force $O_1 = 1 \text{ N}$, then $O_2 = 1/15 \text{ N}$, then taking into account that we used a superposition method which will make reaction at the support $B$ a sum of reactions calculated for force $O_1$ and $O_2$ separately.

$$R_B = R_{B1} + R_{B2}$$  \hspace{1cm} (3.81)

Using advantage of the software MATLAB®, we can construct a preliminary bending on the shaft for both forces applied.

![Figure 3-31 Bending distribution on the shaft with $O_1 = 1\text{ N}$ and $O_2 = O_1/15$](image)

From the Figure 3-31 we can clearly see that the maximum moment is situated at the shaft position $l_1$, which is a product of the centrifugal force $O_1$.

$$M_{\text{max}} = O_1 \cdot l_1$$

$$\sigma_{\text{all}} = \frac{(O_{\text{all}} \cdot l_1)}{I_1}$$

$$O_{\text{all}} = \frac{l_1}{r_1} \cdot \frac{1}{r_1} \cdot \sigma_{\text{all}}$$

$$O_{\text{all}} = \frac{201288.96}{45} \cdot \frac{1}{100} \cdot 100 = 8946.17 \text{ [N]}$$

$$y_{1,\text{all}} = \frac{O_{\text{all}}}{k_{11}} = \frac{8946.17}{19479.329} = 0.46 \text{ [mm]}$$  \hspace{1cm} (3.82)
Figure 3-32 $y_1(\omega)$ with showing allowable deflection. For both mass attached.

Adding the mass of a shaft

Mass of the shaft was read from the 3D CAD model and it is $M_S = 2.16$ kg. It is made from aluminum and has density $2700$ kg/m$^3$. In order to simplify the task, I am not going to take a shaft mass as a distributed thing. I am going to take it as a concentrated mass in the “dangerous” points, which are the places of a applied centrifugal forces $O_1$ and $O_2$. To be on the safety side of the problem I will assume shaft as a 3 kg and will add this mass to each attached mass. It will make $M_1 = 60+3=63$ kg and $M_2 =4+3=7$ kg. Results are:

$$\Omega_s = \begin{bmatrix} \Omega_{s_1} \\ \Omega_{s_2} \end{bmatrix} = \begin{bmatrix} 17.5 \\ 250.8 \end{bmatrix} \text{ [rad/s]}$$

(3.83)

Compare these results with results we’ve got in (3.79) we can say that the difference in $\Omega_1$ is 0.5 rad/s which is 4.77 rpm. The velocity, in this case, that we should not reach is 167 rpm. It satisfies our requirement maximum speed should be bigger than 120 rpm.
3.4. Motor mounting

We chose that we would drive the centrifugal machine using timing belt transmission and every belt transmission requires belt pre-tension, this can be two methods.

First allowing the pulley distance to be varied.

![Figure 3-33 Center distance adjusting](image-url)

Second method: Including an adjustable idler pulley or roller which may be inside or outside the belt loop.

![Figure 3-34 Adjustable idler pulley](image-url)

From this point beyond we will focus on the first method.

![Figure 3-35 Motor adjusting through screw (preliminary design)](image-url)
On the Figure 3-35 is shown, that motor adjusting will be provided throughout screw mechanism. By rotating a Nut B in clock wise direction, the motor will move to the direction of the bolt along the screw rod. By this motion we can crate suitable pretension of the belt.

3.5. Lug and pin design
For the assembling capsule to the centrifuge arm, there should be special connection that I choose to be lug and pin.

3.5.1. Lug design
Loading
First, we have to find what loading will appear on the lug or what loading the lug has to handle. Since I choose that so called “female” lug will be installed on the centrifugal arm, i.e. on the 45 x 45 profile and the loading will be transferred the through the pin we can find forces that will appear on the lug by finding reaction on beam’s supports.

Reactions A and B will be the load that lug has to handle. Since, force from the capsule is situated in the middle of the pin, reactions are

\[ A = B = \frac{F_c}{2} \]  

(3.84)

Force \( F_c \) was found in the paragraph §3.1.2 p.14, it is the resultant force of mass force and centrifugal. Its value is:

\[ F_c = \sqrt{G_c^2 + C_c^2} = \sqrt{150^2 + 1500^2} = 1507.5 \approx 1510 \, [N] \]  

(3.85)

Note that, the real values of the \( G_c \) is 147.15 [N] and centrifugal force is \( C_c = 1471.5 \, [N] \) for acceleration of \( a = 10g \), but to be on the safety side we have to be conservative.

Finally, force in the supports and at the same time forces that lug has to handle is

\[ A = B = \frac{1510}{2} = 755 \, [N] \]
Lug design

The material the lug will be made from has a significant influence on the lug strength (lug geometry). For now we will continue to use the same material as the profile is made from, which is AlMgSi0.5F25 or also known as Al 6063A. Its mechanical properties were shown in the Table 13. There just has to be added information about its ultimate tensile stress, which is $\sigma_{\text{ult}} = 130$ [MPa]. Using method that was described in the book “Airframe stress analyses and sizing” by Michael C. Y. Niu [13, p. 321] we can check if chosen dimensions are valid or not.

As the first step we have to guess what values we will use. They are:

![Figure 3-37 Lug dimensions](image)

 Exists 3 types of lug loads: axial, transverse and oblique.

![Figure 3-38 Types of lug loads](image)

**Case I – Axial load**

The lug failure modes for this load case are **Share – Bearing failure and Tension failure**.

**Share – Bearing Failure**

Failure consists of shear tear-out of the lug along a 40° angle on both sides of the pin (see Figure 3-39), while bearing failure involves the crushing of the lug by the pin bearing.

![Figure 3-39 Lug tension and shear-tear out failures](image)
The ultimate load for this type of failure is given by the equation

\[ P_{bru} = k_{br} \times \sigma_{tux} \times A_{br} \]  (3.86)

Where \( P_{bru} \) – ultimate load for shear – tear out and bearing failure; \( k_{bru} \) – Shear – bearing efficiency factor see Figure 3-40; \( A_{br} \) – Projected bearing area \( A_{br} = D \times t \); \( \sigma_{tux} \) – Ultimate tensile stress in x – direction of the material.

<table>
<thead>
<tr>
<th>D</th>
<th>18  [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>2  [mm]</td>
</tr>
<tr>
<td>a</td>
<td>15  [mm]</td>
</tr>
<tr>
<td>W</td>
<td>30  [mm]</td>
</tr>
</tbody>
</table>

Table 24 Lug dimensions

![Figure 3-40 Shear-Bearing efficiency factor [13]](image)

**Tension failure**

Tension failure is given by:

\[ P_{tu} = k_t \times \sigma_{tux} \times A_t \]  (3.87)

Where \( P_{tu} \) – Ultimate load for tension failure; \( k_t \) – Net tension efficiency factor, see ; \( \sigma_{tux} \) – Ultimate tensile stress in x – direction of the material; \( A_t \) – Minimum net section for tension \( A_t = (W - D) t \)
Yield Failure – lug

Lug yield failure attributable to shear – bearing is given by:

\[ P_{Y_B} = C \cdot \left( \frac{\sigma_{txy}}{\sigma_{txx}} \right) \cdot P_{u_{min}} \]  

(3.88)

Where \( P_Y \) – Yield load; \( C \) – Yield factor see Figure 3-42; \( \sigma_{txy} \) – Tensile yield stress of lug material in load direction; \( P_{u_{min}} \) - The smaller \( P_{bru} \) or \( P_{tu} \)

Yield Failure – bushing

Bushung yield bearing load attributable to shear – bearing is given by:

\[ P_{bry} = 1.85 \cdot \sigma_{cy} \cdot A_{brb} \]  

(3.89)

Where \( P_{bry} \) – Bushing yield bearing load; \( \sigma_{cy} \) – Compression yield stress of bushing material, \( A_{brb} \) – the smaller of the bearing areas of bushing on pin or bushing on lug.
Case II – Transverse Load ($\alpha = 90^\circ$)

For further calculation we have to compute: Projecting bearing area $A_{br}$ and Average area $A_{av}$

$$A_{br} = Dt$$

$$A_{av} = \frac{6}{\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}} \quad (3.90)$$

The ultimate load is obtained using next equation:

$$P_{tru} = k_{tru} \times A_{br} \times \sigma_{tuy} \quad (3.91)$$

Where: $P_{tru}$ – Ultimate transverse load; $k_{tru}$ - Efficiency factor for transverse ultimate load see Figure 3-44; $\sigma_{tuy}$ – Ultimate tensile stress of lug material in y – direction.
The Yield load can be obtained then by next equation.

$$P_{Y,90} = k_{try} \cdot A_{br} \cdot \sigma_{tyy}$$  \hspace{1cm} (3.92)

Where: $P_Y$ – Yield transverse load; $k_{try}$ – Efficiency factor for transverse yield load, see Figure 3-44; $\sigma_{tyy}$ – Tensile yield stress of lug material in y – direction.

**Case III – Oblique Load**

For Ultimate load

$$j = \frac{1}{\left(R_{3.6}^{1.6} + R_{1.6}^{1.6}\right)^{0.625}}$$  \hspace{1cm} (3.93)

For Yield load

$$j = \frac{1}{\left(R_{0.6}^{1.6} + R_{1.6}^{1.6}\right)^{0.625}}$$  \hspace{1cm} (3.94)
Where: \( j \) – Reserve factor; \( R_{a(tr),u(y)} \) – axial or transverse component (indices “a” or “tr” respectively) of applied ultimate or yield (limit) load (indices “u” or “y” respectively) divided by smaller of \( P_{bru} \) or \( P_{tu} \) from equation (3.86) or (3.87) respectively, or \( P_{Y,0} \) (3.88) or \( P_{Y,90} \) (3.92) for yielding.

**Application**

**Axial load**

Using material information and guessed dimensions written above, in the beginning of this part we have following ratios:

<table>
<thead>
<tr>
<th></th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D/t )</td>
<td>( 9 )</td>
</tr>
<tr>
<td>( a/D )</td>
<td>0.83</td>
</tr>
<tr>
<td>( W/D )</td>
<td>1.67</td>
</tr>
</tbody>
</table>

*Table 25 lug dimension ratios*

Following factors \( k_{bru} \), \( k_t \) and C were read from Figure 3-40, Figure 3-41 and Figure 3-42 respectively.

