Aerospace Department

Centrifuge for small experiments

A CubeSat-class satellites

Master's Thesis

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Annotation List

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Abstract:

This diploma work deals with construction of centrifuge machine for hypergravity simulation for samples of CubeSAT sizes. First, a small overview of centrifuges is presented, afterwards will be performed calculation of needed performance according to requirements and given conditions. Last, design of the centrifuge machine will be performed using known methods and technics, like analytical hands calculations, analytical calculations with the help of MATLAB® software and Finite Element Method (FEM).

Declaration:

I declare that I wrote my master (diploma) thesis independently and I used only references that are mentioned in attached list

Prohlášení:

Prohlašuji, že jsem svou bakalářskou práci vypracoval samostatně a použil jsem pouze podklady uvedené v přiloženém seznamu.

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Attachment's content

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2. DVD

    CAD files

    Drawings is PDF

    Excel file

    FEM files

    Matlab files

    Word and PDF document of thesis

List of used software

Siemens NX 11.0

Microsoft Excel/Word 2013

Beta CAE Ansa 16.0

Siemens FEMAP v11.3

Matlab R2014b

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Nomenclature, Abbreviations, Acronyms

1B – one-beam centrifuge's head variant/ simple one-beam centrifuge variant
1B-AWR – one-beam centrifuge with all possible windows reinforced variant
1B-R – one-beam centrifuge with one reinforced frame window variant
2B – AWR – two-beam centrifuge with all possible windows reinforced variant
2B – two-beam centrifuge's head variant/ simple two-beam centrifuge variant
2B-R – two-beam centrifuge with one reinforced frame window variant
A – area
CHP – centrifuge head's plate
C_c – centrifugal force of a capsule
C_D, c_D – drag coefficient [-]
CG – centre of gravity
D_P – pitch diameter [mm]
e – eccentricity
E – Young's modulus [MPa]
F_C – Capsule force, for (3.85) p. 47
F_C – centrifugal force [N]
F_D, F_d – drag force [N]
G_P – the weight load of 45x45 profile
g – gravitational acceleration, g = 9.81 [m.s^{-2}]
G – shear modulus [MPa]
Gc – the weight loading of a capsule
I – mass moment of inertia [kg.m^2]
j – reserve factor [-]
J – second polar moment of area [m^4], [mm^4]
k – stiffness [N/m], [N/mm]
k_{bru} – shear – bearing efficiency factor
k_t – net tension efficiency factor
k_{try} – efficiency factor for transverse yield load
M, M(x) – moment, Bending moment [N.m], [N.mm]
MBP – motor base plate
M_D, M_d – moment caused by drag force/ Drag moment [N.mm], [N.mm]
m – mass [kg]
m_C – mass of a capsule [kg]
m_P – mass of a 45x45 profile [kg]
n – number of revolutions
O – centrifugal force on a rotating shaft
p – contact pressure (feather key)
P_Y – yield transverse load
P_{bru} – ultimate load for shear – tear out and bearing failure
P_{bry} – bushing yield bearing load
P_{tu} – ultimate load for tension failure
q – distributed force [N.m^{-1}], [N.mm^{-1}]
r – radius/ rotational distance [m], [mm]
Re – Reynolds number [-]
RF – reserve factor [-]
Rp0.2 – yield strength [MPa]
rpm – number of revolutions per minute
rps – number of revolutions per second

t – time [s]
T – torque [N.m], [N.mm]
TB – taper bush
v – circumferential velocity/ translation velocity [m/s]
V, V(x) – shearing force [N]
w – distributed force [$N.m^{-1}$], [$N.mm^{-1}$]
W – section modulus [m^3], [cm^3], [mm^3]
λ – fitting factor [-]
Ω – eigen frequency/ critical angular velocity
L – angular momentum [$kg.m^2.s^{-1}$]
f – safety factor [-]
ρ – density [$kg.m^{-3}$]
σ – stress [MPa]
σ_Y – yield stress [MPa]
σ_{cy} – compression yield stress of bushing material
σ_{red} – reduced stress [MPa]
σ_{tux} – ultimate tensile stress in x – direction of the material
σ_{tyy} – tensile yield stress of lug material in y – direction
τ – shear stress [MPa]
v - deflection [mm]
ω – angular velocity [rad/s]

1. Introduction

A centrifuge is a machine, respectively device, which uses a rotation of an object around fixed axis to produce a centripetal force, and due to reaction – centrifugal force.

There are variety of the applications of a centrifuge. It is widely used in pharmaceutical, food and general chemistry industry (using sedimentation principle). Centrifuge that can provide very high rotational speed can separate particles to an extremely low scale, down to molecules for example. Washing machines, pumps also uses centrifuge for their work. It can be used for geotechnical purposes. For example geotechnical centrifuge C60 (Figure 1-1), which is designed by Actidyn, can simulate behavior of structures on foundation and of soil mechanics, with the help of reduced scale model.

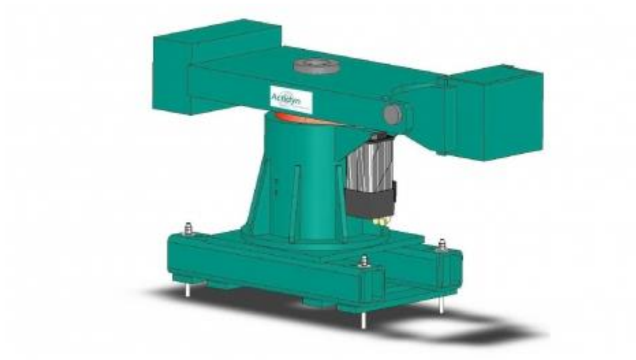


Figure 1-1 C60, geotechnical centrifuge [1]

Another industries, where centrifuge is used are aviation and space technologies. Using centrifuge machines we can simulate flight phases, loading factors from maneuvers and so on. This can be very helpful for pilots who just have begun their professional path. For example, using a Phoenix centrifuge (Figure 1-2) of NASTAR Center we can simulate human performance under high accelerations, space training, high risk maneuvers training. [2]

Figure 1-2 Phoenix centrifuge [2]

Purpose of this work is to design a centrifuge that can carry a payload of 15 kg and can accelerate up to 10g. Normally to simulate a space launch for the small centrifuges, for example CubeSat, required acceleration of 5g, however it is advantageous to use a centrifuge for a larger scope of work, that's why was made a decision to design a centrifuge for 10g.

2. Preliminary Performance Calculation

The assignment of this part is to find the optimum performance characteristic as well as the optimum arm length of the arm-centrifuge machine.

The demand is to obtain the gravity acceleration of the value of 10 g, by using an advantage of the centrifugal force. We have given preliminary design parameters of the capsule, where will be hold the experimental unit. The capsule has the shape of the cylinder with the height of **H = 0.65 m**, diameter **D = 0.34 m** and mass **m = 15 kg**.

2.1. The Centrifugal and Drag Force

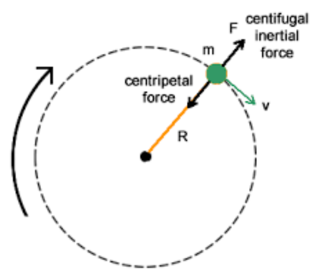


Figure 2-1 Centrifugal force [3]

According to Newton's Law of Inertia, an object in motion tends to follow a straight line. Applying a sideways force on the object can overcome the inertia and cause the object to take a curved path. That force is called a centripetal force.

Newton's Third Law or Action-Reaction Law states that for every applied force, there is an equal and opposite force. In other words, when you apply a force on a rope in swinging an object around you, you will feel an equal and opposite force pulling the object away from you. This force is the centrifugal force. [3]

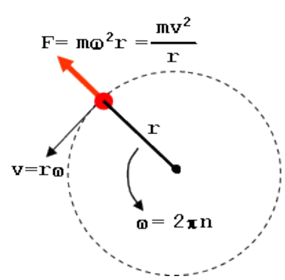


Figure 2-2

The centrifugal force is:

$$F_C = m\omega^2 r \rightarrow v = \omega r \rightarrow \frac{mv^2}{r} \tag{2.1}$$

Required gravitational acceleration is 10 g. Then:

$$m * 10g = m\omega^2 r = \frac{mv^2}{r} \tag{2.2}$$

Where m is the mass of the capsule with experimental unit.

We can see that the mass is located on the boss sides of the equation, so we can using advantage of the equation calculation and cut them.

$$10g = \omega^2 r = (2\pi n)^2 r \quad or \quad 10g = \frac{v^2}{r} \tag{2.3}$$

The task is to find the most useful and the most advantageous angular velocity (rotations) and arm radius.

Limitation parameters are the electric motor performances, where the one of the most important parameter is the torque of a motor. The values of torque are represented as the dependency diagram of Torque [N.m] versus speed [rps].

The given electric motor is ES-MH 342200 3-phase steepen motor. Torque-speed characteristic are shown on the Figure 2-3

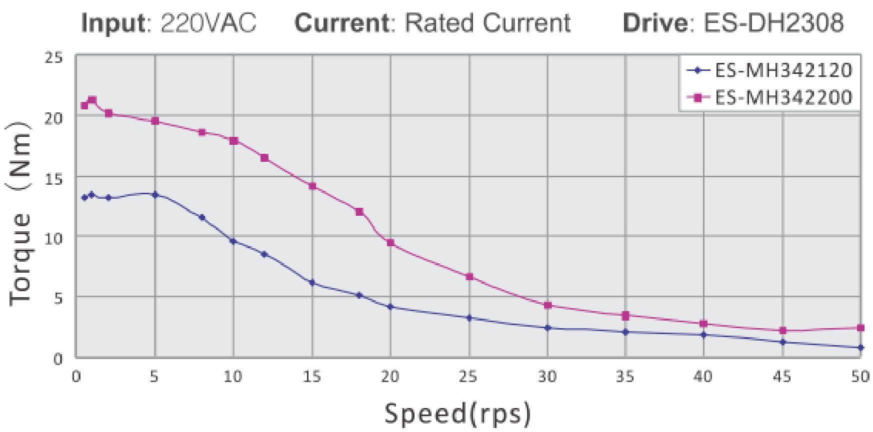


Figure 2-3 Torque vs speed char, of the ES-MH 34xx series [4]

Very important thing that have to be considered during calculations of optimum values of speed and arm length is the drag force that will act opposite direction of the circumferential velocity of the structure, other words, the drag force will cause a resistance force and moment to the working structure. It will directly effect on the ability of structure to reach the maximum angular speed. If the moment caused by drag force will be lower than a toque of a motor, the centrifugal machine will have ability to increase the revolution speed. The machine cannot overcome the specific value of angular velocity, because t=otherwise the drag moment will be higher than motor torque. This specific value is the maximum speed that is reached when the drag moment is equal to the torque of the given motor. Note that the torque is different for every different rotation speed.

The equation of the drag force is:

$$F_D = \frac{1}{2}\rho v^2 A c_D \tag{2.4}$$

Where ρ is the density of the working fluid, which is the air in our case. For the first shoot of the calculation will be used the value of 1.225 kg/m^3, that represents the air density on the sea level at 15 $°C$. More correct value will be obtained from ambient condition of the laboratory.

C_D is the drag coefficient. Its value depends on the working fluid, shape of streamlining body (cylinder) and Reynolds number. See Figure 2-4.

A is the normal cross-section area of the streamlined body. Specifically in our case:

$$A = H * D = 0.65 * 0.34 = 0.221\ [m^2] \quad (2.5)$$

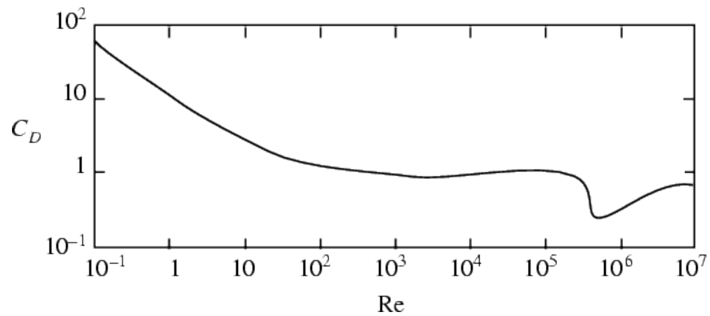


Figure 2-4 Dependency of the Drag Coefficient on the Re [5]

2.2. Preliminary calculation of the performance and arm length

As it was mentioned in the beginning, the required load should reach 10 g = g_{req}. As the first step we will try to guess the arm length. Let's say it will be r = 0.5 m. Then using this value of radius we will find the appropriate angular velocity.

$$\omega_{req} = \sqrt{\frac{g_{req}}{r}} = \sqrt{\frac{9.81}{0.5}} = 14.00714\ \left[\frac{rad}{s}\right] \quad (2.6)$$

Transform it to the revolutions speed:

$$rps = \frac{\omega}{2\pi} = \frac{14}{2\pi} = 2.2293\ [rps]$$

Or

$$rpm = 60 * \frac{\omega}{2\pi} = 60 * \frac{14}{2\pi} = 133.7583\ [rpm] \quad (2.7)$$

The circumferential speed on the tip of the arm is:

$$v = \omega r = 14 * 0.5 = 7\ \left[\frac{m}{s}\right] \quad (2.8)$$

Respective Reynolds number for obtained speed and cylindrical speed is:

$$Re = v * \frac{D}{\nu} = 7 * \frac{0.34}{1.46 * 10^{-5}} = 163097\ [-] \quad (2.9)$$

The suitable drag coefficient for this Reynolds number is $c_D = 1.1\ [-]$. The drag force acted on the capsule is then:

$$F_{D_C} = \frac{1}{2}\rho v^2 * A * c_D = \frac{1}{2} * 1.225 * 7^2 * 0.221 * 1.1 = 7.3035\ [N] \quad (2.10)$$

Moment caused by drag force of capsule. Note that this moment caused only by one loaded arm.

$$M_{D_C} = F_{D_C} * r = 7.3035 * 0.5 = 3.6517\ [N.m] \tag{2.11}$$

Applying same method for other radiuses, we will construct tables.

Arm [m]	0.5	0.6	0.75	1	1.25	1.5	1.75
ω [rad/s]	14.007	12.786	11.436	9.904	8.859	8.087	7.487
rps	2.229	2.035	1.8202205	1.576	1.409937	1.28709	1.192
rpm	133.758	122.1041	109.21323	94.581	84.5962	77.22542	71.497
Angle [^{O}]	84.29	84.29	84.29	84.29	84.29	84.29	84.29
v_{circum} *[m/s]*	7.004	7.672	8.577	9.904	11.073	12.131	13.102
F_{D_C} *[N]*	7.303	8.76	10.955	14.61	18.25	21.91	25.56
M_D *[N.mm]*	3.6517	5.258	8.216	14.61	22.823	32.866	44.733
Re [-]	163096	178663	199752	230653	257878	282492	305126

Arm [m]	2	2.25	2.5
ω [rad/s]	7.003	6.603	6.264
rps	1.115	1.051	0.997
rpm	66.88	63.05	59.82
Angle [^{O}]	84.29	84.29	84.29
v_{circum} *[m/s]*	14.007	14.856	15.66
F_{D_C} *[N]*	29.21393	32.86568	36.517
M_D *[N.mm]*	58.43	73.95	91.29
Re [-]	326193	345980	364695

Table 1 Finding optimum values

Angle is the angle of inclination that is denoted as a on a figure below.

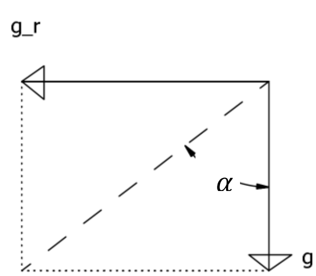


Figure 2-5 Inclination angle

For easier comparison of obtained values, we will construct dependency diagrams.

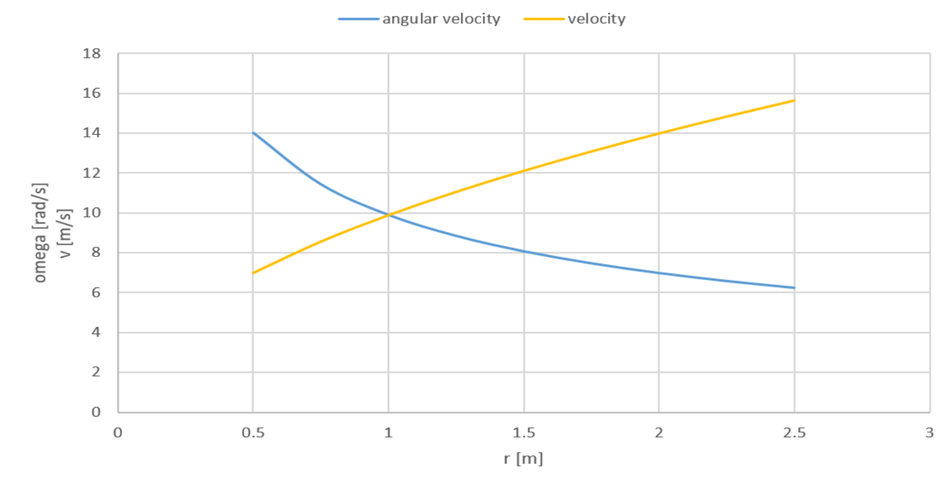


Figure 2-6 Velocity versus arm length r

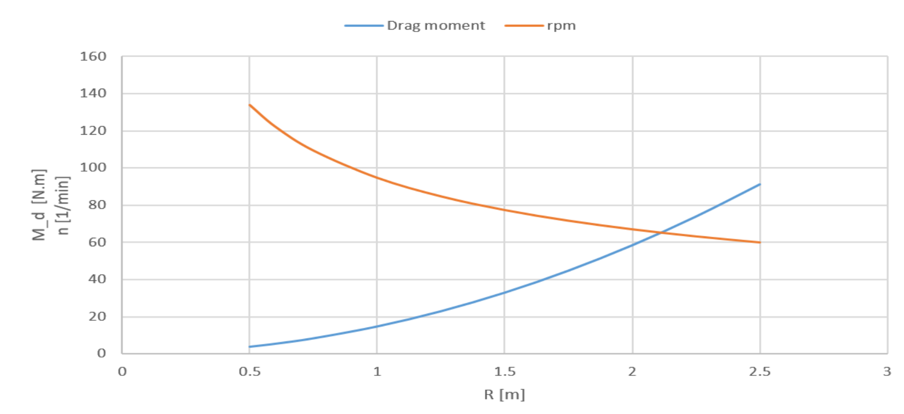


Figure 2-7 Drag moment and revolution with the changing arm length r

The less the moment caused by drag force, the better. The smallest one is convenient for smallest radius, but for smallest radius, we have to have the higher speed. As my personal view, the optimum solution is the arm length of 1m.

Arm [m]	1
ω *[rad/s]*	9.904
rps	1.576
rpm	94.581
Angle $[^{O}]$	84.29
v_{circum} *[m/s]*	9.904
F_{D_C} *[N]*	14.61
M_D *[N.mm]*	14.61
Re [-]	230653

Table 2 Selected parameters

For these chosen values will be continued further calculations.

The essential difference between a centrifuge with a swinging capsule and a centrifuge with a fixed one is the way how the centrifugal force applies on the capsule. The bottom of the capsule is constantly perpendicular to the sum of the earth gravitation vector and of the centrifuge acceleration vector and the distance between centre of gravity of the capsule and center of rotation increases and this causes the increment of acceleration of a particular point, like centre of gravity. In order to find a gradient of the acceleration of this point, including the effect of an angle increasing (increasing a distance between point and rotation centre) we will use an assumption that arm length is 1m and the distance between arm's tip and centre of gravity of the tested capsule is 425 mm, see Figure 3-6, p.17. Dependency between centrifugal acceleration of a point distanced from rotation centre and rotational speed can be found using equation (2.1), where radius r is:

$$r = R + a = R + 0.425 * \sin\alpha \tag{2.12}$$

Where R = 1m, is the chosen arm length, a is an additional distance due to capsule displacement, α is a deflection angle and determined by the equation (2.13).

$$\tan\alpha = \frac{g_r}{g} = \frac{x * g}{g} = x$$
$$\alpha = \operatorname{atan} x \tag{2.13}$$

Where x is a coefficient describing a centrifugal acceleration. For example, if we are talking about 10g acceleration, then x = 10.

Using that logic we can state that:

$$x * g = \omega^2 * (R + a * \sin\alpha)$$
$$x * g = \omega^2 * (R + a * \sin(\operatorname{atan} x)) \tag{2.14}$$

The expression shown in (2.14) can be solved iteratively.

1) $x_i * g = \omega_i^2 * (R + a * \sin\alpha_i)$
2) $a_{i+1} = \operatorname{atan}(x_i)$
3) $x_{i+1} * g = \omega_i^2 * (R + a * \sin(\alpha_{i+1}))$
4) $x_i = x_{i+1};\ \ \alpha_i = \alpha_{i+1};\ \omega_i = \omega_{i+1}$ (2.15)

Initial conditions are:

$$\alpha(\omega_0) = 0, \qquad \omega_0 = 0.001 \left[\frac{rad}{s}\right] \tag{2.16}$$

Then the gradient of centrifugal acceleration can be constructed as a graph, that is shown on the Figure 2-8. From this figure, we can see that in fact we will reach 10g acceleration in CG of a capsule when arms will rotate at 1.33 rotations per second, 1.2 rps for the bottom and 1.5 rps for the top. Even though we found a rotation speed for which preliminary CG of a tested cylindrical capsule of 650x350 mm dimension reaches 10g, and this value is less than the one mention in Table 2, we will use for further calculations the speed of 1.567 rps. The reason is that the centre gravity point is not necessarily located in middle of a capsule or tested sample might be located in different point than the capsule geometrical centre, but rotation speed 1.567 rps corresponds to the 10g acceleration at 1m distance far from rotation centre.

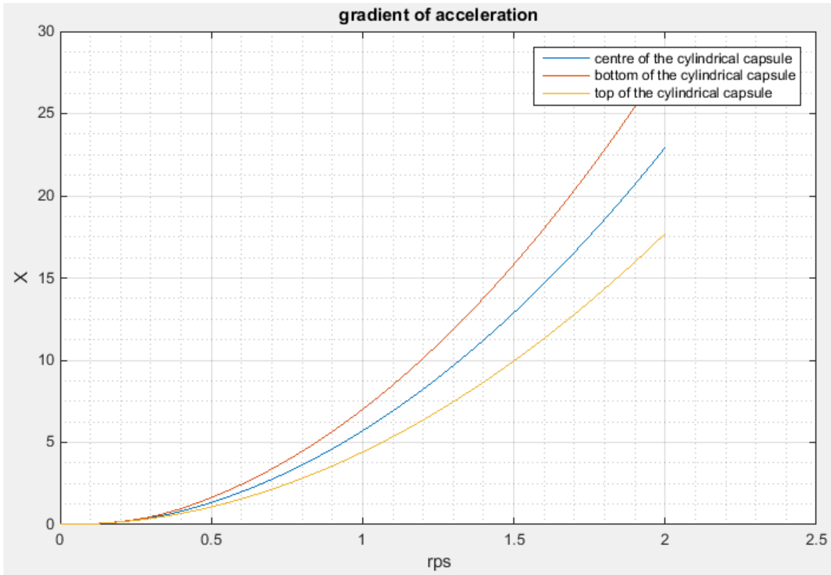


Figure 2-8 Acceleration gradient, dependency of a acceleration on rps of the capsule's centre of gravity.

2.3. Acceleration and time when the required speed will be reached.

Preliminary

To find the angular acceleration of the structure as the first step we have to find the angular momentum.

$$L = I * \omega \left[\frac{kg}{s} m^2\right] \tag{2.17}$$

Where the I is the moment of inertia and ω is the angular speed.

To find the moment of inertia there was created the very first preliminary design of arms together with the capsules on the both sides of it. Using properties function of CAD software Inventor 2016® we have found the preliminary moment of inertias. I have created two variants: 1st when it is on static and when we have reached our required 10g.

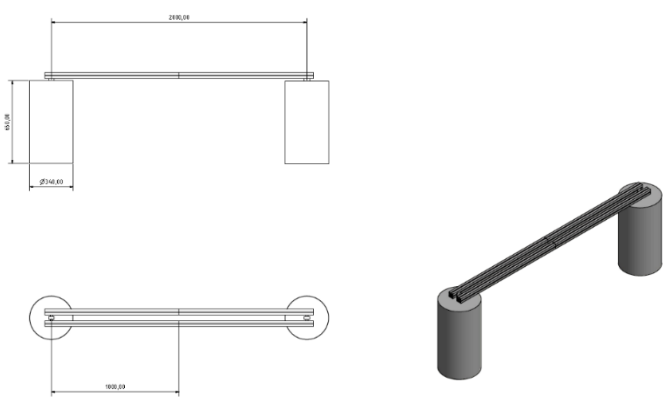


Figure 2 – 9 a

I_{max} = 36127435.887 [kg.mm^2] = I_{yy} ≈ 40 [kg.m^2]

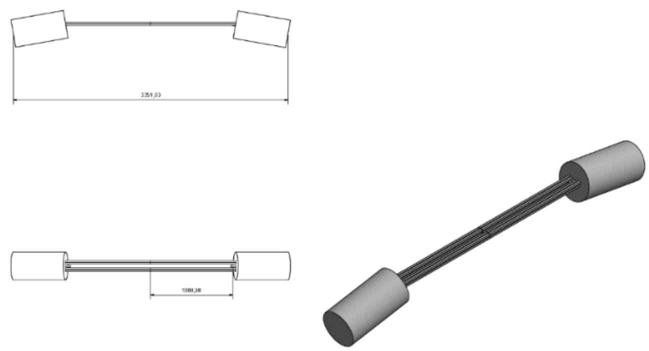


Figure 2-9 b Preliminary moment of Inertia

I_{max} = 60548333.883 [kg.mm^2] ≈ 60 [kg.m^2].

The next calculation will be provided for I = 60 [kg.m^2]. Using (2.17) we find L = 594.2726 [kg.m^2/s]

If torque is given from the motor characteristics, using formula (2.18) we can find a required time until the machine reach required angular speed.

$$T = \frac{dL}{dt}$$
$$Tdt = dL$$
$$\int Tdt = \int dL \tag{2.18}$$

If we assume that torque is constant from 0 to 2 rps, it value will be 20 [N.m].

$$20 \int dt = L$$
$$\int dt = \frac{L}{20} = \frac{594.2726}{20} = 29.7136$$
$$\int_o^t dt = t = 29.7136 \approx 30\ [s] \tag{2.19}$$

Calculation within drag force resistance will be performed with assumption that drag force will be a constant value.

$$T' = T - M_{D_C}$$
$$T' = 20 - 14.6069 = 5.3930\ [N.m] \tag{2.20}$$

Using (2.19) we can recalculate the time.

$$t = \frac{L}{T'} = \frac{594.2726}{5.3930} = 110.1927\ [s] \approx 115\ [s] \tag{2.21}$$

For results that are more accurate, we will no longer assume the torque of a motor as a constant value and drag moment too. Next table will help to create it numerically.

T [N.m]	ω[rad/s]	v [m/s]	Re[-]	M_D[N.m]	α[rad/s^2]	ΔT [N.m]	α' [rad/s2]	**t'[s]**
20	0	0	0	0	0.33	20	0.33	**0**
20.5	2.325	2.325	54139	0.805	0.342	19.695	0.328	**7.082**
21.2	5.97	5.97	139004	5.305	0.353	15.895	0.265	**22.532**
20.44	9.425	9.425	219481	13.587	0.341	6.853	0.114	**82.515**
20.35	9.905	9.905	230653	15.005	0.34	5.345	0.09	**111.19**
20.05	12.566	12.566	292641	23.513	0.334	-3.463	-0.058	**-217.72**

Table 3 Time calculation

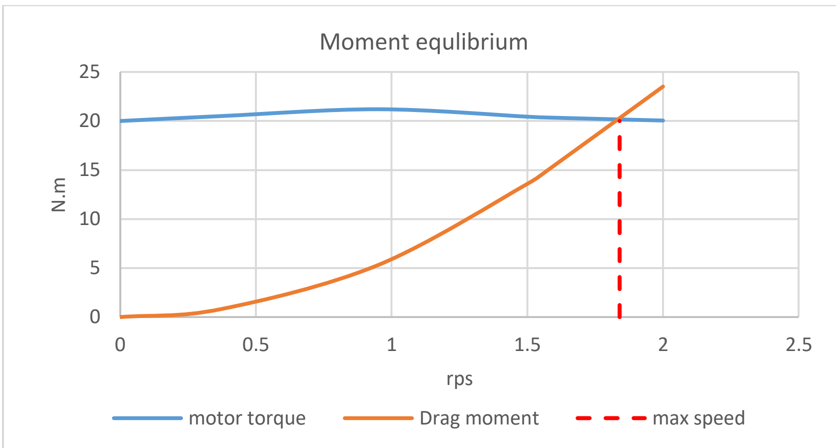


Figure 2-10 Moment equilibrium diagram (For only one arm loaded)

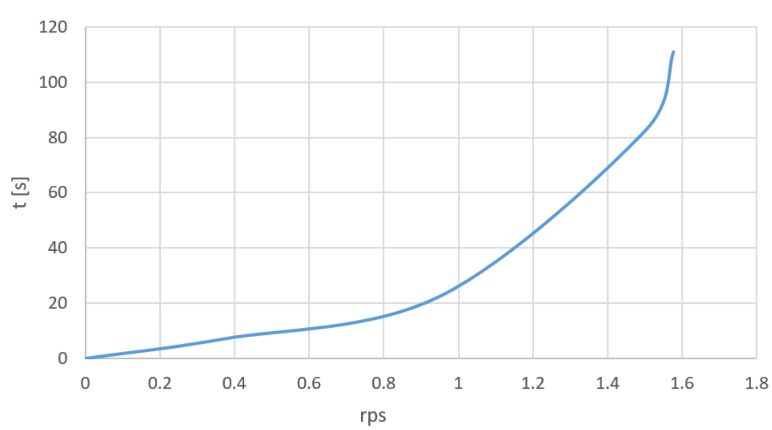


Figure 2-11 Time versus rps

If we will remain the arm length as 1m, but there will be additionally added some more materials it will increase the mass of the structure, which follows into increasing of inertia moment. This will have consequences like time increasing and increasing loading.

I [kg.m^2]	α' [rad/s2]	t[s]	t[min]
20	0.267	37.063	0.618
30	0.1782	55.595	0.9266
50	0.107	92.658	1.544
80	0.067	148.25	2.471
100	0.053	185.32	3.1
120	0.0445	222.38	3.71
150	0.036	277.975	4.633
200	0.027	370.633	6.18
250	0.0214	463.291	7.72
300	0.0178	555.95	9.266
400	0.0134	741.266	12.35

Table 4 Inertia vs Time

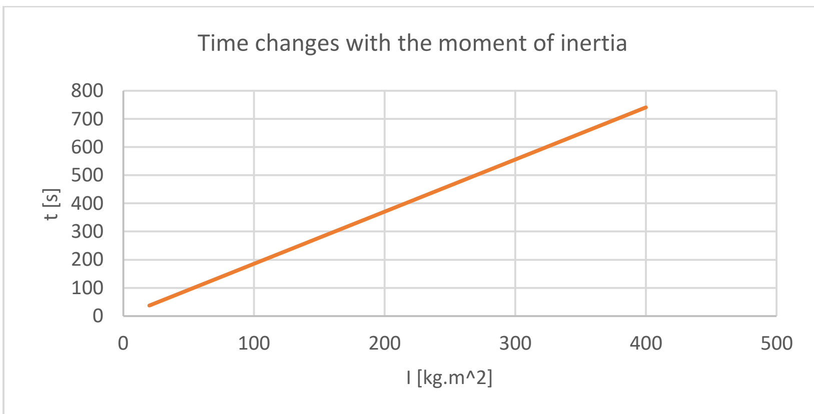


Figure 2-12 Time increasing with moment of inertia

3. Design

This chapter is dedicated for the general design steps of specific centrifuge's parts, such as: Centrifuge arms; Transmission design; Shaft design; Motor mounting; Lug and Pin design; Centrifuge Head and Frame.

3.1. Centrifuge arms strength calculation

Centrifuge arms are the part of the centrifuge machine that carry two significant loadings, which are mass load of the tasted sample (cylindrical capsule) and centrifugal force appeared on this sample due to rotation around fixed axis. Generally, we can divide loadings for two separate cases that arms will carry, those are Static case (when no rotation of the centrifuge happens) and Motion case (with rotation).

3.1.1. Static case

Static case it is the case when the structure of the centrifugal machine is loaded only by weight of capsule and own weight.

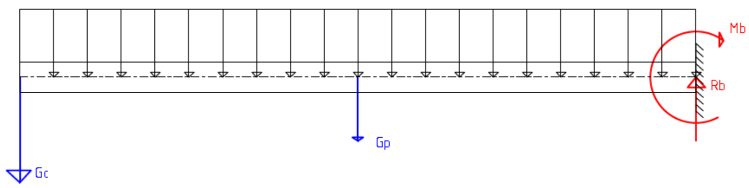


Figure 3-1 Static case loading

Where Gc represents the weight loading of a capsule. G_P represents the weight load of 45x45 profile and can be represented as distributed load w. R_B and Mb are reaction at the fixed connection, which is the center of rotation. Their values are:

$$w = m_P * \frac{g}{L} = \frac{G_P}{L} = 4 * \frac{9.81}{1} \approx 40 \left[\frac{N}{m}\right]$$
$$G_C = m_c * g = 15 * 9.81 \approx 150 \, [N] \tag{3.1}$$

Calculation

$$R_B = G_C + G_P = 150 + 40 = 190 \, [N] \tag{3.2}$$
$$M_B = G_C * L + w * L * \frac{L}{2} = L * \left(G_C + \frac{1}{2} * G_P\right) = 1 * \left(150 + \frac{1}{2} * 40\right) = 170 \, [N.m] \tag{3.3}$$

Shearing force and bending moment:

$$V(x) = -R_B + w * x$$
$$M(x) = -M_B + R_B * x - w * \frac{x^2}{2} \tag{3.4}$$

Where x is from 0 to L (=1m).

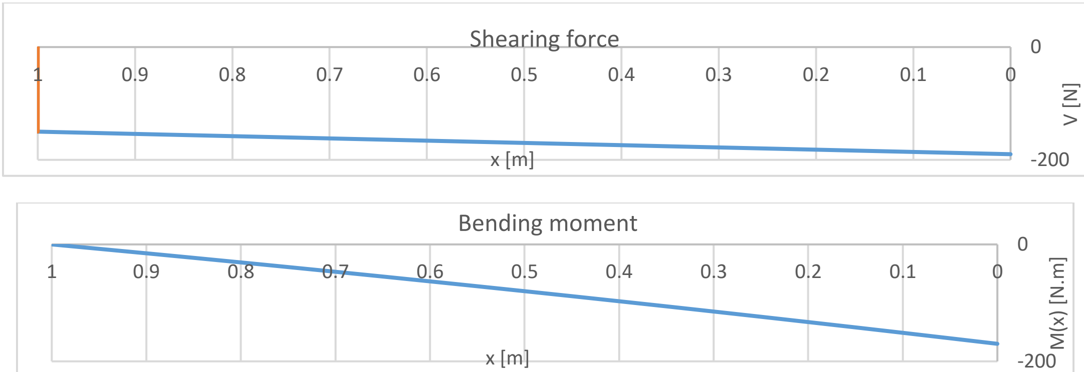


Figure 3-2 Shear Force and bending moment distribution along arm

Stress calculation

$$\sigma_{\max} = \frac{f * M(x)_{\max}}{I} * r = \frac{f * M(x)_{\max}}{W} \tag{3.5}$$

Where W is the section modulus. f –is the safety factor. The maximum stress can be said as the yielding stress of the material. Material is AlMgSi0.5F25 and its strength parameters are:

Rp02	200	[MPa]
E	70000	[MPa]
µ	0.33	[-]
G	27000	[MPa]
ro	2700	[kg/m3]

Table 5 Material property

In order to find the needed amount of profiles to be assembled to handle the loading of the structure, we will find the section modulus W and compare it with the W of the given profile, which is 45 x 45 Alutec k&k.