<table>
<thead>
<tr>
<th></th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{bru} )</td>
<td>0.6</td>
</tr>
<tr>
<td>( k_t )</td>
<td>0.74</td>
</tr>
<tr>
<td>C</td>
<td>1.1</td>
</tr>
</tbody>
</table>

*Table 26 Axial case factors*

Using equation (3.86) we find ultimate \( P_{bru} \)

\[
P_{bru} = 0.6 \times 130 \times (18 \times 2) = 2808 \, [N]
\]

(3.95)

By equations (3.87) we can compute \( P_{tu} \)

\[
P_{tu} = 0.74 \times 130 \times (30 - 18) \times 2 = 2308.8 \, [N]
\]

(3.96)

Yield axial load with the use of equation (3.88)

\[
P_{Y,0} = 1.1 \times \left( \frac{100}{130} \right) \times 2 \times 2308.8 = 1953.6 \, [N]
\]

(3.97)

Theoretically, this Yield load satisfies given conditions, when the arm of the centrifuge is loaded by weight of the capsule only, which makes in ideal case pure axial load. Numerically, female lug it has to stand load of \( \frac{150}{2} = 75 \, [N] \), where the reserve factor is then \( j = \frac{1953.6}{75 \times f (f=2) \times \lambda (\lambda = 1.15)} = 11.3 \, [-] \), of course it is more than we need, but we have to leave chosen dimension, because we have to calculate further transverse load and oblique one, which’s loads are much bigger than axial one.

**Transverse Load**

Projecting bearing area \( A_{br} = 18 \times 2 = 36 \, [mm^2] \) and Average area specifically for our case

\[
A_{av} = \frac{6}{\frac{1}{17.2721} + \frac{1}{12} + \frac{1}{12} + \frac{1}{17.2721}} = \frac{1}{17.2721} + \frac{1}{12} + \frac{1}{12} + \frac{1}{17.2721} = 15.066 \, [mm^2]
\]

The areas ratio is then \( \frac{A_{av}}{A_{br}} = 0.4185 \, [-] \), for the curve (1) of Figure 3-44 the efficiency factor for transverse ultimate load \( k_{bru} \approx 0.5 \, [-] \), so as the same factor for yield load \( k_{try} \approx 0.5 \, [-] \).

The ultimate load is computed by equation (3.91) and yield load by equation (3.92)
\[ P_{\text{tru}} = 0.5 \times (18 \times 2) \times 130 = 2340 \text{ [N]} \]

\[ P_{Y.90} = 0.5 \times (18 \times 2) \times 100 = 1800 \text{ [N]} \]

**(Oblique load)**

The reserve factor for the ultimate load for oblique case:

\[
j = \frac{1}{\lambda \times \left( \left( \frac{f \times 75}{2308.9} \right)^{1.6} + \left( \frac{f \times 750}{2340} \right)^{1.6} \right)^{0.625}} = 1.33 [-]
\]

Where \( \lambda \) is fitting factor and \( \lambda = 1.15 \), \( f \) is safety factor, \( f = 2 \)  \hspace{1cm} (3.99)

If we want to be safe, we have to be on the conservative side of the problem. This means, that we will use yield characteristics to calculate reserve factor, with the usage of ultimate loads, which are *limit loads * safety factor, where limit load are 150/2 N for axial load and 1500/2 N for transversal load.

\[
j = \frac{1}{\lambda \times \left( \left( \frac{f \times 75}{1953.6} \right)^{1.6} + \left( \frac{f \times 750}{1800} \right)^{1.6} \right)^{0.625}} = 1.02
\]

\hspace{1cm} (3.100)

If we change the orientation of the lug the way that axial force on one lug will be 1500/2 N, and transversal force will 150/2 N, the reserve factor is then

\[
j = \frac{1}{\lambda \times \left( \left( \frac{f \times 75}{1953.6} \right)^{1.6} + \left( \frac{f \times 750}{1800} \right)^{1.6} \right)^{0.625}} = 1.11
\]

\hspace{1cm} (3.101)

Changing the orientation of the lug, we have increased reserve factor for the yield load, but it will decrease reserve factor for ultimate load for 1.33 to 1.31

Reserve factor \( j \) should be bigger than 1, \( j > 1 \) and we have satisfied the obligatory requirement that the reserve factor for yield load should be bigger than 1, \( j = 1.11 > 1 \). Note that the calculation above were done for the orientation of lug where centrifugal force will be applied in the direction of axial load, there were used safety factor \( f = 2 \), fitting factor \( \lambda = 1.15 \) and external load that this lug has to stand were artificially increased for little bit, so we can conclude that final dimensions of the lug

In case we need that one lug will stay full scope of the loading, i.e. that axial loading that will be applied on the one lug will be no longer divided by two, \( F_a = 1500 \text{ N} \), not \( F_a = 1500/2 = 759 \text{ N} \). Then lug has to be modified by further steps: increase the thickness for 1 mm (\( t = 3 \text{ mm} \)), increase distance “a” from 15 mm to 16 mm and width “W” from 30 mm to 32 mm (see Figure 3-37). Output parameters are then shown in Table 27
Table 27 Strength parameters for one lug mode

Results in the Table 27 are acceptable since reserve factors for yielding stress and ultimate stress are bigger than one, which means they satisfies necessary requirements. Thickness of the lug for this result is 3mm, but bearings of width 3mm of SKF manufacturer can not carry the load of 755 N. The closest bearing that can carry the load and being in geometrical limits of the lug that can be mounted to the beam of 45x45 profile is SKF 61801. It has next parameters:

Figure 3-45 SKF 61801 bearing parameters [14]

Because of this bearing we need to increase the bore diameter up to 12 mm, which will be also pin diameter in further calculation. Thickness should increase minimum to 5 mm (for the construction purposes it will be increased to 7 mm). In that case strength parameters of the lug that has parameters are shown in the

Table 28 Lug strength result, geometry modified for bearing SKF 61801
### 3.5.2. Pin design

**Pin shear – off failure** [13, pp. 326-329]

Pin single shear – off failure is given by:

\[
P_{p,s} = \tau_{ult} \left( \frac{\pi D^2}{4} \right)
\]

Where: \( P_{p,s} \) = Ultimate load for pin shear – off failure; \( \tau_{ult} \) – ultimate shear stress of the pin material.

**Pin Bending Failure**

If the pin used in the lug is not big enough to resist the bending that appeared from the load, the bend of the pin can participate failure in the lug. Because, as the pin bends, the stress distribution action on the inner side of the lug tends to peak rather than form an even distribution, as shown on Figure 3-46.

![Figure 3-46 Peak Pin Load](image)

For the pin bending failure check, we will use two different modes of the pin loading and which of these methods will be less dangerous being close to the failure will be used as the final version.

- **I mode** – one male cylindrical lug of width 43 mm
- **II mode** – two inner lug of 3 mm thickness

![Figure 3-47 First mode of the pin loading](image)
As simple free body diagram it can be shown as following:

Using equation (3.102) we can find ultimate load for the pin shear.

\[ P_{ps} = 100 \times \left( \frac{\pi \times 12^2}{4} \right) = 11309.73 [N] \]  

(3.103)

Where \( D = 12 \text{ mm} \), \( \tau = 100 [MPa] \). The reason why \( D = 12 \text{ mm} \) is in inner diameter of the bearing that was chosen in the paragraph §3.5.1.

\[ R_1 = R_2 = \frac{q \times (54 - 5.5 \times 2) \times \left( 54 - 5.5 \times 2 + 5.5 \right)}{54} = 755 [N] \]  

(3.104)

\[ M(x) = \begin{cases} R_1 \times x, & \text{for } x \text{ from } 0 \text{ to } 5.5 \text{ mm} \\ R_1 \times x - q \times \frac{(x - 5.5)^2}{2}, & \text{for } x \text{ from } 5.5 \text{ to } 27 \left( \frac{54}{2} \right), \end{cases} \]

since problem is symmetrical, it should be enough  

(3.105)

Shearing force and Bending moment diagrams are then shown on the figures below, with the peak for Shearing force 755 N (or -755 N) and the peak for bending moment 12268,75 [N.mm] at \( x = 27 \text{ mm} \).

![Figure 3-48 Free body diagram of the Mode I](image)

![Figure 3-49 Shearing Force distribution force, loading of Mode I](image)
The normal stress is then:

\[
\sigma_{\text{max}} = \frac{M_{\text{max}}}{I} = \frac{12268.75}{\pi \frac{D}{32}} = 72.32 \text{ [MPa]}
\]  

(3.106)

Shear stress

\[
\tau = \frac{4}{3} \frac{V_{\text{max}}}{\pi D^2/4} = \frac{4}{3} \frac{755}{\pi \frac{12^2}{4}} = 8.9 \text{ [MPa]}
\]  

(3.107)

HMH hypothesis for equivalent stress

\[
\sigma_{\text{eq}} = \sqrt{\sigma^2 + (3 \tau)^2} = 77.1 \text{ [MPa]}
\]  

(3.108)

Allowable stress for the material mentioned in the Table 13, for safety factor \(f = 2\), is 100 MPa, the reserve factor in this case is:

\[
j = \frac{100}{77.1} = 1.29 > 1 [-], \text{which is acceptable}
\]  

(3.109)

Maximum deflection is expected to be in the middle of the pin, so we apply there the unit force. Using the Mohr’s integral (equation (3.110)) we can find the deflection at the required point

\[
y_i = \int_{(L)} \frac{M(x)}{EI} m_i \, dx
\]  

(3.110)

Since we have isotropic material with constant cross-section of Young’s modulus and second moment of inertia can be considered as constants, hence may be taken out of integral .

\[
y_i = \frac{1}{EI} \int_0^L M(x) \cdot m_i \, dx = \frac{1}{EI} \left[ \int_0^{5.5} \left( R_5 x \cdot \frac{1}{3} x^2 \right) \, dx + \int_{5.5}^{27} \left( R_5 (x - \frac{q(54 - 11)}{4}) - \frac{q}{4} (x - 5.5)^2 x \right) \, dx \right] \times 2^{(*)} =
\]

\[
= 2 \times \left[ q \left( \frac{54 - 11}{4} \right) \frac{5.5^3}{3} + \frac{1}{EI} \int_{5.5}^{27} q \left( \frac{54 - 11}{4} \right) x^2 - \frac{q}{4} (x - 5.5)^2 x \, dx \right] =
\]

\[
= 2 \times 0.00029 + \frac{2}{70000 \times 1017.876} \frac{q}{4} \left[ 43 \left( \frac{x^3}{3} \right)_{5.5}^{27} - \left( \frac{x^4}{4} \right)_{5.5}^{27} + 2 \times 5.5 \left( \frac{x^3}{3} \right)_{5.5}^{27} - 5.5^2 \left( \frac{x^2}{2} \right)_{5.5}^{27} \right] =
\]

\[
y_L = (0.000294 + (-0.02564)) \times 2 = -0.051 [\text{mm}]
\]  

(3.111)

\((*)\) Times 2 due to symmetricity of the problem
This mode of the loading transfer can be represented as free body diagram as shown on the figure below.

\[ R_1 = R_2 = \frac{qa \left(2 \left(\frac{a}{2} + a_1\right) + \left(b + 2 \cdot \frac{a}{2}\right)\right)}{L}, \text{if } b = L - 2(a_1 + a), \text{then} \]
\[ R_1 = R_2 = \frac{qa \left(2 \left(\frac{a}{2} + a_1\right) + (L - 2a_1 - 2a + a)\right)}{L} = \frac{q \cdot a \cdot L}{L} = q \cdot a \]
\[ R_1 = R_2 = 251.667 \times 3 = 755 \text{ [N]} \]

\[ M(x) = R_1 \cdot x, \quad \text{for } x \text{ from } 0 \text{ to } a_1 \]
\[ M(x) = R_1 \cdot x - q \cdot \frac{(x - a_1)^2}{2}, \quad \text{for } x \text{ from } a_1 \text{ to } a_2 \]
\[ M(x) = R_1 \cdot x - q \cdot a \cdot \left(x - \left(a_1 + \frac{a}{2}\right)\right), \quad \text{for } x \text{ from } a_2 \text{ to } \frac{L}{2} \]

Shearing force and Bending moment diagrams are then shown on the diagrams below, with their peaks 755 N (or -755 N) for Shearing force and 5285 [N.mm] for bending moment.
As we can see, the bending moment, in the loading of the mode II, decreased more than two time, compare to the mode I. Taking into account that shearing force remain the same in both cases, shearing stress will not be changed, hence equivalent stress will decrease, which will make pin more safety. Bending stress is:

\[
\sigma_{\text{max}} = \frac{M_{\text{max}}}{r} = \frac{5285}{\pi \times \frac{D^3}{32}} = 31.15 \text{ [MPa]}
\]

(3.114)

\[
\sigma_{\text{eq}} = \sqrt{\sigma^2 + (3 \times \tau)^2} = 41.03 \text{ [MPa]}
\]

(3.115)

\[
J = \frac{100}{41.03} = 2.43 > 1 [-], \text{which is acceptable}
\]

(3.116)

Comparing reserve factors from the mode I [(3.112)] and II [(3.116)], we can make a statement, that from the strengthen point of view mode II is more desirable. It is 1.88 time safer. To find how different maximum deflections are, we, again, will use Mohr’s integral, which formula was shown in (3.107).

Since the problem is a little complex, compare to the mode I, we will use an advantage of the super position method. Which is:

\[
y_i = y_i(f_1) + y_i(f_2)
\]

(3.117)

Where \(f_1\) and \(f_2\) are cases, when the pin is only loaded by left or right distributed load, \(f_1\) represents left one, \(f_2\) represents the right one, from the view that is shown on the Figure 3-52. But due to symmetricity both deflection will be the same, so the final form of the equation (3.117) is \(y_i = 2 \times y_i(f_1) = 2 \times y_i(f_2)\)
Figure 3-55 A. Mode II, $f_1$

Bending moments from loading $f_1$

\[
x \in [0; a_1] \rightarrow M(x) = R_1 x;
\]

\[
x \in [a_1; a_2] \rightarrow M(x) = R_1 x - \frac{q(x - a_1)^2}{2}
\]

\[
x \in [a_2; L] \rightarrow M(x) = R_1 x - qa \left(x - a_1 - \frac{a_2}{2}\right)
\]

or \( \bar{x} \in [0; 0.5L] \rightarrow M(\bar{x}) = R_2 \bar{x} \)

Bending moment from Unite force “1”

\[
x \in \left[0; \frac{L}{2}\right] \rightarrow m_1(x) = \frac{1}{2} x
\]

\[
x \in \left[\frac{L}{2}; L\right] \rightarrow m_1(x) = \frac{L}{2} - \frac{1}{2} x
\]

or \( \bar{x} \in \left[0; \frac{L}{2}\right] \rightarrow m_1(\bar{x}) = \frac{1}{2} \bar{x} \)

Due to symmetry follows:

\[
R_1 = 97.9 \ [N]; \quad R_2 = 677 \ [N]
\]

Bending moments from the load $f_2$

\[
\bar{x} \in [0; a_1] \rightarrow M(\bar{x}) = R_2 \bar{x}
\]

\[
\bar{x} \in [a_1; a_2] \rightarrow M(\bar{x}) = R_2 \bar{x} - \frac{q(\bar{x} - a_1)^2}{2}
\]

\[
x \in [0; 0.5L] \rightarrow M(x) = R_1 x
\]

Bending moments from unite force

\[
\bar{x} \in \left[0; \frac{L}{2}\right] \rightarrow m_1(\bar{x}) = \frac{1}{2} \bar{x}
\]

\[
x \in \left[\frac{L}{2}; \frac{L}{2}\right] \rightarrow m_1(x) = \frac{1}{2} x
\]

Type equation here. \hspace{1cm} (3.118)

From obtained equations we can solve the Mohr’s integral for the load $f_1$ and then multiply it by 2, because of symmecicity.
$$y_L(f_1) = \int_{(L)}^1 \frac{1}{EI} * M(x) * m_1(x) dx \rightarrow$$

$$\frac{1}{EI} \left[ \int_0^{a_1} R_1 x \cdot \frac{1}{2} \cdot x dx + \int_{a_1}^{a_2} \left( R_1 x - q \cdot \frac{(x - a_1)^2}{2} \right) \cdot \frac{1}{2} \cdot x dx + \int_{a_2}^{L \over 2} \left( R_1 x - qa \left( x - a_1 - \frac{a_2}{2} \right) \right) \cdot \frac{1}{2} \cdot x dx \right]$$

For the simplification each part of the integral will be solved individually and then summed

$$\int_0^{a_1} R_1 x \cdot \frac{1}{2} \cdot x dx = \frac{R_1}{2} \cdot \frac{x^3}{3} \bigg|_0^{a_1} = \frac{R_1}{2} \cdot \left( \frac{a_1^3}{3} \right) = \frac{657}{2} \cdot \frac{5.5^3}{3} = 18221.657 \text{ [N.mm]} \quad (1)$$

$$\int_{a_1}^{a_2} \left( R_1 x - q \cdot \frac{(x - a_1)^2}{2} \right) \cdot \frac{1}{2} \cdot x dx$$

$$= \frac{R_1}{2} \cdot \left( \frac{a_2^3}{3} - \frac{a_1^3}{3} \right) - \frac{q}{4} \cdot \left[ \left( \frac{a_2^4}{4} - \frac{a_1^4}{4} \right) - 2a_1 \left( \frac{a_2^3}{3} - \frac{a_1^3}{3} \right) + a_1^2 \left( \frac{a_2^2}{2} - \frac{a_1^2}{2} \right) \right]$$

$$= 657 \cdot \frac{8.5^3}{3} - 5.5^3 \cdot \frac{251.667}{4} \cdot \left( \frac{8.5^4}{2} - 5.5^4 \right) - 2 \cdot 5.5 \cdot \left( \frac{8.5^3}{3} - 5.5^3 \right) + 5.5^2 \cdot \left( \frac{8.5^2}{2} - 5.5^2 \right)$$

$$= 44649.861 \text{ [N.mm]} \quad (2)$$

$$\int_{a_2}^{L \over 2} \left( R_1 x - qa \left( x - a_1 - \frac{a_2}{2} \right) \right) \cdot \frac{1}{2} \cdot x dx = 556684.63 \text{ [N.mm]} \quad (3)$$

$$\int_0^{L \over 2} R_2 \cdot \frac{1}{2} \cdot x \cdot \frac{1}{2} \cdot x dx = 979.9 \cdot \frac{27^3}{3} = 321063.75 \text{ [N.mm]} \quad (4)$$

$$y_L(f_1) = \frac{1}{EI} \cdot \left[ 18221.657 + 44649.861 + 556684.63 + 321063.75 \right]$$

$$= \frac{1}{70000 \cdot 1017.876} \cdot 940619.9 = 0.0132 \text{ [mm]} \quad (5)$$

(3.119)

The positive meaning of the deflection is in the direction of the applied force.

Since we mentioned that due to symmetricity of the case, the complete deflection will be twice more than it has been calculated in (3.119).

$$y_L = 0.0132 + 0.0132 = 0.0264 \text{ or } 0.0264 \text{[mm]} \quad (3.120)$$
Comparing the deflection results of the mode I and mode II, we can see that: $-0.051 > 0.0264$, in almost two times (1.93).

From the obtained results of the stress safety and deflection, I conclude, that the more reliable and loading mode is the mode II. This mode will be used for further calculations.

In reality, the loading of the pin during arms rotation will not be symmetric and trivial as it was represented Figure 3-52. There was not mentioned the presence of the extra forces that will appear with moment from the drag force. The drag force can be determined through classic formula $F_D = \frac{1}{2} \rho v^2 * S * c_D$, where $c_D$ is the drag coefficient of the cylinder. The velocity $v$ is circumferential that is the function of the rotational speed and radius of rotation. For the 10g loading, for our case of machine, corresponds angular velocity of 9.9045 rad/s and arm can is 1m plus distance to the drag force concentration point. The distance to the drag force concentration point from the lug’s eye center as shown on the Figure 3-56:

![Figure 3-56 Point of force applied on capsule, Fd is a drag force, Fc is a centrifugal force](image)

\[
r = \frac{L_c}{2} + \frac{s}{2} = \frac{650}{2} + 60 = 385 [\text{mm}]
\]

\[
a = r \cos(90 - \alpha) = 385 \cos(90 - 84) = 382.89 = 383 [\text{mm}]
\]

\[
b = r \sin(90 - \alpha) = 385 \sin(90 - 84) = 40.24 [\text{mm}]
\]

Therefore, the drag force is:

\[
F_D = \frac{1}{2} * 1.225 * \left(9.9045 * \left(1 + \frac{0.65}{2} + 0.06\right)^2\right) * 0.65 * 0.35 * 1 = 20.2 [\text{N}]
\]

The drag force will have effect of moment to the pin. The moment has value of:

\[
M = F_D * r = 20.2 * 385 = 7777 [\text{N.mm}]
\]

The moment M will be transferred to the pin by inner lugs as forces (distributed forces), which values are:
\[ M = F_{M1} \cdot r_{p1} + F_{M2} \cdot r_{p2} \]

if \( r_{p1} = r_{p2} = r_p = \frac{a}{2} + \frac{b}{2} + \frac{a}{2} = 20 \, [mm] \) and \( F_{M1} = F_{M2} \)

\[ M = 2 \cdot F_M \cdot r_p \rightarrow F_M = \frac{M}{2r_p} \]

\[ F_M = \frac{7777}{2 \times 20} = 194.425 \, [N] \]

(3.124)

The next step is just to add appeared forces on the pin from drag according to the logic shown on the Figure 3-57

\[ F_1 = \frac{1510}{2} + F_M = 755 + 194.425 = 949.425 \, [N] \approx 950 \, [N] \]

(3.125)

\[ F_2 = \frac{1510}{2} - F_M = 755 - 194.425 = 560.575 \, [N] \approx 561 \, [N] \]

So, the final form of the pin loading looks like as how on the figure below, if we assume distributed force as the point force (if we imagine 3mm distance as the point).