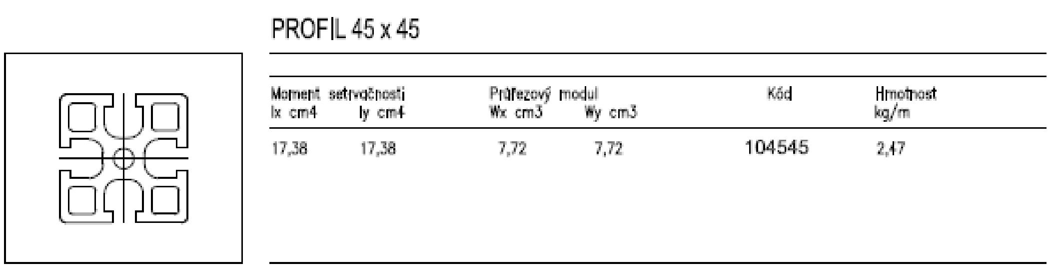


Figure 3-3 Profile parameters [6]

$$W = f * \frac{M(x)_{\max}}{R_{P_{02}}} = 2 * \frac{170}{200 * 10^6} = 1.7 * 10^{-6}\,[m^3] = 1.7[cm^3] \tag{3.6}$$

Compare it with the profile W: $\frac{7.72}{1.7} = 4.5$ times it is bigger than we are required. It means that one profile is enough to handle the loading stress.

Deformation calculation:

We will use Mohr's integral for that.

$$v_I = \int_0^L \frac{M(x)}{EI} * m(x)dx = 0.004520\ [m] = 4.52\ [mm]$$

where m(x) = x-1 (3.7)

Which is acceptable.

Reserve factor

Reserve factor "j" for chosen one profile will follow from next calculations:

$$\sigma_{\max} = f * \frac{M(x)_{\max}}{W} = 2 * \frac{170}{7.72 * 10^{-6}} = 22020725\ [Pa] = 22.0207\ [MPa] \quad (3.8)$$

Shear stress:

$$\tau_{\max} = f * \frac{V(x)_{\max}}{A_P} = 2 * \frac{190}{9.096 * 10^{-4}} = 417786.3\ [Pa] = 0.4178\ [MPa] \quad (3.9)$$

Where A_P is the cross-section area of the 45 x 45 profile.

Using HMH (von Mises) stress criterion we will find a reduced stress, also known as equivalent stress.

$$\sigma_{red} = \sqrt{\sigma^2 + (\sqrt{3} * \tau)} \quad (3.10)$$

In our case it will be:

$$\sigma_{red} = \sqrt{22.0207^2 + 3 * 0.4178^2} = 22.0326\ [MPa] \quad (3.11)$$

Comparing this value with Yielding stress of the material we will find out what is the reserve factor for this kind of load. Comparison is providing by dividing yielding stress to the maximum applicable stress on the structure, and the value of the reserve factor must be bigger than one, j(=RF)>1.

$$\boldsymbol{j = \frac{R_{P02}}{\sigma_{red}} = \frac{200}{22.0326} \approx 9\ [-] > 1} \quad (3.12)$$

3.1.2. Motion case

In this case will be calculated strength criterion for the load that will be applied on the structure during the required motion, which is 10g load on the capsule and 94.6 rpm.

Now the applied forces have been changed relative to the previous loading case. Besides G_C and G_P (w) will appear:

Centrifugal force on capsule: $\boldsymbol{C_C} = m_c\omega^2 r = 15 * 9.9045 * 1 = \mathbf{1471.5}\ [\boldsymbol{N}]$

Centrifugal force on profile (calculated by dividing arm for 10 parts, calculating for each part the C force, and then summarize them, since they are acting on the same axis on the same direction. We will neglect the force couple (moment) from the place where centrifugal force acts to the central axis of profile, since the deformation distance is very small) using formula (2.1) $\boldsymbol{C_P = 135\ [N]}$ **.**

We have to also consider the effect of the drag force appeared by the air resistance during motion.

Drag force on applied in the capsule have already been calculated in Table 2 and its value is: $\boldsymbol{F_{D_C}} = \mathbf{14.6070}\ [\boldsymbol{N}]$

The calculation of the drag force on profile will be more complicated since circumferential velocity depends on radius ($v = \omega r$) and drag force according to (2.10) is function of area and square of circumferential velocity. This makes some complication the calculation procedure.

For easier and simplify, but relatively accurate calculation, I have decided to divide the arm for 10 equal parts, which is Δr=1/10 = 0.1m each part and I assume that velocity is constant for each part, which is the velocity appeared in the center of its part. For better understanding see Figure 3-4. Velocity is calculated by:

$$v_i = \omega * \left((i-1) + \frac{1}{2}\right) dr \quad \dots i = 1 \dots 10 \tag{3.13}$$

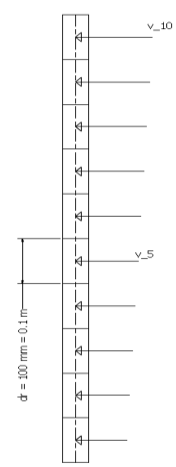


Figure 3-4 Velocities in arm for each element

Since Δr is constant, the cross-section area required for drag force calculation will be also constant.

$$\Delta A = \Delta r * (45 * 10^{-3}) = 0.0045\ [m^2] \tag{3.14}$$

The drag force will be calculated by equation (2.10):

$$F_{D_{P_i}} = \frac{1}{2} \rho v_i^2 \Delta A * c_{D_p} \tag{3.15}$$

Where $c_{D_P} = 2.5\ [-]$ is the drag coefficient for rectangular shapes, since there is no information provided about aerodynamic characteristics of this profile.

Provided calculation were tabulated, see Table 6.

			$F_{D_{P_i}}$	
v1	0.495227	[m/s]	0.00142	[N]
v2	1.485682	[m/s]	0.012776	[N]
v3	2.476136	[m/s]	0.035488	[N]
v4	3.466591	[m/s]	0.069557	[N]
v5	4.457045	[m/s]	0.114983	[N]
v6	5.447499	[m/s]	0.171764	[N]
v7	6.437954	[m/s]	0.239902	[N]
v8	7.428408	[m/s]	0.319396	[N]
v9	8.418863	[m/s]	0.410246	[N]
v10	9.409317	[m/s]	0.512453	[N]

Table 6 Drag forces on profile

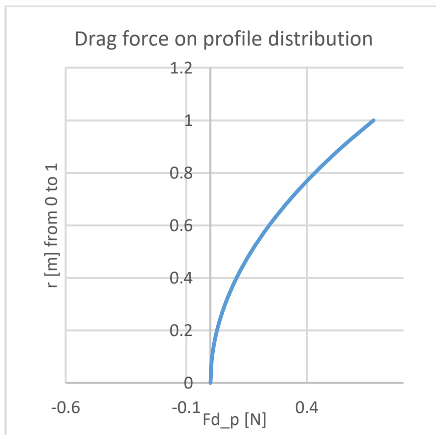


Figure 3-5 Drag force on profile

Let us assume this drag force as the distributed force as function of distance r, →q(r). We need to find this function q(r). The easiest way to find analytical function of this force is to make a trend line with analytical function by special function of Microsoft Office Excel® software. This function is:

$$y = 0.676x^2 \tag{3.16}$$

If we try to find analytically by ourselves we will obtain:

$$k = \frac{1}{2} * \rho * (45 \cdot 10^{-3}) * \omega^2 * c_{D_P} * \frac{\Delta r}{L} = \frac{1}{2} \cdot 1.225 * (45 \cdot 10^{-3}) \cdot 9.9045^2 \cdot 2.5 \cdot \frac{0.1}{1} = \\ = 0.67597 \left[\frac{N}{m^3}\right] \tag{3.17}$$

$$q(r) = k * r^2 = 0.67597 * r^2 \tag{3.18}$$

In order to provide strength calculation we should know external applied forces and geometrical values. Geometrical values for this load case were taken from Figure 3-6.

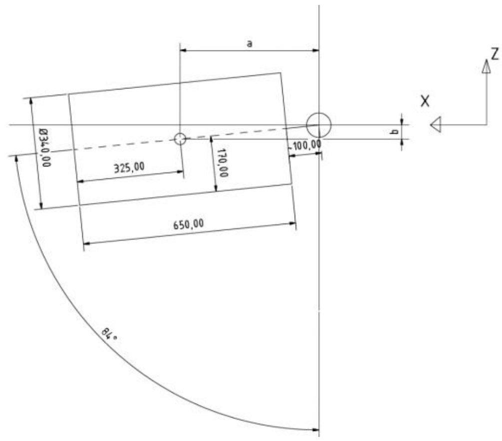


Figure 3-6 Displaced of preliminary tested capsule. Dimensions are in mm

If we assume that forces, that were mentioned above, acts in the center of gravity (centroid) and inclination angle is 84°, we can analytically calculate required distances *a* and *b*.

$$a = (0.325 + 0.1) * \sin 84° \approx 0.4\ [m]$$
$$b = (0.325 + 0.1) * \cos 84° \approx 0.04\ [m] \tag{3.19}$$

Calculation of strength

X-Y plane

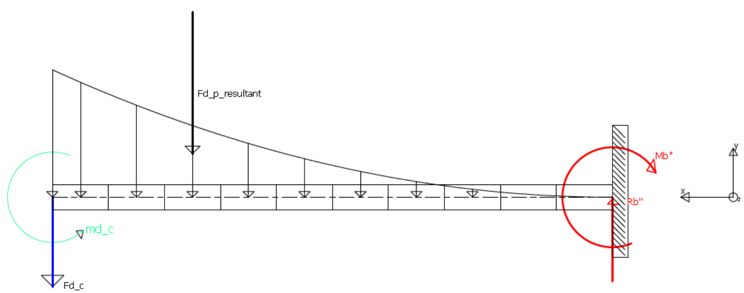


Figure 3-7 Y-X view load

Drag force:

$$F_{Dp_{resultant}} = \int_0^L q(r)dr = \int_0^L 0.676 * r^2 dr = 0,676 * \frac{L^3}{3} = 0.2253\ [N] \tag{3.20}$$

Place where drag force (3.20) is applied:

$$x_C = \frac{\int_0^L q(x) * x\,dx}{\int_0^L q(x)dx} = \frac{0.676 * \frac{L^4}{4}}{0.676 * \frac{L^3}{3}} = \frac{3}{4}L \tag{3.21}$$

Using relative force transformation theory from analytical static mechanics the centrifugal arm will be loaded by next load:

Moment caused by capsule's drag force:

$$m_{F_{Dc}} = F_{Dc} * a = 14.607 * 0.4 = 5.843 \ [N.m] \quad (3.22)$$

The transformed drag force itself $F_{D_C} = 14.607 \ [N]$.

Reactions:

$$M_b^{II} = F_{Dc} * L + m_{F_{Dc}} + F_{Dp} * x_c$$
$$M_b^{II} = 14.607 * 1 + 5.843 + 0.2253 * \frac{3}{4} = 20.6 \ [N.m] \quad (3.23)$$

$$R_b^{II} = F_{Dc} + F_{Dp} = 14,607 + 0.2253 = 14.832 \ [N] \quad (3.24)$$

Shearing force:

$$V(x) = -R_B^{II} + k * \frac{x^3}{3} = -14.832 + 0.676 * \frac{x^3}{3}, \qquad \ldots x = 0 \ to \ L \quad (3.25)$$

Bending moment:

$$M(x) = R_B^{II} * x - k * \frac{x^3}{3} * \frac{x}{4} - M_B^{II} = 14.832 * x - \frac{0.676x^4}{12} - 20.6 \quad \ldots x = 0 \ to \ L \quad (3.26)$$

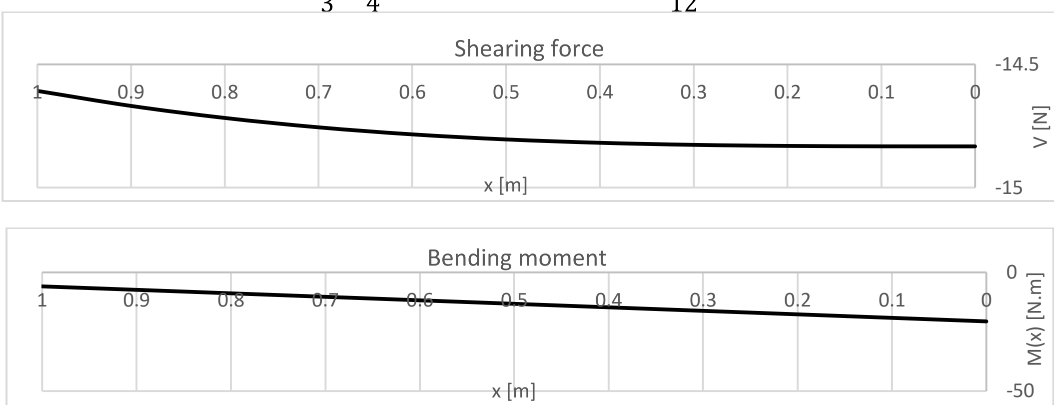


Figure 3-8 Distribution of Shear force and Bending moment

Maximum Shearing force is $V(x)_{\max} = 14.8323 \ [N]$

Maximum Bending moment $M(x)_{\max} = 20.6000 \ [N.m]$

Shear stress:

$$\tau_{\max} = f * \frac{V(x)_{\max}}{A} = 2 * \frac{14.8323}{9.096 * 10^{-4}} = 32614.3543[Pa] = 0.0326 \ [MPa] \quad (3.27)$$

Bending stress:

$$\sigma_{z_{\max}} = f * \frac{M(x)_{\max}}{W} = 2 * \frac{-20.6}{7.72 * 10^{-6}} = -5341644.275 \ [Pa] = -5.34166 \ [MPa] \quad (3.28)$$

Z-Y Plane

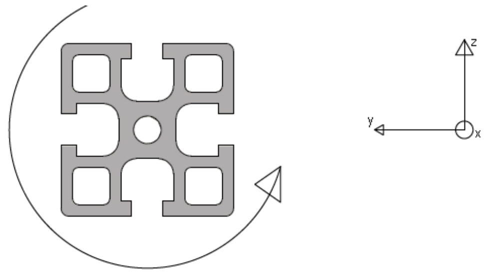


Figure 3-9 Z-Y View. Torque loading

$$T = F_{Dc} * b = 14.607 * 0.04 = 0.5813\ [N.m] \tag{3.29}$$

Shear stress from torque:

$$\tau_T = f * T * \frac{t_{\max}}{J} \tag{3.30}$$

Where $t_{\max}$ is the maximum thickness = 4 mm, J is polar moment of inertia

$$J = I_X + I_Y = 1.738 * 10^{-7} + 1.738 * 10^{-7} = 3.476 * 10^{-7} [m^4] \tag{3.31}$$

$$\tau_T = 2 * 0.5813 * \frac{0.004}{3.476 * 10^{-7}} = 13380.41\ [Pa] = 0.013\ [MPa] \tag{3.32}$$

X-Z Plane

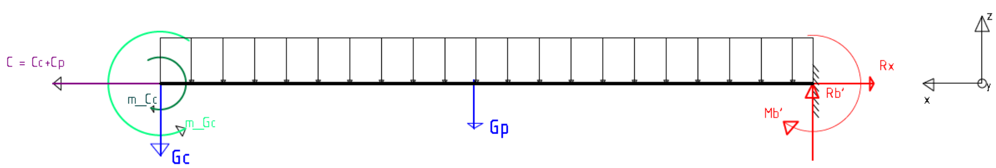


Figure 3-10 X-Z view load

External loads: $m_{G_C} = G_C * a = 60\ [N.m]$; $m_{C_C} = C_C * b = 58.56789\ [N.m]$

Reactions:

$$R_X = C_C + C_P = 1471.5 + 135 = 1606.5\ [N] \tag{3.33}$$

$$M_B^I = G_C * L + G_P * \frac{L}{2} - m_{Cc} + m_{Gc} = 171.4321\ [N.m]$$

$$R_B^I = M_B^I + G_P * \frac{L}{2} - m_{Gc} + m_{Cc} = 192.8642\ [N] \tag{3.34}$$

Shearing force:

$$V(x) = -R_B^I + wx = 192.8642 + 40 * x \quad ... x = 0\ to\ L \tag{3.35}$$

Bending Moment:

$$M(x) = R_B^I * x - M_B^I - w * \frac{x^2}{2} \quad ... x = 0\ to\ L \tag{3.36}$$

Normal internal force (neglecting temperature expansion)

$$N(x) = C = C_C + C_P = 1606.5 \tag{3.37}$$

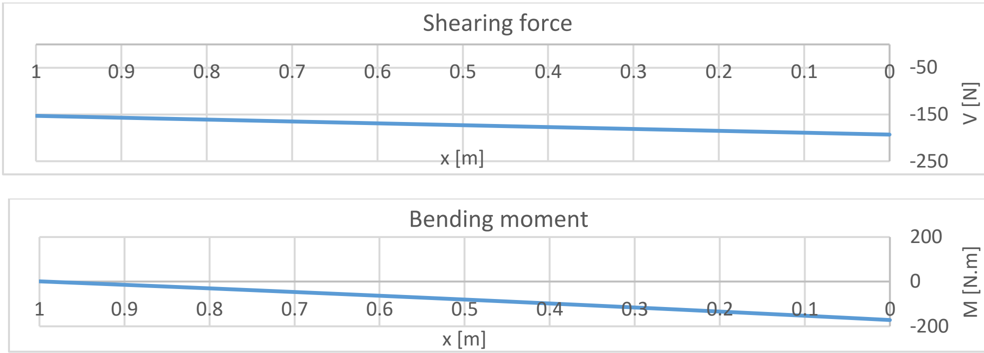


Figure 3-11 Shearing force and Bending moment distribution

Tensile stress:

$$\sigma_x = \frac{N(x)}{A} = \frac{f * 1606.5}{9.096 * 10^{-4}} = 4098483.2160\ [Pa] = 4.0984\ [MPa] \tag{3.38}$$

Bending Stress:

$$\sigma_y = f * \frac{M(x)_{\max}}{W} = 2 * -\frac{171.432}{7.72 * 10^{-7}} = -44412463.99\ [Pa] = -44.4124\ [MPa] \tag{3.39}$$

Shear stress:

$$\tau = f * \frac{V(x)_{\max}}{A} = 2 * \frac{-192.8642}{9.096 * 10^{-4}} = -42.4084.3268\ [Pa] = 0.4241\ [MPa] \tag{3.40}$$

HMH stress criterion and reserve factor

Von Misses (HMH) stress criterion reduce all stress components to one total normal stress.

$$\sigma_{red} = \sqrt{\frac{1}{2} * \left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6 * \left(\tau_x^2 + \tau_y^2 + \tau_z^2\right)\right]} \tag{3.41}$$

Specifically in our load case:

$$\sigma_{red} = \sqrt{\frac{1}{2}\left[\left(4.1 - (-44.41)\right)^2 + (-44.41 - 5.34)^2 + (-5.34 - 4.1)^2 + 6(0.42^2 + 0.013^2 + 0.033^2)\right]}$$

$$\boldsymbol{\sigma_{red} = 49.59 \approx 50\ [MPa]} \tag{3.42}$$

Using relation (3.12) for finding reserve factor:

$$\boldsymbol{j = \frac{R_{P02}}{\sigma_{red}} = \frac{200}{50} = 4}\ [-] > 1 \tag{3.43}$$

3.1.3. Summary

Calculations that were provided shows us that it is enough to use one 45 x 45 profile of 1m length, because it satisfies stress requirements for static case load (when centrifuge is not rotating) and motion case load (for maximum rotational speed). Even though using one 45x45 profile is enough for centrifugal arm, the machine, specifically the rotating head (=centrifuge head) will be designed in order to have an option to use 2-beam of 45x45 profile for one arm, preliminary design if this variant was shown on the Figure 2-9. Since one "centrifuge head" will support both variants, it is good to create an identifications for each variant of a centrifuge. One-beam variant will have a designation 1B, two-beam variant will have a designation 2B.

3.2. Belt transmission

Since the centrifugal machines will be driven by electric motor, it is sufficient and reliable to design a machine the way, that electric motor will transfer its torque and revolutions by belt transmission. It will help to avoid problems with rotation axis misalignment that can negatively affect work of the motor by damaging a motor bearing and decreasing efficient of the motor performance. For this purpose a timing belt (teeth belt) transmission was chosen, because this kind of a belt has better transmission characteristics due to lack of slipping.

Traditionally belt transmission consists of sprockets and belt. Considering that manufacturing abilities and budget are limited, the best solution here will be to choose desirable sprockets and belts that are already manufactured and can be easily found in the market.

The supplier of the belt transmission components is chosen Pikron s.r.o.

Sprockets

As the point to start form, we will use information about shaft diameter of the motor. Diameter of the electric motor ES-MH342200 [7] is 19 [mm], feather key length is s=30 [mm], feather key width is b =6 [mm]. Motor torque will be assumes constant in the level of 20 [Nm].

Sprocket for timing belt requires a special taper bush. From the web page of the supplier of a belt transmission, we can barely choose from:

TB	1210	1610	2012
Dmax [mm]	47.5	57	70
L [mm]	25.4	25.4	31.8
Clamping moment [Nm]	20	20	31
Prize [CZK]	100	105	140

Table 7 Taper bushes parameters

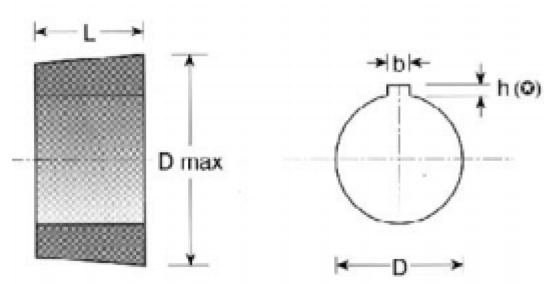


Figure 3-12 Taper bush dimensions [8]

For the tapper bushes that are mentioned in the Table 7 matches pulleys from the Table 8.

TB1210	z	Belt width [mm]	Type	D_pitch [mm]	Pitch [mm]	Prize [CZK]	F_Te [N]
HTD 5M	48	15	8F	76.39	5.0	180	0.523629
	56	15	8F	89.13	5.0	220	0.448783
	64	15	8F	101.72	5.0	280	0.393236
HTD/GT 8M	28	30	5F	71.3	8.0	195	0.56101
	28	50	5F	71.3	8.0	580	0.56101
L	30	100	5F	90.96	9.5	245	0.439754
	32	100	3F	97.02	9.5	270	0.412286
TB1610	z	Belt width [mm]	Type	D_pitch [mm]	Pitch [mm]	Prize [CZK]	F_Te [N]
HTD 5M	72	15	8F	114.59	5.0	320	0.349071
	80	15	8F	127.32	5.0	370	0.314169
	90	15	7A	143.24	5.0	475	0.279252
	112	15	7A	178.25	5.0	730	0.224404
HTD/GT 8M	32	20	5F	81.49	8.0	195	0.490858
	34	20	5F	86.58	8.0	215	0.462
	38	20	5F	96.77	8.0	255	0.413351
	40	20	5F	101.86	8.0	270	0.392696
TB 2012	z	Belt width [mm]	Type	D_pitch [mm]	Pitch [mm]	Prize [CZK]	F_Te [N]
HTD/8M	44	20	8F	112.05	8.0		0.356983
	48	20	8F	122.23	8.0		0.327252
	56	20	8F	142.6	8.0		0.280505
	64	20	8WF	162.96	8.0		0.245459
	72	20	8WF	183.35	8.0		0.218162
	80	20	8W	203.72	8.0		0.196348

Table 8 Pulleys

Where F_{Te} is the effective tension force which is calculated by:

$$F_{Te} = 2 * \frac{T}{D_p} = 2 * \frac{20}{D_p}\ [N] \tag{3.44}$$

Gear ratio for the toothed pulleys can be calculated by next equation:

$$i = \frac{D_{P_1}}{D_{P_2}} \tag{3.45}$$

For example, I marked in the Table 8 by blue colour pulleys for motor shaft (driver) and centrifuge shaft (driven), with the gear ratio of **1.33**, by green line I marked fitted pulleys with the gear ratio **1.25**.

Different gear ratios needs for different torque requirements. For example, for fixed driving torque of 20 N.m the bigger gear ratio will make bigger torque for driven pulley, with relation shown in equation (3.50)

Ratios that we can reach using items from the chosen supplier are 1.25, 1.375, 1.4, 1.6, 1.75, and 2. In order to compare them and to choose the optimum one, we will construct equilibrium diagram, where we will compare drag moment from rotation and the torque.

$$T\ 1{:}i = T_{output} = T_{input}(= T\ 1{:}1) * i * \eta \tag{3.46}$$

where η = 0.95.

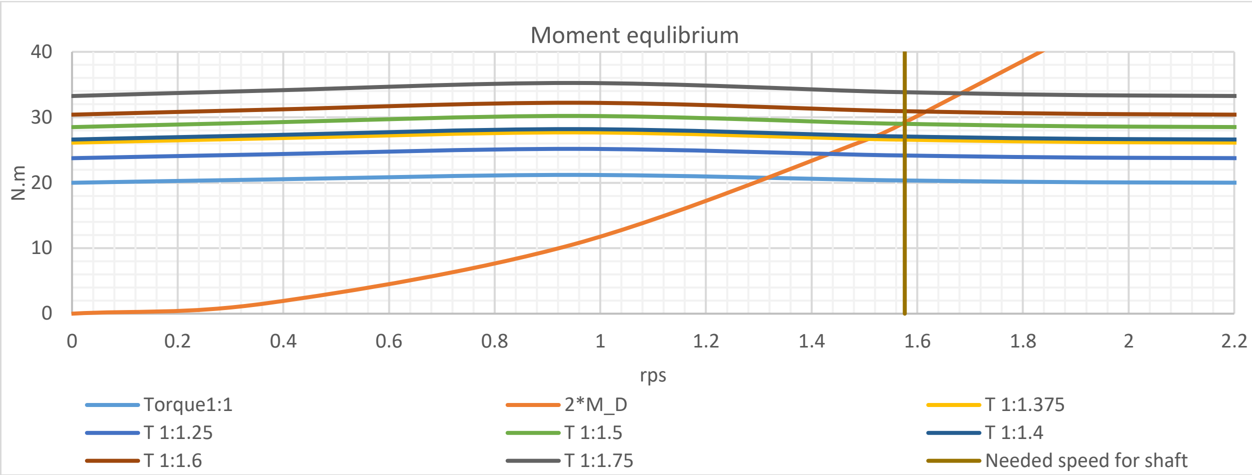


Figure 3-13 Equilibrium of moments for various gear ratios, for two loaded arms

Previously we stated that the required rotational speed is 1.58 [rps = 1/s] (see paragraph §2.2), with this speed our shaft has to rotate in order to reach desired acceleration of 10g. In this case rotor of an electric motor has to rotate with rotational speed of 1.58 * i. See table below.

Ratio [-]	1.25	1.375	1.4	1.5	1.6	1.75
Rotor speed [1/s]	1.9704465	2.167491153	2.20690008	2.3645358	2.522172	2.7586251

Table 9 Table of desired speed

We have to reference these values of the rps when working with Figure 3-13. For gearing ratio i = 1.25 rotor speed should be 1.97 [1/s] and according to the Figure 3-13, maximum rotational speed of centrifuge arms that we can reach is about 1.437 [1/s], which does not satisfy desired task. From the diagram also we can read that maximum reachable speed is approximately the same as needed rotational speed, that is 1.58 [1/s] for ratio i = 1.5.

For example if we choose i = 1.5, then we can find what excess capacity of a torque, that will help us to decrease time to reach needed rotational speed.

$$L = I * \omega;\ T = \frac{dL}{dt} \rightarrow dt = \frac{d(I\omega)}{dT} \rightarrow \Delta t = \frac{\Delta L}{\Delta T} \tag{3.47}$$

Equation (3.47) states, that the more excess of the torque we have the less time we need, as well as the less angular momentum makes less time. ΔT is shown on the Figure 3-14. This figure shows, that to rotate shaft by transmitting force using a timing belt with efficiency η = 0.95 with the angular speed ω_{out} = 1.58 rad/s, we need to reach the speed of the electric motor rotor ω_{in} = 2.3645 rad/s, where the torque transmitted by the shaft to the centrifuge's arms is approximately 28.4 N.m. The difference between this torque and resistive drag moment relative to the ω_{out} angular speed will be our excess capacity of the torque that will influence the time to reach desired angular velocity.

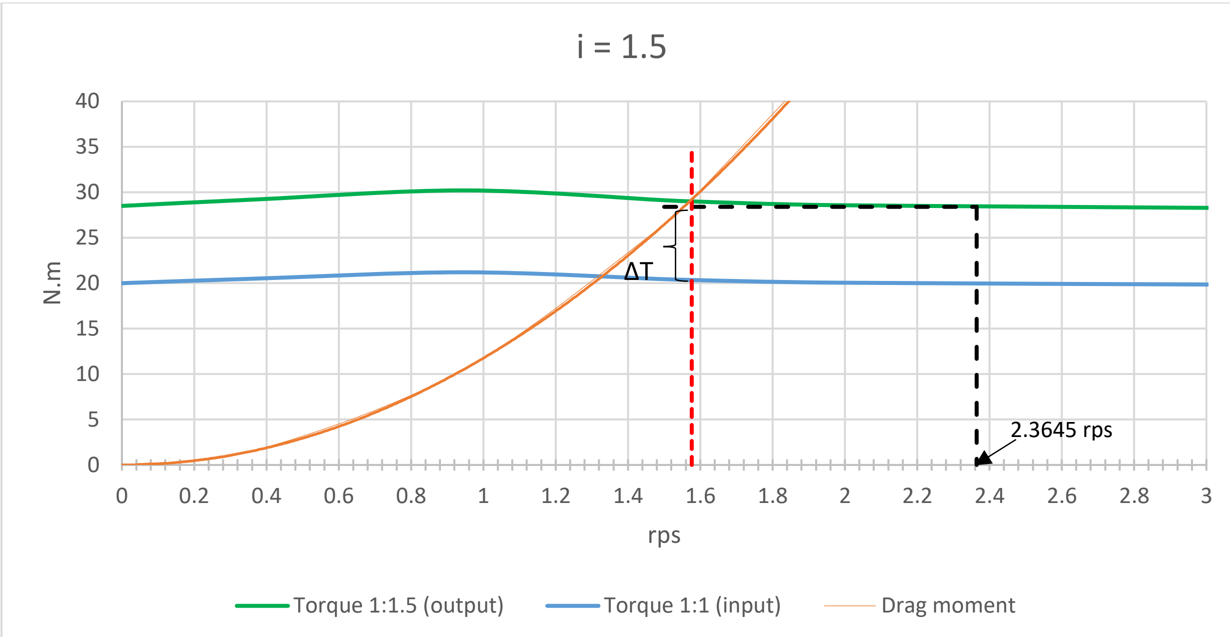


Figure 3-14 Equilibrium for gear ratio 1.5, for only one resisting moment (second arm free)

As the final decision, there were chosen next items, which combination will make gearing ratio i = 1.6:

	Driving	Driven
Taper bush	TB 1615 - 19	TB 2012 - 50
Sprocket	HTD/GT 40 – 8M – 20	HTD/GT 64 – 8M – 20

Table 10 Transmission combination, i = 1.6

3.3. Rotating Shaft

A shaft is an important part of a centrifuge machine that transfers rotation from a motor (or any other driver) to the centrifuge's head and connects centrifuge's head to a frame. Hence, a shaft should be designed not only to carry static load from the centrifuge's head (arms, tested sample under the 10g load for symmetric and asymmetric case etc.), but also rotates at desired speed with required mass attached safely. That is why this chapter is divided for two parts: Static Calculation and Calculation of a Critical Speed

3.3.1. Shaft. Static

For shaft, design there was draw schematic diagram of it with relatable loadings on it on the Figure 3-15. Preliminary shaft design is shown on the Figure 3-16, it just shows tendency of how is shaft needed to be design acceding to the constructions requirements, diameters can be changed.

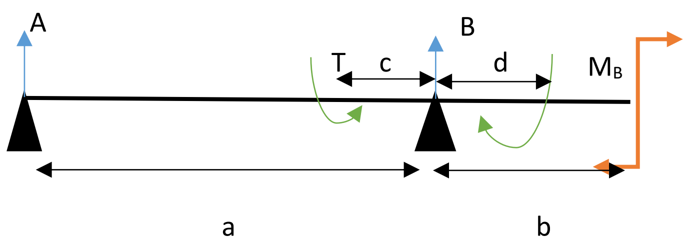


Figure 3-15 Schematic diagram of the loaded shaft

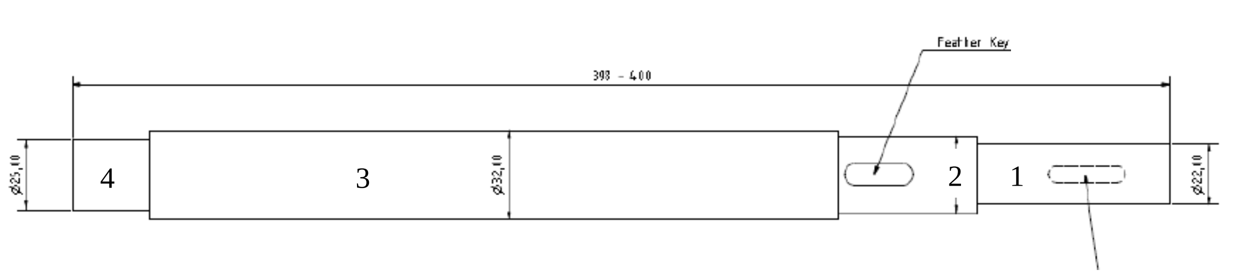


Figure 3-16 First version of the shaft; 1,2,3 and 4 are so called "sections"

L_1	100 [mm]	d_1	22 [mm]	a	322 [mm]
L_2	60 [mm]	d_2	28 [mm]	b	63 [mm]
L_3	210 [mm]	d_3	32 [mm]	c	37.5 [mm]
L_4	30 [mm]	d_4	25 [mm]	d	40 [mm]

Table 11 Shaft dimensions

Note that, diameters are temporary, after provided calculations all diameter values will be changed.

Support reactions are:

$$B * a + M_B = 0$$

$$B = \frac{-M_B}{a} = -\frac{-600000}{322} = 1863.45\ [N] = -A \tag{3.48}$$

Bending moment distribution is shown on the Figure 3-17, where the maximum absolute value of the bending moment is equal to 600000 N.mm.

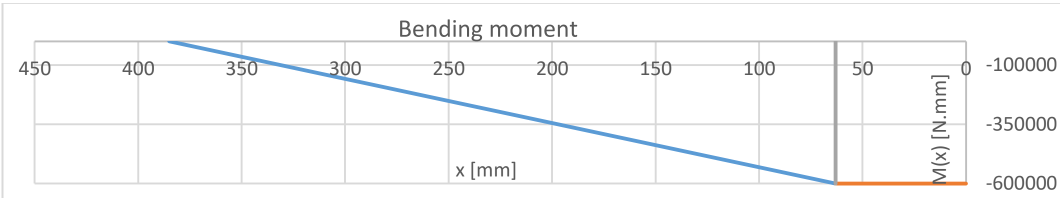


Figure 3-17 Bending moment of the shaft

If we apply for all section (1,2,3 and 4) this maximum bending moment, normal stresses on these sections are (using equation $\sigma = \frac{M}{Z}, where\ Z = \frac{J}{r} = \frac{\pi d^3}{32}$):

i [-]	*4*	*3*	*2*	*1*
σ [MPa]	391.14	186.51	278.41	573.96

Table 12 Normal stresses from bending moment in each section

If we use the same material to produce the shaft as material that are Alutek profiles are made from, than its properties are:

Rp02=σ_y	200	[MPa]
E	70000	[MPa]
μ	0.33	[-]
G	27000	[MPa]
ρ	2700	[kg/m3]

Table 13 AlMgSi0.5F25 general properties

Then, from known material properties we will find allowable stress and comparing it with the values from Table 12, we can say if this design is safe or not.

$$\sigma_{all} = \frac{\sigma_y}{f} = \frac{200}{2} = 100\ [MPa] \tag{3.49}$$

It is clearly seen, that no values from the Table 12 are smaller than 100 MPa, i.e. that we have to redesign shaft diameters.

We also have to take into account the fact of the presence of shearing stress from the torque T, which is located between sections *1* and *2*. To consider the affection of the shearing stress we will use HMH hypothesis to reduce normal and shearing stresses into one equivalent stress, and according to designing philosophy, this value has to be less than allowable stress σ_{all}. The value of the torque for the gear ratio i = 1.6 is 32 224 N.mm.

$$\sigma_{red} = \sqrt{\sigma^2 + 3 * \tau^2} \le \sigma_{all} \tag{3.50}$$

$$\sigma_{all}^2 \ge \sigma^2 + 3 * \tau^2 = \frac{M_B^2}{Z^2} + \frac{T^2}{W^2},\ where\ Z = \frac{\pi d^3}{32};\ W = \frac{\pi d^3}{16}, \tag{3.51}$$

$$d \geq \sqrt[6]{\frac{((32M_B)^2 + (16T)^2)}{(\pi\sigma_{all})^2}}$$

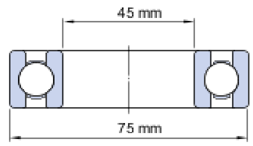
$$d \geq 39.39\ [mm], \text{and we choose } \boldsymbol{d} = \mathbf{45}\ [\boldsymbol{mm}]$$

This diameter is minimum required one, so it is referred to the **d₄**, but according to the chosen design, it is minimum dimeter of the shaft, so we will reference this value as the minimum one.

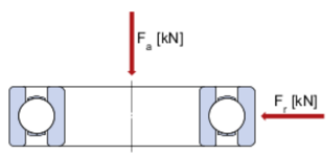
Since, we have change the diameter, we have to recalculate bearing for the shaft at section *4*, tapper bush and sprocket for shaft's section 3 and for both sections recalculate and re-choose feather keys.

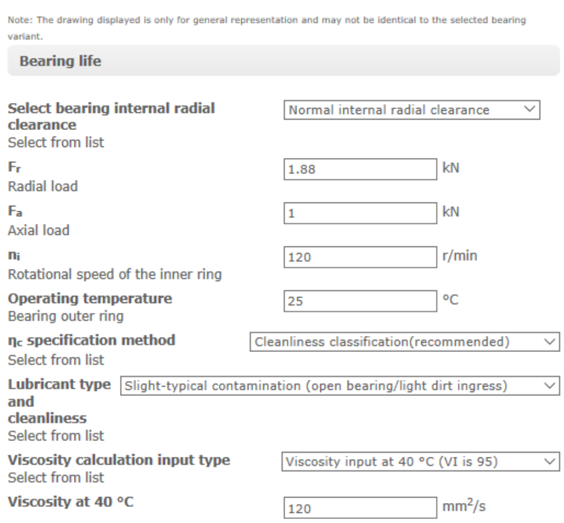
Bearings

Using very useful tool for bearing life calculation from bearing producer itself (SKF Engineering tool: http://www.skf.com/group/knowledge-centre/engineering-tools/skfbearingcalculator.html), we can calculate, if specific bearing satisfies given conditions. We choose bearing SKF with designation 16009



Designation	16009 *
d	45 mm
D	75 mm
B	10 mm
C	16.5 kN
C_0	10.8 kN
Type	Deep groove ball bearing





Note: The drawing displayed is only for general representation and may not be identical to the selected bearing variant.

Bearing life

Select bearing internal radial clearance Select from list	Normal internal radial clearance	
F_r Radial load	1.88	kN
F_a Axial load	1	kN
n_i Rotational speed of the inner ring	120	r/min
Operating temperature Bearing outer ring	25	°C
η_c specification method Select from list	Cleanliness classification(recommended)	
Lubricant type and cleanliness Select from list	Slight-typical contamination (open bearing/light dirt ingress)	
Viscosity calculation input type Select from list	Viscosity input at 40 °C (VI is 95)	
Viscosity at 40 °C	120	mm²/s

Result

L_{10mh} SKF rating life	77200 hour
a_{SKF} SKF life modification factor a_{SKF}	2
κ Viscosity ratio	3.06
P Equivalent dynamic bearing load	2.53 kN
η_c Factor for contamination level	0.19
v_1 Required kinematic viscosity for κ=1	96.2 mm²/s
L_{10h} Basic rating life	38500 hour
C/P Load ratio	6.5

Figure 3-18 Bearing life calculation

Analogical bearing to SKF 16009 is CSN 024630 SKF. To be sure if provided bearing calculation using SKF bearing calculator tool are correct, we will provide calculation of the bearing life using Autodesk Inventor® 2016 Professional, special bearing designer tool. Results are shown in Table 14. Results are very similar, so we can surely state that the CSN 024640 SKF bearing with inner diameter of 45 mm and width 10 mm is sufficient to fulfill needed requirements.

Results

Basic rating life	L_{10}	77467 hr
Adjusted rating life	L_{na}	77467 hr
Calculated static safety factor	s_{0c}	8.00000[-]
Power lost by friction	P_z	0.66268 W
Necessary minimum load	F_{min}	108 N
Static equivalent load	P_0	1950 N
Dynamic equivalent load	P	1880 N
Over-revolving factor	k_n	72.000 [-]
Equivalent speed	n_e	120 rpm
Strength Check	**Positive**	

Table 14 Autodesk Inventor Bearing calculation for CSN 024630 SKF

Feather Key

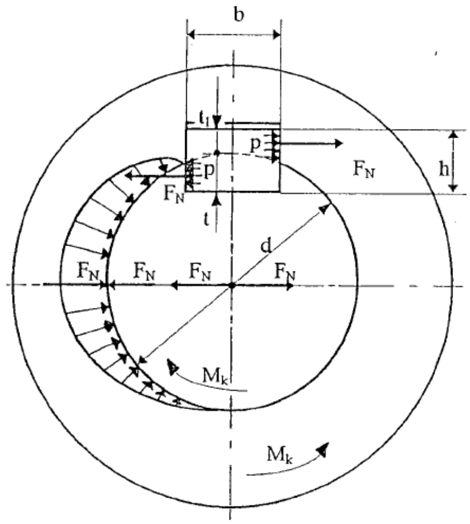


Figure 3-19 Loads on the shaft and hub with feather key. [9, p. 38]

Contact pressure is

$$p = \frac{4M_k}{d * h * l_a} \tag{3.52}$$

Where M_k is the torque applied to a shaft, which is in our case is T_2. $l_a = l - b$ is the effective feather key length.

Centrifuge head

In the section *4*, for the shaft diameter 45 mm, according to standards [10] we will choose feather key CSN 02 2562 A 14 x 9 x l.

Suitable, l can be determined using equation (3.52), if we assume p as allowable pressure $p_{all} \approx \tau_{all} = 70\ [MPa]$. Then $l_{a_{\min}} = 15.6\ [mm]$. To be suitable to the standard we call $l_a = 26[mm]$, so that $l = 40\ [mm]$. Therefore, needed feather key is *CSN 02 2562A 14 x 9 x 40.*

Contact pressure is:

$$p = \frac{4T}{d * h * l_a} = \frac{4 * 32224}{45 * 9 * (40 - 14)} = 12.2\ [MPa] \tag{3.53}$$

Value of contact pressure of 12.2 MPa is several times less, than allowable one. Required safety j > 5.

In the section *4*, for the shaft diameter 45 mm, according to standards [10] we will choose feather key *CSN 02 2562 A 14 x 9 x 40.*

Tupper bush

The taper bush is manufactured for usage with a feather key and the shaft diameter calculation should take into account this fact.

For the chosen diameter of the shaft of 28 mm, desirable taper bush TB2012, according to manufacturer, has a pocket with the width of b = 14mm, see Figure 3-20.

D	b	h (✪)	1008 A	1108 A	1210 A	1215 A	1310 A	1610 A	1615 A	2012 A
9	3	1,4	●	●						
10	3	1,4	●	●						
11	4	1,8	●	●	●	●				
12	4	1,8	●	●	●	●	●	●	●	
14	5	2,3	●	●	●	●	●	●	●	●
15	5	2,3	●	●	●	●	●	●	●	●
16	5	2,3	●	●	●	●	●	●	●	●
18	6	2,8	●	●	●	●	●	●	●	●
19	6	2,8	●	●	●	●	●	●	●	●
20	6	2,8	●	●	●	●	●	●	●	●
22	6	2,8	●	●	●	●	●	●	●	●
24	8	3,3(1,3)	✪	●	●	●	●	●	●	●
25	8	3,3(1,3)	✪	●	●	●	●	●	●	●
28	8	3,3(1,3)		✪	●	●	●	●	●	●
30	8	3,3			●	●	●	●	●	●
32	10	3,3			●	●	●	●	●	●
35	10	3,3(1,3)					✪	●	●	●
38	10	3,3						●	●	●
40	12	3,3						●	●	●
42	12	3,3(1,3)						●	✪	●
45	14	3,8								●
48	14	3,8								●
50	14	3,8								●

Figure 3-20 TB list. Pikron s.r.o

From the Czech standard tables, suitable feather key can be *CSN 02 2562A 14 x 9 x 40*.

Contact pressure is:

$$p = \frac{4T}{d * h * l_a} = \frac{4 * 32224}{50 * 9 * (40 - 14)} = 11.016\ [MPa] \tag{3.54}$$

Value of contact pressure of 11.016 MPa is several times less, than allowable one. Required safety j > 4.

Conclusion of the shaft static design

Section *2* of the shaft will have diameter d_3 = 55 mm, so it will refer as the pulley support. Shaft diameter of the section *1*, will be 45 mm, which was calculated for maximum bending moment of absolute value 600 000 N.mm. For this diameter is suitable thrust bearing and its housing – UCF209 by GISS.

Final table of the shaft dimensions is:

Section	Length	Diameter
1	100	45
2	60	50
3	210	55
4	30	45

Table 15 Modified shaft dimensions

3.3.2. Critical speed of the shaft

Critical angular velocity of a rotating shaft is the angular velocity that will reach the value of Eigen frequency (Eigen angular velocity) of the shaft with all masses attached. If the angular velocity of the shaft will be the same value as the Eigen velocity of the working shaft will appear the phenomena called Resonant, which means that the deformation at some point of the shaft will be infinity big value that, obviously, will cause a failure of a part.

In order to find Eigen frequencies of the shaft, we are going to solve this problem with consideration that the shaft is *massless* and *ideal*, i.e. has no eccentricity (e = 0). The shaft has two masses attached to it, first one is the pulley from the belt transmission, second is the combination of several parts (arms, working capsule or another test examples, fasteners, bearings, lugs etc.), assumed as one solid. For the more precise calculation, we will respect non-homogeneity of shaft's cross-section (resp. second polar moment of area) and its character of the support (resp. boundary conditions). See Figure 3-21.

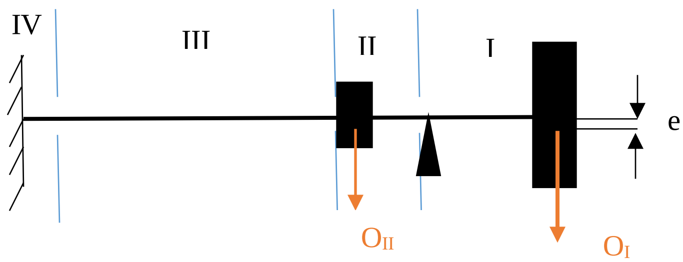


Figure 3-21 Simplified diagram of the shaft for Eigen frequency calculation, I II III and IV represents each section. Black rectangles represents mass wheels

3.3.2.1. *Massless shaft, zero eccentricity and one mass attached*

If there is no eccentricity (e =0 mm), then the location of the centrifugal force $O_{I,II}$ from the attached mass exists on the shaft's axis of rotation.

First, we will find Eigen frequency for the first mass mounted to the shaft (mass on the section I). If we assume that the material of the shaft will have linear behavior according and will behave like a spring, then the next equation will be valid

$$F = k * y \qquad (3.55)$$

Where k is the stiffness of the shaft at certain point, y is the deflection and F is the force needed to provide on the shaft the deflection y at certain point.

If we apply these assumptions and knowing equation of the centrifugal force, then we will obtain next:

$$m_I * y * \omega^2 = k * y \qquad [11] \qquad (3.56)$$

The deflection y as a function of angular velocity is then:

$$m_I y \omega^2 - ky = 0$$

$$y(m_I \omega^2 - k) = 0$$

$$y = \frac{0}{m_I \omega^2 - k} = \frac{0}{\omega^2 - \frac{k}{m_I}} \qquad (3.57)$$

From the equation (3.57), we can say that the deflection y will reach any value (infinity) as soon as the value of angular will tend to the value of $\frac{k}{m}$. From here, we can state, that $y(\Omega) \rightarrow \pm\infty$, where Ω is:

$$\Omega^2 = \frac{k}{m_I} \rightarrow \Omega = \sqrt{\frac{k}{m_I}} \tag{3.58}$$

Same method is valid for the mass attached on the section II.

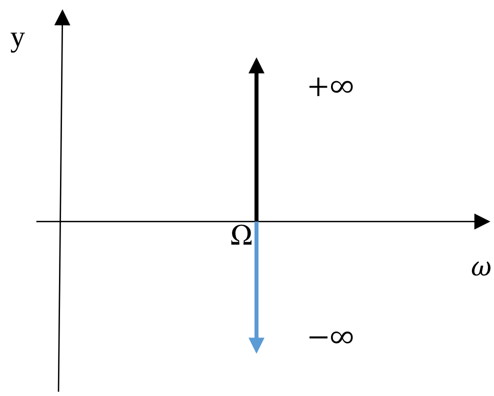


Figure 3-22 Defection y as a function of angular velocity, e=0

3.3.2.2. Massless shaft, with non-zero eccentricity and one mass attached

For the case of non-zero eccentricity (e≠0), the equation (3.56) transforms onto the next form:

$$m_I * (y + e) * \omega^2 = k * y \tag{3.59}$$

The deflection as function of angular velocity, now, has this form

$$m_I y\omega^2 + me\omega^2 - ky = 0$$

$$y(m\omega^2 - k) = -me\omega^2$$

$$y = \frac{-me\omega^2}{m\omega^2 - k} = \frac{me\omega^2}{k - m\omega^2} = \frac{e\omega^2}{\frac{k}{m} - \omega^2} \tag{3.60}$$

Again, as we mentioned for the equation (3.57), the deflection y will tend to be infinity when angular velocity ω will tend to the value of $\frac{k}{m}$, but now significant difference is in numerator. In case of (3.57), we had 0, which means, that the deflection will be zero at any angular velocity except, when it reaches the value of Ω. For the non-zero eccentricity numerator of the last equation in (3.60) is non-zero value, but the function of angular velocity and eccentricity. The presence of eccentricity represents more realistic rotor behavior.

If we provide the limit of the equation (3.60), where angular velocity tends to infinity we will find that the deflection will be the same as eccentricity

$$\lim_{\omega\to\infty} y(\omega) = \frac{e\omega^2}{\frac{k}{m} - \omega^2} = \frac{e}{\frac{\frac{k}{m}}{\omega^2} - 1} = \frac{e}{0 - 1} = -e \tag{3.61}$$

Same method is valid for the mass attached on the section II.

`

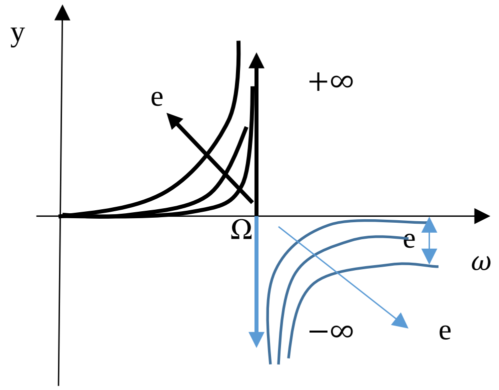


Figure 3-23 Deflection of the shaft as a function of angular velocity, e ≠0

3.3.2.3. Massless shaft, with two masses attached

For two masses attached to the shaft without their separation, the next equation is valid

$$k_{11} * y_1 + k_{12} * y_1 + k_{22} * y_2 + k_{21} * y_2 = O_1 + O_2$$

Transform to another form

$$y_1 = \frac{O_1}{k_{11}} + \frac{O_2}{k_{12}}; \quad y_2 = \frac{O_1}{k_{21}} + \frac{O_2}{k_{22}} \tag{3.62}$$

Since centrifugal forces O_1 and O_2 are also functions of the deflection y_1 and y_2 respectively, the dependency of the deflection y from angular velocity ω can be determined by following equations:

$$y_1 = \frac{m_1(y_1 + e_1)\omega^2}{k_{11}} + \frac{m_2(y_2 + e_2)\omega^2}{k_{12}}$$

$$y_2 = \frac{m_1(y_1 + e_1)\omega^2}{k_{21}} + \frac{m_2(y_2 + e_2)\omega^2}{k_{22}}$$

$$y_1 - \frac{m_1 y_1 \omega^2}{k_{11}} - \frac{m_2 y_2 \omega^2}{k_{12}} = \frac{m_1 e_1 \omega^2}{k_{11}} + \frac{m_2 e_2 \omega^2}{k_{12}}$$

$$y_2 - \frac{m_1 y_1 \omega^2}{k_{21}} - \frac{m_2 y_2 \omega^2}{k_{22}} = \frac{m_1 e_1 \omega^2}{k_{21}} + \frac{m_2 e_2 \omega^2}{k_{22}}$$

Let's say that $\frac{m_1 e_1 \omega^2}{k_{11}} + \frac{m_2 e_2 \omega^2}{k_{12}} = S_1$ and $\frac{m_1 e_1 \omega^2}{k_{21}} + \frac{m_2 e_2 \omega^2}{k_{22}} = S_2$

(3.63)

$$y_1\left(1-\frac{m_1\omega^2}{k_{11}}\right)-y_2*\frac{m_2\omega^2}{k_{12}}=S_1$$

$$y_2\left(1-\frac{m_2\omega^2}{k_{22}}\right)-y_1*\frac{m_1\omega^2}{k_{21}}=S_2$$

$$y_1\left(1-\frac{m_1\omega^2}{k_{11}}\right)-y_2*\frac{m_2\omega^2}{k_{12}}=S_1$$

$$-y_1*\frac{m_1\omega^2}{k_{21}}+y_2\left(1-\frac{m_2\omega^2}{k_{22}}\right)=S_2$$

Let's say that $\left(1-\frac{m_1\omega^2}{k_{11}}\right)=a_{11};\ \frac{m_2\omega^2}{k_{12}}=a_{12};\ \ \frac{m_1\omega^2}{k_{21}}=a_{21};\ \ \left(1-\frac{m_2\omega^2}{k_{22}}\right)=a_{22}$

$$y_1a_{11}-y_2a_{12}=S_1$$

$$-y_1a_{21}+y_2a_{22}=S_2\rightarrow -/-*(-1)$$

$$y_1a_{11}-y_2a_{12}=S_1$$

$$y_1a_{21}-y_2a_{22}=-S_2$$

We can create now system of linear algebraic evacuation in matrix form:

$$\begin{bmatrix}a_{11} & -a_{12}\\ a_{21} & -a_{22}\end{bmatrix}*\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}S_1\\ S_2\end{bmatrix}$$

If $\begin{bmatrix}a_{11} & -a_{12}\\ a_{21} & -a_{22}\end{bmatrix}=A,\ \begin{bmatrix}y_1\\ y_2\end{bmatrix}=y\ and\ \begin{bmatrix}S_1\\ S_2\end{bmatrix}=S,$ then

$$y=A^{-1}*S \tag{3.64}$$

In order to find Eigen frequencies, we can assume that the matrix S, where eccentricities are, is equal to 0. Then the equation (3.64) has the form:

$$\begin{bmatrix}a_{11} & -a_{12}\\ a_{21} & -a_{22}\end{bmatrix}*\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \tag{3.65}$$

This equation has two solutions, first it is when deflections are zero, which is not interesting for us, because it gives us nothing, and when system is linearly dependent and deflections y can be anything. Which means matrix A is singular:

$$\det\begin{bmatrix}a_{11} & -a_{12}\\ a_{21} & -a_{22}\end{bmatrix}=0$$

$$a_{11}*(-a_{22})+a_{12}*a_{21}=0 \tag{3.66}$$

Values for the angular velocity that we will obtain from the equation (3.66) are Eigen values, respectively, Eigen frequencies, Ω_1 and Ω_2.

3.3.2.4. Applying calculation

In order to find Eigen frequency $\Omega_{I \text{ or } II}$ it is significantly important to calculate shaft's bending stiffness in the place of centrifugal force from the attached mass.

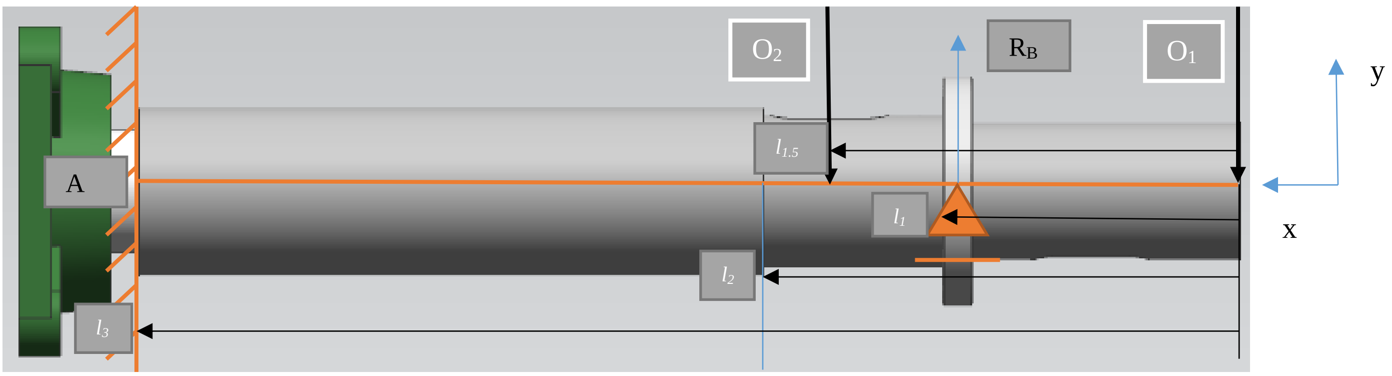


Figure 3-24 Shaft, calculation model

Section I – Mass 1 – massless shaft

To find deflection at the place where centrifugal force O_1 is applied, we have to find reactions in supports (bearings) and solve Mohr's integral. However, the way that shaft is supported is so called *statically indeterminate (3 unknowns for 2 equilibrium equations),* so we have to apply deformation conditions in order to find missing equation. The needed deformation condition is that the vertical deflection y_B, at the position of the support B is equal to zero, $y(l_1) \approx y_B = 0\, mm$. Knowing that boundary condition we can state, that:

$$y(l_1) = 0 = \int_0^{l_3} \frac{M(x)}{EI_x} m_i(x)dx \tag{3.67}$$

Where $m_i(x)$ is the bending from the unit load applied at l_1.

Since our shaft does not have constant cross-section along its length and not distributes within x by some specific function, we can rewrite Mohr's integral (3.67) as the sum of integrals of each section.

$$0 = \int_0^{l_1} \frac{M(x)}{EI_1} m_i(x)dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_i(x)dx \tag{3.68}$$

Where, if we say $O_1 = F_1$ and using the method of section we can write:

x	$[0, l_1]$	$[l_1, l_2]$	$[l_2, l_3]$
$M(x)$	$-F_1 * x$	$-F_1 * x + R_{B_1} * (x - l_1)$	
$m_i(x)$	0	$-(x - l_1)$	
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$	$\frac{\pi d_3^4}{64}$

Table 16 Mohr's integral values for the shaft at loading O₁, for unit load at support B

By solving equation (3.68) we find the reaction force R_{B_1}. Using software MATLAB® we can fast and easy find the needed solution. The script with the solution can be found in the attachments.

$$R_{B_1} = -\frac{F * \left(\frac{2l_2^3 - 3l_1l_2^2 - 2l_3^3 + 3l_1l_3^2}{I_3} - \frac{(l_1 + 2l_2) * (l_1 - l_2)^2}{I_2}\right)}{\frac{-6l1^2l_2 + 6l1^2l3 + 6l_1l_2^2 - 6l_1l_3^2 - 2l_2^3 + 2l_3^3}{I_3} - \frac{(l_1 - l_2)^3 * 2}{I_2}} \tag{3.69}$$

We assumed that the centrifugal force O_1 approaches to the free end of the shaft's section I, in order to be on the safety side of the problem. So the stiffness will be at this place will be $k_{11} = \frac{O_1}{y_{11}} = \frac{F_1}{y_{11}}$, where the first index shows the place and second index shows the force applied, so the stiffness k_{11} is saying that this stiffness is the stiffness of the shaft at the place 1 (where the first force is applied) from the force O_1. So the k_{12} will be the stiffnes at the place 1, that will be caused by the force O_2.

To find stiffness k_{11} again, we have to use Mohr's integral to find the deflection y_{11}, with applying unit load at the same place as centrifugal force O_1. Again using an advantage of the MATLAB®, we can solve the integral equation (3.70), where all needed values are written in the

$$y_{11} = \int_0^{l_1} \frac{M(x)}{EI_1} m_i(x)dx + \int_{l_1}^{l_2} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_i(x)dx \tag{3.70}$$

x	$[0, l_1]$	$[l_1, l_2]$	$[l_2, l_3]$
$M(x)$	$-F_1 * x$	$-F_1 * x + R_{B_1} * (x - l_1)$	
$m_i(x)$	$-1 * x$	$-1 * x + r_{b_1} * (x - l_1)$	
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$	$\frac{\pi d_3^4}{64}$

Table 17 Mohr's integral values for the shaft at loading O1, for unit load at the place of O_1

Where r_{b_1} is the reaction at the support B from the unit load at the place of force $O_1 = F_1$. Symbolic result for y_{11} you can find in the attachments.

From the equation (3.55) we can find the stiffness k_{11}, which was solved by MATLAB® with the using values for our shaft. Symbolic result, again, in the attachments.

$$k_{11} = \frac{F_1}{y_{11}} = 1.9479329E + 04 = 19\,479.329 \left[\frac{N}{mm}\right] \tag{3.71}$$

Preliminary mass was read from the CAD file, assuming geometry and density for all parts involved at the place 1 according to the Figure 3-24. For "One Beam" (1B) modification the mass is m_{1_0} = 40 kg and we will take a reserve 10 kg for future sensors and for additional masses if appears, so the 1B mass is $m_1 = 50$ kg. "Two Beam" (2B) modification has mass of $M_{1_0} = 50$ kg and with extra mass 2B is $M_1 = 60$ kg. Using equation (3.58) we obtain:

Modification	1B	2B
Eigen angular velocity, Ω_{01} [rad/s]	19.73	18.018
Eigen frequency, f_{01} [1/s]	3.14	2.86
Eigen angular velocity, n_{01} [rpm]	188.40	171.98

Table 18 Eigen values for case 1

To reach 10g acceleration for the chosen 1m arm requires 9.905 [rad/s] or 94.582 [rpm], or 1.577 [rps=1/s]. In order to be conservative let us assume that maximum required speed is 2 [1/s] and the related Eigen value should not be lower than this value. Results from Table 18 shows that we are okay.

If we add the effect of eccentricity, then with the help of equation (3.60) it is possible to construct the graph $y(\omega)$ for eccentricities $e \in [0.1; 1]$

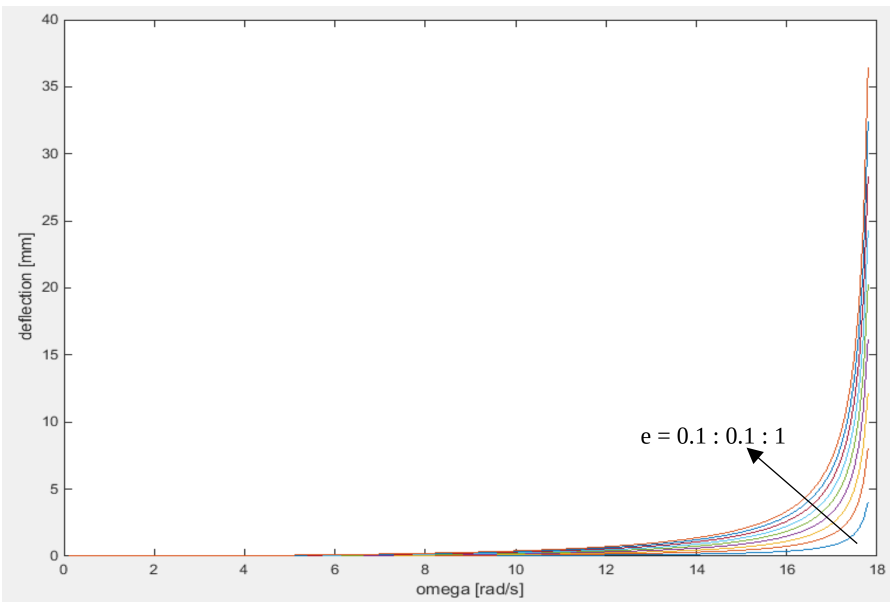


Figure 3-25 y(ω) diagram for M_1

The maximum allowable deflection can be calculated from the allowable stress (bending stress).

$$\sigma_{all} = \frac{M_{\max}(F_{all})}{\frac{I}{r}} \to F_{all} = f(\sigma_{all}) \to y_{11_{all}} = \frac{F_{all}}{k_{11}} \tag{3.72}$$

In order to clearly know where is the maximum bending moment along the shaft, let's say that the F =1 N so we can construct the diagram for the bending moment distribution M(x). Using information from the Table 16 and equation (3.69), we can construct next diagram.

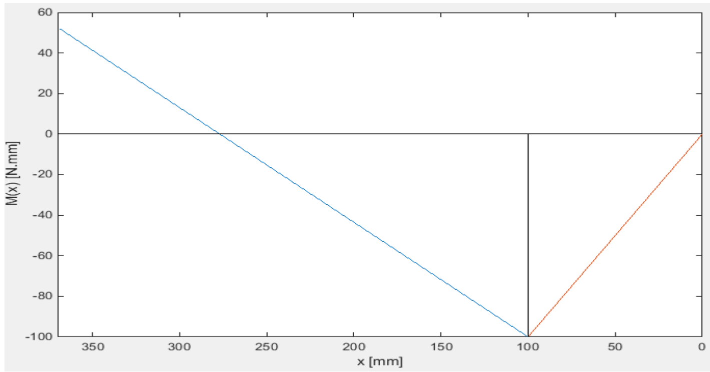


Figure 3-26 Bending moment distribution for the load of 1N at the place of force O_1

From the Figure 3-26 it is seen that the maximum absolute value of the bending moment is situated when x = 100 mm or x = l_1. In that case the maximum bending moment is $M_{\max\ all} = F_{all} * l_1$. Then follows:

$$\sigma_{all} = \frac{F_{all} * l_1}{\frac{I_1}{r_1}} \rightarrow F_{all} = \sigma_{all} * \frac{I_1}{\frac{d_1}{2}} * \frac{1}{l_1} = 100 * \frac{\pi * 45^3}{32} * \frac{1}{100} = 8946.176\ [N] \tag{3.73}$$

$$y_{11all} = \frac{F_{all}}{k_{11}} = \frac{8946.176}{19479.329} = 0.459\ [mm] \tag{3.74}$$

Therefore, the Figure 3-25 will finally look as follows.

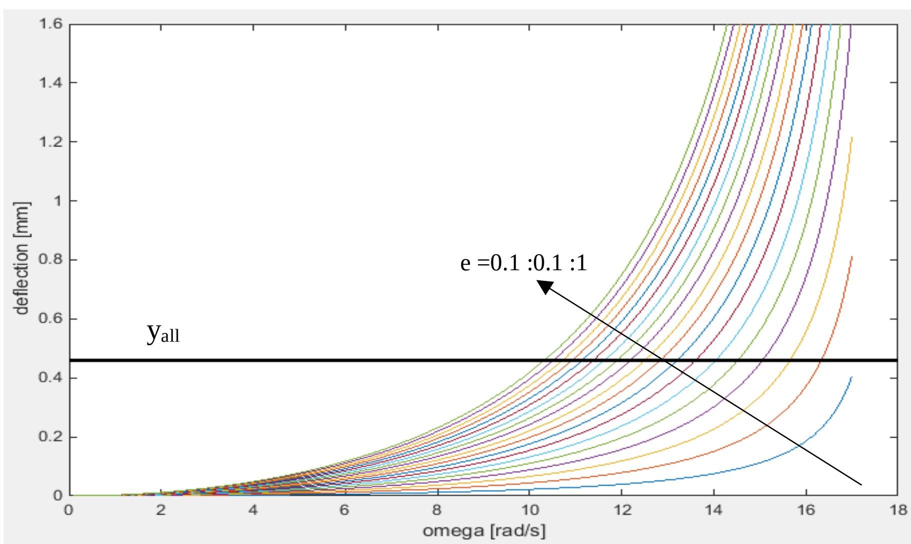


Figure 3-27 y(ω) for M_1 with allowable deflection

Section II – Mass 2 – massless shaft

For the section II as well as for the first section we need to find reaction force in the bearing and then use it to find stiffness and after the Eigen frequency. Again, to find reaction force in the support B, we need to apply there a boundary condition that is saying that the vertical deflection at the support B is equal to zero. General equations to find the reaction at the support B for the section II will be same as for the section I, i.e. equation (3.67). Equation (3.68) will have another view, since the force O_2 is assumed to be from the sprocket that has 32 mm width and assuming that the centrifugal force O_2 situates exactly in the center, hence its position is in l_2 - 32/2 mm = $l_{1.5}$.

$$y_b = 0 = \int_0^{l_1} \frac{M(x)}{EI_1} m_i(x)dx + \int_{l_1}^{l_{1.5}} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_{1.5}}^{l_2} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_i(x)dx \tag{3.75}$$

Values for the equation (3.75) are mentioned in the Table 19.

x	$[0;l_1]$	$[l_1; l_{1.5}]$	$[l_{1.5}; l_2]$	$[l_2; l_3]$
$M(x)$	0	$R_{B_2} * (x - l_1)$	$R_{B_2} * (x - l_1) - F_2 * (x - l_{1.5})$	
$m(x)$	0	$-1 * (x - l_1)$		
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$		$\frac{\pi d_3^4}{64}$

Table 19 Integral values for the shaft at loading $O_2=F_2$, for unit load at support B (apply for (3.75))

Using MATLAB® we can solve this integral equation and find needed value R_{B_2} (the symbolic result for it you can find in attachments).

Finding a deflection at the place where force $O_2 = F_2$ appeared we can find a related stiffness $k_{22} = \frac{F_2}{y_{22}}$.

$$y_{22} = \int_0^{l_1} \frac{M(x)}{EI_1} m_i(x)dx + \int_{l_1}^{l_{1.5}} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_{1.5}}^{l_2} \frac{M(x)}{EI_2} m_i(x)dx + \int_{l_2}^{l_3} \frac{M(x)}{EI_3} m_i(x)dx \tag{3.76}$$

Where all functions are:

x	$[0;l_1]$	$[l_1; l_{1.5}]$	$[l_{1.5}; l_2]$	$[l_2; l_3]$
$M(x)$	0	$R_{B_2} * (x - l_1)$	$R_{B_2} * (x - l_1) - F_2 * (x - l_{1.5})$	
$m(x)$	0	$r_{b_2} * (x - l_1)$	$r_{b_2} * (x - l_1) - 1 * (x - l_{1.5})$	
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$		$\frac{\pi d_3^4}{64}$

Table 20 Integral values for equation (3.76)

Where r_{b_2} is the reaction at the support B from the unit load at the place of force F_2. Symbolic result for y_{22} you can find in the attachments. The stiffness is then:

$$k_{22} = \frac{F_2}{y_{22}} = 2.82637293E + 05 = 282\ 637.293 \left[\frac{N}{mm}\right] \tag{3.77}$$

Mass of the complete sprocket is 3.45 kg, but to be on the conservative side let us assume that this mass will be M_2 = 4 kg. Using equation (3.58) we find Eigen values.

Eigen angular velocity, Ω_{02} [rad/s]	265.8
Eigen frequency, f_{02} [1/s]	42.3
Eigen angular velocity, n_{02} [rpm]	2538.2

Table 21 Eigen values for mass M_2

Results from the Table 21 shows that they are far beyond than required speed for 10g acceleration, which is I remind from conservative point of view is 120 [rpm] or 2 [1/s].

If we add the effect of eccentricity, then with the help of equation (3.60) it is possible to construct the graph $y(\omega)$ for eccentricities $e \in [0.1; 1]$. See Figure 3-28.

MATLAB® shows that deflection for the angular velocity $2 * 2\pi = 12.566 \left[\frac{rad}{s}\right]$ is 2.04E-04, so for my point of view it is very small deflection, there is no point to solve the shaft for the allowable deflection for mass M_2.