![Figure 3-57 Pin loading, according drag effect (1B modification)](image)

\[ F_1 \cdot a + F_2 \cdot (a + b) - R_2 \cdot l = 0 \]

\[ R_2 = \frac{F_1a + F_2(a + b)}{l} = 611.426 \, [N] \]

\[ R_1 = F_1 + F_2 - R_2 = 899.6 \, [N] \]

(3.126)

\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{l} = \frac{R_1a}{\pi D^3} = 37.12 \, [MPa] \]

\[ \tau = \frac{4}{3} \cdot \frac{V_{\text{max}}}{\pi D^2} = 11.2 \, [MPa] \]

\[ \sigma_{\text{eq}} = \sqrt{\sigma^2 + (3\tau)^2} = 50 \, [MPa] \]

\[ j = \frac{\sigma_{\text{all}}}{\sigma_{\text{eq}}} = \frac{100}{50} = 2 \, [-] \]

(3.127)
3.6. Centrifuges head

As is it was mentioned on the page 21 there will be two major variant of the centrifuge’s head. One-Beam variant – when only one beam on each arm carries a tested sample, its designation is 1B. Two-Beam variant – when two beams on each arm carries a load from the tested sample accelerated up to 10g. Has a 2B designation. This variant needs in order to decrease the risk of failure and decrease deflection of the beams under the load.

The centrifuges head should be designed the way that it will be easier in future to disassemble one variant and assemble another one. In order to complete this task, the cheapest and easiest way is to make a part, where beams are assembled, the same for both variants. After several iteration and design experiment I came to the next solution:

Centrifuge Head will consist from:

1) Hub – the part that is assembled to the rotating shaft
2) Head’s Plate – the part, which is connected to the Hub and will carry beams.
3) Fasteners – connection elements.

All of the mentioned should be the same for 1B or 2B variant, and made of steel, in order to make it more rigid, stiffer.

Final solution are shown on the following figures.

Figure 3-58 Centrifuge Head, 1B variant
As we see from Figure 3-58 and Figure 3-59, there is used one Head’s Plate (pink) and one Hub (grey). Using Finite Element Method we will check their dimensions (thicknesses), in the paragraph §4.9, p. 97.

On the Figure 3-60 shown Centrifuge’s Head Plate (CHP) dimensions. Holes that are situated on the upper part (72.5 mm far from the center of the 190 mm side) and lower part belongs for the 2B variant, when holes on the middle (center of the 190 mm side) belongs for 1B variant. Holes that are allocated as a radial pattern feature of 82 mm diameter are for the fastening CHP and Hub.

Hub transfers torque from shaft through the feather key. Contact pressure of the hub is similar to the contact pressure of the shaft of the relatable section, that was calculated in equation (3.53) p. 30. Contact pressure $p = 12.2$ MPa, and reserve factor is $j > 10$. 
3.7. Frame

The frame of the centrifuge is primarily consist of 45x45 Alutec profile, its material parameters are mentioned on the Table 13. Connection elements were also chosen from the same supplier, which is Alutec KK s.r.o. Rigidity of the Frame is checked in the paragraph §4.7 and §4.8. Generally the frame was designed the way that it will balance centrifuge in any arms position and will have as minimum as it possible frame members deflection.

*Figure 3-61 Frame of the Centrifuge machine, 1B or 2B var., without reinforcement*
4. FEM Analysis

To check if separate parts, assemblies and complete object respond to the strength and rigidity requirements we will use Finite Element Method with the help of pre-processor Siemens FEMAP 11.3 and Beta CAE ANSA 16.0. Processor (solver) was used NX Nastran, integrated to the FEMAP.

4.1. Requirements

Maximum stress appeared on the part should not accede allowable stress of the part.

\[ \sigma_{\text{max}} \leq \sigma_{\text{all}} \]  

(4.1)

Where \( \sigma_{\text{all}} \) is allowable stress and can be calculated by

\[ \sigma_{\text{all}} = \frac{\sigma_y}{f} \]  

(4.2)

Where \( f \) is a safety factor, \( f = 2[-] \) and \( \sigma_y \) is the Yield stress of the material.

For every case reserve factor \( j \) should be bigger than 1.

\[ j = \frac{\sigma_{\text{all}}}{\sigma_{\text{max}}} > 1 \]  

(4.3)

For the static stability cases (for the columns, frame members loaded for compression)

\[ F \leq F_{\text{crit}} \]  

(4.4)

Where \( F \) is the compression force applied on the member and \( F_{\text{crit}} \) is the critical force.

However it is better to design frame the way that it will fit deflection requirements that are written in the acceptable standard. For our case I use ISO 4356:1977. This standard contains large scope of requirements, however to simplify the work I choose the smallest from them all which is

\[ \Delta_i = \frac{L}{500} \]  

(4.5)

Where \( \Delta_i \) is the suggested limited value for deflection of specific \( i \) – member of length \( L \), both in millimeters.
4.2. Lug

How it was mentioned before lug is made of same material as 45 x 45 profile, which is AlMgSi0.5F25, and its mechanical parameters were mentioned in Table 13.

Loading for the lug was mentioned in paragraph §3.5.1. Which is 755 N in the direction of a resultant force, which is for 10g acceleration of machine is \(\approx 84^\circ\) from the direction of a transverse load. Analysis FEM model is shown on Figure 4-1. Thickness of the shell is 7mm. Constraints are defined by constraining translation motion of the lug in x, y and z direction in the place of lug fasteners.

There was used static linear analysis using NASTRAN solver (NX Nastran), sol 101.

Results shown in Figure 4-2, shows equivalent stress appeared on a model using Von Misses hypothesis, and appeared total deformation in scale that is 1000 times bigger than actual deformation (white color behind the model shows unreformed state). As we can see, maximum stress there is 7.5 MPa, when allowable stress for this material is 100 MPa. Reserve factor here is \(j = \frac{\sigma_{all}}{7.5} = \frac{100}{7.5} = 13.3\). Maximum deformation is 0.00285 mm, this value can be neglected. This results shows us that we are too safe, which means we can decrease the material use on the model, but this design approach is done due to bearing requirements. Smaller bearing will not be safe to use for this loading. All results and model information is represented as the table after the result figure. This way of representing result will be used for this and for further models.

![Figure 4-1 Lug FEM analysis model](image)

![Figure 4-2 Results for Lug, fringe = Von Misses stress, deformation shows resultant translation deformation (1000x bigger than actual deformation)](image)
### Max. stress [MPa]
- Value: 8

<table>
<thead>
<tr>
<th>Max. deformation [mm]</th>
<th>0.00285</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>12.5</td>
</tr>
</tbody>
</table>

*Table 29 Lug results*

#### 4.3. Pin

**One beam modification (1B)**

It was already performed hand calculation in paragraph §3.5.2 with results of stress shown in equation (3.127). We will check here the correctness of job done.

The total length of the pin used for 1B modification is 70 mm (see), but as it was shown on the Figure 3-52, the functional length is 54 mm.

![Figure 4-3 Pin drawing for 1B modification.](image)

**Figure 4-3 Pin drawing for 1B modification.**

**Figure 4-4 Pin (1B) results**

Results in Figure 4-4 shows deformation of the pin in scale of 10% of total length of the model and bending stress as a fringe. As we can see maximum bending stress there is 37.12 MPa. In hand calculation bending stress was 37.12 MPa (see eq. (3.126)), FEM model confirms that provided hand calculation were done correctly. Max. Shear stress is same as in hand calculation, which is 11.2 MPa, than equivalent stress is again 50 MPa (see eq. (3.127)).
Deformation is 0.0285 mm, which acceptable. Design fits all requirements.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. bend. stress [MPa]</td>
<td>37.12</td>
</tr>
<tr>
<td>Max. shear stress [MPa]</td>
<td>11.2</td>
</tr>
<tr>
<td>Max. deformation [mm]</td>
<td>0.028</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>2</td>
</tr>
</tbody>
</table>

*Table 30 Pin 1B results*

For the loading shown in Figure 3-52, results are shown in Figure 4-5

FEA results shows, that maximum bending stress is 29 [MPa], when result is eq. (3.114) gives maximum bending stress as 31.15 [MPa]. Deformation from FEA is 0.0247 mm, when hands calculation in eq. (3.120) gives us 0.0264 mm, difference in $1.7 \times 10^{-3}$ I would count as a neglected one.

*Two beam modification*

*Figure 4-5 Pin (1B) no-drag loading*

*Figure 4-6 CAD model of 2B modification, view to the lugs*
Figure 4.7 Pin for 2B modification, drawing

Determination of a precise loading applied on male lugs from implementation of 10g force on 15kg capsule that is fastened to the blue plate from Figure 4-6 (named as a Lug out plate) is relatively complex task, but thanks to FEM we can solve this problem by combining pin model and lug out plate model. Resultant force from centrifugal force and mass force form the capsule is equal to 1510 N (see eq. (3.85)) and this force is normal to the lug out plate. During rotation on the capsule will appear drag force that will act against the rotation. Its value was calculated in the equation (3.122). This force will be transferred to the place of connection that will be assumed to be in the center of the plate. After force translation will appear a moment form the drag force on plate at the place of connection. This force is:

$$M_{Fd} = F_D \frac{L_c}{2} = 20.2 * 325 = 6565 \approx 6600 \, [N.mm] \quad (4.6)$$

Figure 4.8 Force applied on the capsule (red) and its value translated to the plate (green)

FEM analysis model then looks like:
Results for pin only:

On the Figure 4-10, maximum absolute value of the stress appeared there is 0.5 MPa. From free body diagram we can see what reaction forces are on supports (bearing SKF 61801) see Figure 4-11. Maximum reaction force is 1093.8 N ≈ 1095 N. For SKF bearing with maximum loading 1.7 kN this value is sufficient. Maximum shear stress can be found by eq.(3.110), and its value is 13MPa. Using HMH hypothesis:

\[
\sigma_{eq} = \sqrt{(-0.5)^2 + (3 \times 13)^2} = 40 \text{ [MPa]}
\]
### 4.4. Lug out plate

#### 4.4.1. Two-beam plate (2B)

Since there was already preformed calculation of lug out plate of 2B variant on previous case, here will be shown only its results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. bend. stress [MPa]</td>
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</tr>
<tr>
<td>Max. shear stress [MPa]</td>
<td>13</td>
</tr>
<tr>
<td>Max. deformation [mm]</td>
<td>0.00395</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>2.5</td>
</tr>
</tbody>
</table>

*Table 31 Pin 2B result table*

![Free body diagram of pin (2B var.)](image1)

**Figure 4-11** Free body diagram of pin (2B var.)

![Lug out plate (2B var.) FEM results, colourbar represents Von Misses stress, deformation = Total deformation (100s bigger than actual deformation)](image2)

**Figure 4-12a** Lug out plate (2B var.) FEM results, colourbar represents Von Misses stress, deformation = Total deformation (100s bigger than actual deformation)
It is expected, that maximum stress appeared at the place of force implementation. Value of the stress is 72.5 MPa, which is less than allowable stress of the material used (which is AlMgSi0.5F25) 100 MPa. Another important point where we have to check stress is the place of the connection of lugs to the plate. Lugs are connected by the welding. Strength of the aluminum welding connection, according to the welding company ESAB, for the closest related material to ours AL 6061-T6 is $\sigma_u = 185 \text{ MPa}$ [15]. With the safety factor 2, allowable stress for the weld is then:

$$\sigma_{all\,weld} = \frac{185}{2} = 92.5 \text{ [MPa]} \approx 90 \text{ [MPa]}$$

(4.8)

Limiting FEM stress result only for the weld parts, results shows us that the maximum stress at the place where should be welding connection, we can see that maximum stress there is $\approx 40 \text{ [MPa]}$ and it is more than 2 time less than allowable stress for the weld. see Figure 4-13.

Deformation of the plate in the place of applied force is 0.22 mm, which I count as acceptable one, since at this place this small deformation will not affect stability a centrifuge laboratory testing results. Deformation of the inner lugs of 2B variant Lug-out plate is 0.0211 mm, which is also very small so we can count the lug as a relatively rigid.
4.4.2. One–beam plate

For the same loadings that are shown on the Figure 4-9 and same material there are next results:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. stress [MPa]</td>
<td>73</td>
</tr>
<tr>
<td>Max. stress (weld) [MPa]</td>
<td>40</td>
</tr>
<tr>
<td>Max. deformation (plate) [mm]</td>
<td>0.22</td>
</tr>
<tr>
<td>Max. deformation (lug) [mm]</td>
<td>0.02</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>1.37</td>
</tr>
<tr>
<td>Stress reserve factor, j (weld)[-]</td>
<td>2.25</td>
</tr>
</tbody>
</table>

*Table 32 Lug-out part result, 2B*

As we can see, the deformation of the plate itself for 1B variant has significantly decreased compared to the 2B variant, from 0.217 mm to 0.0482 mm. This happened because the length of the plate has decreased
when all thicknesses remain the same, by decreasing the length we have increased the slenderness ratio, that will follow into the higher deformation to force ratio.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Max. stress [MPa]</td>
<td>49</td>
</tr>
<tr>
<td>Max. stress (weld) [MPa]</td>
<td>20</td>
</tr>
<tr>
<td>Max. deformation (plate) [mm]</td>
<td>0.05</td>
</tr>
<tr>
<td>Max. deformation (lug) [mm]</td>
<td>0.02</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>2</td>
</tr>
<tr>
<td>Stress reserve factor, j (weld)[-]</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 33 Lug-out part result, IB

4.5. Motor base plate
The main loading that will carry “Motor base plate” (MBP) is the weight of the motor. Till this step I was assuming that will use steepen electric motor ES-MH 342200 with the weight 12.8 kg. In order to be on a safety side, let us increase the weight of the motor to 20 kg, so the force will be:

\[ F_{\text{motor}} = 20 \times 9.81 = 196.2 \approx 200\,[\text{N}] \] (4.9)

Motor is fastened to the plate by 4 M8 bolts, each is located on the same distance from the centre of gravity of the motor, so we can divide \( F_{\text{motor}} \) to 4 equal parts (50 N) and apply each to the place of the connection.

Motor base is made of 6mm steel plate (ČSN 411523 (old 11 523) see ) and connected to the frame members at 4 points. These points will be assumed as constraints of the MBP. Another constraints that are not so obvious at the first sight, it is the touch between inner borders of the frame members (45x45 profile) and MBP. This connection will limit the motion of the plate in the Y direction (see Figure 4-16) at the point where the profile edge is. The FEM model is then looks like as it shown on the Figure 4-17.
Figure 4-17 FEM model of the MBP

Table 34 Material parameters for steel [16]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f$ [MPa]</td>
<td>275</td>
</tr>
<tr>
<td>$\sigma_u$ [MPa]</td>
<td>500</td>
</tr>
<tr>
<td>$E$ [MPa]</td>
<td>210000</td>
</tr>
<tr>
<td>$\mu$ [–]</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho$ [kg m$^{-3}$]</td>
<td>7850</td>
</tr>
</tbody>
</table>

Results are then for stress tensor:

Maximum appeared stress is at the place of bolt connections, as expected. The value of maximum stress is very small 6 MPa. Allowable stress for used steel is 130 MPa (see eq.(4.10)), the reserve factor is then $j=130/6 = 21.6 [-]$. Which is unnecessary big, however, strength isn’t the most important parameter for this part. Rigidity has here decisive character, because it is required for a motor to be on the same position in order to not deflect the axis of rotation or torque transfer to the from driving pull to driven. Deformation result is shown on Figure 4-19.
$$\sigma_{ult,steel} = \frac{\sigma_Y}{f} = \frac{275}{2} = 137.5 \approx 130 \ [MPa]$$

(4.10)

As we see, maximum deformation is in the center of a Motor base plate, its value is 0.013 mm. This deformation can be neglected.

If in future will be decided to increase the motor torque by changing motor, I assume that this kind of motor will have bigger weight than the one we use. For example if motor will have a weight of 40 kg ≈ 400 N → 4 * 100 N, then MBP’s stress and deformation will be as it shown in the Figure 4-20.

<table>
<thead>
<tr>
<th>Max. stress (200N load) [MPa]</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. stress (400N load) [MPa]</td>
<td>12</td>
</tr>
<tr>
<td>Max. deformation (200N) [mm]</td>
<td>0.013</td>
</tr>
<tr>
<td>Max. deformation (400N) [mm]</td>
<td>0.026</td>
</tr>
<tr>
<td>Stress reserve factor, j (200N) [-]</td>
<td>21.6</td>
</tr>
<tr>
<td>Stress reserve factor, j (400N)[-]</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Table 35 MBP result table, plate thickness 6mm
If we decrease plate thickness from 6mm to 5mm, the results will be next:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. stress (200N load) [MPa]</td>
<td>8.5</td>
</tr>
<tr>
<td>Max. stress (400N load) [MPa]</td>
<td>17</td>
</tr>
<tr>
<td>Max. deformation (200N) [mm]</td>
<td>0.022</td>
</tr>
<tr>
<td>Max. deformation (400N) [mm]</td>
<td>0.045</td>
</tr>
<tr>
<td>Stress reserve factor, j (200N) [-]</td>
<td>15.2</td>
</tr>
<tr>
<td>Stress reserve factor, j (400N)[-]</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Table 36 MBP result table, plate thickness 5mm

Results from Table 36 shows that actually we can use 5mm plate instead of 6 mm plate, because I count 0.05 mm deformation as an acceptable one.

4.6. Bearing house

In the paragraph §3.3.1 we were designing shaft, for this we calculated reactions in supports A and B (see Figure 3-15. Support B is the bearing SKF 16009 and this bearing should to be mounted to the specific place named bearing housing, and I have designed it in the way as it shown on the Figure 4-21. Part where is a bearing placed is connected to the steel plate of 5 mm thickness by four M12 bolts. Plate is assembled to the frame members on the corners, this connection will be assumed as a constraint. Both parts are made of steel which parameters were mentioned in the Table 34. The loading will be the reaction force in the support, that will appear of asymmetric loading of centrifuge arms by 60 kg on one side and zero on the other, this loading will call a reaction in a support with the value of $\approx 1880$ N, see paragraph §3.3.1 eq. (3.48). This force will be directed in the FEM model to the z-axis direction.

*Figure 4-21 CAD model of a Bearing house*
Maximum stress there is 18 MPa, which is significantly small for a steel material. However, as I mentioned before in calculation Motor base plate, decisive parameter here is a deformation. Maximum deformation is appeared to be 0.056 mm, see Figure 4-24. For this plate it sufficient, because maximum allowable deformation should be less than 0.1 mm.

<table>
<thead>
<tr>
<th>Max. stress [MPa]</th>
<th>18 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. deformation [mm]</td>
<td>0.056</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>7</td>
</tr>
</tbody>
</table>

*Table 37 Bearing house plate results*
Figure 4-24 Deformation of Bearing house plate (800x of actual deformation)

Bearing house

Figure 4-25 Bearing house results made of steel (colourbar - Von Misses stress, deformation - total deformation 10 000x bigger than actual deformation)

Figure 4-25 shows us that stress and deformation is so small, that we can state that this part is too rigid and strengthen unnecessarily. It means it has much bigger weight than it could be. One of the way to decrease weight is to switch material from steel to aluminum, from ČSN 411523 to AlMgSi0.5F25, and then by decreasing density with remaining volume (geometry) we will decrease weight for 65% from initial weight. If this part made of steel had a weight 1.1 kg then the same detail made of aluminum alloy will have weight 0.385 grams. FEM solution (Figure 4-26) shows that the maximum stress value hasn’t change at all (because we didn’t change geometry of a part). Deformation has increased, from 0.0007 mm to 0.002 mm, which is still very small distance. Finally, we can say that it is more effective to use aluminum alloy made part than steel made, because deformation that appears on Al made part is acceptable. Results for both cases are represented in the Table 38. Results meets requirements, so a designed part can be safely used.

| Max. stress [MPa] | 3 |
| Max. deformation (steel) [mm] | 0.0007 |
| Max. deformation (Al) [mm] | 0.002 |
| Stress reserve factor, j (steel) [-] | 43 |
| Stress reserve factor, j (400N)[-] | 33 |

Table 38 Bearing house results
For the load of 4 000 N there are results shown in the Table 39, gives us an understanding that if we increase the load 2 times still we are on the safe side.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. stress [MPa]</td>
<td>6</td>
</tr>
<tr>
<td>Max. deformation (Al) [mm]</td>
<td>0.0045</td>
</tr>
<tr>
<td>Stress reserve factor, j (400N)[-]</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 39 Bearing house results made of aluminum alloy, with the loading of 4000 N

4.7. Frame, 2B variant
In order to provide precise FEM analysis for a frame, I decided to create a full centrifuge model and to apply all loads that I know (neglecting the own weight) on it. I divided these loads for 3 types:

1) Load from tested capsule – Centrifuge force + Capsule mass (see Figure 4-8, Figure 4-9)
2) Load from the motor (see paragraph §4.5)
3) Mass load transferred by the shaft – mass of the centrifuge arms (with capsules on both sides), lugs, plates, fasteners. As it was calculated before, maximum weight of a centrifuge head is 60 kg, in order to be on a conservative side of a task, we will take that the maximum weight will be 100 kg, pointed downward on the shaft.