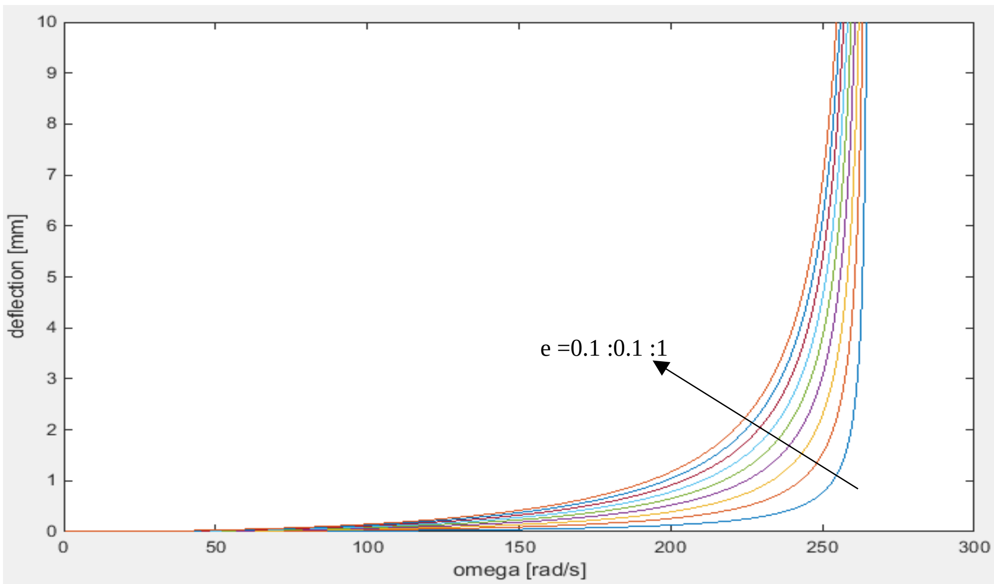


Figure 3-28 y(ω) for the mass M_2

Shaft with both mass attached – massless shaft

To find Eigen values for this case we have to calculate equation (3.66), which is determinant of matrix that contains masses, stiffness and angular velocities. This matrix was mentioned in (3.63) and (3.64). Stiffness k_{11} and k_{22} were found in previous parts of this paragraph. Our task now is to find stiffness k_{12} and k_{21} which for isotropic material should be the same values, $k_{12} = k_{21}$. To find stiffness k_{12} we need to find deflection caused by force F_2 at the place of applied force F_1, which is y_{12}. $k_{12} = \frac{F_2}{y_{12}}$. To find deformation y_{12}, will be used general equation (3.76), but will contain another functions for bending moment from an applied force and bending moment from a unit load. Table for this integral equation is written below.

x	$[0;l_1]$	$[l_1; l_{1.5}]$	$[l_{1.5}; l_2]$	$[l_2; l_3]$
$M(x)$	0	$R_{B_2}(x - l_1)$	$R_{B_2}(x - l_1) - F_2 * (x - l_{1.5})$	
$m(x)$	$-x$	$-x + r_{b_1}(x - l_1)$		
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$		$\frac{\pi d_3^4}{64}$

Table 22 Integral values to solve (3.76) to find y_{12}

To find deflection at the place where F_2 applies but from the force F_1, y_{21} we need to use again general integral equation (3.76) and put there next values:

x	$[0;l_1]$	$[l_1;l_{1.5}]$	$[l_{1.5};l_2]$	$[l_2;l_3]$
$M(x)$	$-F_1 * x$	$-F_1 * x + R_{B_1}(x-l_1)$		
$m(x)$	0	$r_{b_2}(x-l_1)$	$r_{b_2} * (x-l_1) - 1 * (x-l_{1.5})$	
I	$\frac{\pi d_1^4}{64}$	$\frac{\pi d_2^4}{64}$		$\frac{\pi d_3^4}{64}$

Table 23 Integral values to solve (3.76) to find y_{21}

Dividing each force by related deflection, we can find all needed stiffness. Constructing something like a stiffness matrix specifically for a designed shaft, will look like:

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 19479.329 & -124295.99 \\ -124295.99 & 282637.293 \end{bmatrix} \text{ [N/mm]} \tag{3.78}$$

This matrix confirms the theory that $k_{12} = k_{21}$.

Solving the equation (3.66) will give us next:

$$\det\begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} = \det\begin{bmatrix} \left(1-\frac{m_1\omega^2}{k_{11}}\right) & -\frac{m_2\omega^2}{k_{12}} \\ \frac{m_1\omega^2}{k_{21}} & -\left(1-\frac{m_2\omega^2}{k_{22}}\right) \end{bmatrix} = \det\begin{bmatrix} \left(1-\frac{\omega^2}{\Omega_{01}}\right) & -\frac{m_2\omega^2}{k_{12}} \\ \frac{m_1\omega^2}{k_{21}} & -\left(1-\frac{\omega^2}{\Omega_{02}}\right) \end{bmatrix} =$$

$$\det\begin{bmatrix} \left(1-\frac{\omega^2}{18.018}\right) & -\frac{4*\omega^2}{-124295.99} \\ \frac{60*\omega^2}{-124295.99} & -\left(1-\frac{\omega^2}{265.8}\right) \end{bmatrix} = 0 \quad \rightarrow \quad \Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix} = \begin{bmatrix} 18 \\ 331.6 \end{bmatrix} \text{ [rad/s]} = \begin{bmatrix} 171.8 \\ 3166.5 \end{bmatrix} \text{ rpm} \tag{3.79}$$

Example in (3.79) was calculated for two beams modification (2B). Generally, we can exceed the rotational speed Ω_1, if it will be done quickly, but specifically for the designed centrifugal machine we don't need to reach angular speed more than 120 [rpm] = 12.556 [rad/s]. From this point of view we are satisfied with the designed shaft.

Adding eccentricity $e \in [0.05:1]$ to the shaft the function y(ω) which is (3.64) the dependency for the deflection y_1 will be then as it shows Figure 3-29. For the y_2 see Figure 3-30.

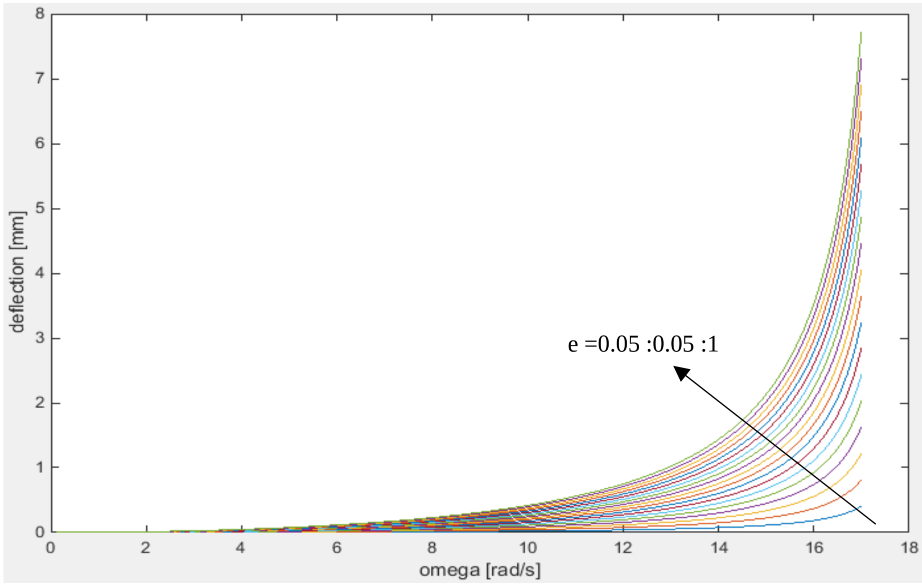


Figure 3-29 $y_1(\omega)$ for the both mass attached

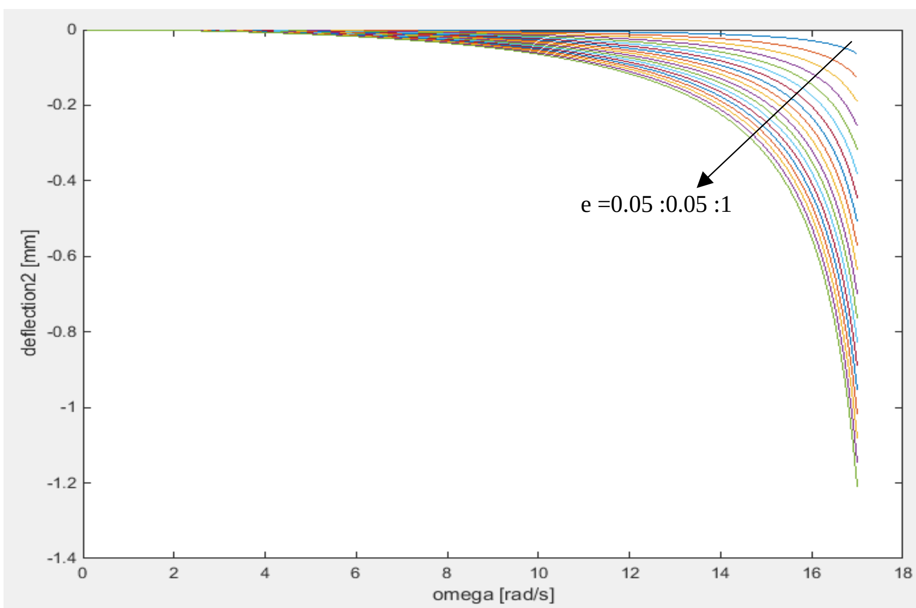


Figure 3-30 $y_2(\omega)$ for the both mass attached

To find allowable deflection, first we need to find a maximal bending moment on the shaft, preliminary to find a place of it and its value we can use several assumptions in order to simplify a work. We know that $M_1 = 60$ kg and $M_2 = 4$ kg, which makes $M_2 = M_1 / 15$ and assuming that $y_2 = y_1$ then we can write next:

$$O_1 = M_1 * y_1 * \omega^2$$

$$O_2 = M_2 * y_2 * \omega^2 = \frac{M_1}{15} * y_1 * \omega^2 = \frac{O_1}{15} \tag{3.80}$$

If we say that centrifugal force O_1 = 1 N, then O_1 =1/15 N, then taking into account that we used a super position method which will make reaction at the support B a sum of reactions calculated for force O_1 and O_2 separately.

$$R_B = R_{B_1} + R_{B_2} \tag{3.81}$$

Using advantage of the software MATLAB®, we can construct a preliminary bending on the shaft for both forces applied.

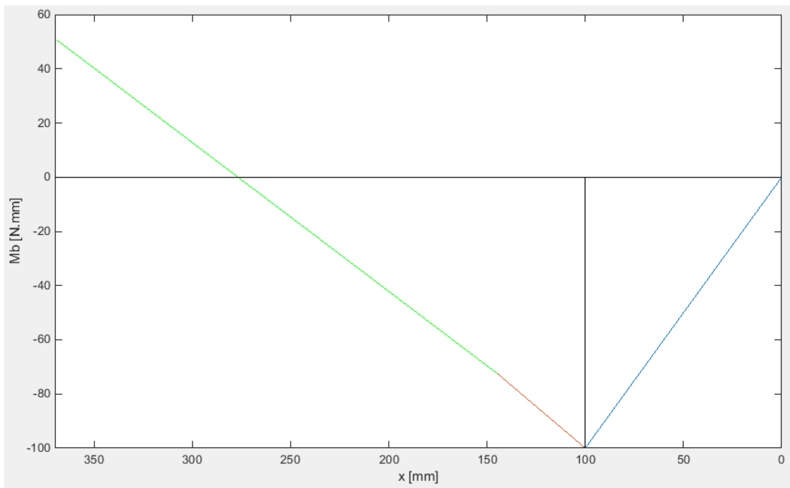


Figure 3-31 Bending distribution on the shaft with O_1=1N and O_2 = O_1/15

From the Figure 3-31 we can clearly see that the maximum moment is situated at the shaft position l_1, which is a product of the centrifugal force O_1.

$$M_{\max} = O_1 * l_1$$

$$\sigma_{all} = \frac{(O_{all} * l_1)}{\frac{I_1}{r_1}}$$

$$O_{all} = \frac{I_1}{r_1} * \frac{1}{l_1} * \sigma_{all}$$

$$O_{all} = \frac{201288.96}{\frac{45}{2}} * \frac{1}{100} * 100 = 8946.17\ [N]$$

$$y_{1_{all}} = \frac{O_{all}}{k_{11}} = \frac{8946.17}{19479.329} = 0.46\ [mm] \tag{3.82}$$

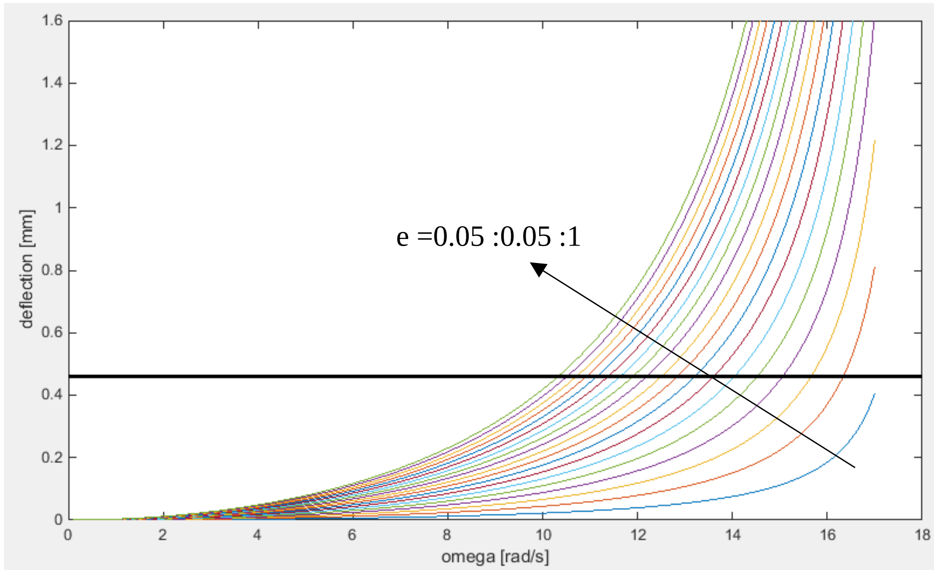


Figure 3-32 $y_{1.}(\omega)$ with showing allowable deflection. For both mass attached.

Adding the mass of a shaft

Mass of the shaft was read from the 3D CAD model and it is $M_S = 2.16$ kg. It is made from aluminum and has density 2700 kg/m^3. In order to simplify the task, I am not going to take a shaft mass as a distributed thing. I am going to take it as a concentrated mass in the "dangerous" points, which are the places of a applied centrifugal forces O_1 and O_2. To be on the safety side of the problem I will assume shaft as a 3 kg and will add this mass to each attached mass. It will make M_1 = 60+3=63 kg and M_2 =4+3=7 kg. Results are:

$$\Omega_s = \begin{bmatrix} \Omega_{1_s} \\ \Omega_{2_s} \end{bmatrix} = \begin{bmatrix} 17.5 \\ 250.8 \end{bmatrix} \text{ [rad/s]} \tag{3.83}$$

Compare these results with results we've got in (3.79) we can say that the difference in Ω_1 is 0.5 rad/s which is 4.77 rpm. The velocity, in this case, that we should not reach is 167 rpm. It satisfies our requirement maximum speed should be bigger than 120 rpm.

3.4. Motor mounting

We chose that we would drive the centrifugal machine using timing belt transmission and every belt transmission requires belt pre-tension, this can be two methods.

First allowing the pulley distance to be varied.

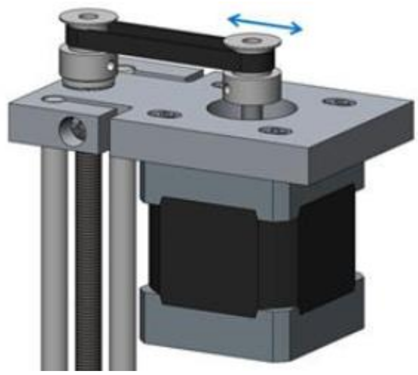


Figure 3-33 Center distance adjusting [12]

Second method: Including an adjustable idler pulley or roller which may be inside or outside the belt loop.

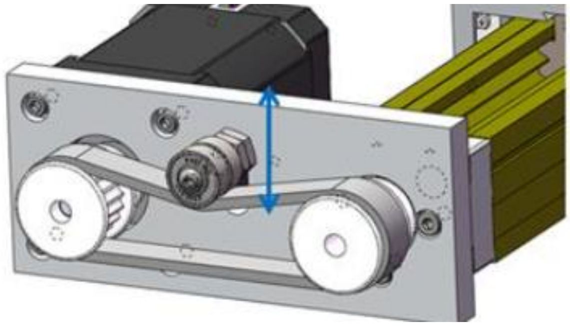


Figure 3-34 Adjustable idler pulley [12]

From this point beyond we will focus on the first method.

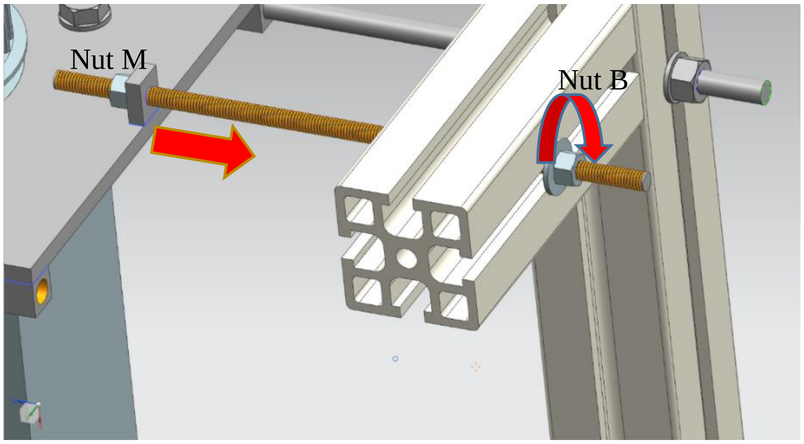


Figure 3-35 Motor adjusting through screw (preliminary design)

On the Figure 3-35 is shown, that motor adjusting will be provided throughout screw mechanism. By rotating a Nut B in clock wise direction, the motor will move to the direction of the bolt along the screw rod. By this motion we can crate suitable pretension of the belt.

3.5. Lug and pin design

For the assembling capsule to the centrifuge arm, there should be special connection that I choose to be lug and pin.

3.5.1. Lug design

Loading

First, we have to find what loading will appear on the lug or what loading the lug has to handle. Since I choose that so called "female" lug will be installed on the centrifugal arm, i.e. on the 45 x 45 profile and the loading will be transferred the through the pin we can find forces that will appear on the lug by finding reaction on beam's supports.

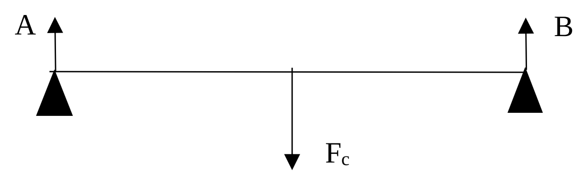


Figure 3-36 Pin loading

Reactions A and B will be the load that lug has to handle. Since, force from the capsule is situated in the middle of the pin, reactions are

$$A = B = \frac{F_c}{2} \tag{3.84}$$

Force F_C was found in the paragraph §3.1.2 p.14, it is the resultant force of mass force and centrifugal. Its value is:

$$F_c = \sqrt{G_c^2 + C_c^2} = \sqrt{150^2 + 1500^2} = 1507.5 \approx 1510\ [N] \tag{3.85}$$

Note that, the real values of the G_c is 147.15 [N] and centrifugal force is C_c = 1471.5 [N] for acceleration of a = 10g, but to be on the safety side we have to be conservative.

Finally, force in the supports and at the same time forces that lug has to handle is

$$\boldsymbol{A = B = \frac{1510}{2} = 755\ [N]}$$

Lug design

The material the lug will be made from has a significant influence on the lug strength (lug geometry). For now we will continue to use same material as the profile is made from, which is AlMgSi0.5F25 or also known as Al 6063A. Its mechanical properties were shown in the Table 13. There just has to be added information about its ultimate tensile stress, which is σ_{ult} = 130 [MPa]. Using method that was described in the book "Airframe stress analyses and sizing" by Michael C. Y. Niu [13, p. 321] we can check if chosen dimensions are valid or not.

As the first step we have to guess what values we will use. They are:

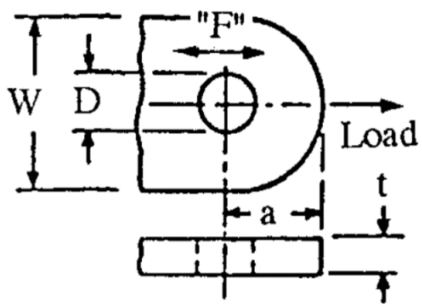


Figure 3-37 Lug dimensions

Exists 3 types of lug loads: axial, transverse and oblique.

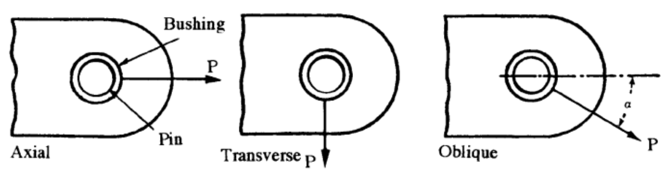


Figure 3-38 Types of lug loads

Case I – Axial load

The lug failure modes for this load case are *Share – Bearing failure and Tension failure.*

Share – Bearing Failure

Failure consists of shear tear-out of the lug along a 40^0 angle on both sides of the pin (see Figure 3-39), while bearing failure involves the crushing of the lug by the pin bearing.

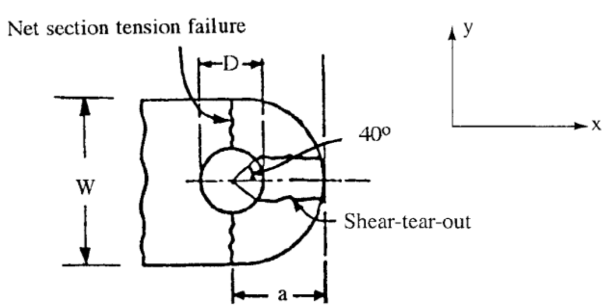


Figure 3-39 Lug tension and shear-tear out failures

The ultimate load for this type of failure is given by the equation

$$P_{bru} = k_{br} * \sigma_{tux} * A_{br} \tag{3.86}$$

Where P_{bru} – ultimate load for shear – tear out and bearing failure; k_{bru} – Shear – bearing efficiency factor see Figure 3-40; A_{br} – Projected bearing area $A_{br} = D * t$; σ_{tux} – Ultimate tensile stress in x – direction of the material.

D	18	[mm]
t	2	[mm]
a	15	[mm]
W	30	[mm]

Table 24 Lug dimensions

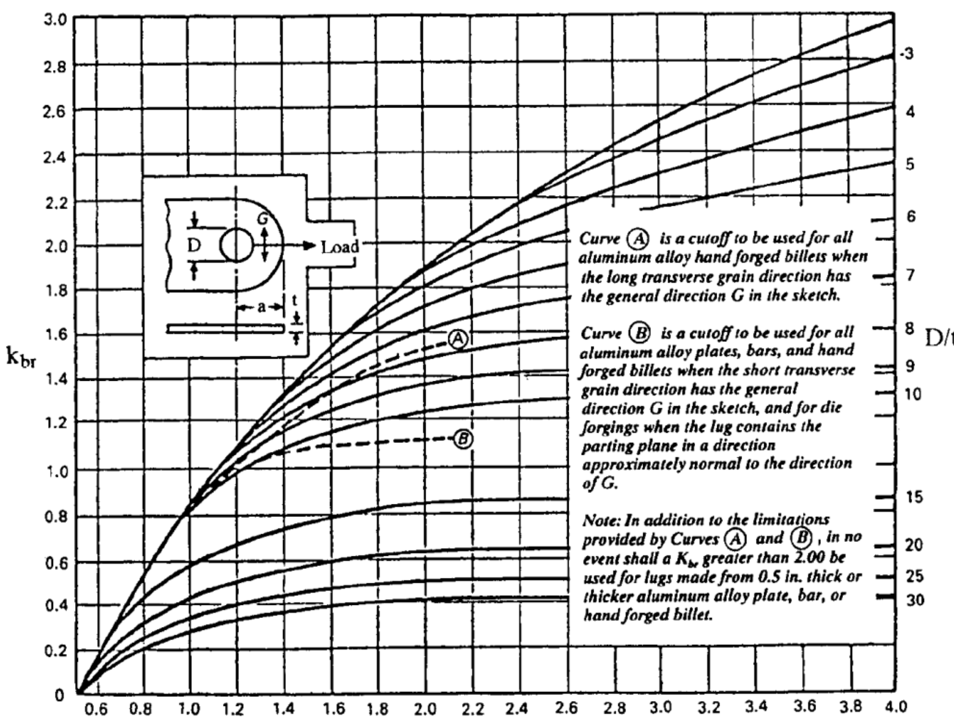


Figure 3-40 Shear-Bearing efficiency factor [13]

Tension failure

Tension failure is given by:

$$P_{tu} = k_t * \sigma_{tux} * A_t \tag{3.87}$$

Where P_{tu} – Ultimate load for tension failure; k_t – Net tension efficiency factor, see ; σ_{tux} – Ultimate tensile stress in x – direction of the material; A_t – Minimum net section for tension $A_t = (W - D)t$

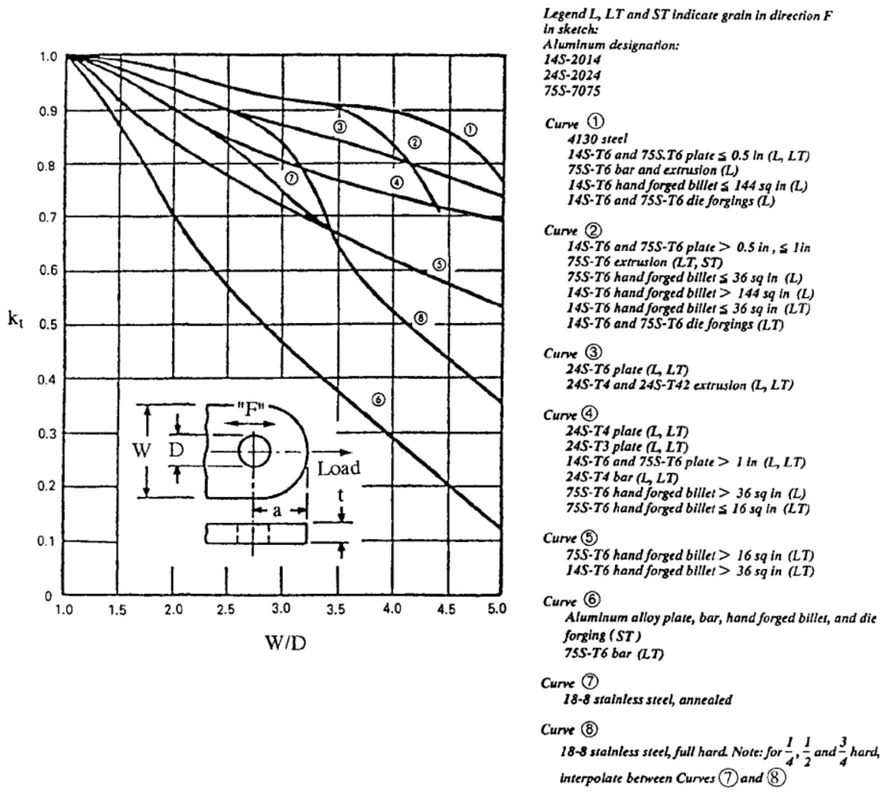


Figure 3-41 Lug efficiency factor for tension [13]

Yield Failure – lug

Lug yield failure attributable to shear – bearing is given by:

$$P_{Y,0} = C * \left(\frac{\sigma_{tyx}}{\sigma_{tux}}\right) * P_{u_{\min}} \tag{3.88}$$

Where P_Y – Yield load; C – Yield factor see Figure 3-42; σ_{tyx} – Tensile yield stress of lug material in load direction; $P_{u_{\min}}$ - The smaller P_{bru} or P_{tu}

Yield Failure – bushing

Bushing yield bearing load attributable to shear – bearing is given by:

$$P_{bry} = 1.85 * \sigma_{cy} * A_{brb} \tag{3.89}$$

Where P_{bry} – Bushing yield bearing load; σ_{cy} – Compression yield stress of bushing material, A_{brb} – the smaller of the bearing areas of bushing on pin or bushing on lug.

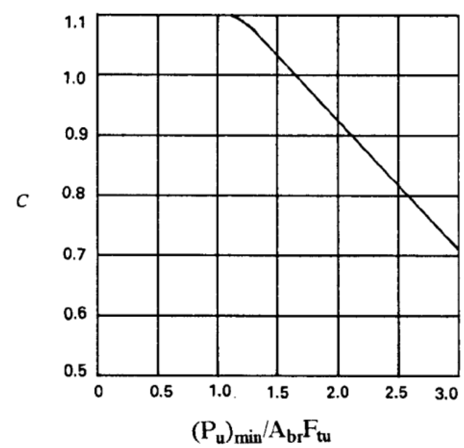


Figure 3-42 Yield factor

Case II – Transverse Load (α = 90°)

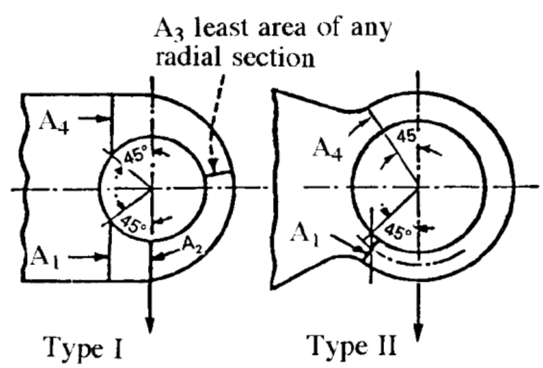


Figure 3-43 Lugs subjected to transverse load

For further calculation we have to compute: Projecting bearing area A_{br} and Average area A_{av}

$$A_{br} = Dt$$

$$A_{av} = \frac{6}{\frac{3}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}} \quad (3.90)$$

The ultimate load is obtained using next equation:

$$P_{tru} = k_{tru} * A_{br} * \sigma_{tuy} \quad (3.91)$$

Where: P_{tru} – Ultimate transverse load; k_{tru} -Efficiency factor for transverse ultimate load see Figure 3-44; σ_{tuy} – Ultimate tensile stress of lug material in y – direction.

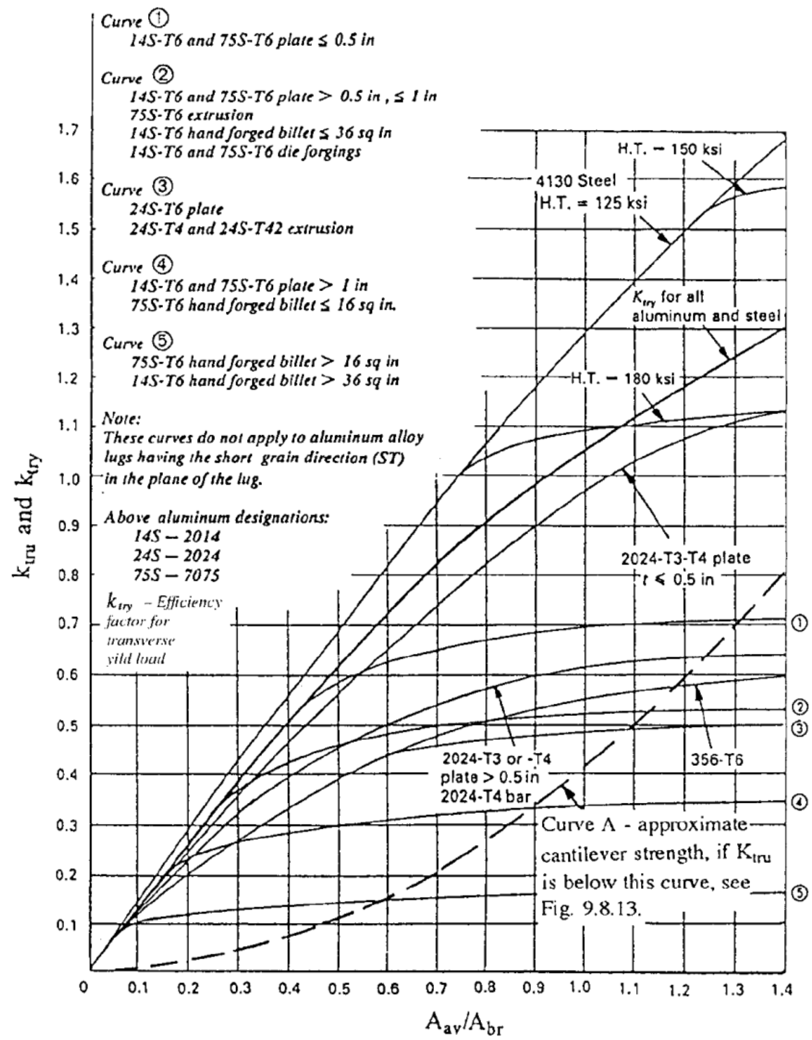


Figure 3-44 Efficiency factor for transverse load [13]

The Yield load can be obtained then by next equation.

$$P_{Y,90} = k_{try} * A_{br} * \sigma_{tyy} \tag{3.92}$$

Where: P_Y – Yield transverse load; k_{try} – Efficiency factor for transverse yield load, see Figure 3-44; σ_{tyy} – Tensile yield stress of lug material in y – direction.

Case III – Oblique Load

For Ultimate load

$$j = \frac{1}{\left(R_{a,u}^{1.6} + R_{tr,u}^{1.6}\right)^{0.625}} \tag{3.93}$$

For Yield load

$$j = \frac{1}{\left(R_{a,y}^{1.6} + R_{tr,y}^{1.6}\right)^{0.625}} \tag{3.94}$$

Where; j –Reserve factor; $R_{a(tr),u(y)}$ – axial or transverse component (indices "a" or "tr" respectively) of applied ultimate or yield (limit) load (indices "u" or "y" respectively) divided by smaller of P_{bru} or P_{tu} from equation (3.86) or (3.87) respectively, or $P_{Y,0}$ (3.88) or $P_{Y,90}$ (3.92) for yielding.

Application

Axial load

Using material information and guessed dimensions written above, in the beginning of this part we have following ratios:

D/t	9
a/D	0.83
W/D	1.67

Table 25 lug dimension ratios

Following factors k_{bru}, k_t and C were read from Figure 3-40, Figure 3-41 and Figure 3-42 respectively.

k_{bru}	0.6
k_t	0.74
C	1.1

Table 26 Axial case factors

Using equation (3.86) we find ultimate P_{bru}

$$P_{bru} = 0.6 * 130 * (18 * 2) = 2808\ [N] \tag{3.95}$$

By equations (3.87) we can compute P_{tu}

$$P_{tu} = 0.74 * 130 * (30 - 18) * 2 = 2308.8\ [N] \tag{3.96}$$

Yield axial load with the use of equation (3.88)

$$P_{Y,0} = 1.1 * \left(\frac{100}{130}\right) * 2308.8 = 1953.6\ [N] \tag{3.97}$$

Theoretically, this Yield load satisfies given conditions, when the arm of the centrifuge is loaded by weight of the capsule only, which makes in ideal case pure axial load. Numerically, female lug it has to stand load of $\frac{150}{2} = 75\ [N]$, where the reserve factor is then $j = \frac{1953.6}{75*f,(f=2)*\lambda,(\lambda=1.15)} = 11.3\ [-]$, of course it is more than we need, but we have to leave chosen dimension, because we have to calculate further transverse load and oblique one, which's loads are much bigger than axial one.

Transverse Load

Projecting bearing area $A_{br} = 18 * 2 = 36\ [mm^2]$ and Average area specifically for our case

$$A_{av} = \frac{6}{\frac{3}{t(\frac{W}{2}-\frac{D}{2}*\sin 45)}+\frac{1}{(\frac{W}{2}-\frac{D}{2})t}+\frac{1}{(\frac{W}{2}-\frac{D}{2})t}+\frac{1}{t(\frac{W}{2}-\frac{D}{2}*\sin 45)}} = \frac{6}{\frac{1}{17.2721}+\frac{1}{12}+\frac{1}{12}+\frac{1}{17.2721}} = 15.066\ [mm^2]$$

The areas ratio is then $\frac{A_{av}}{A_{br}} = 0.4185\ [-]$, for the curve (1) of Figure 3-44 the efficiency factor for transverse ultimate load $k_{tru} \approx 0.5\ [-]$, so as the same factor for yield load $k_{try} \approx 0.5\ [-]$.