Calculation will be provided for several cases:

Case I – Symmetric loading, when both arms are loaded equally.
Subcase 1 – 0 degree (reference position of centrifuge arms)
Subcase 2 – 45 degree
Subcase 3 – 90 degree

Case II – Asymmetric loading, when only one arm is loaded.
Subcase 1 – 0 degree
Subcase 2 – 45 degree
Subcase 3 – 90 degree
Results for all cases and frame variants are shown in the Table 40 in the paragraph §4.7.1 on the page 94.

**Case I**

**Subcase 1**

As a reference centrifuge arm position, “0 deg”, will be taken arms position that are shown on the Figure 4-27. Loading from centrifuge force, testing capsule’s mass, moment from a drag force are located on both arms, specifically in Lug-out plates. Model is constrained by fixing 6 frame legs in 6 degree of freedom, also in for the FEM calculation I had to constraint a shaft in rotation about its own axis, otherwise model won’t be calculable. FEM model is shown on the

![Figure 4-27 0 deg. centrifuge's arm position, 2B var.](image)

![Figure 4-28 Case1.I, 2B var. 0deg. FEM model](image)

Full model results for the deformation only is shown on the next figure.
Maximum deflection of 2.4 mm is located on the tip of the centrifuge arm, which is expected place. For the frame only, the maximum deflection is 0.0265 mm; its location is in the place of shaft connection to the frame (thrust bearing connected to the frame), see Figure 4-30. According to the requirement (4.5), deformation of a frame member should not exceed the value of its length divided by 500. 

\[ \Delta_l = \frac{200}{500} = 0.4 \text{ mm} \]

and 0.0265 < 0.4 mm, i.e. results according to this requirement is acceptable.

**Subcase 2**

For the arms position of 45 degree from reference position results are: arms deflection 2.359 mm and maximum frame deflection is 0.0265 mm, at the same place as it was for previous subcase for both, see Figure 4-31. Compare to the previous subcase, deflection of the beams has increased only for 0.003 mm, so we can say that there was relatively no change happened, for the frame, literally there is no difference between deflections. Because both sides have same loadings located at the equal distances from the axis of a rotation, on the place of axis of a rotation appears two equal moments with different meaning (different signs), which cancels each other. In the end there is only one force left, which is 1000N on the shaft. This

* Values for the length of frame members are taken from the CAD file, also optionally can be taken from drawings.
force makes deformation of 0.0265 mm on those frame members where shaft is mounted by thrust bearing housing. All deformations are acceptable.

Subcase 3

For the arms position of 45 degree from reference position results are: arms deflection 2.288 mm and maximum frame deflection is 0.0265 mm, at the same place as it was for previous subcase for both, see Figure 4-32 and Figure 4-33.

With the frame happens no changes, compare to the Case I.1 and Case I.2.

All deflections are acceptable.

Generally we can say that for the symmetric loading of the centrifuge there are no major changes between different position of centrifuge arms during rotation. All frame members have small deflections and fits requirements. Regarding stresses that appears on the frame members by carrying loads, they are so small that there is no point to mention them and searching for their reserve factors. For instance, the maximum stress that has a frame member from all subcases of the Case I is equal to 0.07 MPa. It is located at the same place where is the maximum deflection of the frame or - 0.6 MPa at the constraints.
Case II

Subcase 1

Reference centrifuge’s arm positions was shown on the Figure 4-27. Applying load only for the one end, we can solve extreme case, when only one arm is under the load. In real practice, of course it is always better to put some counterweight on the other side in order to decrease a moment on the center. In order to be on the safe side of the question we are going to solve this problem only for the one arm loading. FEM model for analyzing is shown on the Figure 4-34.

Results from Figure 4-35 shows that deformation of the arm significantly increased, from 2.3 mm to 9.5 mm, however deflection for 9.5 mm of the beam is still acceptable, since this deflection will not affect results of the test.

Frame deformation is shown on the Figure 4-36. The place of the maximum deformation has changed from frame members that supports a shaft to the members that are connected to the Bearing house plate. The maximum deformation is 0.551 mm. According to requirements (4.5), allowable deflection is $\Delta_i =$
allowable deflection is $\Delta_2 = \frac{352.5}{500} = 0.7 \text{ [mm]}$. Maximum deformation 0.551 mm is less than limit value of 1.47 mm and 0.7 mm. Even though obtained results from the calculation fits requirements I decided reinforce the frame by adding one diagonal frame member to the place where this member will not worsen the work of the centrifuge machine. How this reinforcement looks, you can see on the Figure 4-37.

Figure 4-34 Case II.1, 2B var. FEM analyzing model

Figure 4-35 Case II.1, 2B var. Deformation results (10x of actual scale, black is undeformed state)

Figure 4-36 Case II.1, 2B var. Frame deformation result (100x actual deformation)
Deformation result of the reinforced frame model of the two-beam variant of the centrifuge (2B-R) is shown on the Figure 4-38. Maximum deflection now is 0.25 mm, which is twice smaller compare to the frame without reinforcement.

If we reinforce other “windows” except the one, where a belt goes through, then frame looks like as it is on the Figure 4-39.
As we can see from the Figure 4-40, the maximum deflection hasn’t changed that much (from 0.25 mm to 0.236 mm) and the its place hasn’t changed at all, however, this type of reinforcement can help to decrease the deformation from the load Case II.2 and Case II.3.

**Subcase 2**

For the arms position at 45 degrees from reference frame and frame model without any reinforcement there are next results: maximum deformation of the arm beams is 9.1 mm (Figure 4-41), which is slightly smaller than for the Case I.1 of the same frame variant; maximum frame member deflection is 0.43 mm. Allowable deflection is \( \Delta_2 = \frac{352.5}{500} = 0.7 \) [mm]. 0.43 < 0.7mm. It is seen that this result fits the requirement given by ISO 4356:1977.
For 2B-R variant of the frame, maximum deflection is 0.232 mm (Figure 4-43). Compare to the result from the 2B variant this result is 46% smaller, which state that frame is more rigid.

For 2B-AWR variant of the frame, maximum deflection is 0.198 mm (Figure 4-44). Compare to the result from the 2B variant and 2B-R variant this result is 54% and 14% less, respectively for 2B and 2B-R.
**Subcase 3**

When the centrifuge arms are located on the 90 degrees from reference position, arms deformation from the asymmetric load for 2B variant is 8.4 mm. For the centrifuge arms it is an acceptable deformation.

![Figure 4-45 Case II.3 2B var. Deformation result (10x actual deformation)](image)

![Figure 4-46 Case II.3 2B var. Frame deformation (100x actual deformation)](image)

Maximum deformation of the frame is $\approx 0.22$ mm, as it is seen from the Figure 4-46. Since the place of the deformation is the same, then it has similar requirements as it was for previous subcases, which is $\Delta_1 = 1.47$ mm and $\Delta_2 = 0.7$ mm. Definitely $0.213 < 0.7$ mm, which means that obtained results for the asymmetric load case fits requirements with enough margin till the limited deflection given by ISO standard.

For the frame variant 2B-R the maximum deflection is in the same place as for other variants, its value is 0.212 mm (Figure 4-47), compare to the 2B var. it is 0.47% less than 0.213 mm, or no change by other words.

For the frame variant 2B-AWR the maximum deflection is in the same place as for other variants, its value is 0.15 mm (Figure 4-48), compare to the 2B var. it is 31% less than 0.22 mm, and for 2B-R var. it is 29.2% less than 0.212 mm.
As it was mentioned before, on the frame we check mostly deformation, since we are interested in the rigidity of the frame. However, if we check maximum combined stress of the beam element in the FEM model we will see that maximum appeared stress from all solution of Case II is $\approx 3$ MPa.
4.7.1. Results, Frame 2B variants

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case I.1</th>
<th>Case I.2</th>
<th>Case I.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arms deformation [mm]</td>
<td>2.356</td>
<td>2.355</td>
<td>2.355</td>
</tr>
<tr>
<td>Frame deformation [mm]</td>
<td>0.027</td>
<td>0.027</td>
<td>0.026</td>
</tr>
<tr>
<td>$\Delta_{\text{min}}$ [mm]</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Safety margin [mm]</td>
<td>0.374</td>
<td>0.374</td>
<td>0.374</td>
</tr>
<tr>
<td>Safety margin [-]</td>
<td>15.094</td>
<td>15.094</td>
<td>15.504</td>
</tr>
<tr>
<td>Frame def. 1 [%]</td>
<td>100.00</td>
<td>0.00</td>
<td>-2.64</td>
</tr>
<tr>
<td>Frame def. 2 [%]</td>
<td>0.00</td>
<td>100.00</td>
<td>-2.64</td>
</tr>
</tbody>
</table>

Table 40 Frame deformation results for 10g accelerated mass of 15kg for two-beam modification of the centrifuge machine

For the symmetric case of loading, Case I.1, Case I.2 and Case I.3 there is no point to reinforce a frame, since there is no major changes between different variants neither in arms deflection nor in frame. For the asymmetric loading cases Case II.1, Case II.2 and Case II.3, of course, the best variant to use is the one with all available windows reinforced (see Figure 4-39), however the effectiveness of its use is not that big. If we compare difference in frame deformations between 2B-AWR variant and 2B-R, we will see that in percentage difference especially for the Case II.1 is very small. In other hand, in the loading Case II.3 percentage of the difference of the maximum frame deformation seems to be high, at the first sight, however the deformation itself of the initial variant 2B is relatively small and already has similar value to the maximum frame deformations of 2B-R frame variant.

In order to use material with higher efficiency, I would recommend to use frame variant 2B-R, where, if we compare with 2B variant only one frame member is added (see Figure 4-37). In this way, we reduce the weight of the frame, the time demand to assemble/disassemble machine (compare to 2B-AWR var.); the risk of the stability loss, exceeding structural limits given by ISO 4356:1977 (compare to 2B variant); and finally we will have relatively same value of deformation on every position of the centrifuge arms, which means small fluctuation of the frame members that will provide relatively static (quasi-static) condition of the centrifuge machine. Frame deformation: 0.25 → 0.231 → 0.212; Arms deformation: 8.673 → 8.6 → 8.531.