The ultimate load is computed by equation (3.91) and yield load by equation (3.92)

$$P_{tru} = 0.5 * (18 * 2) * 130 = 2340\ [N]$$
$$P_{Y,90} = 0.5 * (18 * 2) * 100 = 1800\ [N] \tag{3.98}$$

Oblique load

The reserve factor for the ultimate load for oblique case:

$$j = \frac{1}{\lambda * \left(\left(\frac{f * 75}{2308.9}\right)^{1.6} + \left(\frac{f * 750}{2340}\right)^{1.6}\right)^{0.625}} = 1.33\ [-]$$

Where λ is fitting factor and λ=1.15, *f* is safety factor, *f* = 2 (3.99)

If we want to be safe, we have to be on the conservative side of the problem. This means, that we will use yield characteristics to calculate reserve factor, with the usage of ultimate loads, which are *limit loads * safety factor*, where limit load are 150/2 N for axial load and 1500/2 N for transversal load.

$$j = \frac{1}{\lambda * \left(\left(\frac{f * 75}{1953.6}\right)^{1.6} + \left(\frac{f * 750}{1800}\right)^{1.6}\right)^{0.625}} = 1.02 \tag{3.100}$$

If we change the orientation of the lug the way that axial force on one lug will be 1500/2 N, and transversal force will 150/2 N, the reserve factor is then

$$j = \frac{1}{\lambda * \left(\left(\frac{f * 75}{1953.6}\right)^{1.6} + \left(\frac{f * 750}{1800}\right)^{1.6}\right)^{0.625}} = 1.11 \tag{3.101}$$

Changing the orientation of the lug, we have increased reserve factor for the yield load, but it will decrease reserve factor for ultimate load for 1.33 to 1.31

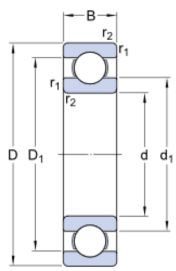
Reserve factor *j* should be bigger than 1, *j>1* and we have satisfied the obligatory requirement that the reserve factor for yield load should be bigger than 1, **j = 1.11> 1**. Note that the calculation above were done for the orientation of lug where centrifugal force will be applied in the direction of axial load, there were used safety factor *f = 2*, fitting factor λ = 1.15 and external load that this lug has to stand were artificially increased for little bit, so we can conclude that final dimensions of the lug

In case we need that one lug will stay full scope of the loading, i.e. that axial loading that will be applied on the one lug will be no longer divided by two, F_a = 1500 N, not F_a = 1500/2 =759 N. Then lug has to be modified by further steps: increase the thickness for 1 mm (t = 3 mm), increase distance "*a"* from 15 mm to 16 mm and width "*W"* from 30 mm to 32 mm (see Figure 3-37). Output parameters are then shown in Table 27

Geometry			Axial			Transversal			Reserve factors for oblique		
D/t	6	[-]	k_{bru}	0.8	[-]	k_{tru}	0.57	[-]	j_{ult}	1.22	[-]
a/D	0.889	[-]	k_t	0.785	[-]	k_{try}	0.578	[-]	j_y	**1.03**	**[-]**
W/D	1.778	[-]	C	1.1	[-]	P_{tru}	4001.4	[N]			
			P_{bru}	5616	[N]	$P_{y,90}$	3121.2	[N]			
			P_{tu}	4286.1	[N]						
			$P_{y,0}$	3626.7	[N]						

Table 27 Strength parameters for one lug mode

Results in the Table 27 are acceptable since reserve factors for yielding stress and ultimate stress are bigger than one, which means they satisfies necessary requirements. Thickness of the lug for this result is 3mm, but bearings of width 3mm of SKF manufacturer can not carry the load of 755 N. The closest bearing that can carry the load and being in geometrical limits of the lug that can be mounted to the beam of 45x45 profile is SKF 61801. It has next parameters:



d		12	mm
D		21	mm
B		5	mm
d_1	≈	14.8	mm
D_1	≈	18.3	mm
$r_{1,2}$	min.	0.3	mm

Calculation data

Basic dynamic load rating	C	1.74	kN
Basic static load rating	C_0	0.915	kN
Fatigue load limit	P_u	0.039	kN
Reference speed		70000	r/min
Limiting speed		43000	r/min
Calculation factor	k_r	0.015	
Calculation factor	f_0	13.4	

Mass

Mass bearing	0.0063	kg

Figure 3-45 SKF 61801 bearing parameters [14]

Because of this bearing we need to increase the bore diameter up to 12 mm, which will be also pin diameter in further calculation. Thickness should increase minimum to 5 mm (for the construction purposes it will be increased to 7 mm). In that case strength parameters of the lug that has parameteers are shown in the

Geometry			Axial			Transversal			Reserve factors for oblique		
D/t	1.71	[-]	k_{bru}	1.3	[-]	k_{tru}	1.1	[-]	j_{ult}	13.31	[-]
a/D	1.375	[-]	k_t	0.75	[-]	k_{try}	1.2	[-]	j_y	11.55	[-]
W/D	3.75	[-]	C	1.08	[-]	P_{tru}	23100	[N]			
			P_{bru}	27300	[N]	P_y	20160	[N]			
			P_{tu}	43313	[N]						
			P_y	23587	[N]						

Table 28 Lug strength result, geometry modified for bearing SKF 61801

3.5.2. Pin design

Pin shear – off failure [13, pp. 326-329]

Pin single shear – off failure is given by:

$$P_{p,s} = \tau_{ult} * \left(\frac{\pi D^2}{4}\right) \tag{3.102}$$

Where: $P_{p,s}$ = Ultimate load for pin shear – off failure; τ_{ulit} – ultimate shear stress of the pin material.

Pin Bending Failure

If the pin used in the lug is not big enough to resist the bending that appeared from the load, the bend of the pin can participate failure in the lug. Because, as the pin bends, the stress distribution action on the inner side of the lug tends to peak rather than form an even distribution, as shown on Figure 3-46.

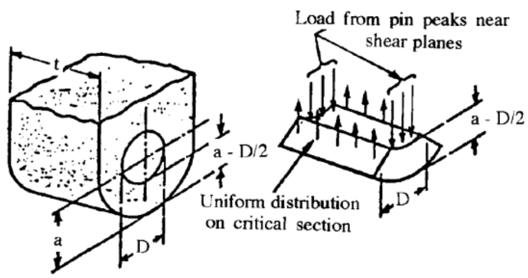


Figure 3-46 Peak Pin Load

For the pin bending failure check, we will use two different modes of the pin loading and which of these methods will be less dangerous being close to the failure will be used as the final version.

I mode – one male cylindrical lug of width 43 mm

II mode – two inner lug of 3 mm thickness \

I mode)

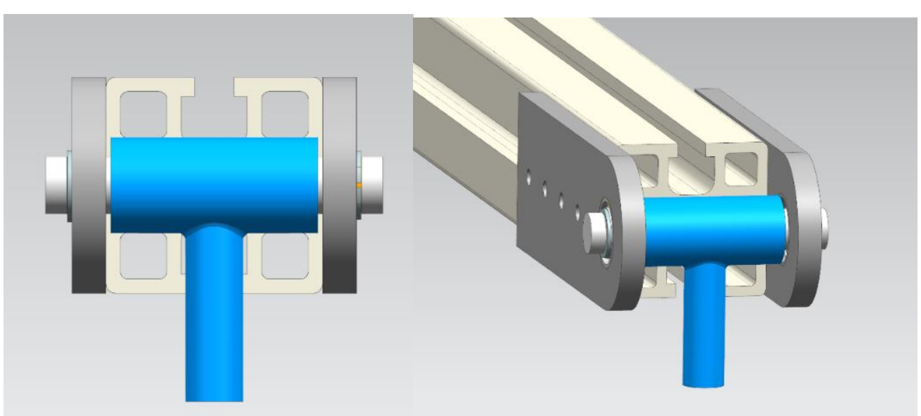


Figure 3-47 First mode of the pin loading

As simple free body diagram it can be shown as following:

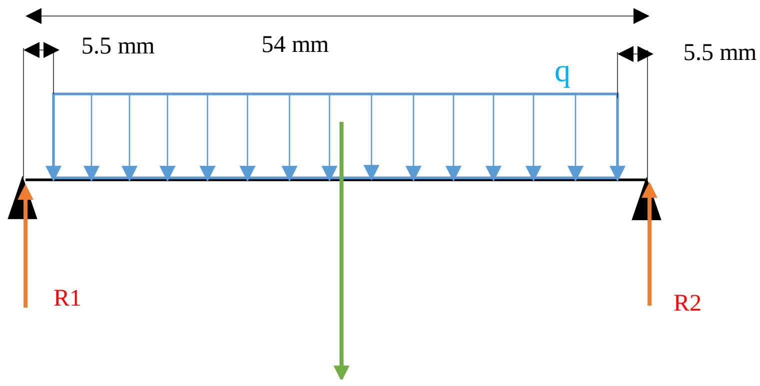


Figure 3-48 Free body diagram of the Mode I

Using equation (3.102) we can find ultimate load for the pin shear.

$$P_{p,s} = 100 * \left(\frac{\pi 12^2}{4}\right) = 11\,309.73\ [N] \tag{3.103}$$

Where D = 12 mm, $\tau = 100\ [MPa]$. The reason why D = 12 mm is in inner diameter of the bearing that was chosen in the paragraph §3.5.1.

$$R_1 = R_2 = \frac{q * (54 - 5.5 * 2) * \left(\frac{54 - 5.5 * 2}{2} + 5.5\right)}{54} = 755\ [N] \tag{3.104}$$

$$M(x) = R_1 * x\,, \qquad for\ x\ from\ 0\ to\ 5.5\ mm$$
$$M(x) = R_1 * x - q * \frac{(x - 5.5)^2}{2}, \qquad for\ x\ from\ 5.5\ to\ 27\left(= \frac{54}{2}\right),$$
$$since\ problem\ is\ symmetrical, it\ should\ be\ enough \tag{3.105}$$

Shearing force and Bending moment diagrams are then shown on the figures below, with the peak for Shearing force 755 N (or -755 N) and the peak for bending moment 12268,75 [N.mm] at x =27 mm.

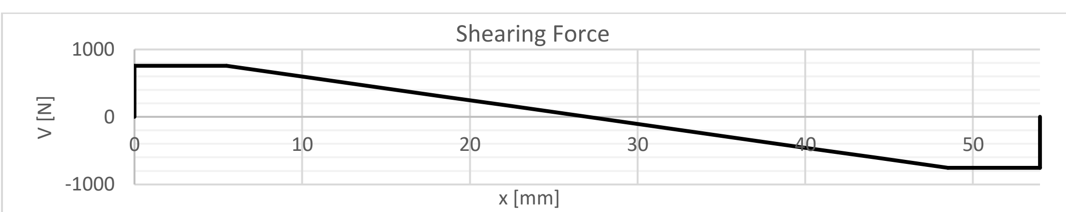


Figure 3-49 Shearing Force distribution force, loading of Mode I

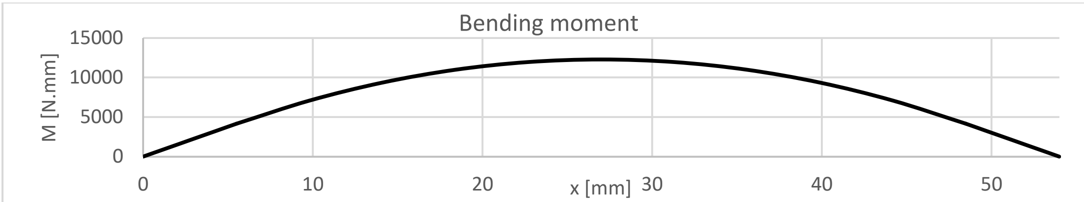


Figure 3-50 Bending moment distribution, Mode I

The normal stress is then:

$$\sigma_{\max} = \frac{M_{\max}}{\frac{I}{r}} = \frac{12268.75}{\pi * \frac{D^3}{32}} = 72.32\ [MPa] \tag{3.106}$$

Shear stress

$$\tau = \frac{4}{3} * \frac{V_{\max}}{\frac{\pi D^2}{4}} = \frac{4}{3} * \frac{755}{\pi * \frac{12^2}{4}} = 8.9\ [MPa] \tag{3.107}$$

HMH hypothesis for equivalent stress

$$\sigma_{eq} = \sqrt{\sigma^2 + (3 * \tau)^2} = \mathbf{77.1}\ [\boldsymbol{MPa}] \tag{3.108}$$

Allowable stress for the material mentioned in the Table 13, for safety factor $f = 2\ [-]$, is 100 MPa, the reserve factor in this case is:

$$\boldsymbol{j} = \frac{\mathbf{100}}{\mathbf{77.1}} = \mathbf{1.29}\ > 1\ [-], \textit{which is acceptable} \tag{3.109}$$

Maximum deflection is expected to be in the middle of the pin, so we apply there the unit force. Using the Mohr's integral (equation (3.110)) we can find the deflection at the required point

$$y_i = \int_{(L)} \frac{M(x)}{EI} m_i\ dx \tag{3.110}$$

Since we have isotropic material with constant cross-section of Young's modulus and second moment of inertia can be considered as constants, hence may be taken out of integral .

$$y_i = \frac{1}{EI}\int_0^L M(x) * m_i\, dx = \frac{1}{EI}\left[\int_o^{5.5}\left(R_1 x * \frac{1}{2}x\right)dx + \int_{5.5}^{27}\left(\left(R_1 x - \frac{q(x-5.5)^2}{2}\right) * \frac{1}{2}x\right)dx\right] * 2^{(1)} =$$

$$= 2 * \left[\frac{q(54-11)}{4EI} * \frac{5.5^3}{3} + \frac{1}{EI}\int_{5.5}^{27}\left(\frac{q(54-11)}{4}x^2 - \frac{q}{4}(x-5.5)^2 x\right)dx\right] =$$

$$= 2 * 0.00029 + \frac{2}{70000 * 1017.876}\frac{q}{4}\left[43 * \left(\frac{x^3}{3}\right)_{5.5}^{27} - \left(\frac{x^4}{4}\right)_{5.5}^{27} + 2 * 5.5\left(\frac{x^3}{3}\right)_{5.5}^{27} - 5.5^2\left(\frac{x^2}{2}\right)_{5.5}^{27}\right] =$$

$$= y_{\frac{L}{2}} = \left(0.000294 + (-0.02564)\right) * 2 = -\mathbf{0.051}\ [\boldsymbol{mm}]$$

(3.111)

---

(1) Times 2 due to symmetricity of the problem

II mode)

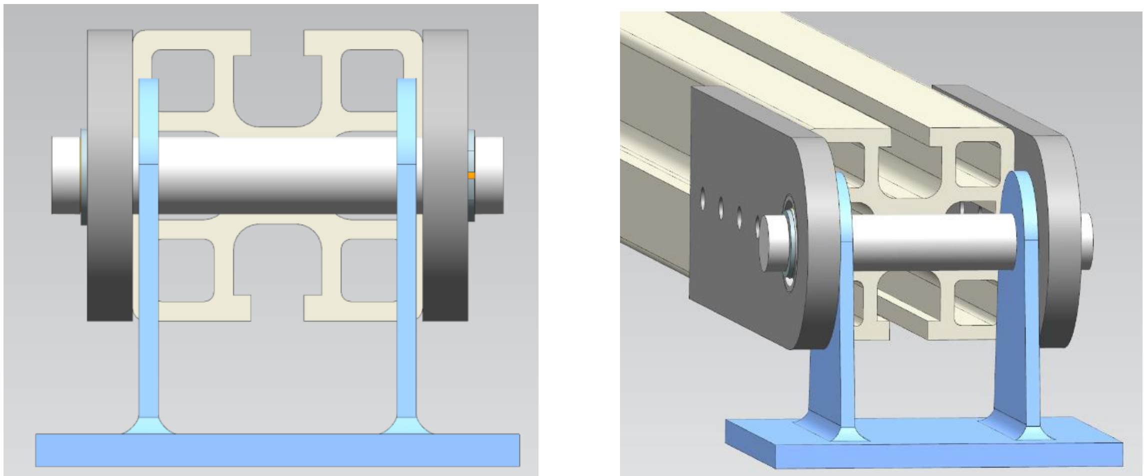


Figure 3-51 II mode of pin loading

This mode of the loading transfer can be represented as free body diagram as shown on the figure below.

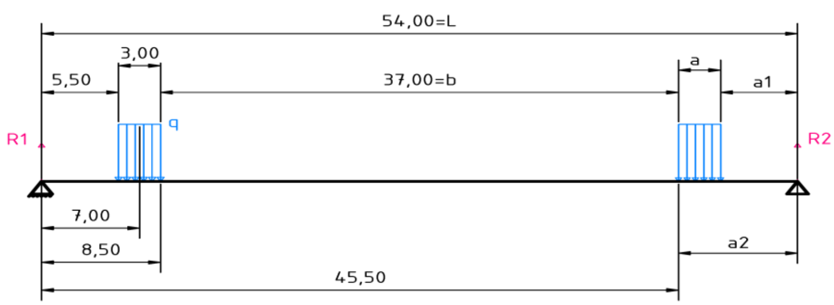


Figure 3-52 Free body diagram, mode II

$$R_1 = R_2 = \frac{q * a * \left(2\left(\frac{a}{2} + a_1\right) + \left(b + 2 * \frac{a}{2}\right)\right)}{L}, if\ b = L - 2(a_1 + a), then$$

$$R_1 = R_2 = \frac{qa\left(2 * \left(\frac{a}{2} + a1\right) + (L - 2a_1 - 2a + a)\right)}{L} = \frac{q * a * L}{L} = q * a$$

$$\boldsymbol{R_1 = R_2 = 251.667 * 3 = 755}\ [\boldsymbol{N}] \tag{3.112}$$

$$M(x) = R_1 * x, \qquad for\ x\ from\ 0\ to\ a_1$$

$$M(x) = R_1 * x - q * \frac{(x - a_1)^2}{2}, \qquad for\ x\ from\ a_1 to\ a_2$$

$$M(x) = R_1 * x - q * a * \left(x - \left(a_1 + \frac{a}{2}\right)\right), \qquad for\ x\ from\ a_2\ to\ \frac{L}{2} \tag{3.113}$$

Shearing force and Bending moment diagrams are then shown on the diagrams below, with their peaks 755 N (or -755 N) for Shearing force and 5285 [N.mm] for bending moment.

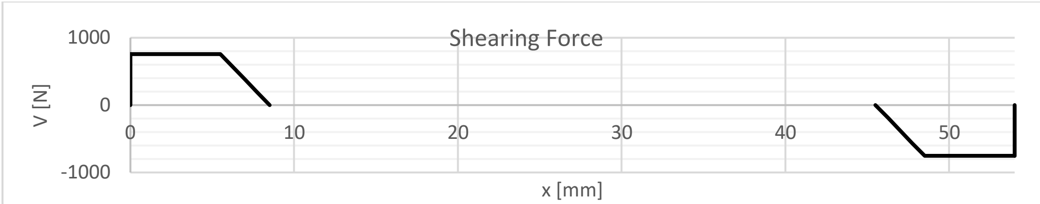


Figure 3-53 Shearing force distributions, Mode II

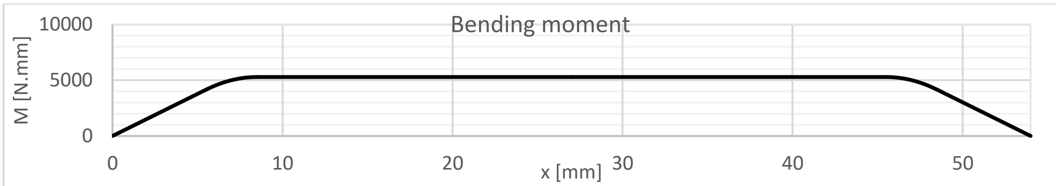


Figure 3-54 Bending moment, Mode II

As we can see, the bending moment, in the loading of the mode II, decreased more than two time, compare to the mode I. Taking into account that shearing force remain the same in both cases, shearing stress will not be changed, hence equivalent stress will decrease, which will make pin more safety. Bending stress is:

$$\sigma_{\max} = \frac{M_{\max}}{\frac{I}{r}} = \frac{5285}{\pi * \frac{D^3}{32}} = 31.15\ [MPa] \tag{3.114}$$

$$\sigma_{eq} = \sqrt{\sigma^2 + (3 * \tau)^2} = \mathbf{41.03}\ [\boldsymbol{MPa}] \tag{3.115}$$

$$\boldsymbol{j} = \frac{\mathbf{100}}{\mathbf{41.03}} = \mathbf{2.43}\ > 1\ [-], which\ is\ acceptable \tag{3.116}$$

Comparing reserve factors from the mode I [(3.112)] and II [(3.116)], we can make a statement, that from the strengthen point of view mode II is more desirable. It is 1.88 time safer. To find how different maximum deflections are, we, again, will use Mohr's integral, which formula was shown in (3.107).

Since the problem is a little complex, compare to the mode I, we will use an advantage of the super position method. Which is:

$$y_i = y_i(f_1) + y_i(f_2) \tag{3.117}$$

Where f_1 and f_2 are cases, when the pin is only loaded by left or right distributed load, f_1 represents left one, f_2 represents the right one, from the view that is shown on the Figure 3-52. But dew to symmetricity both deflection will be the same, so the final form of the equation (3.117) is $y_i = 2 * y_i(f_1) = 2 * y_i(f_2)$

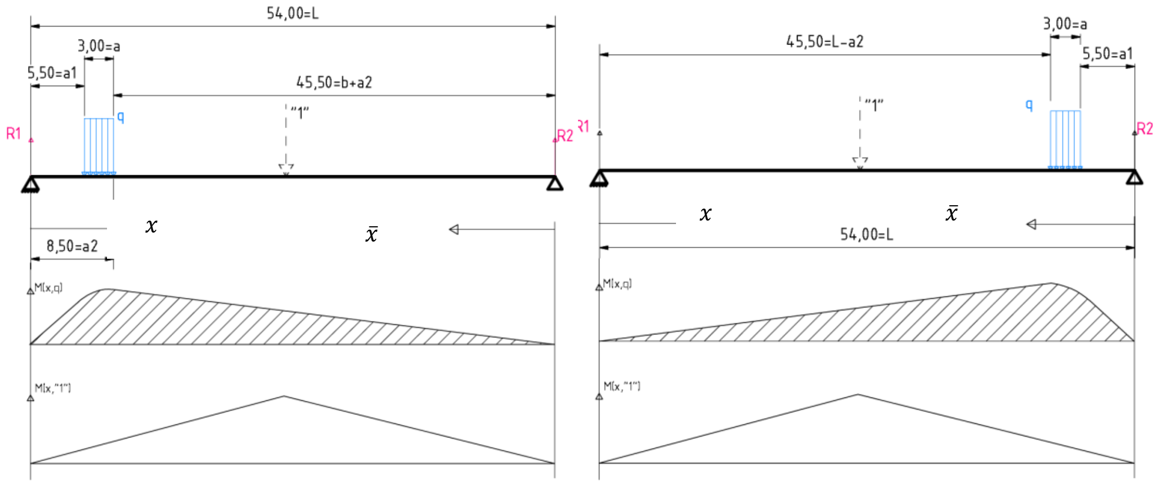


Figure 3-55 A. Mode II, f_1   *B. Mode II, f_2*

$$R_2 = \frac{qa\left(\frac{a}{2}+a1\right)}{L} = 97.9\ [N]\ ,\quad R_1 = \frac{qa\left(\frac{a}{2}+b+a_2\right)}{L} = 657\ [N]$$

Bending moments from loading f_1

$$x \in [0; a_1] \rightarrow M(x) = R_1 x;$$

$$x \in [a_1; a_2] \rightarrow M(x) = R_1 x - \frac{q(x-a_1)^2}{2}$$

$$x \in [a_2; L] \rightarrow M(x) = R_1 x - qa\left(x - a_1 - \frac{a}{2}\right)$$

$$or \quad \bar{x} \in [0; 0.5L] \rightarrow M(\bar{x}) = R_2 \bar{x}$$

Bending moment form Unite force "1"

$$x \in \left[0; \frac{L}{2}\right] \rightarrow m_i(x) = \frac{1}{2}x$$

$$x \in \left[\frac{L}{2}; L\right] \rightarrow m_i(x) = \frac{L}{2} - \frac{1}{2}x$$

$$or \quad \bar{x} \in \left[0; \frac{L}{2}\right] \rightarrow m_i(x) = \frac{1}{2}\bar{x}$$

Due to symmetricity follows:

$$R_1 = 97.9\ [N]; \quad R_2 = 677\ [N]$$

Bending moment from the load f_2

$$\bar{x} \in [0; a_1] \rightarrow M(\bar{x}) = R_2 \bar{x}$$

$$\bar{x} \in [a_1; a_2] \rightarrow M(\bar{x}) = R_2 \bar{x} - \frac{q(\bar{x}-a_1)^2}{2}$$

$$x \in [0; 0.5L] \rightarrow M(x) = R_1 x$$

Bending moments from unite force

$$\bar{x} \in \left[0; \frac{L}{2}\right] \rightarrow m_i(x) = \frac{1}{2}\bar{x}$$

$$x \in \left[0; \frac{L}{2}\right] \rightarrow m_i(x) = \frac{1}{2}x$$

Type equation here. (3.118)

From obtained equations we can solve the Mohr's integral for the load f_1 and then multiply it by 2, because of symmetricity.

$$y_{\frac{L}{2}}(f_1) = \int_{(L)} \frac{1}{EI} * M(x) * m_i(x)dx \;\rightarrow$$

$$\frac{1}{EI} * \left[\int_0^{a_1} R_1 x * \frac{1}{2}x\, dx + \int_{a_1}^{a_2} \left(R_1 x - q * \frac{(x-a_1)^2}{2}\right) * \frac{1}{2}x\; dx + \int_{a_2}^{\frac{L}{2}} \left(R_1 x - qa\left(x - a_1 - \frac{a}{2}\right)\right) * \frac{1}{2}x\; dx \right.$$
$$\left. + \int_0^{\frac{L}{2}} R_2 * \bar{x} * \frac{1}{2} * \bar{x}\; d\bar{x}\right]$$

For the simplification each part of the integral will be solved individually and then summed

$$\int_0^{a_1} R_1 x * \frac{1}{2} x\, dx = \frac{R_1}{2} * \left(\frac{x^3}{3}\right)_{0_1}^{a} = \frac{R_1}{2} * \left(\frac{a_1^3}{3}\right) = \frac{657}{2} * \frac{5.5^3}{3} = 18221.657\; [N.mm^3] \qquad (1)$$

$$\int_{a_1}^{a_2} \left(R_1 x - q * \frac{(x-a_1)^2}{2}\right) * \frac{1}{2}x\; dx$$
$$= \frac{R_1}{2}\left(\frac{a_2^3}{3} - \frac{a_1^3}{3}\right) - \frac{q}{4} * \left[\left(\frac{a_2^4}{4} - \frac{a_1^4}{4}\right) - 2a_1\left(\frac{a_2^3}{3} - \frac{a_1^3}{3}\right) + a_1^2\left(\frac{a_2^2}{2} - \frac{a_1^2}{2}\right)\right]$$
$$= \frac{657}{2}\left(\frac{8.5^3}{3} - \frac{5.5^3}{3}\right) - \frac{251.667}{4}\left[\left(\frac{8.5^4}{2} - \frac{5.5^4}{2}\right) - 2 * 5.5\left(\frac{8.5^3}{3} - \frac{5.5^3}{3}\right) + 5.5^2\left(\frac{8.5^2}{2} - \frac{5.5^2}{2}\right)\right]$$
$$= 44649.861\; [N.mm^3] \qquad (2)$$

$$\int_{a_2}^{\frac{L}{2}} \left(R_1 x - qa\left(x - a_1 - \frac{a}{2}\right)\right) * \frac{1}{2}x\; dx = 556684.63\; [N.mm^3] \qquad (3)$$

$$\int_0^{\frac{L}{2}} R_2 * \bar{x} * \frac{1}{2} * \bar{x}\; d\bar{x} = \frac{97.9}{2} * \left(\frac{27^3}{3}\right) = 321063.75\; [N.mm^3\,] \qquad (4)$$

$$\boldsymbol{y_{\frac{L}{2}}(f_1)} = \frac{1}{EI} * [18221.657 + 44649.861 + 556684.63 + 321063.75]$$
$$= \frac{1}{70000 * 1017.876} * 940619.9 = \mathbf{0.0132}\; [\boldsymbol{mm}]$$

(5)

(3.119)

The positive meaning of the deflection is in the direction of the applied force.

Since we mentioned that due to symmetricity of the case, the complete deflection will be twice more than it has been calculated in (3.119).

$$\boldsymbol{y_{\frac{L}{2}} = 0.0132 + 0.0132 = 0.0264\; or - 0.0264[mm]} \qquad (3.120)$$

Comparing the deflection results of the *mode I* and *mode II*, we can see that: $-0.051 > 0.0264$, in almost two times (1.93).

From the obtained results of the stress safety and deflection, I conclude, that the more reliable and loading mode is the *mode II*. This mode will be used for further calculations.

In reality, the loading of the pin during arms rotation will not be symmetric and trivial as it was represented Figure 3-52. There was not mentioned the presence of the extra forces that will appear with moment from the drag force. The drag force can be determined through classic formula $F_D = \frac{1}{2}\rho v^2 * S * c_D$, where c_D is the drag coefficient of the cylinder. The velocity v is circumferential that is the function of the rotational speed and radius of rotation. For the 10g loading, for our case of machine, corresponds angular velocity of 9.9045 rad/s and arm can is 1m plus distance to the drag force concentration point. The distance to the drag force concentration point from the lug's eye center as shown on the Figure 3-56:

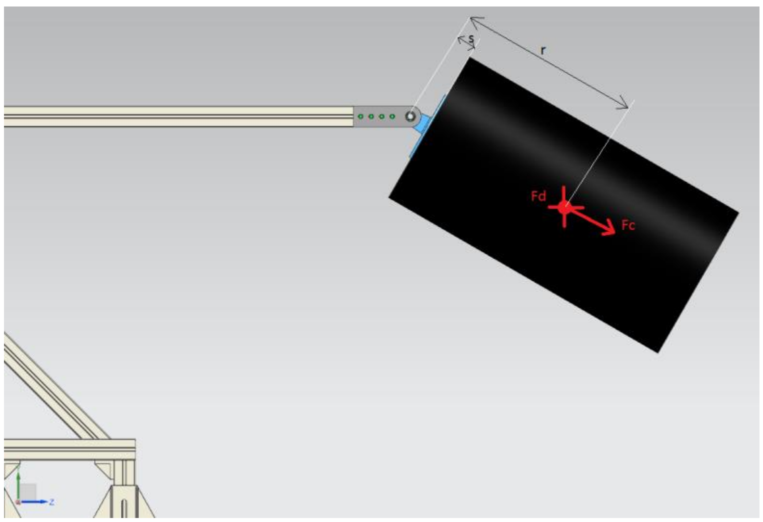


Figure 3-56 Point of force applied on capsule, Fd is a drag force, Fc is a centrifugal force

$$r = \frac{L_c}{2} + s = \frac{650}{2} + 60 = 385\ [mm]$$
$$a = r * \cos(90 - \alpha) = 385 * \cos(90 - 84) = 382.89\ \approx 383\ [mm] \tag{3.121}$$
$$b = r * \sin(90 - \alpha) = 385 * \sin(90 - 84) = 40.24\ [mm]$$

Therefore, the drag force is:

$$F_D = \frac{1}{2} * 1.225 * \left(9.9045 * \left(1 + \frac{0.65}{2} + 0.06\right)^2\right) * 0.65 * 0.35 * 1 = 20.2\ [N] \tag{3.122}$$

The drag force will have effect of moment to the pin. The moment has value of:

$$M = F_D * r = 20.2 * 385 = 7777\ [N.mm] \tag{3.123}$$

The moment M will be transferred to the pin by inner lugs as forces (distributed forces), which values are:

$$M = F_{M1} * r_{p_1} + F_{M2} * r_{p_2}$$

$$if\ r_{p_1} = r_{p_2} = r_p = \frac{a}{2} + \frac{b}{2} + \frac{a}{2} = 20\ [mm]\ and\ F_{M1} = F_{M2}$$

$$M = 2 * F_M * r_p \rightarrow F_M = \frac{M}{2r_p}$$

$$F_M = \frac{7777}{2 * 20} = 194.425\ [N]$$

(3.124)

The next step is just to add appeared forces on the pin from drag according to the logic shown on the Figure 3-57

$$F_1 = \frac{1510}{2} + F_M = 755 + 194.425 = 949.425\ [N] \approx 950\ [N]$$

$$F_2 = \frac{1510}{2} - F_M = 755 - 194.425 = 560.575\ [N] \approx 561\ [N]$$

(3.125)

So, the final form of the pin loading looks like as how on the figure below, if we assume distributed force as the point force (if we imagine 3mm distance as the point).

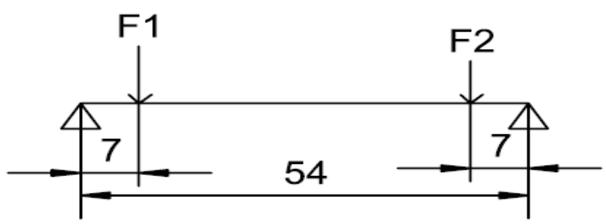


Figure 3-57 Pin loading, according drag effect (1B modification)

$$F_1 * a + F_2 * (a + b) - R_2 * l = 0$$

$$R_2 = \frac{F_1 a + F_2(a + b)}{l} = 611.426\ [N]$$

$$R_1 = F_1 + F_2 - R_2 = 899.6\ [N]$$

(3.126)

$$\sigma_{\max} = \frac{M_{\max}}{\frac{J}{r}} = \frac{R_1 a}{\frac{\pi D^3}{32}} = 37.12\ [MPa]$$

$$\tau = \frac{4}{3} * \frac{V_{\max}}{\frac{\pi D^2}{4}} = 11.2\ [MPa]$$

$$\sigma_{eq} = \sqrt{\sigma^2 + (3\tau)^2} = 50\ [MPa]$$

$$j = \frac{\sigma_{all}}{\sigma_{eq}} = \frac{100}{50} = 2\ [-]$$

(3.127)

3.6. Centrifuges head

As is it was mentioned on the page 21 there will be two major variant of the centrifuge's head. One-Beam variant – when only one beam on each arm carries a tested sample, its designation is 1B. Two-Beam variant – when two beams on each arm carries a load from the tested sample accelerated up to 10g. Has a 2B designation. This variant needs in order to decrease the risk of failure and decrease deflection of the beams under the load.

The centrifuges head should be designed the way that it will be easier in future to disassemble one variant and assemble another one. In order to complete this task, the cheapest and easiest way is to make a part, where beams are assembled, the same for both variants. After several iteration and design experiment I came to the next solution:

Centrifuge Head will consist from:

1) Hub – the part that is assembled to the rotating shaft
2) Head's Plate – the part, which is connected to the Hub and will carry beams.
3) Fasteners – connection elements.

All of the mentioned should be the same for 1B or 2B variant, and made of steel, in order to make it more rigid, stiffer.

Final solution are shown on the following figures.

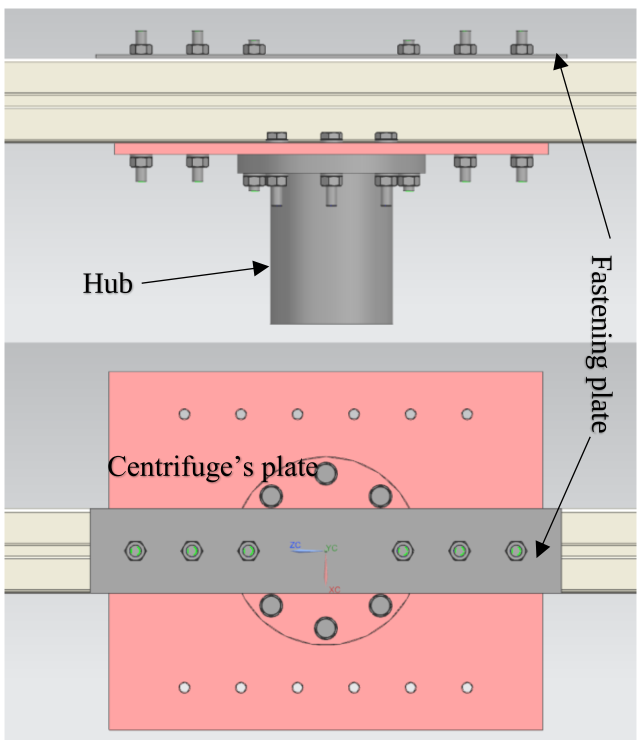


Figure 3-58 Centrifuge Head, 1B variant

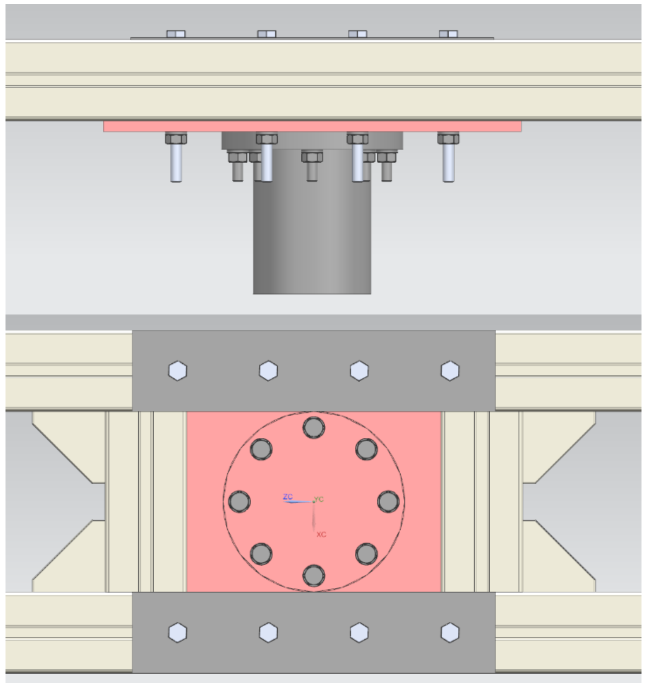


Figure 3-59 Centrifuge Head, 2B variant

As we see from Figure 3-58 and Figure 3-59, there is used one Head's Plate (pink) and one Hub (grey). Using Finite Element Method we will check their dimensions (thicknesses), in the paragraph §4.9, p. 97.

On the Figure 3-60 shown Centrifuge's Head Plate (CHP) dimensions . Holes that are situated on the upper part (72.5 mm far from the center of the 190 mm side) and lower part belongs for the 2B variant, when holes on the middle (center of the 190 mm side) belongs for 1B variant. Holes that are allocated as a radial pattern feature of 82 mm diameter are for the fastening CHP and Hub.

Hub transfers torque from shaft through the feather key. Contact pressure of the hub is similar to the contact pressure of the shaft of the relatable section, that was calculated in equation (3.53) p. 30. Contact pressure p = 12.2 MPa, and reserve factor is j >10.

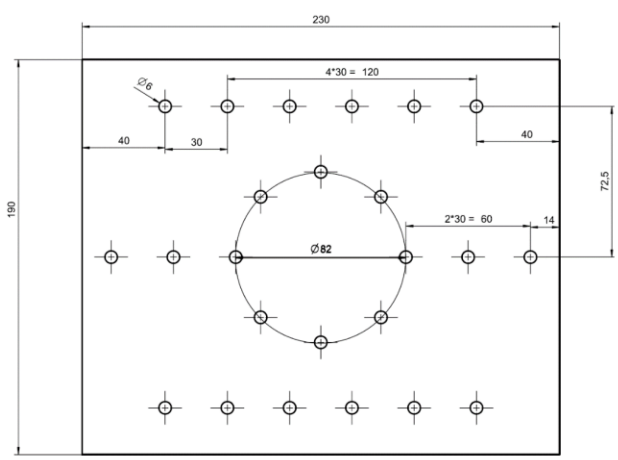


Figure 3-60 Centrifuge's Head Plate dimensions.

3.7. Frame

The frame of the centrifuge is primary consist of 45x45 Alutec profile, its material parameters are mentioned on the Table 13. Connection elements were also chosen from the same supplier, which is Alutec KK s.r.o. Rigidity of the Frame is checked in the paragraph §4.7 and §4.8. Generally the frame was designed the way that it will balance centrifuge in any arms position and will have as minimum as it possible frame members deflection.

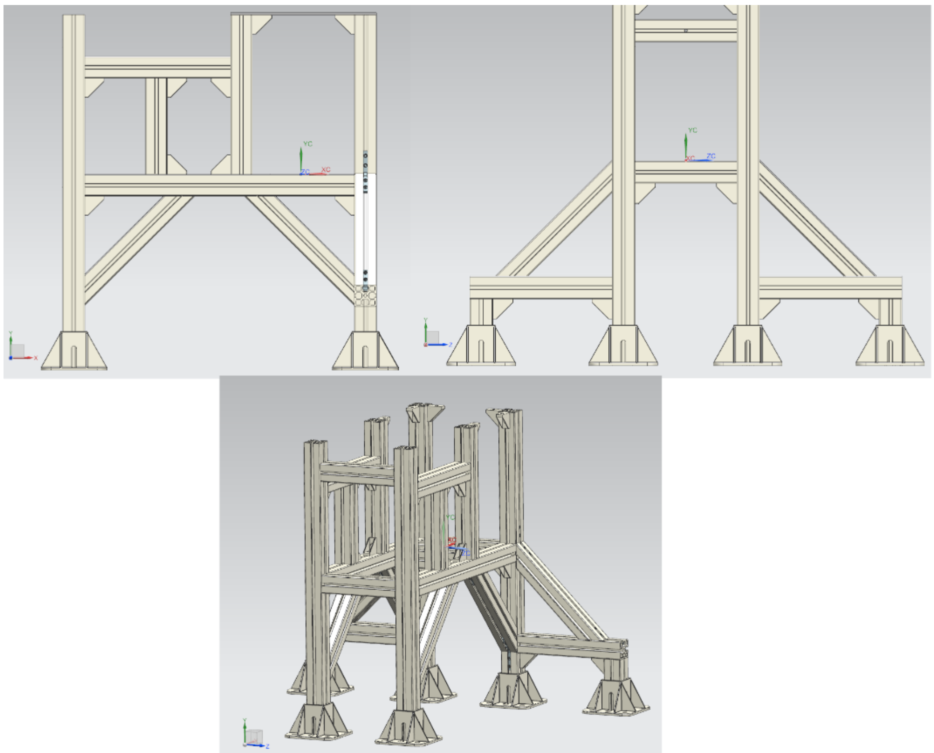


Figure 3-61 Frame of the Centrifuge machine, 1B or 2B var., without reinforcement

4. FEM Analysis

To check if separate parts, assemblies and complete object respond to the strength and rigidity requirements we will use Finite Element Method with the help of pre-processor Siemens FEMAP 11.3 and Beta CAE ANSA 16.0. Processor (solver) was used NX Nastran, integrated to the FEMAP.

4.1. Requirements

Maximum stress appeared on the part should not accede allowable stress of the part.

$$\sigma_{\max} \leq \sigma_{all} \tag{4.1}$$

Where σ_{all} is allowable stress and can be calculated by

$$\sigma_{all} = \frac{\sigma_y}{f}, \tag{4.2}$$

Where f is a safety factor, $f = 2[-]$ and σ_y is the Yield stress of the material.

For every case reserve factor j should be bigger than 1.

$$j = \frac{\sigma_{all}}{\sigma_{\max}} > 1 \tag{4.3}$$

For the static stability cases (for the columns, frame members loaded for compression)

$$F \leq F_{crit} \tag{4.4}$$

Where F is the compression force applied on the member and F_{crit} is the critical force.

However it is better to design frame the way that it will fit deflection requirements that are written in the acceptable standard. For our case I use ISO 4356:1977. This standard contains large scope of requirements, however to simplify the work I choose the smallest from them all which is

$$\Delta_i = \frac{L}{500} \tag{4.5}$$

Where Δ_i is the suggested limited value for deflection of specific i – member of length L, both in millimeters.

4.2. Lug

How it was mentioned before lug is made of same material as 45 x 45 profile, which is AlMgSi0.5F25, and its mechanical parameters were mentioned in Table 13.

Loading for the lug was mentioned in paragraph §3.5.1. Which is 755 N in the direction of a resultant force, which is for 10g acceleration of machine is $\approx 84°$ from the direction of a transverse load. Analysis FEM model is shown on Figure 4-1. Thickness of the shell is 7mm. Constraints are defined by constraining translation motion of the lug in x,y and z direction in the place of lug fasteners.

There was used static linear analysis using NASTRAN solver (NX Nastran), sol 101.

Results shown in Figure 4-2, shows equivalent stress appeared on a model using Von Misses hypothesis, and appeared total deformation in scale that is 1000 times bigger than actual deformation (white color behind the model shows unreformed state). As we can see, maximum stress there is 7.5 MPa, when allowable stress for this material is 100 MPa. Reserve factor here is $j = \frac{\sigma_{all}}{7.5} = \frac{100}{7.5} = 13.3\ [-]$. Maximum deformation is 0.00285 mm, this value can be neglected. This results shows us that we are too safe, which means we can decrease the material use on the model, but this design approach is done due to bearing requirements. Smaller bearing will not be safe to use for this loading. All results and model information is represented as the table after the result figure. This way of representing result will be used for this and for further models.

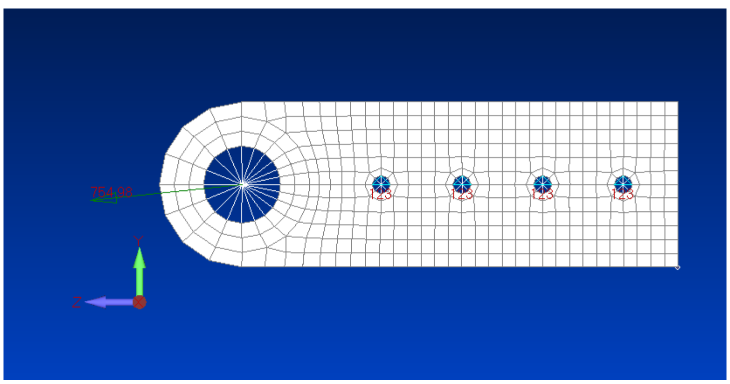


Figure 4-1 Lug FEM analysis model

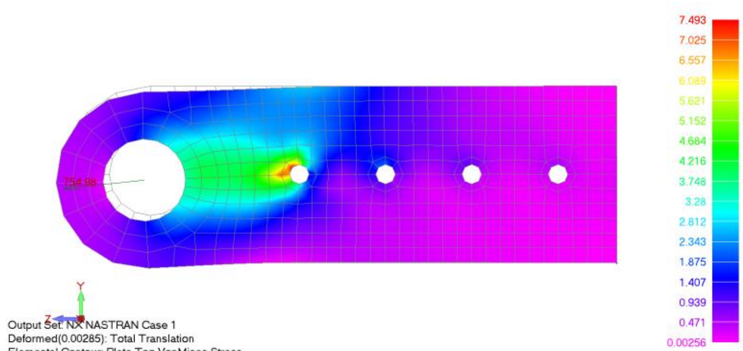


Figure 4-2 Results for Lug, fringe = Von Misses stress, deformation shows resultant translation deformation (1000x bigger than actual deformation)

Max. stress [MPa]	8
Max. deformation [mm]	0.00285
Stress reserve factor, j [-]	12.5

Table 29 Lug results

4.3. Pin

One beam modification (1B)

It was already performed hand calculation in paragraph §3.5.2 with results of stress shown in equation (3.127). We will check here the correctness of job done.

The total length of the pin used for 1B modification is 70 mm (see), but as it was shown on the Figure 3-52, the functional length is 54 mm.

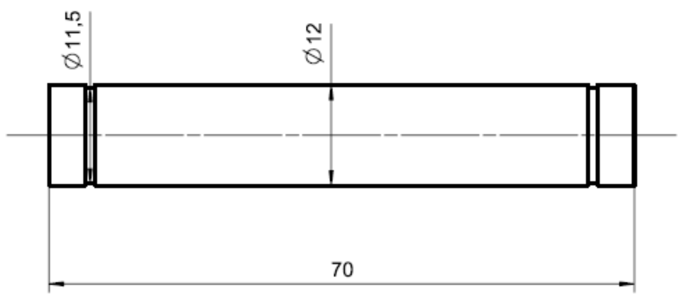


Figure 4-3 Pin drawing for 1B modification.

Figure 4-4 Pin (1B) results

Results in Figure 4-4 shows deformation of the pin in scale of 10% of total length of the model and bending stress as a fringe. As we can see maximum bending stress there is 37.12 MPa. In hand calculation bending stress was 37.12 MPa (see eq. (3.126)), FEM model confirms that provided hand calculation were done correctly. Max. Shear stress is same as in hand calculation, which is 11.2 MPa, than equivalent stress is again 50 MPa (see eq. (3.127)).

Deformation is 0.0285 mm, which acceptable. Design fits all requirements.

Max. bend. stress [MPa]	37.12
Max. shear stress [MPa]	11.2
Max. deformation [mm]	0.028
Stress reserve factor, j [-]	2

Table 30 Pin 1B results

For the loading shown in Figure 3-52, results are shown in Figure 4-5

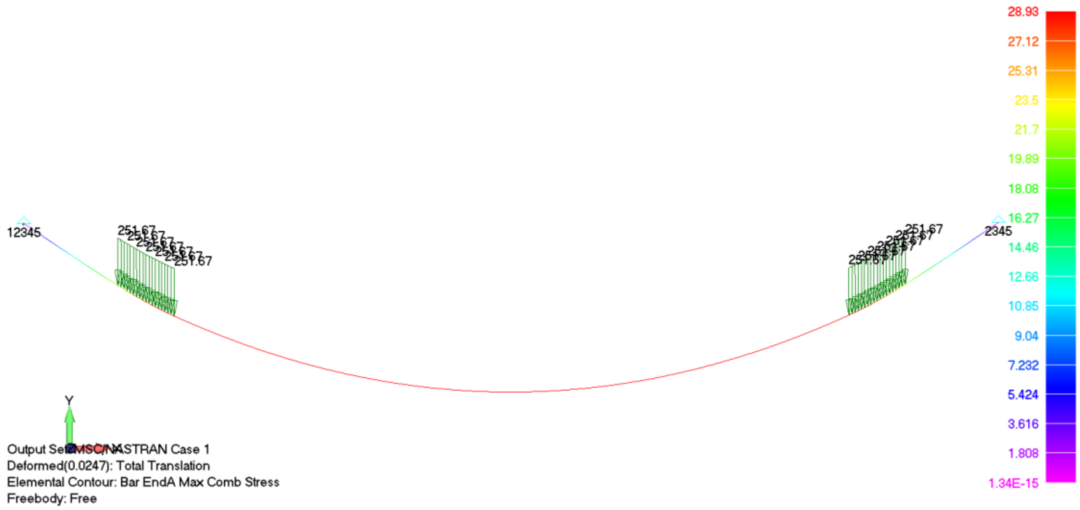


Figure 4-5 Pin (1B) no-drag loading

FEA results shows, that maximum bending stress is 29 [MPa], when result is eq. (3.114) gives maximum bending stress as 31.15 [MPa]. Deformation from FEA is 0.0247 mm, when hands calculation in eq. (3.120) gives us 0.0264 mm, difference in $1.7*10^{-3}$ I would count as a neglected one.

Two beam modification

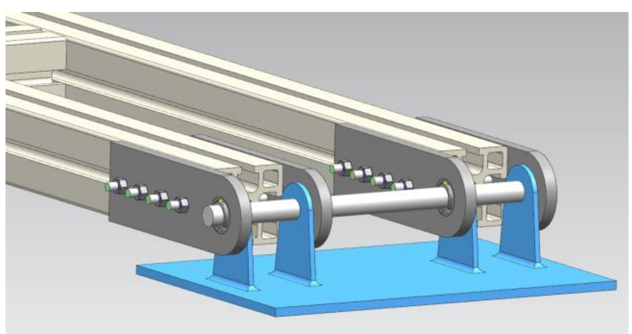


Figure 4-6 CAD model of 2B modification, view to the lugs

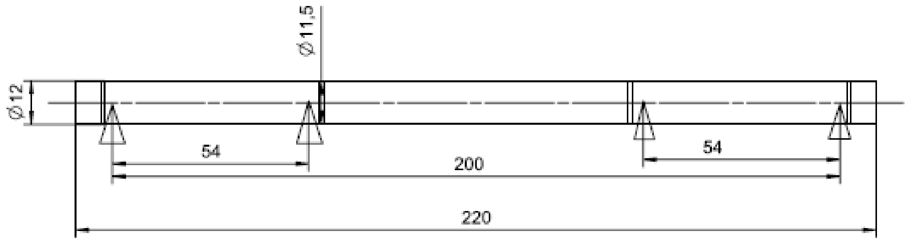


Figure 4-7 Pin for 2B modification, drawing

Determination of a precise loading applied on male lugs from implementation of 10g force on 15kg capsule that is fastened to the blue plate from Figure 4-6 (named as a Lug out plate) is relatively complex task, but thanks to FEM we can solve this problem by combining pin model and lug out plate model. Resultant force from centrifugal force and mass force form the capsule is equal to 1510 N (see eq. (3.85)) and this force is normal to the lug out plate. During rotation on the capsule will appear drag force that will act against the rotation. Its value was calculated in the equation (3.122). This force will be transferred to the place of connection that will be assumed to be in the center of the plate. After force translation will appear a moment form the drag force on plate at the place of connection. This force is:

$$M_{F_D} = F_D * \frac{L_c}{2} = 20.2 * 325 = 6565 \ \approx 6600 \ [N.mm] \tag{4.6}$$

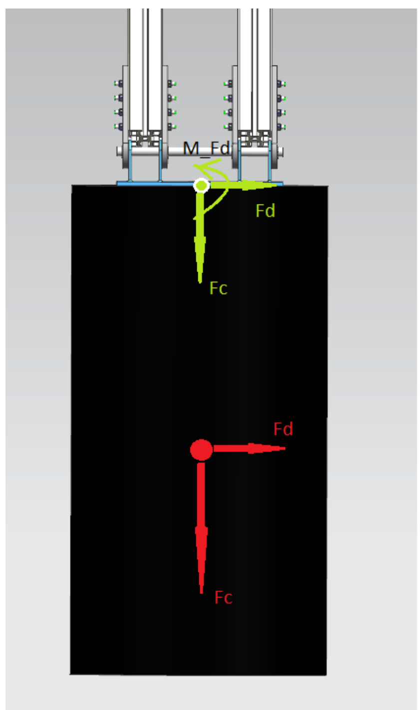


Figure 4-8 Force applied on the capsule (red) and its value translated to the plate (green)

FEM analysis model then looks like:

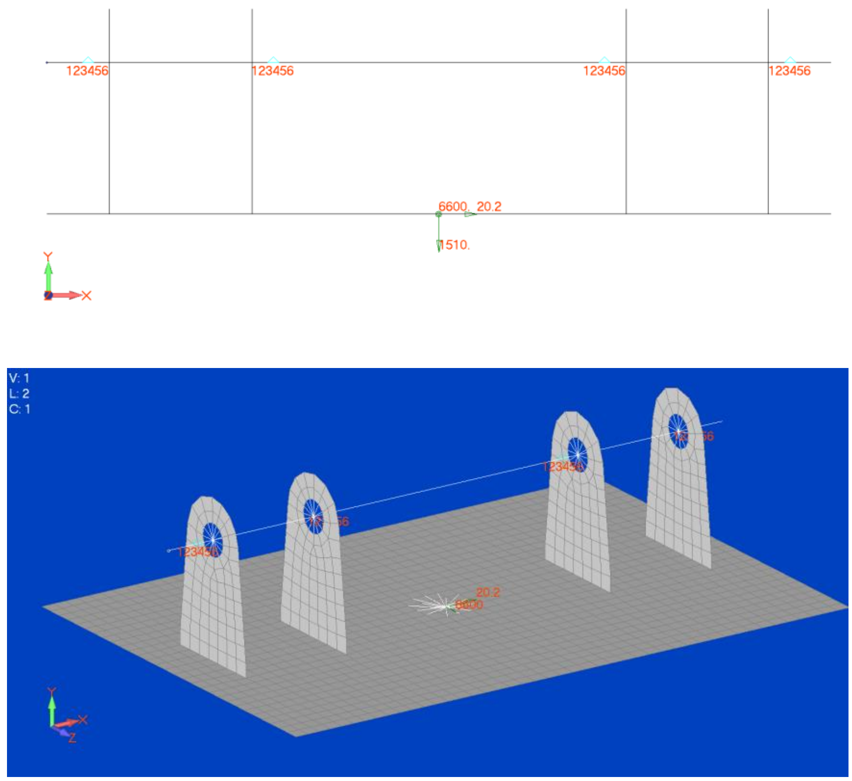


Figure 4-9 FEM model, Lug out plate

Results for pin only:

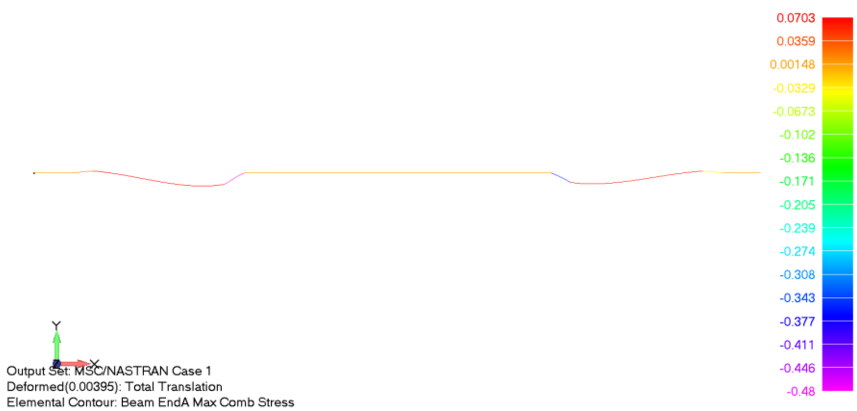


Figure 4-10 Pin 2B variant results, color = beam stress, deformation = 1000x total deformation

On the Figure 4-10, maximum absolute value of the stress appeared there is 0.5 MPa. From free body diagram we can see what reaction forces are on supports (bearing SKF 61801) see Figure 4-11. Maximum reaction force is 1093.8 N ≈ 1095 N. For SKF bearing with maximum loading 1.7 kN this value is sufficient. Maximum shear stress can be found by eq.(3.110), and its value is 13MPa. Using HMH hypothesis:

$$\sigma_{eq} = \sqrt{(-0.5)^2 + (3 * 13)^2} = 40\ [MPa] \tag{4.7}$$

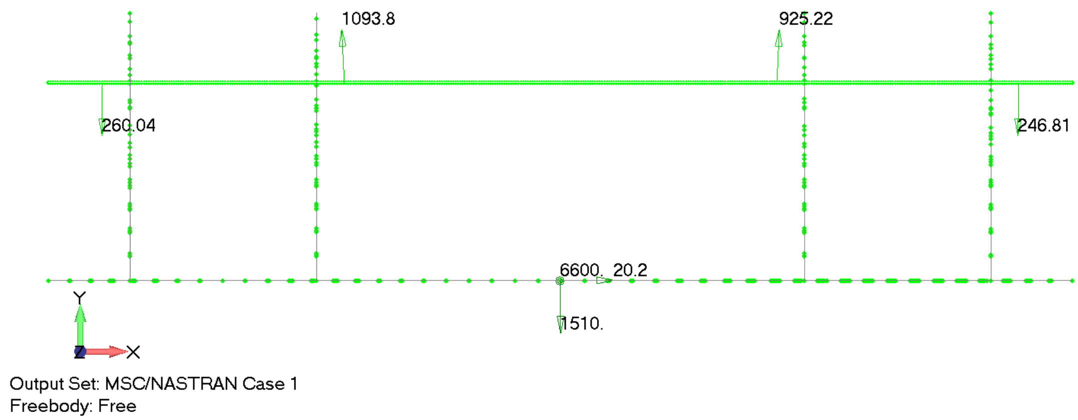


Figure 4-11 Free body diagram of pin (2B var.)

Max. bend. stress [MPa]	0.5
Max. shear stress [MPa]	13
Max. deformation [mm]	0.00395
Stress reserve factor, j [-]	2.5

Table 31 Pin 2B result table

4.4. Lug out plate

4.4.1. Two-beam plate (2B)

Since there was already preformed calculation of lug out plate of 2B variant on previous case, here will be shown only its results.

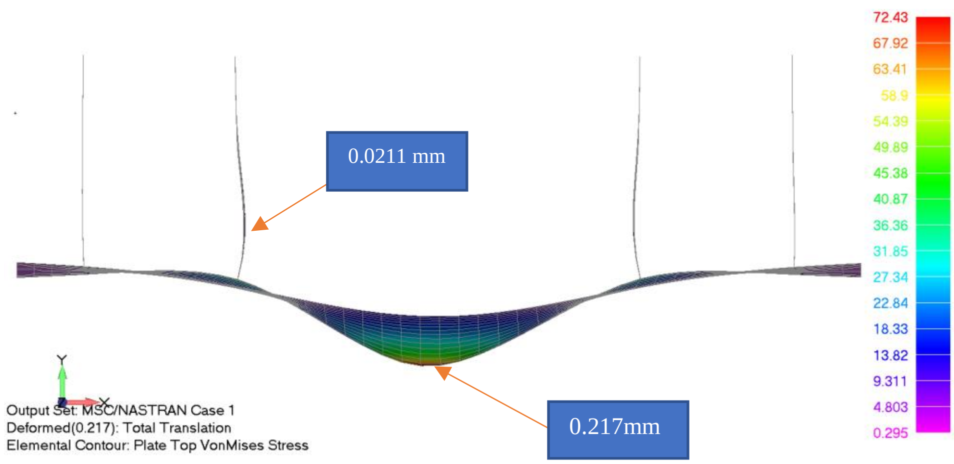


Figure 4-12a Lug out plate (2B var.) FEM results, colourbar represents Von Misses stress, deformation = Total deformation (100x bigger than actual deformation)

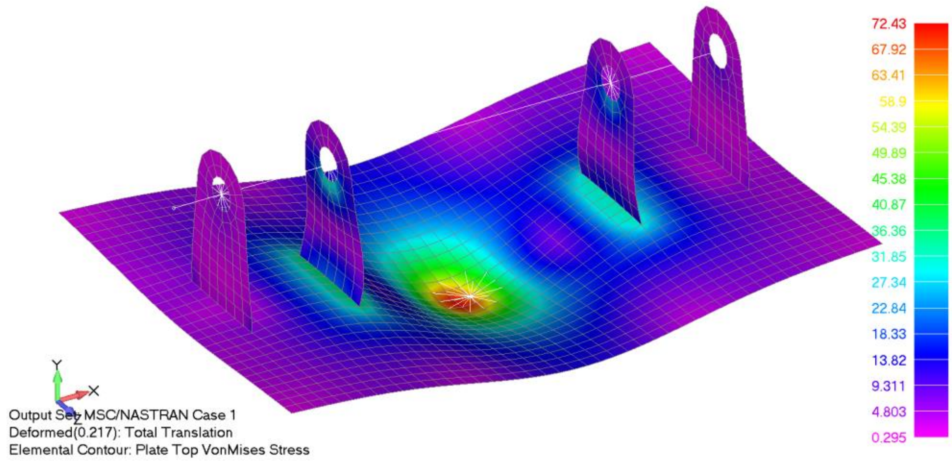


Figure 51b Lug out plate (2B var.) FEM results, trimetric view.

It is expected, that maximum stress appeared at the place of force implementation. Value of the stress is 72.5 MPa, which is less than allowable stress of the material used (which is AlMgSi0.5F25) 100 MPa. Another important point where we have to check stress is the place of the connection of lugs to the plate. Lugs are connected by the welding. Strength of the aluminum welding connection, according to the welding company ESAB, for the closest related material to ours AL 6061-T6 is σ_u =185 MPa [15]. With the safety factor 2, allowable stress for the weld is then:

$$\sigma_{all_{weld}} = \frac{185}{2} = 92.5\ [MPa]\ \approx 90\ [MPa] \tag{4.8}$$

Limiting FEM stress result only for the weld parts, results shows us that the maximum stress at the place where should be welding connection, we can see that maximum stress there is $\approx 40\ [MPa]$ and it is more than 2 time less than allowable stress for the weld. see Figure 4-13.

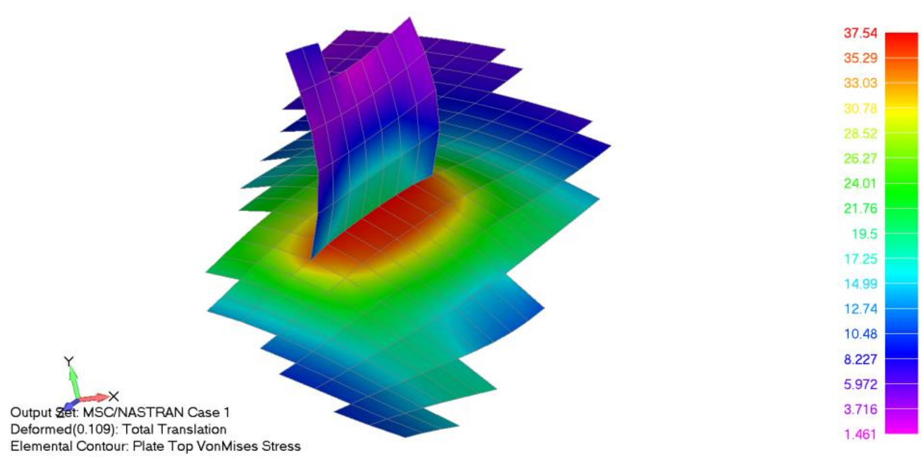


Figure 4-13 Lug out plate (2B var.), weld area.

Deformation of the plate in the place of applied force is 0.22 mm, which I count as acceptable one, since at this place this small deformation will not affect stability a centrifuge laboratory testing results. Deformation of the inner lugs of 2B variant Lug-out plate is 0.0211 mm, which is also very small so we can count the lug as a relatively rigid.

Max. stress [MPa]	73
Max. stress (weld) [MPa]	40
Max. deformation (plate) [mm]	0.22
Max. deformation (lug) [mm]	0.02
Stress reserve factor, j [-]	1.37
Stress reserve factor, j (weld)[-]	2.25

Table 32 Lug-out part result, 2B

4.4.2. One – beam plate

For the same loadings that are shown on the Figure 4-9 and same material there are next results:

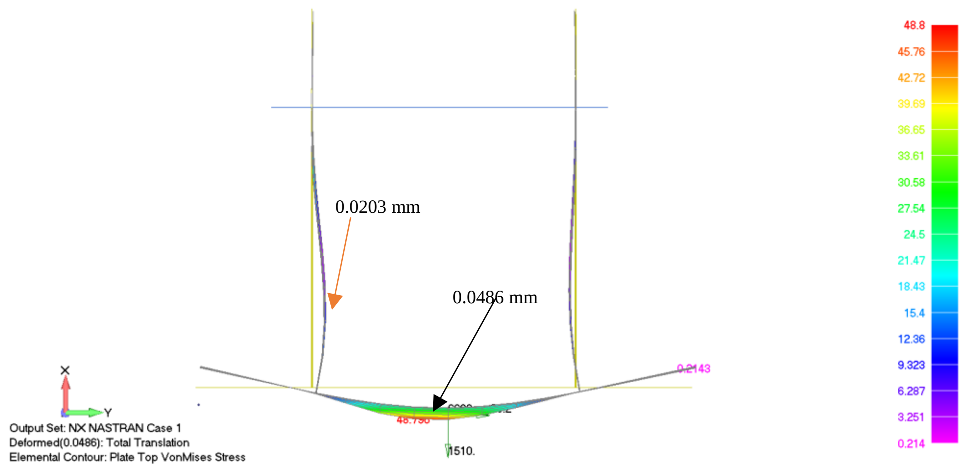


Figure 4-14 Lug-out plate, 1B variant, Deformation results (100x actual deformation, white is undeformed)

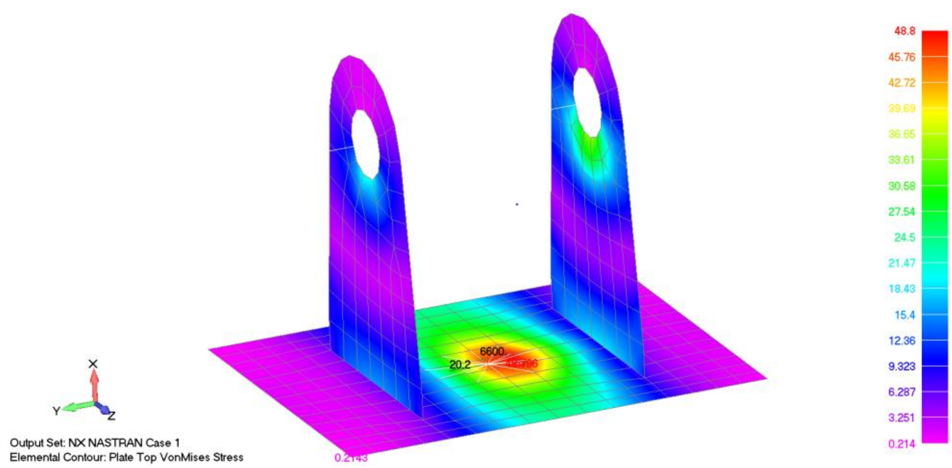


Figure 4-15 Lug-out plate, 1B variant, Von Misses stress results

As we can see, the deformation of the plate itself for 1B variant has significantly decreased compare to the 2B variant, from 0.217 mm to 0.0482 mm. This happened because the length of the plate has decreased when all thicknesses remain the same, by decreasing the length we have increased the slenderness ratio, that will follow into the higher deformation to force ratio.

Max. stress [MPa]	49
Max. stress (weld) [MPa]	20
Max. deformation (plate) [mm]	0.05
Max. deformation (lug) [mm]	0.02
Stress reserve factor, j [-]	2
Stress reserve factor, j (weld)[-]	4.5

Table 33 Lug-out part result, 1B

4.5. Motor base plate

The main loading that will carry "Motor base plate" (MBP) is the weight of the motor. Till this step I was assuming that will use steepen electric motor ES-MH 342200 with the weight 12.8 kg. In order to be on a safety side, let us increase the weight of the motor to 20 kg, so the force will be:

$$F_{motor} = 20 * 9.81 = 196.2 \approx 200[N] \tag{4.9}$$

Motor is fastened to the plate by 4 M8 bolts, each is located on the same distance from the centre of gravity of the motor, so we can divide F_{motor} to 4 equal parts (50 N) and apply each to the place of the connection.

Motor base is made of 6mm steel plate (ČSN 411523 (old 11 523) see) and connected to the frame members at 4 points. These points will be assumed as constraints of the MBP. Another constraints that are not so obvious at the first sight, it is the touch between inner borders of the frame members (45x45 profile) and MBP. This connection will limit the motion of the plate in the Y direction (see Figure 4-16) at the point where the profile edge is. The FEM model is then looks like as it shown on the Figure 4-17.