From the diagram shown on the Figure 4-49, we can read that the amplitude of the frame maximum deformation is:

$$A_{F_{\text{def}}} = 0.25 - 0.212 = 0.038 \text{ [mm]}$$

(4.11)

For example, amplitude for the 2B var. is 0.33 mm, for 2B-ARW var. is 0.086 mm.
For the maximum arms deformation amplitude is:

\[ A_{\text{arm def}} = 8.673 - 8.531 = 0.142 \]  

\[ \text{Fig. 4-49 Deformation vs time, 10g load, 2B-R var., Frame} \]

\[ \text{Fig. 4-50 Deformation vs time, 10g load, 2B-R var., Arms} \]

### 4.8. Frame, 1B variant

Calculation approach will be similar as it was done for 2B variant, with exactly same loading values and their cases, as it was described in paragraph §4.7 on the page 83. There will be used same variant modifications as for 2B variant, which means that 3 variants will be calculated 1B, 1B-R and 1B-AWR. Difference between these modifications you can see on the picture Figure 4-37 for 2B-R variant (but instead of two beams arm will be used one beam arm), Figure 4-39 for 2B-AWR (same for 1B-AWR). For example, 1B variant of asymmetric loading at 0 degree position of centrifuge arms (Case II.1) is shown on the Figure 4-51.

Since deformation profiles of arms and frame are similar to the 2B variant, I found no need to put figures here with of deformations. Instead, I will write just a result table analogic to the result table for 2B variant.

All deformations results and requirements you can find on the Table 41 on the page 96.
4.8.1. Results, Frame 1B variants

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case I.1</th>
<th>Case I.2</th>
<th>Case I.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var.</td>
<td>1B</td>
<td>1B-R</td>
<td>1B-AWR</td>
</tr>
<tr>
<td>Frame deformation [mm]</td>
<td>0.027</td>
<td>0.028</td>
<td>0.027</td>
</tr>
<tr>
<td>$\Delta_{\text{min}}$ [mm]</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Safety margin [mm]</td>
<td>0.373</td>
<td>0.373</td>
<td>0.373</td>
</tr>
<tr>
<td>Frame def. 1 [%]</td>
<td>100.00</td>
<td>1.48</td>
<td>-1.11</td>
</tr>
<tr>
<td>Frame def. 2 [%]</td>
<td>-1.45</td>
<td>100.00</td>
<td>-2.55</td>
</tr>
</tbody>
</table>

Table 41: Table of deformation results for 1B variants

From the result table we can say that, again optimum variant to use is the 1B-R, because of same reasons as it was described for the Table 40 on the page 94.

Another thing, which is interesting is the difference in deformations between 1B and 2B variants. For symmetric load cases, arms deformations of 1B variant are approximately twice bigger than analogical values of 2B variant. This result however is expected somehow, however deformations for asymmetric load cases are similar and deformation for 1B variants are slightly smaller than for 2B variants. For both cases were used same frame models, same materials, same CBEAM characteristics and was used the same solver. From this point of view we can state the 1B – R variant is better than 2B – R variant due to less material usage and assembly time. Less material usage follows in to the less weight of the centrifuge’s head that will decrease frame member deformation at the place of a shaft mounting to the frame by thrust bearing housing. At the same time for the symmetric cases, 2B-R variant is better, from the deformation point of view.

Generally, I strongly recommend using a counterweight if only one sample is tested and it is better to make counterweight as close as possible to tested sample by mass characteristic and shape.
4.9. Centrifuge’s head

4.9.1. Centrifuge’s Head Plate

1B Case I

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

![Figure 4-52 CHP, 1B, Case I, steel made](image1)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. stress [MPa]</td>
<td>31 MPa</td>
</tr>
<tr>
<td>Max. deformation [mm]</td>
<td>0.068</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>4.19</td>
</tr>
</tbody>
</table>

*Table 42 CHP, 1B, Case I result table*

1B Case II

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

![Figure 4-53 CHP, 1B, Case II, steel made](image2)
Max. stress [MPa] | 42 MPa  
---|---
Max. deformation [mm] | 0.13  
Stress reserve factor, j [-] | 3.1  

*Table 43 CHP, 1B, Case II result table*

### 2B Case I

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

**Figure 4-54 CHP, 2B, Case I, steel made**

Max. stress [MPa] | 12 MPa  
---|---
Max. deformation [mm] | 0.05  
Stress reserve factor, j [-] | 10.8  

*Table 44 CHP, 2B, Case I, result table*

**Figure 4-55 CHP, 2B, Case II, steel made**
Summary

Again, we can see that for the symmetric case it is better to use a 2B variant, however when the load is completely asymmetric 1B variant is better for the stress and for the deformation.

Very big value of the stress of the 2B Case II can be explained. In the FEM model holes are modulated as squares, which due to presence of right angle increase the stress peak much higher than it should be in reality, this is so called local notch. In addition the force transmitted from the Lug-out part to the Lug (female lug) in 2B variant is higher than for the 1B variant, see Figure 4-11.

4.9.2. Hub

The most critical load case is the asymmetrical load, Case II for 1B variant. Its stress and deformation are shown on the Figure 4-56.

<table>
<thead>
<tr>
<th>Max. stress [MPa]</th>
<th>120 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. deformation [mm]</td>
<td>0.5</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>1.1</td>
</tr>
</tbody>
</table>

*Table 45 CHP, 2B, Case II, result table*

<table>
<thead>
<tr>
<th>Max. stress [MPa]</th>
<th>47 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. deformation [mm]</td>
<td>0.008</td>
</tr>
<tr>
<td>Stress reserve factor, j [-]</td>
<td>2.76</td>
</tr>
</tbody>
</table>

*Table 46 Hub, 1B, Case II, table result*

All results satisfies all necessary condition.
4.10. Shaft’s critical speed

It was already provided a hand calculation for this problem in the paragraph §3.3.2, p. 32. There was made an assumption that the shaft is massless, which is not true, it was done to simplify the problem. The advantage of the FEA will help to find the critical shaft speed with proper mass model of the shaft. For this purpose was used MSC Patran software as pre-processor and MSC Nastran software as a calculator.

Shaft was designed as 1D element, namely CBEAM. Shaft was divided into 5 sections. Additional masses, as a centrifuge’s head of 60 kg mass and driven pulley of 4 kg mass were modeled as a shaft section with relatable cross-section. Driven pulley was modeled as a beam with circular cross-section of radius \( R_{M_2} = 84 \text{ mm} \) and density \( \rho_{M_2} = 6.445 \times 10^{-6} \text{ kg/mm}^3 \) in order to have a proper mass character in this section. Centrifuge’s head was modeled as a thin rectangle of thickness \( t = 5\text{ mm} \), height \( H = 230 \text{ mm} \) and width \( W = 190 \text{ mm} \), density of this section is \( \rho_{M_1} = 2.746 \times 10^{-4} \text{ kg/m}^3 \). FEM shaft model is shown on the Figure 4-57.

![Figure 4-57 Shaft FEM model (CBEAM)](image)

Using rotor dynamic tool in MSC Patran, we have defined rotational orientation and speed unit. The solution method was chosen to be Complex Eigenvalue solver with direct formulation, SOL 107. The solution result is chosen to be given as a Campbell diagram (plot represents a system's response spectrum as a function of its oscillation regime; Eigen frequencies as a function of the shaft's rotation speed. This
case is also called "whirl speed map" [17] [18]). In addition, FEM will help to show what shape of each mode will be.

From the Campbell diagram, Figure 4-58, it is seen that first critical speed is 2.52 rps (very first intersection between first speed and Eigen frequency of Mode 1 = Mode 2), which makes 151.2 rpm.

Comparing results from FEM calculation with hand calculation (see eq. (3.83)) where the first critical speed was 17.5 rad/s = 2.7 rps, we can see that the result given by numerical solution is relatively smaller than the one we calculated by hand. It is because the FEM solution was solving with another method and with proper mass model of the shaft.

Even though the FEM result is less, still it is bigger than the chosen minimum required speed, which is 2 rps or 120 rpm. The shaft design satisfies requirements.
In order to find a critical force of the shaft in buckling, we have to use non-linear Nastran solver SOL106. We apply on the free end of the shaft FEM model that is shown on the Figure 4-57 a nodal force 1 000 000 N. When a deformation of the shaft (of specific node), under the percentage of the applied load, will start to be non-linear, then this load will be counted as a critical load.

**4.11. Shaft’s force (Buckling)**

In order to find a critical force of the shaft in buckling, we have to use non-linear Nastran solver SOL106. We apply on the free end of the shaft FEM model that is shown on the Figure 4-57 a nodal force 1 000 000 N. When a deformation of the shaft (of specific node), under the percentage of the applied load, will start to be non-linear, then this load will be counted as a critical load.

*Figure 4-59* Shaft rotational shape, Mod 1 = Mod 2 (f=2.52 Hz), Mod 3 = Mod 4 (f = 5.55 Hz) and Mod 5 = Mod 6 (f = 6.65 Hz)

*Figure 4-60* Shaft's node deformation vs Force
From the Figure 4-60 it is seen that the last point when deformation has a linear dependency on the force is approximately 27.5% of 1 000 000 N, which is 275 000 N.

\[ F_{\text{crit}} = 275 000 \text{ [N]} \]

\[ F_{\text{all}} = \frac{F_{\text{crit}}}{f} = \frac{275 000}{2} = 137 500 \text{ [N]} \approx 135 000 \text{[N]} \]

(4.13)

Allowable compression force of the shaft is then 135 000 N, which is \( \approx 13500 \text{ kg} \). Which is far beyond of needs, and shaft seems to be oversized for the buckling. However, due to bending and critical speed, we can not reduce the diameter in order to remain sufficient margin safety.