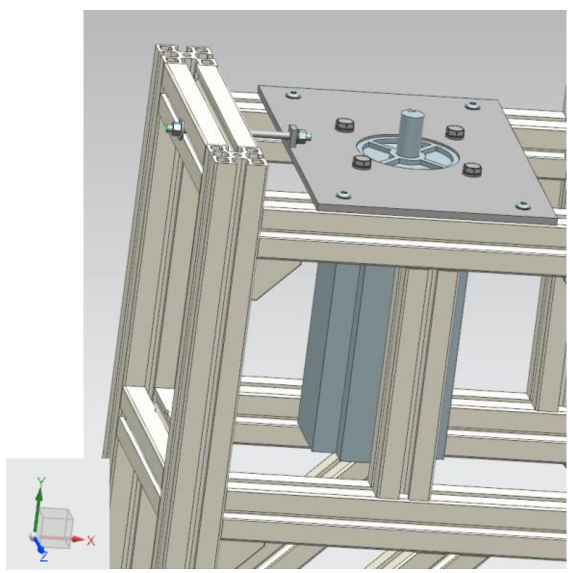


Figure 4-16 CAD model of the motor connected to the MBP and frame

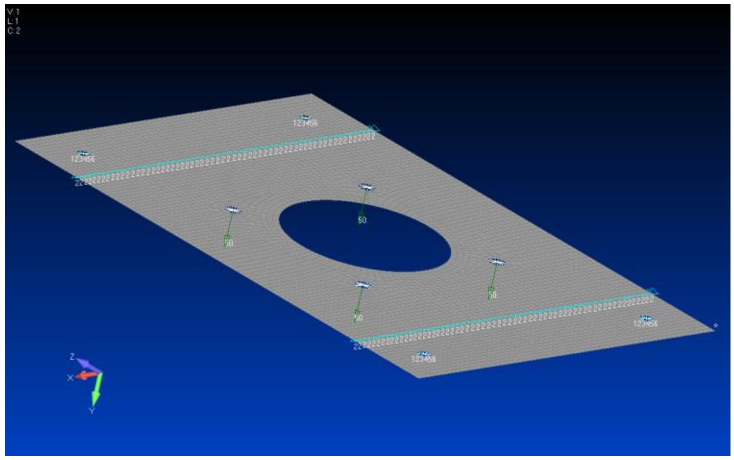


Figure 4-17 FEM model of the MBP

ČSN 411523 or EN 10025-90 (Fe510)	
σ_Y $[MPa]$	275
σ_U $[MPa]$	500
E $[MPa]$	210000
μ $[-]$	0.3
ρ $[kg.m^{-3}]$	7850

Table 34 Material parameters for steel [16]

Results are then for stress tensor:

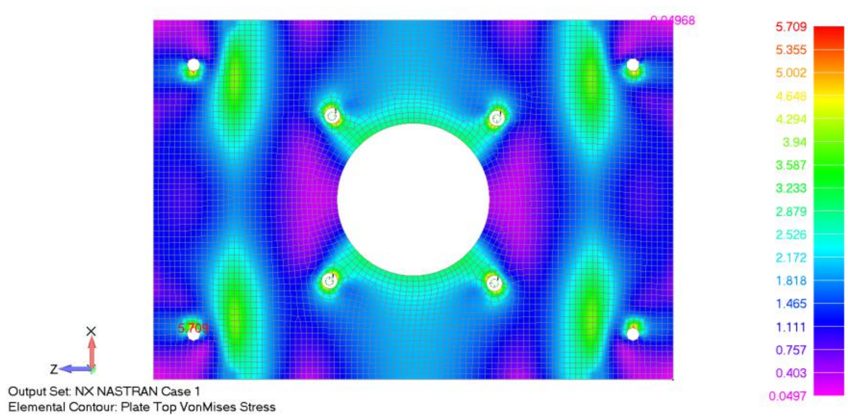


Figure 4-18 MBP Von Misses Stress result

Maximum appeared stress is at the place of bolt connections, as expected. The value of maximum stress is very small 6 MPa. Allowable stress for used steel is 130 MPa (see eq.(4.10)), the reserve factor is then j=130/6 = 21.6 [-]. Which is unnecessary big, however, strength isn't the most important parameter for this part. Rigidity has here decisive character, because it is required for a motor to be on the same position in order to not deflect the axis of rotation or torque transfer to the from driving pull to driven. Deformation result is shown on Figure 4-19.

$$\sigma_{all_{steel}} = \frac{\sigma_Y}{f} = \frac{275}{2} = 137.5 \ \approx 130 \ [MPa] \quad (4.10)$$

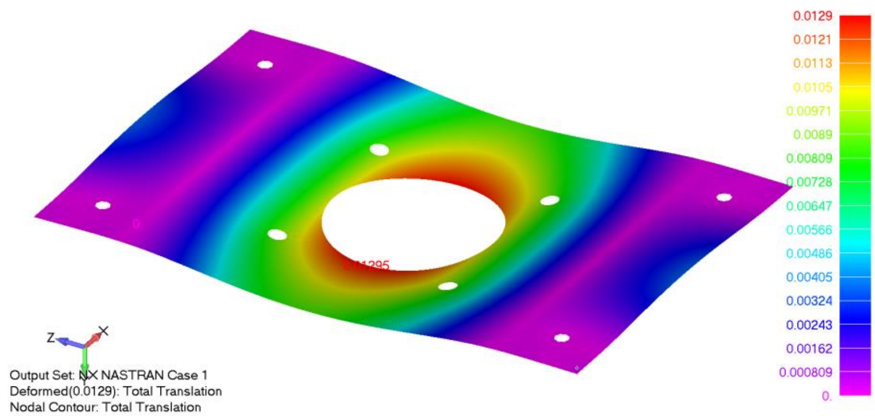


Figure 4-19 Deformation of MBP, 2000x bigger than actual deformation

As we see, maximum deformation is in the center of a Motor base plate, its value is 0.013 mm. This deformation can be neglected.

If in future will be decided to increase the motor torque by changing motor, I assume that this kind of motor will have bigger weight than the one we use. For example if motor will have a weight of 40 kg ≈ 400 N → 4 * 100 N, then MBP's stress and deformation will be as it shown in the Figure 4-20.

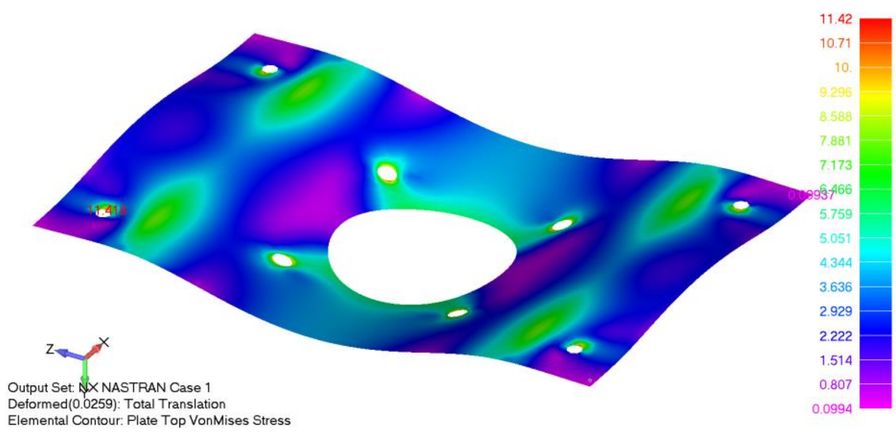


Figure 4-20 MPB stress and deformation results (400 N total load)

Max. stress (200N load) [MPa]	6
Max. stress (400N load) [MPa]	12
Max. deformation (200N) [mm]	0.013
Max. deformation (400N) [mm]	0.026
Stress reserve factor, j (200N) [-]	21.6
Stress reserve factor, j (400N)[-]	10.8

Table 35 MBP result table, plate thickness 6mm

If we decrease plate thickness from 6mm to 5mm, the results will be next:

Max. stress (200N load) [MPa]	8.5
Max. stress (400N load) [MPa]	17
Max. deformation (200N) [mm]	0.022
Max. deformation (400N) [mm]	0.045
Stress reserve factor, j (200N) [-]	15.2
Stress reserve factor, j (400N)[-]	7.6

Table 36 MBP result table, plate thickness 5mm

Results from Table 36 shows that actually we can use 5mm plate instead of 6 mm plate, because I count 0.05 mm deformation as an acceptable one.

4.6. Bearing house

In the paragraph §3.3.1 we were designing shaft, for this we calculated reactions in supports A and B (see Figure 3-15. Support B is the bearing SKF 16009 and this bearing should to be mounted to the specific place named bearing housing, and I have designed it in the way as it shown on the Figure 4-21. Part where is a bearing placed is connected to the steel plate of 5 mm thickness by four M12 bolts. Plate is assembled to the frame members on the corners, this connection will be assumed as a constraint. Both parts are made of steel which parameters were mentioned in the Table 34. The loading will be the reaction force in the support, that will appear of asymmetric loading of centrifuge arms by 60 kg on one side and zero on the other, this loading will call a reaction in a support with the value of ≈ 1880 N, see paragraph §3.3.1 eq. (3.48). This force will be directed in the FEM model to the z-axis direction.

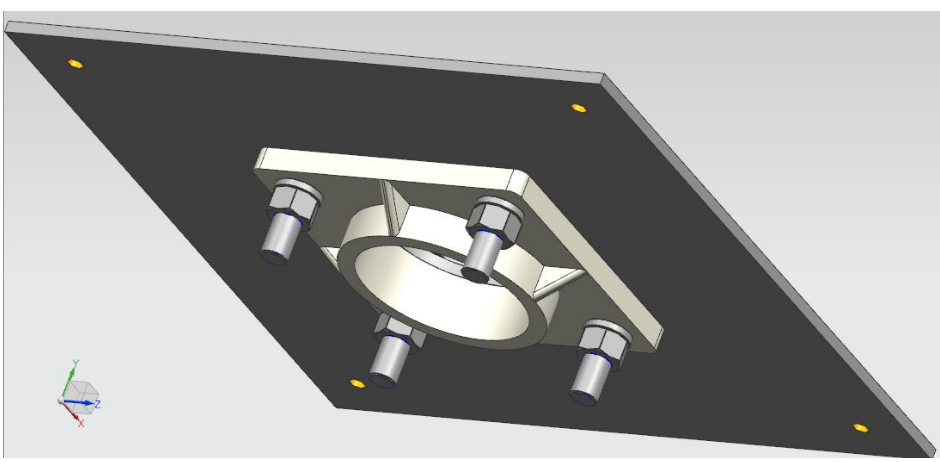


Figure 4-21 CAD model of a Bearing house

Bearing house plate (BHP)

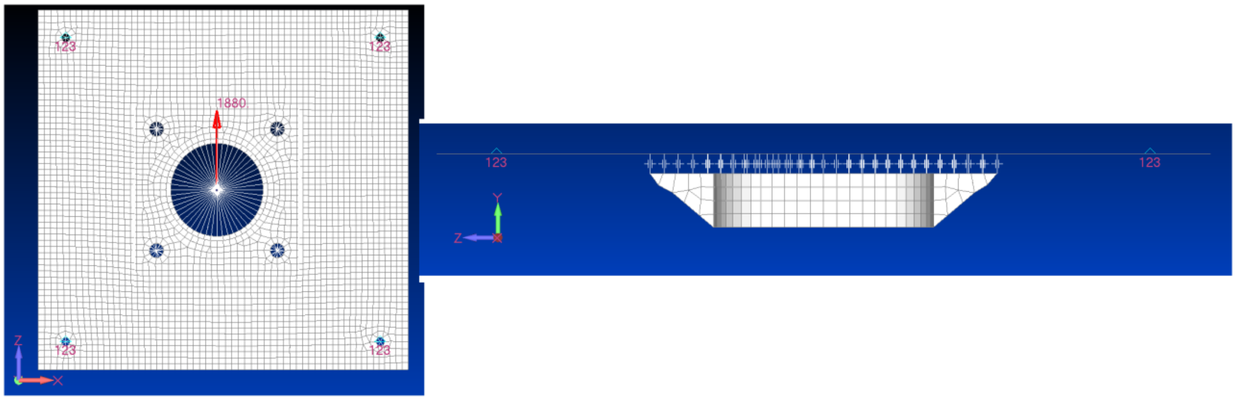


Figure 4-22 Bearing housing FEM model

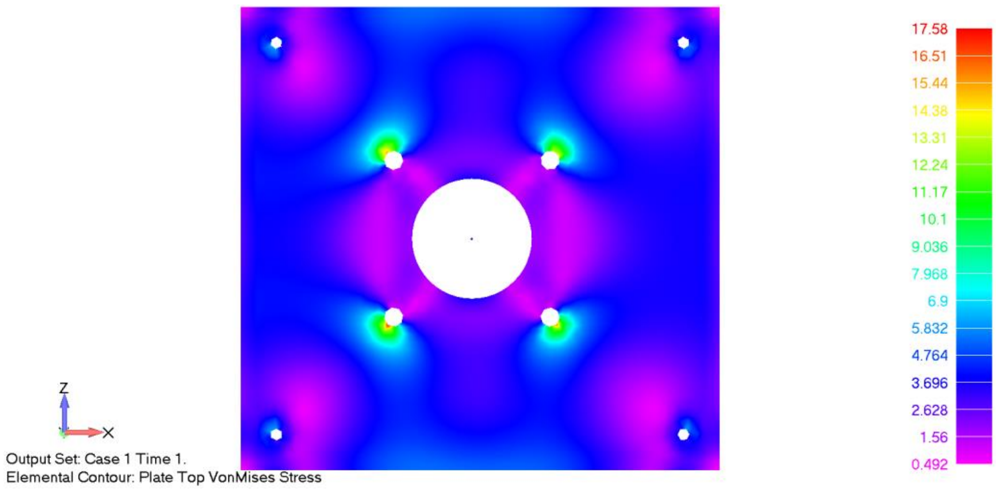


Figure 4-23 Von Misses stress on Bearing house plate (using nonlinear static solver→ no stability loss)

Maximum stress there is 18 MPa, which is significantly small for a steel material. However, as I mentioned before in calculation Motor base plate, decisive parameter here is a deformation. Maximum deformation is appeared to be 0.056 mm, see Figure 4-24. For this plate it sufficient, because maximum allowable deformation should be less than 0.1 mm.

Max. stress [MPa]	18 MPa
Max. deformation [mm]	0.056
Stress reserve factor, j [-]	7

Table 37 Bearing house plate results

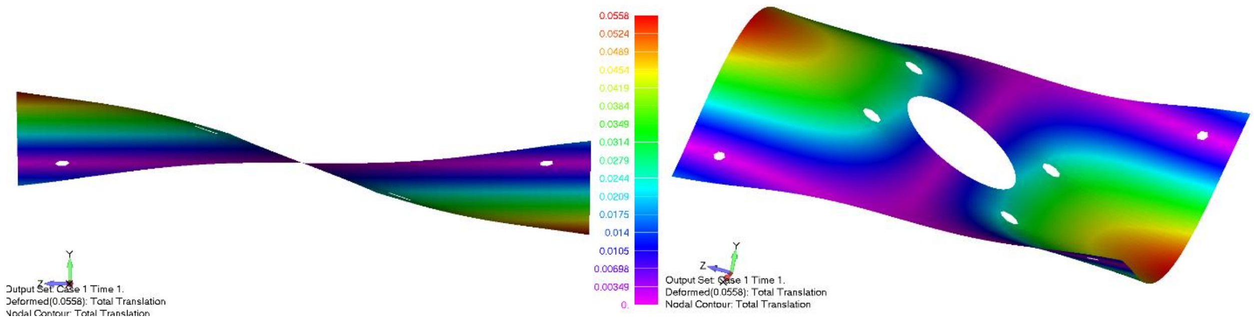


Figure 4-24 Deformation of Bearing house plate (800x of actual deformation)

Bearing house

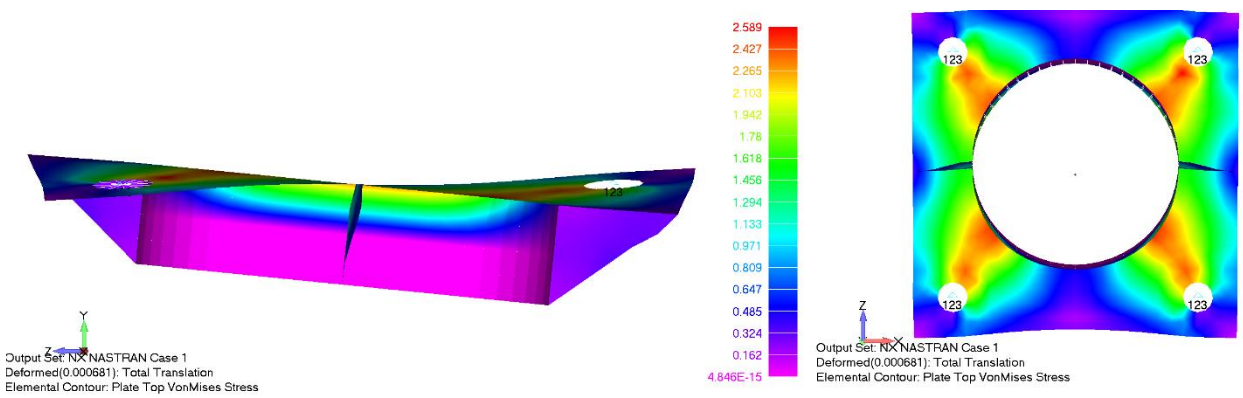


Figure 4-25 Bearing house results made of steel (colourbar - Von Misses stress, deformation - total deformation 10 000x bigger than actual deformation)

Figure 4-25 shows us that stress and deformation is so small, that we can state that this part is too rigid and strengthen unnecessarily. It means it has much bigger weight than it could be. One of the way to decrease weight is to switch material from steel to aluminum, from ČSN 411523 to AlMgSi0.5F25, and then by decreasing density with remaining volume (geometry) we will decrease weight for 65% from initial weight. If this part made of steel had a weight 1.1 kg then the same detail made of aluminum alloy will have weight 0.385 grams. FEM solution (Figure 4-26) shows that the maximum stress value hasn't change at all (because we didn't change geometry of a part). Deformation has increased, from 0.0007 mm to 0.002 mm, which is still very small distance. Finally, we can say that it is more effective to use aluminum alloy made part than steel made, because deformation that appears on Al made part is acceptable. Results for both cases are represented in the Table 38. Results meets requirements, so a designed part can be safely used.

Max. stress [MPa]	3
Max. deformation (steel) [mm]	0.0007
Max. deformation (Al) [mm]	0.002
Stress reserve factor, j (steel) [-]	43
Stress reserve factor, j (400N)[-]	33

Table 38 Bearing house results

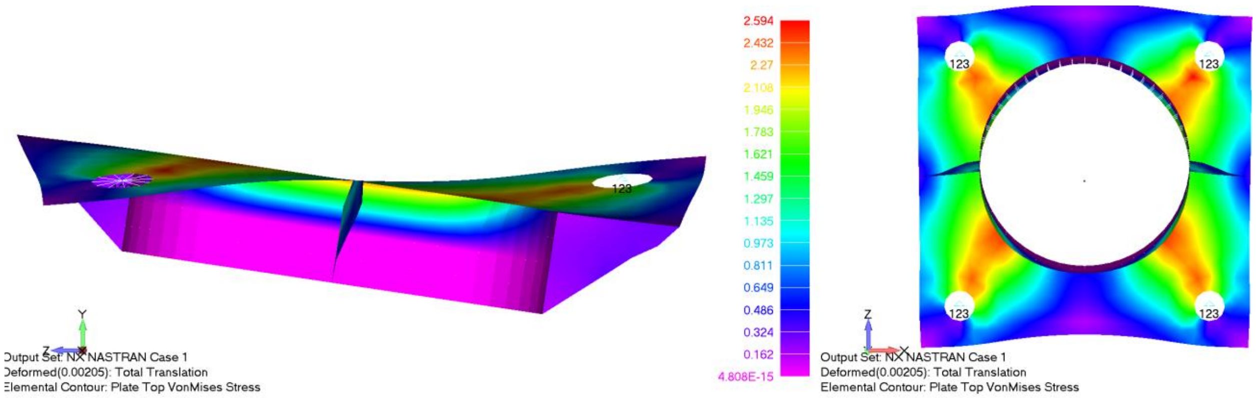


Figure 4-26 FEM results of a Bearing house made of AlMgSi0.5F25 (clourbar - stress, deformation is 5000x bigger than actual one)

For the load of 4 000 N there are results shown in the Table 39, gives us an understanding that if we increase the load 2 times still we are on the safe side.

Max. stress [MPa]	6
Max. deformation (Al) [mm]	0.0045
Stress reserve factor, j (400N)[-]	16

Table 39 Bearing house results made of aluminum alloy, with the loading of 4000 N

4.7. Frame, 2B variant

In order to provide precise FEM analysis for a frame, I decided to create a full centrifuge model and to apply all loads that I know (neglecting the own weight) on it. I divided these loads for 3 types:

1) Load from tested capsule – Centrifuge force + Capsule mass (see Figure 4-8, Figure 4-9)
2) Load from the motor (see paragraph §4.5)
3) Mass load transferred by the shaft – mass of the centrifuge arms (with capsules on both sides), lugs, plates, fasteners. As it was calculated before, maximum weight of a centrifuge head is 60 kg, in order to be on a conservative side of a task, we will take that the maximum weight will be 100 kg, pointed downward on the shaft.

Calculation will be provided for several cases:

Case I – Symmetric loading, when both arms are loaded equally.

Subcase 1 – 0 degree (reference position of centrifuge arms)

Subcase 2 – 45 degree

Subcase 3 – 90 degree

Case II – Asymmetric loading, when only one arm is loaded.

Subcase 1 – 0 degree

Subcase 2 – 45 degree

Subcase 3 – 90 degree

Results for all cases and frame variants are shown in the Table 40 in the paragraph §4.7.1 on the page 94.

Case I
Subcase 1

As a reference centrifuge arm position, “0 deg”, will be taken arms position that are shown on the Figure 4-27. Loading from centrifuge force, testing capsule’s mass, moment from a drag force are located on both arms, specifically in Lug-out plates. Model is constrained by fixing 6 frame legs in 6 degree of freedom, also in for the FEM calculation I had to constraint a shaft in rotation about its own axis, otherwise model won’t be calculable. FEM model is shown on the

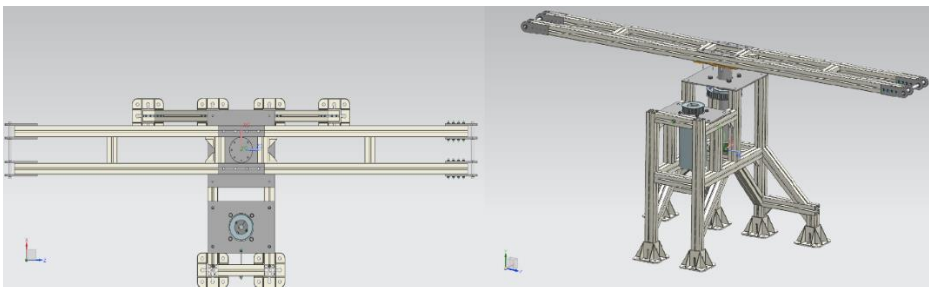


Figure 4-27 0 deg. centrifuge's arm position, 2B var.

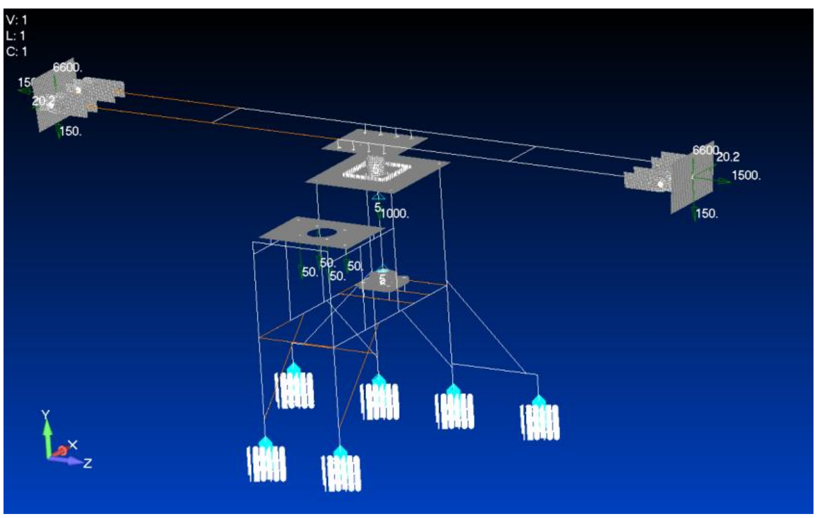


Figure 4-28 CaseI.1, 2B var. 0deg. FEM model

Full model results for the deformation only is shown on the next figure.

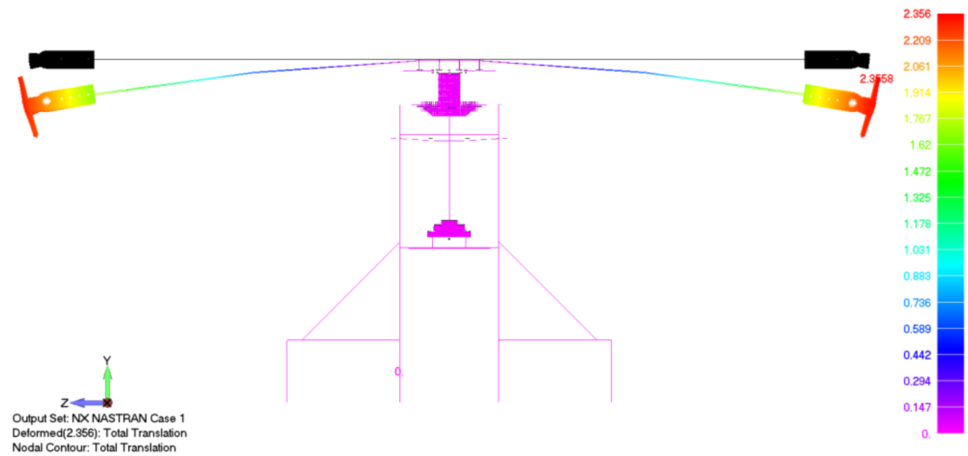


*Figure 4-29 CaseI.1, 2B var. Deformation results (50*x *bigger than actual deformation, white is undeformed state)*

Maximum deflection of 2.4 mm is located on the tip of the centrifuge arm, which is expected place. For the frame only, the maximum deflection is 0.0265 mm; its location is in the place of shaft connection to the frame (thrust bearing connected to the frame), see Figure 4-30. According to the requirement (4.5), deformation of a frame member should not exceed the value of its length divided by 500. $\Delta_i = \frac{200^*}{500} = 0.4\ [mm]$ and 0.0265 < 0.4 mm, i.e. results according to this requirement is acceptable.

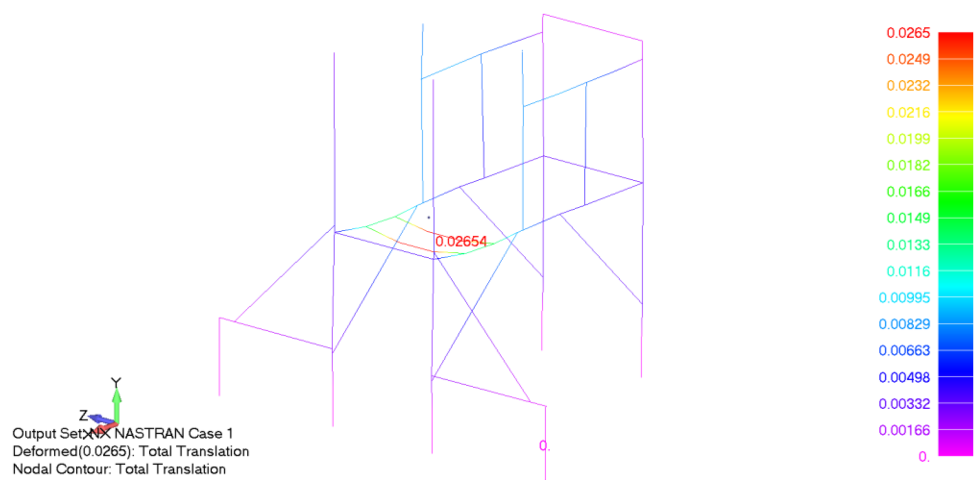


*Figure 4-30 CaseI.1, 2B var. Frame deformation result (1000*x *times bigger than actual deformation)*

Subcase 2

For the arms position of 45 degree from reference position results are: arms deflection 2.359 mm and maximum frame deflection is 0.0265 mm, at the same place as it was for previous subcase for both, see Figure 4-31. Compare to the previous subcase, deflection of the beams has increased only for 0.003 mm, so we can say that there was relatively no change happened, for the frame, literally there is no difference between deflections. Because both sides have same loadings located at the equal distances from the axis of a rotation, on the place of axis of a rotation appears two equal moments with different meaning (different signs), which cancels each other. In the end there is only one force left, which is 1000N on the shaft. This

[*] Values for the length of frame members are taken from the CAD file, also optionally can be taken from drawings.

force makes deformation of 0.0265 mm on those frame members where shaft is mounted by thrust bearing housing. All deformations are acceptable.

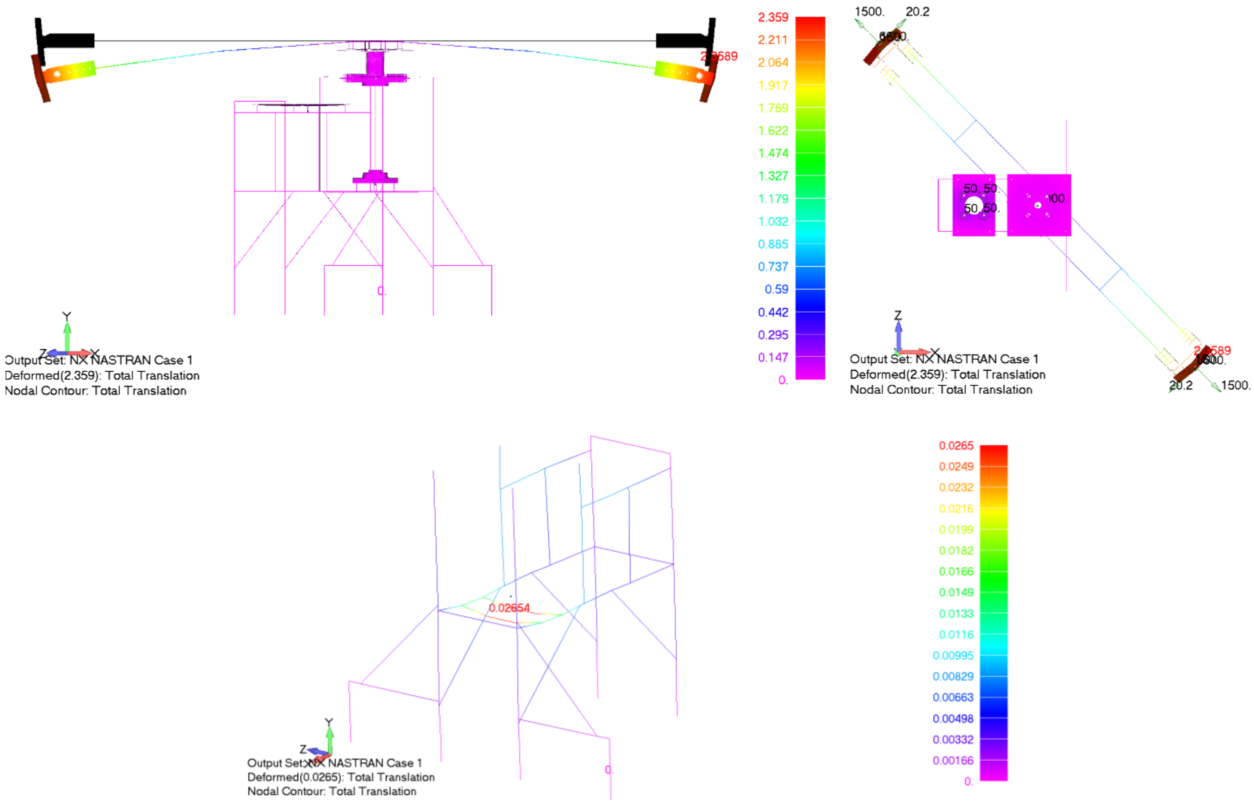


Figure 4-31 CaseI.2, 2B var. Deformation results

Subcase 3

For the arms position of 45 degree from reference position results are: arms deflection 2.288 mm and maximum frame deflection is 0.0265 mm, at the same place as it was for previous subcase for both, see Figure 4-32 and Figure 4-33.

With the frame happens no changes, compare to the Case I.1 and Case I.2.

All deflections are acceptable.

Generally we can say that for the symmetric loading of the centrifuge there are no major changes between different position of centrifuge arms during rotation. All frame members have small deflections and fits requirements. Regarding stresses that appears on the frame members by carrying loads, they are so small that there is no point to mention them and searching for their reserve factors. For instance, the maximum stress that has a frame member from all subcases of the Case I is equal to 0.07 MPa. It is located at the same place where is the maximum deflection of the frame or - 0.6 MPa at the constraints.

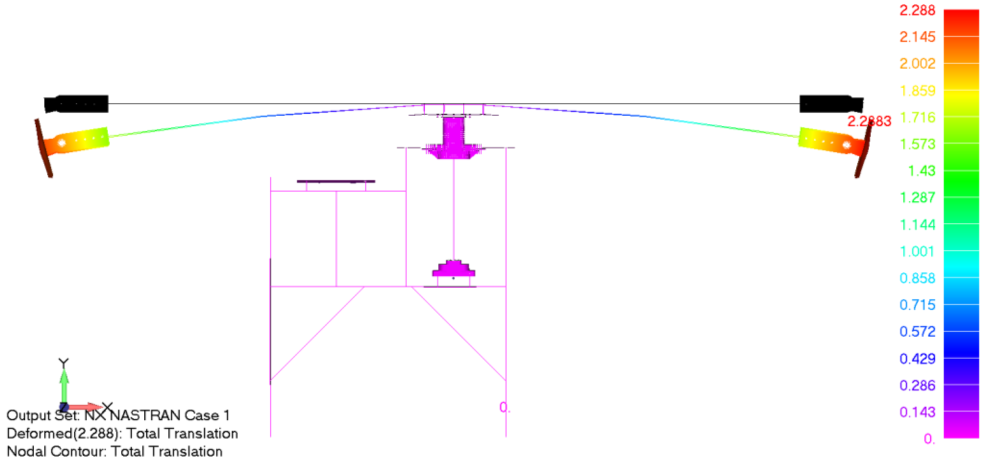


Figure 4-32 CaseI.3, 2B var. Deformation results (50x bigger than actual size, black is undeformed)

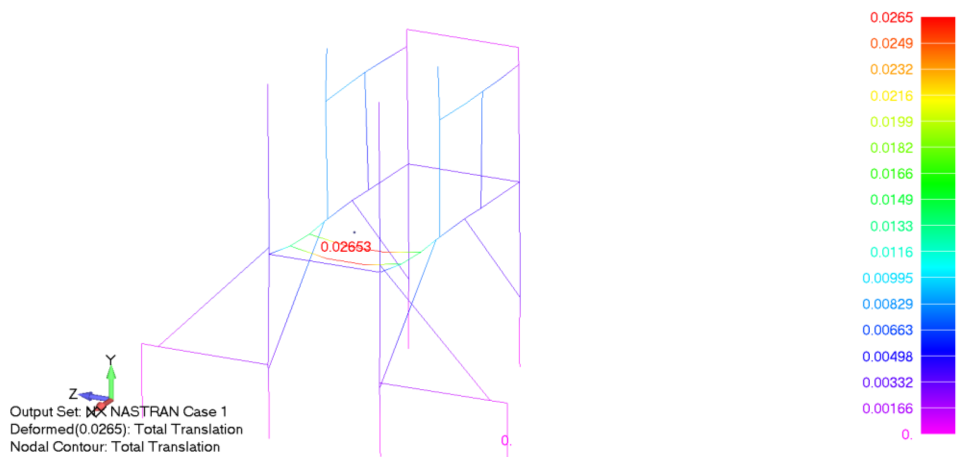


Figure 4-33 Case I.3, 2B var. Frame deformation result (1000x bigger than actual deformation)

Case II

Subcase 1

Reference centrifuge's arm positions was shown on the Figure 4-27. Applying load only for the one end, we can solve extreme case, when only one arm is under the load. In real practice, of course it is always better to put some counterweight on the other side in order to decrease a moment on the center. In order to be on the safe side of the question we are going to solve this problem only for the one arm loading. FEM model for analyzing is shown on the Figure 4-34.

Results from Figure 4-35 shows that deformation of the arm significantly increased, from 2.3 mm to 9.5 mm, however deflection for 9.5 mm of the beam is still acceptable, since this deflection will not affect results of the test.

Frame deformation is shown on the Figure 4-36. The place of the maximum deformation has changed from frame members that supports a shaft to the members that are connected to the Bearing house plate. The maximum deformation is 0.551 mm. According to requirements (4.5), allowable deflection is $\Delta_1 =$

$\frac{735.8}{500} = 1.47\ [mm]$, for another frame member (above it is a red font number is located on the Figure 4-36) allowable deflection is $\Delta_2 = \frac{352.5}{500} = 0.7\ [mm]$ Maximum deformation 0.551 mm is less than limit value of 1.47 mm and 0.7 mm. Even though obtained results from the calculation fits requirements I decided reinforce the frame by adding one diagonal frame member to the place where this member will not worsen the work of the centrifuge machine. How this reinforcement looks, you can see on the Figure 4-37.