### 4.12. Fasteners

All bolts, nuts, washers and threaded rods are chosen from the standards and can easily be purchased on the market. From the global FEM model, I chose the most critical stress that appeared on the fastener. All fasteners were modeled as a beam element with circular cross-section. The result is written in the table, material of standard fasteners are taken from the NX material library.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Parts connected</th>
<th>( \sigma_U ) [Mpa]</th>
<th>( \sigma_{\text{all}} ) [Mpa]</th>
<th>( \sigma_{\text{max}} ) [Mpa]</th>
<th>j [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>rod M5 x 80</td>
<td>Lug - 45x45 profile</td>
<td>275</td>
<td>137.5</td>
<td>10</td>
<td>13.75</td>
</tr>
<tr>
<td>DIN M6 x 35</td>
<td>CHP - Hub</td>
<td>275</td>
<td>137.5</td>
<td>45</td>
<td>3.06</td>
</tr>
<tr>
<td>M6 x 80</td>
<td>45x45 - CHP</td>
<td>275</td>
<td>137.5</td>
<td>65</td>
<td>2.12</td>
</tr>
<tr>
<td>DIN M8 x 55</td>
<td>BHP - 45x45</td>
<td>275</td>
<td>137.5</td>
<td>25</td>
<td>5.50</td>
</tr>
<tr>
<td>321345.1</td>
<td>45x45 - 45x45 (angle connection)</td>
<td>100</td>
<td>50</td>
<td>2</td>
<td>25.00</td>
</tr>
<tr>
<td>DIN M12 x 40</td>
<td>BHP - Bearing House</td>
<td>137</td>
<td>68.5</td>
<td>5</td>
<td>13.70</td>
</tr>
<tr>
<td>DIN M12</td>
<td>UCF 209 - 45x45</td>
<td>137</td>
<td>68.5</td>
<td>5</td>
<td>13.70</td>
</tr>
<tr>
<td>M8</td>
<td>Connection between Frame members</td>
<td>137</td>
<td>68.5</td>
<td>4</td>
<td>17.13</td>
</tr>
</tbody>
</table>

*Table 47 Fasteners FEM result table*

All reserve factors are above than one, which means all designed fasteners satisfies necessary conditions. Some of the fasteners are oversized, however from the design point of view it is not advantageous to redesign these connections.
5. Remarks

In the remarks, I would like to recalculate the time required to reach 10g and 5g with the complete knowledge of the mass moment of inertia and torque on that will be applied on the Centrifuge’s head.

From the CAD software Siemens NX 11\textsuperscript{®} we can read the weight characteristics of the Centrifuge’s head with all lugs, pins, bearings, fastener and attached capsule with 15 kg mass and size of 340 x 650 (D x H in mm). Final result for the mass moment of inertia for 2B centrifuge’s head is:

\[ I_{\text{max}} = 38135230 \text{[kg.m]}^2 = 38.14 \text{[kg.m]}^2 \approx 40 \text{[kg.m]}^2 \]  

(5.1)

Using method described in the paragraph §2.3, p. 8 we can recalculate the time that will be need to reach such a rotation that will create a 10 g loading.

Using gear ratio of \( i = 1.6 \), there will be next values:

<table>
<thead>
<tr>
<th>rps</th>
<th>( T ) [N.m]</th>
<th>( \omega ) [rad/s]</th>
<th>( v ) [m/s]</th>
<th>Re[-]</th>
<th>( 2 \times M_d )</th>
<th>( T_2 ) [N.m]</th>
<th>( \alpha' ) [rad/s2]</th>
<th>t'[s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>30.40</td>
<td>0.76</td>
<td>0.00</td>
</tr>
<tr>
<td>0.37</td>
<td>20.5</td>
<td>2.32</td>
<td>2.32</td>
<td>54138.68</td>
<td>1.61</td>
<td>29.55</td>
<td>0.74</td>
<td>3.15</td>
</tr>
<tr>
<td>0.95</td>
<td>21.2</td>
<td>5.97</td>
<td>5.97</td>
<td>139004.72</td>
<td>10.61</td>
<td>21.61</td>
<td>0.54</td>
<td>11.05</td>
</tr>
<tr>
<td>1.5</td>
<td>20.44</td>
<td>9.42</td>
<td>9.42</td>
<td>219481.13</td>
<td>26.45</td>
<td>4.62</td>
<td>0.12</td>
<td>81.66</td>
</tr>
<tr>
<td>1.576</td>
<td>20.35</td>
<td>9.90</td>
<td>9.90</td>
<td>230653.77</td>
<td>29.21</td>
<td>1.72</td>
<td>0.04</td>
<td>230.60</td>
</tr>
<tr>
<td>2</td>
<td>20.05</td>
<td>12.57</td>
<td>12.57</td>
<td>292641.51</td>
<td>47.03</td>
<td>-16.55</td>
<td>-0.41</td>
<td>-30.37</td>
</tr>
</tbody>
</table>

\textit{Table 48}

Where \( 2 \times M_d \) is the moment from the drag force from the both capsules of 340 x 650 mm size, \( T_2 \) is the torque on the shaft. Time that is required to reach the tested sample of 10g acceleration is 230.6 [s]. In order to reach 5g load we need as rotational speed to be 1.15 rps, which makes approximate 30 seconds.

\[ \text{Figure 5-1 Time demand for specific rotational speed, for } I=40 \text{kg.m}^2 \text{ and a drag from both sides of arms.} \]

Please note that, the drag force, respectively moment is not necessary always equal to the value that is written in the Table 48, since it is the function of the shape of the tested sample by manipulating with it we can decrease drag force, hence a resisting drag moment. The less drag moment is then less time is needed to reach the desired rotational speed. In addition, drag force has not to be always symmetrical, since the second arm can be counterweighted by a mass of different shape.
If we would like to reduce the time that is needed to reach specific rotational speed, also, we can use a motor with better torque characteristics. For example, if we use a motor with constant torque value \( T = 35 \text{ N.m} \) up to \( 2 \text{ rps} \) and transmission with ratio \( i = 1.25 \), the time to reach speed that will load tested sample for \( 10\text{g} \) will be \( t = 41.25 \text{ s} \).

Another way to improve this machine is to add a rope that will be pinned into the centrifuge’s arms and connected to the Holder that. For example, preliminary, as it shown on the Figure 5-2. This will help reduce a deformation of the centrifuge’s arms.

![Figure 5-2 1B centrifuge with rope (or rod) support](image)

Mainly, space rockets accelerates approximately with \( 5\text{g} \) value. For the acceleration the designed centrifuge machine fits perfectly with accomplishing all necessary requirements, however the maximum acceleration, for which the machine is designed is \( 10\text{g} \).

**Technological remarks**

*Lug-out*

Centrifuge’s part Lug-out was meant to be manufactured by welding connection of two aluminum elements, however aluminum welding is the complicated task and requires from the person specific skills, which potentially can lead into the increasing of production cost, because will be needed skillful person who needs higher salary. Steel welding is easier compare to an aluminum one. Changing material from aluminum to steel if we remain geometry similar as it was before we will decrees cost by decreasing labor working time, but the mass of the part will be increased that will lead into the time of reaching the desired rotational speed (mass moment of inertia). Material swapping will simplify the task.

I don’t find it necessary to recalculate the strength and rigidity of the part, since traditionally, steel is more strengthen and more rigid than aluminum. Young’s modulus of the steel is 3 times higher than aluminum (210 GPa against 70 GPa). It would be required to recalculate the task if we decide to change geometry (primarily thickness) of the part, but it is cheaper and easier to use plate of the same thickness for all shell parts of the machine, which is 5 mm. The reason is that it is nearly impossible to by in the market raw
material like a steel metal sheet of small size like 400x400mm; usually nominal size starts from 1000x1000 mm. So all plate look alike parts of the centrifuge machine will be produced form 5mm steel sheet.

Shaft

At the first place shaft had to be manufactured from the aluminum material (AlMgSi0.5F25), however after consultation with the technology staff of Aerospace Department of Czech Technical University, it was stated that it is complicated task to manufacture by turning aluminum bar of 60mm cross-section with necessary tolerances. It was recommended, again, to change material to the designed part from aluminum to steel, so it will be easier for the university staff to manufacture this part. Changing shaft material will increase critical speed, which is advantageous for us, but it will increase the weight of the rotating part, so the time required reaching the desired rotational speed will increased too. The critical speed at the first mod (the first critical speed) is $\approx 180$ rps, which is more than enough.

By changing the shaft material to steel, which has tensile modulus as 210 GPa and Rp02 is 275 MPa, the minimum diameter that can transfer loads that were mentioned in the paragraph §3.3.1 on the page 26 is then 35 mm. We will remain all geometry same as it was on aluminum made shaft except the section 4, the one that is connected to the thrust bearing. By reducing diameter from 45mm to 35mm we can change thrust bearing and its housing from UCF 209 to UCF 207, which will reduce the weight of the structure and cost.

Cover

It is good to cover the frame of the machine by plastic or wooden plate, so the dust will have limited access to the moving parts of a centrifuge, such as belt, shaft, bearings etc. There is no need to calculate the strength of it, because cover will not transfer any loadings and will execute only decorative function and protection from the dust or any other small particles. How preliminary wooden cover would look like you can see on the Figure 5-3.

![Figure 5-3 CAD model of centrifuge with wooden cover](image-url)
6. Conclusion

The main goal of the work was to design a centrifuge machine that can load a tested sample up to 10g, which is 98.1 m.s\(^2\). Tested sample together with its capsule (box, cylinder or any other element that will be connected to the centrifuge arm and will carry a tested sample) should not exceed the mass of 15 kg. In this work, preliminary tested sample was chosen to be a cylindrical capsule of 650mm height and 350mm diameter. Centrifuge performance calculation was made according to chosen capsule that will call a drag force during the rotation. Bearing friction was neglected due to absence of precise date without experiment, as well as a drag force of the arms, due to no data of drag coefficient of a 45 x 45 profile and compare to the drag force and then drag moment to the rotation centre it very small, so it can be neglected.

The frame and centrifuge’s arms were made from the aluminum profile 45 x 45 from Alutec KK supplier. Connection elements of the profile, again, were supplied by the Alutec KK company. Parts of the centrifuge machine that are not standardized, the one that are not selling in the market and requires manufacturing, were designed in the way that it will be cheap to manufacture them by the devices that are in the Aerospace Department workshop.

The centrifuge arm distance, the distance between rotation centre and pin centerline where a capsule with a tested sample is connected, is 1m. The centrifuge’s span is then 2m. The centrifuge’s head was designed in the way that it can be assembled as centrifuge with one-beam arm (1B) or two-beams arm (2B). Calculation showed that it is better to use 1B, due to less usage of a material and smaller deflection in asymmetric cases of load. However, for the symmetric loading cases the deflection of the arms is twice smaller for the same loading, but for 1B variant.

According to the calculation in paragraphs §4.7.1 and §4.8.1 on the page 94 and 96 respectively the optimum variant of the frame is the one reinforced window 2B-R or 1B-R, as it is shown on the Figure 4-37 p. 89.

Preliminary time demand to reach a desired rotation when centrifuge’s arm tip reaches 10g acceleration is 4 minutes, for the gear ratio \(i = 1.6\). The maximum speed of a rotation and torque can be manipulated by changing gear ratio that can be done by changing various pulleys. For details, see Figure 3-13 p.24. All centrifuge elements were designed to safely operate (safety factor \(f=2\)) for the rotational speed 95 rpm, which we can call as a maximum allowable speed of the centrifuge. However, due to high safety margins it can rotate maximum up to 120 rpm.

\[
n_{\text{all}} = 95 \, \text{rpm} \quad (6.1)
\]

All necessary dimensions you can find in the attached assembly drawing for 2B-R, drawing number is 10-02-002.
References