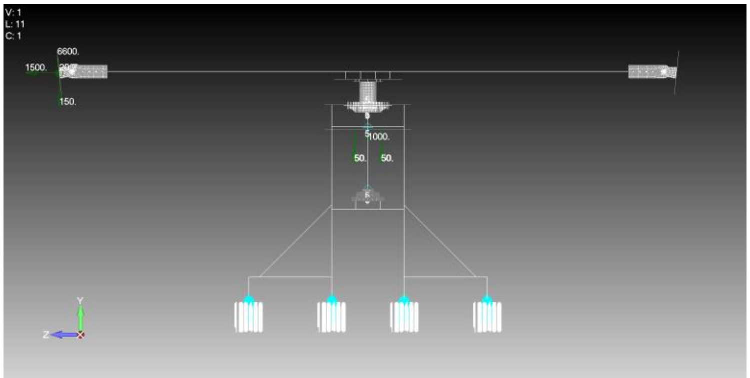


Figure 4-34 Case II.1, 2B var. FEM analyzing model

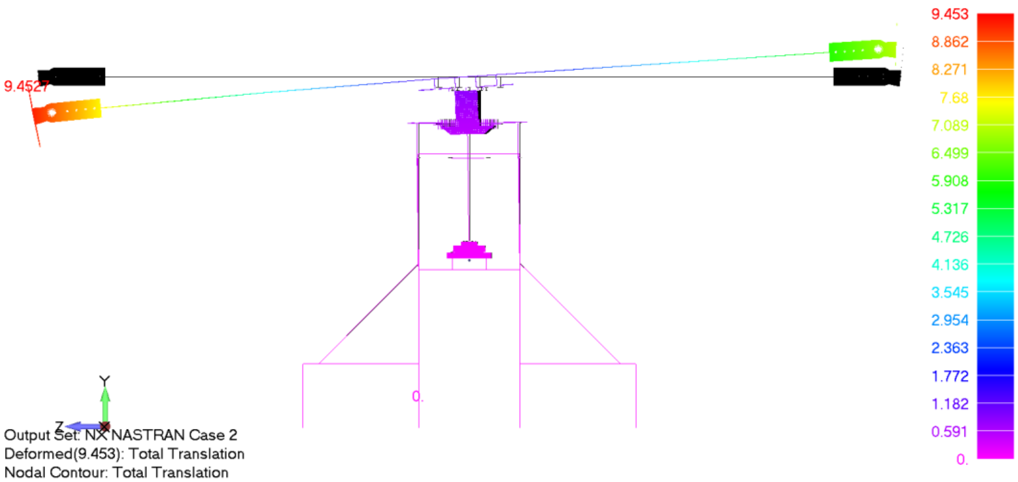


Figure 4-35 Case II.1, 2B var. Deformation results (10x of actual scale, black is undeformed state)

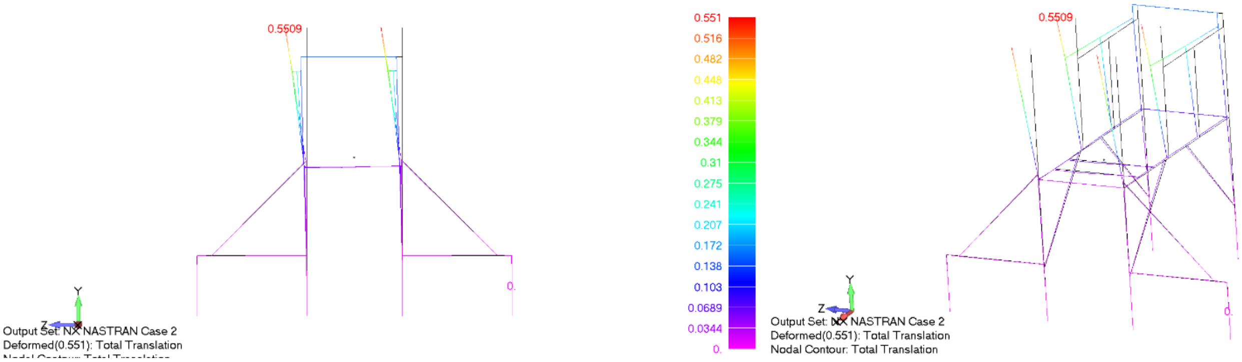


Figure 4-36 Case II.1, 2B var. Frame deformation result (100x actual deformation)

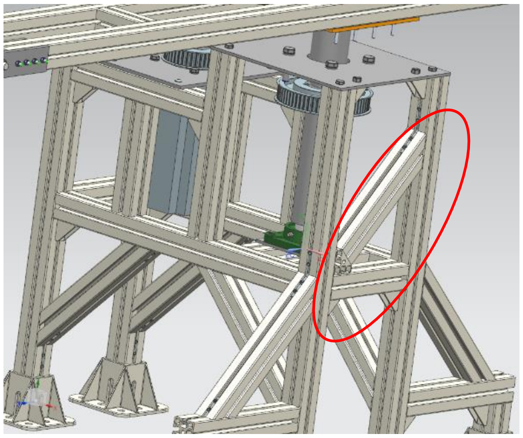


Figure 4-37 CAD model with one reinforced frame member, 2B-R frame variant

Deformation result of the reinforced frame model of the two-beam variant of the centrifuge (*2B-R*) is shown on the Figure 4-38. Maximum deflection now is 0.25 mm, which is twice smaller compare to the frame without reinforcement.

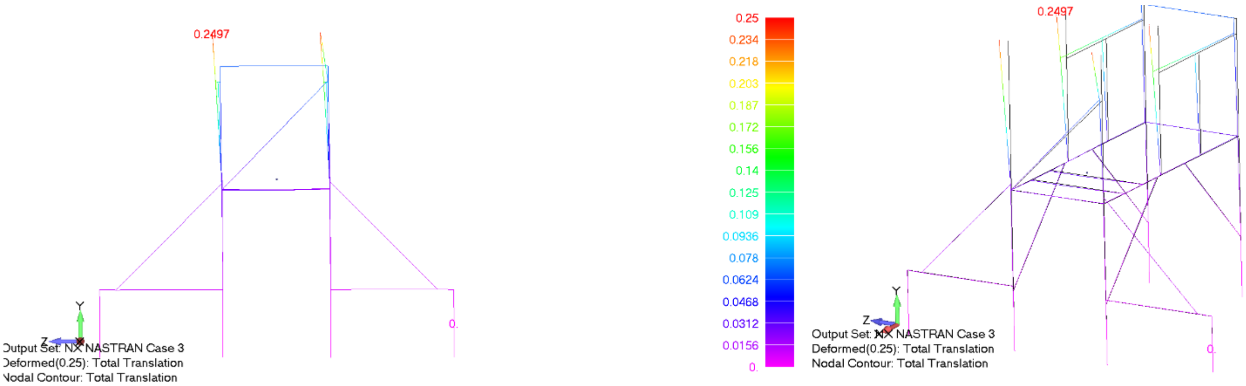


*Figure 4-38 Case II.1 2B-R var. Frame deformation result (100*x *actual deformation, white is undeformed)*

If we reinforce other "windows" except the one, where a belt goes through, then frame looks like as it is on the Figure 4-39.

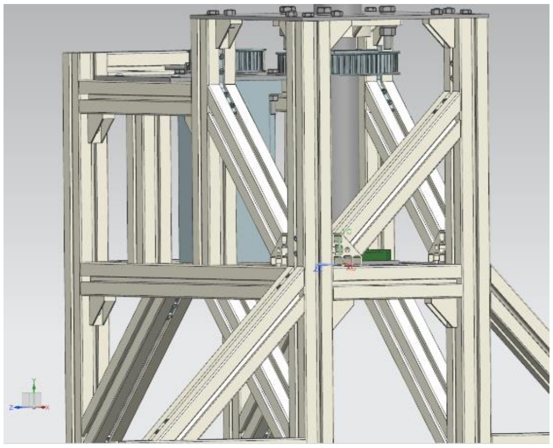


Figure 4-39 All window reinforced (AWR) model of a frame, 2B-AWR frame variant

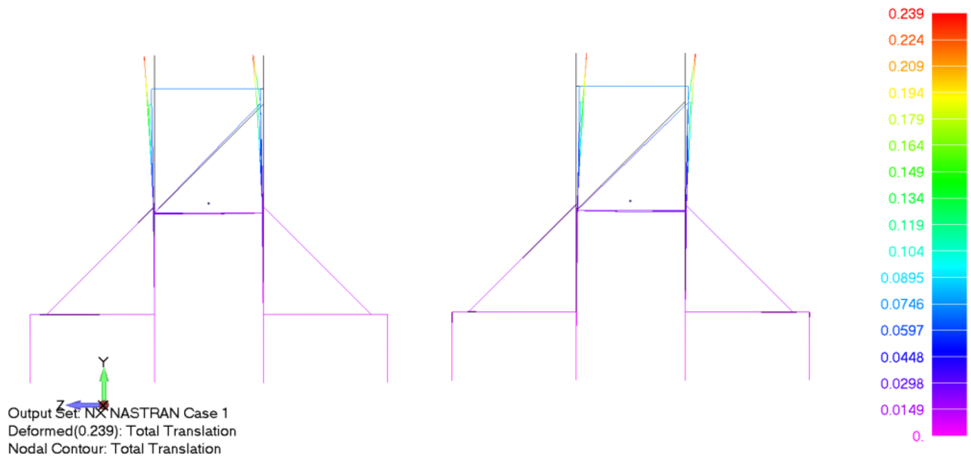


Figure 4-40 Case II.1 2B - AWR version, deformation results

As we can see from the Figure 4-40, the maximum deflection hasn't changed that much (from 0.25 mm to 0.236 mm) and the its place hasn't changed at all, however, this type of reinforcement can help to decrease the deformation from the load Case II.2 and Case II.3.

Subcase 2

For the arms position at 45 degrees from reference frame and frame model without any reinforcement there are next results: maximum deformation of the arm beams is 9.1 mm (Figure 4-41), which is slightly smaller than for the Case I.1 of the same frame variant; maximum frame member deflection is 0.43 mm. Allowable deflection is $\Delta_2 = \frac{352.5}{500} = 0.7\ [mm]$. 0.43 < 0.7mm. It is seen that this result fits the requirement given by ISO 4356:1977.

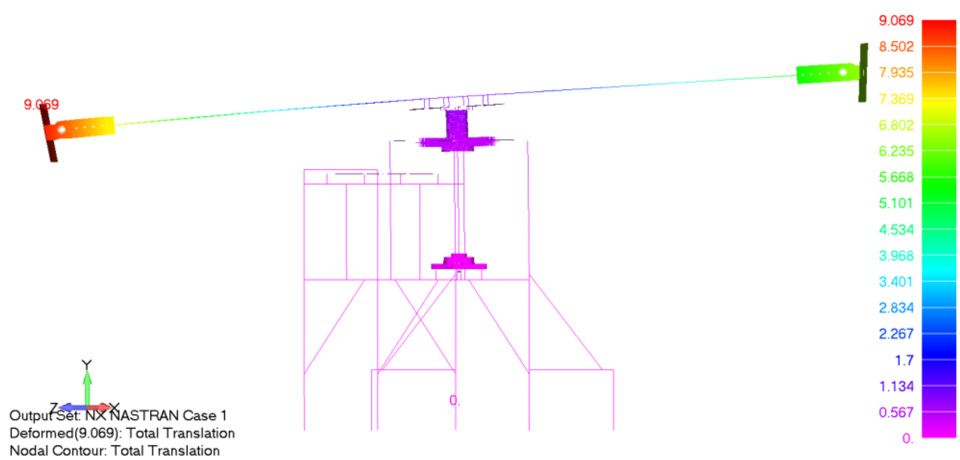


Figure 4-41 Case II.2 2B var. Deformation result (10x actual deformation)

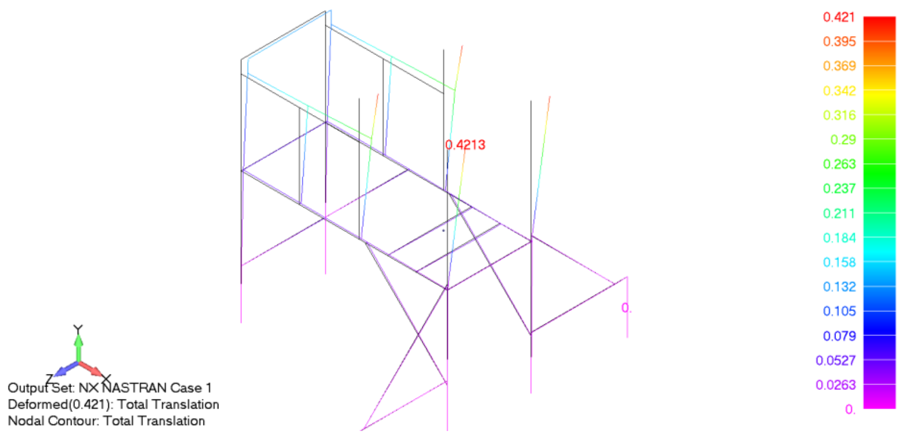


*Figure 4-42 Case II.2 2B var. Frame deformation (100*x *of actual deformation)*

For 2B-R variant of the frame, maximum deflection is 0.232 mm (Figure 4-43). Compare to the result from the 2B variant this result is 46% smaller, which state that frame is more rigid.

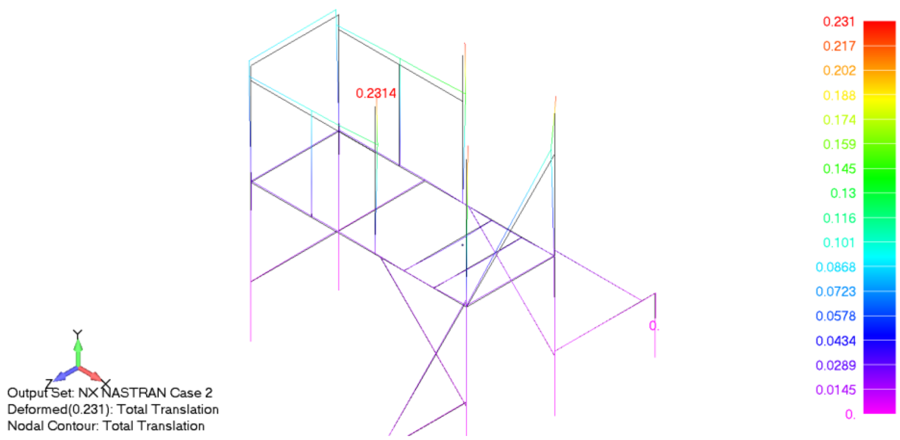


*Figure 4-43 Case II.2 2B-R var. Frame deformation (200*x *actual deformation)*

For 2B-AWR variant of the frame, maximum deflection is 0.198 mm (Figure 4-44). Compare to the result from the 2B variant and 2B-R variant this result is 54% and 14% less, respectively for 2B and 2B-R.

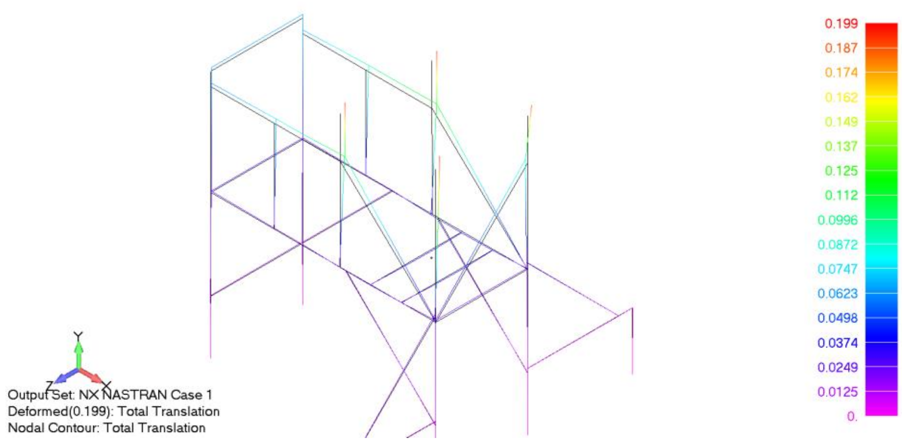


*Figure 4-44 Case II.2 2B-AWR var. Frame deformation (200*x *actual deformation)*

Subcase 3

When the centrifuge arms are located on the 90 degrees from reference position, arms deformation from the asymmetric load for 2B variant is 8.4 mm. For the centrifuge arms it is an acceptable deformation.

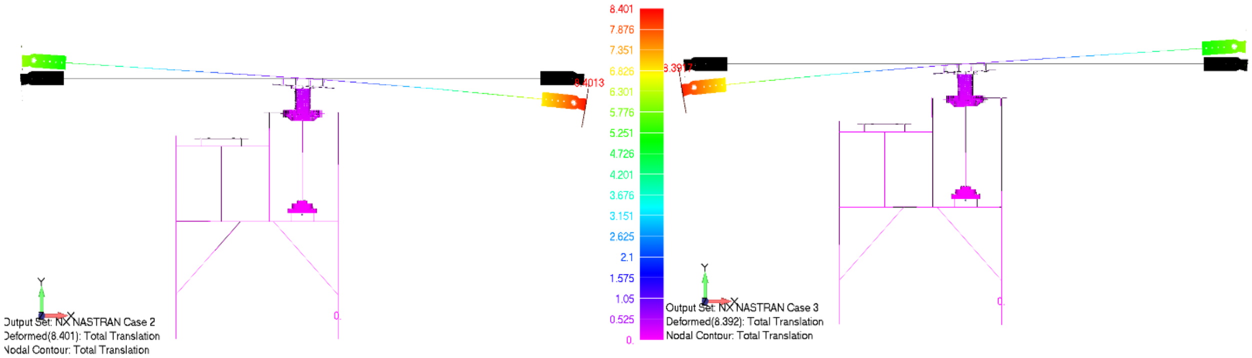


Figure 4-45 Case II.3 2B var. Deformation result (10x actual deformation)

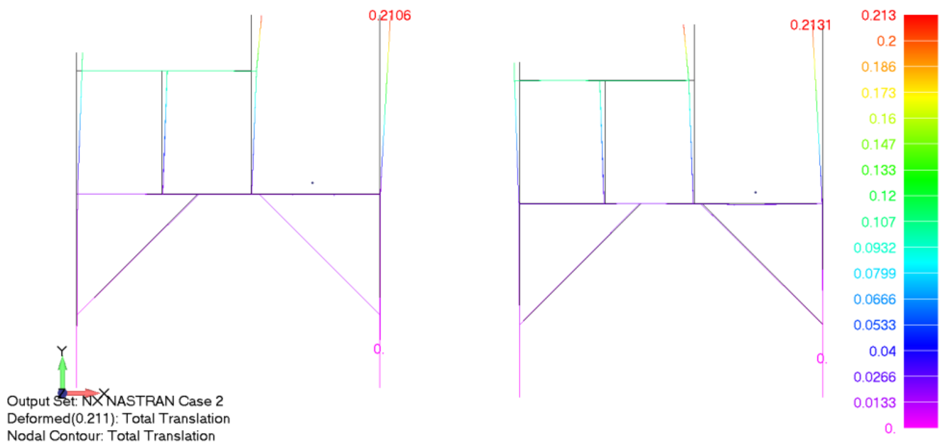


Figure 4-46 Case II.3 2B var. Frame deformation (100x actual deformation)

Maximum deformation of the frame is ≈ 0.22 mm, as it is seen from the Figure 4-46. Since the place of the deformation is the same, then it has similar requirements as it was for previous subcases, which is $\Delta_1 =$ 1.47 mm and $\Delta_2 = 0.7$ mm. Definitely 0.213 <0.7 mm, which means that obtained results for the asymmetric load case fits requirements with enough margin till the limited deflection given by ISO standard.

For the frame variant 2B-R the maximum deflection is in the same place as for other variants, its value is 0.212 mm (Figure 4-47), compare to the 2B var. it is 0.47% less than 0.213 mm, or no change by other words.

For the frame variant 2B-AWR the maximum deflection is in the same place as for other variants, its value is 0.15 mm (Figure 4-48), compare to the 2B var. it is 31% less than 0.22 mm, and for 2B-R var. it is 29.2% less than 0.212 mm.

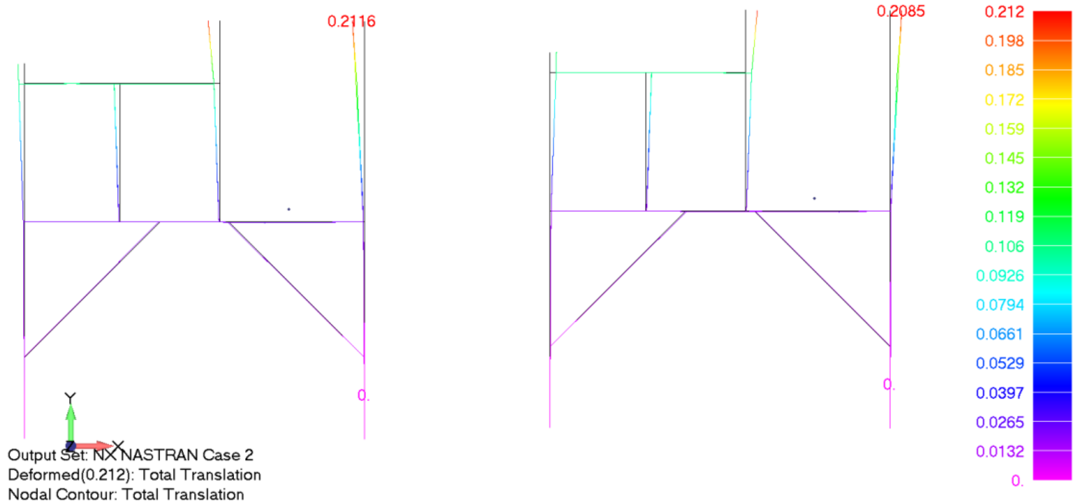


Figure 4-47 Case II.3 2B-R var. Frame deformation (100x actual deformation)

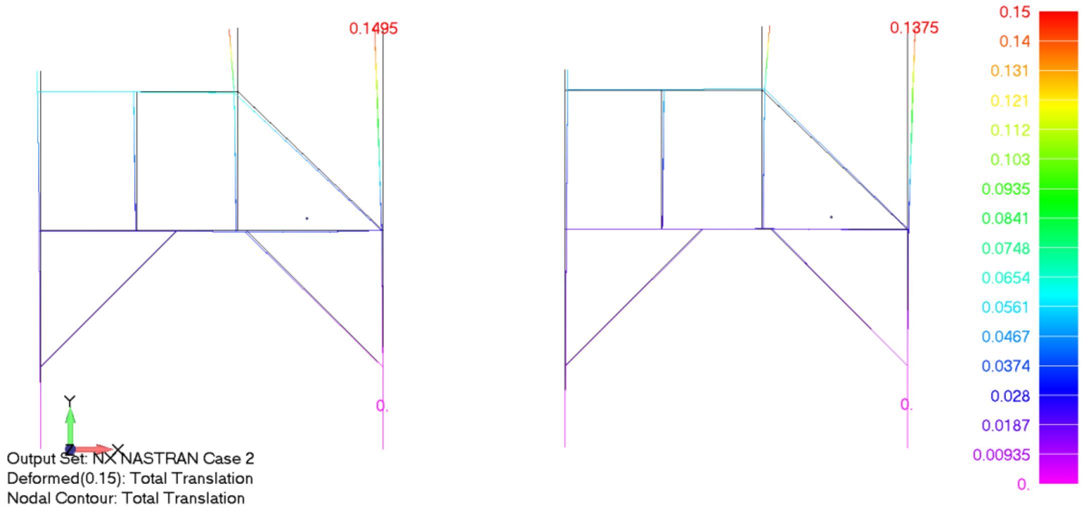


Figure 4-48 Case II.3 2B-AWR var. Frame deformation (100x actual size)

As it was mentioned before, on the frame we check mostly deformation, since we are interested in the rigidity of the frame. However, if we check maximum combined stress of the beam element in the FEM model we will see that maximum appeared stress from all solution of Case II is ≈ 3 MPa.

4.7.1. Results, Frame 2B variants

Cases	*Case I.1*			*Case I.2*			*Case I.3*		
Var.	*2B*	*2B-R*	*2B-AWR*	*2B*	*2B-R*	*2B-AWR*	*2B*	*2B-R*	*2B-AWR*
Arms deformation [mm]	2.356	2.355	2.355	2.359	2.358	2.362	2.288	2.359	2.37
Frame deformation [mm]	0.027	0.027	0.026	0.027	0.027	0.026	0.027	0.027	0.026
Δ_{min_i} [mm]	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Safety margin [mm]	0.374	0.374	0.374	0.374	0.374	0.374	0.374	0.374	0.37
Safety margin [-]	15.094	15.094	15.504	15.094	15.094	15.504	15.094	15.094	15.50
Frame def. 1 [%]	100.00	0.00	-2.64	100.00	0.00	-2.64	100.00	0.00	-2.64
Frame def. 2 [%]	0.00	100.00	-2.64	0.00	100.00	-2.64	0.00	100.00	-2.64
Cases	*Case II.1*			*Case II.2*			*Case II.3*		
Var.	*2B*	*2B-R*	*2B-AWR*	*2B*	*2B-R*	*2B-AWR*	*2B*	*2B-R*	*2B-AWR*
Arms deformation [mm]	9.453	8.673	8.63	9.075	8.6	8.5	8.401	8.531	8.346
Frame deformation [mm]	0.551	0.25	0.236	0.421	0.231	0.199	0.213	0.212	0.15
Δ_{min_i} [mm]	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Safety margin [mm]	0.149	0.450	0.464	0.279	0.469	0.502	0.487	0.488	0.55
Safety margin [-]	1.27	2.80	2.97	1.66	3.03	3.54	3.29	3.30	4.67
Frame def. 1 [%]	100.0	-54.6	-57.2	100.0	-45.1	-53.0	100.0	-0.5	-29.6
Frame def. 2 [%]	120.4	100.0	-5.6	82.3	100.0	-14.3	0.5	100.0	-29.2

Table 40 Frame deformation results for 10g accelerated mass of 15kg for two-beam modification of the centrifuge machine

For the symmetric case of loading, CaseI.1, CaseI.2 and Case I.3 there is no point to reinforce a frame, since there is no major changes between different variants neither in arms deflection nor in frame. For the asymmetric loading cases Case II.1, Case II.2 and Case II.3, of course, the best variant to use is the one with all available windows reinforced (see Figure 4-39), however the effectiveness of its use is not that big. If we compare difference in frame deformations between 2B-AWR variant and 2B-R, we will see that in percentage difference especially for the Case II.1 is very small. In other hand, in the loading Case II.3 percentage of the difference of the maximum frame deformation seems to be high, at the first sight, however the deformation itself of the initial variant 2B is relatively small and already has similar value to the maximum frame deformations of 2B-R frame variant.

In order to use material with higher efficiency, I would recommend to use frame variant 2B-R, where, if we compare with 2B variant only one frame member is added (see Figure 4-37). In this way, we reduce the weight of the frame, the time demand to assemble/disassemble machine (compare to 2B-AWR var.); the risk of the stability loss, exceeding structural limits given by ISO 4356:1977 (compare to 2B variant); and finally we will have relatively same value of deformation on every position of the centrifuge arms, which means small fluctuation of the frame members that will provide relatively static (quasi-static) condition of the centrifuge machine. Frame deformation: 0.25 →0.231→0.212; Arms deformation: 8.673 → 8.6 → 8.531.

From the diagram shown on the Figure 4-49, we can read that the amplitude of the frame maximum deformation is:

$$A_{\mathrm{Fr}_{\mathrm{def}}} = 0.25 - 0.212 = 0.038\ [mm] \tag{4.11}$$

For example, amplitude for the 2B var. is 0.33 mm, for 2B-ARW var. is 0.086 mm.

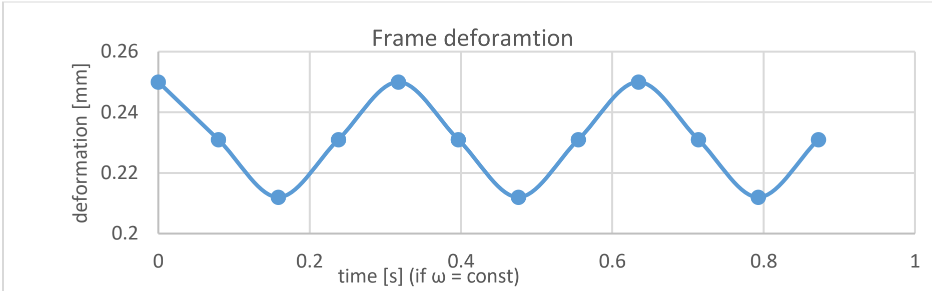


Figure 4-49 Deformation vs time, 10g load, 2B-R var., Frame

For the maximum arms deformation amplitude is:

$$A_{arm_{\mathrm{def}}} = 8.673 - 8.531 = 0.142 \quad (4.12)$$

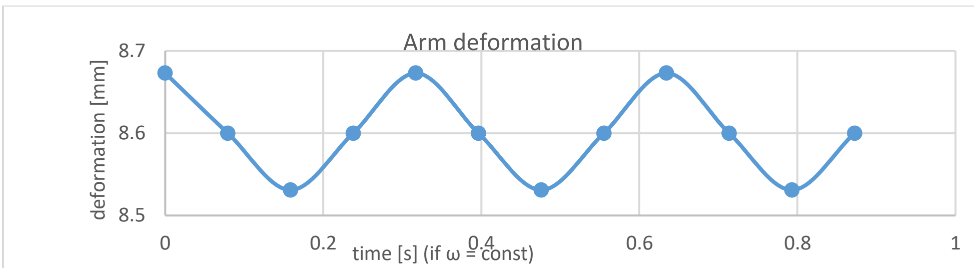


Figure 4-50 Deformation vs time, 10g load, 2B-R var., Arms

4.8. Frame, 1B variant

Calculation approach will be similar as it was done for 2B variant, with exactly same loading values and their cases, as it was described in paragraph §4.7 on the page 83. There will be used same variant modifications as for 2B variant, which means that 3 variants will be calculated 1B, 1B-R and 1B-AWR. Difference between these modifications you can see on the picture Figure 4-37 for 2B-R variant (but instead of two beams arm will be used one beam arm), Figure 4-39 for 2B-AWR (same for 1B-AWR). For example, 1B variant of asymmetric loading at 0 degree position of centrifuge arms (Case II.1) is shown on the Figure 4-51.

Since deformation profiles of arms and frame are similar to the 2B variant, I found no need to put figures here with of deformations. Instead, I will write just a result table analogic to the result table for 2B variant.

All deformations results and requirements you can find on the Table 41 on the page 96.

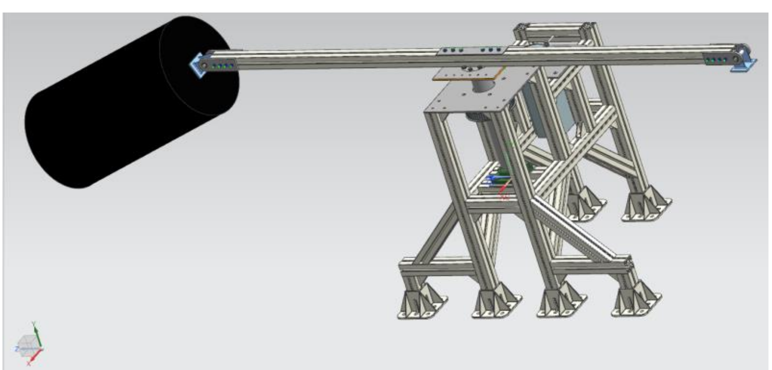


Figure 4-51 Cad model of 1B variant, with loading Case II.1

4.8.1. Results, Frame 1B variants

Cases	***Case I.1***			***Case I.2***			***Case I.3***		
Var.	*1B*	*1B-R*	*1B-AWR*	*1B*	*1B-R*	*1B-AWR*	*1B*	*1B-R*	*1B-AWR*
Arms deformation [mm]	4.834	4.836	4.833	4.836	4.835	4.835	4.837	4.836	4.85
Frame deformation [mm]	0.027	0.028	0.027	0.028	0.028	0.028	0.027	0.028	0.027
Δ_{min_i} [mm]	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Safety margin [mm]	0.373	0.373	0.373	0.372	0.373	0.372	0.373	0.373	0.37
Safety margin [-]	14.760	14.545	14.925	14.493	14.545	14.493	14.760	14.545	14.93
Frame def. 1 [%]	100.00	1.48	-1.11	100.00	-0.36	0.00	100.00	1.48	-1.11
Frame def. 2 [%]	-1.45	100.00	-2.55	0.36	100.00	0.36	-1.45	100.00	-2.55
Cases	***Case II.1***			***Case II.2***			***Case II.3***		
Var.	*1B*	*1B-R*	*1B-AWR*	*1B*	*1B-R*	*1B-AWR*	*1B*	*1B-R*	*1B-AWR*
Arms deformation [mm]	9.459	8.496	8.441	8.96	8.408	8.279	8.337	8.33	8.13
Frame deformation [mm]	0.582	0.267	0.251	0.458	0.245	0.208	0.221	0.22	0.153
Δ_{min_i} [mm]	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Safety margin [mm]	0.118	0.433	0.449	0.242	0.455	0.492	0.479	0.480	0.55
Safety margin [-]	1.20	2.62	2.79	1.53	2.86	3.37	3.17	3.18	4.58
Frame def. 1 [%]	100.0	-54.1	-56.9	100.0	-46.5	-54.6	100.0	-0.5	-30.8
Frame def. 2 [%]	118.0	100.0	-6.0	86.9	100.0	-15.1	0.5	100.0	-30.5

Table 41 Table of deformation results for 1B variants

From the result table we can say that, again optimum variant to use is the 1B-R, because of same reasons as it was described for the Table 40 on the page 94.

Another thing, which is interesting is the difference in deformations between 1B and 2B variants. For symmetric load cases, arms deformations of 1B variant are approximately twice bigger than analogical values of 2B variant. This result however is expected somehow, however deformations for asymmetric load cases are similar and deformation for 1B variants are slightly smaller than for 2B variants. For both cases were used same frame models, same materials, same CBEAM characteristics and was used the same solver. From this point of view we can state the 1B – R variant is better than 2B – R variant due to less material usage and assembly time. Less material usage follows in to the less weight of the centrifuge's head that will decrease frame member deformation at the place of a shaft mounting to the frame by thrust bearing housing. At the same time for the symmetric cases, 2B-R variant is better, from the deformation point of view.

Generally, I strongly recommend using a counterweight if only one sample is tested and it is better to make counterweight as close as possible to tested sample by mass characteristic and shape.

4.9. Centrifuge’s head

4.9.1. Centrifuge’s Head Plate

1B Case I

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

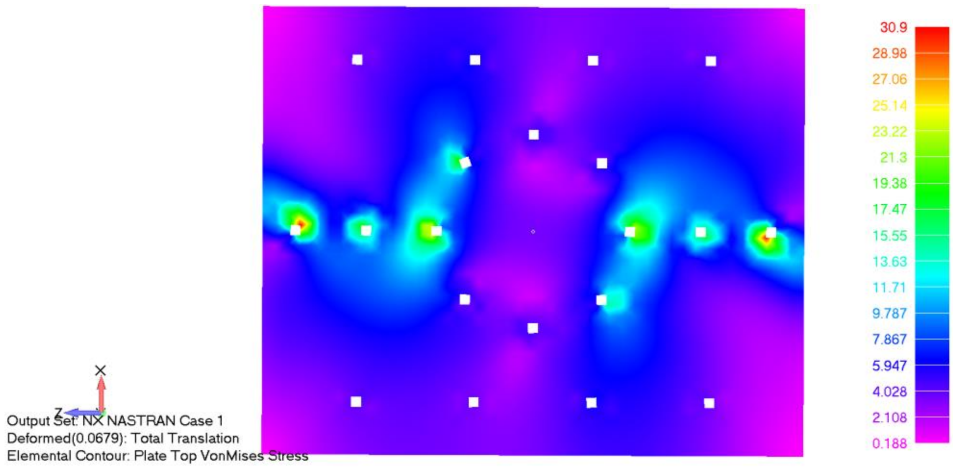


Figure 4-52 CHP, 1B, Case I, steel made

Max. stress [MPa]	31 MPa
Max. deformation [mm]	0.068
Stress reserve factor, j [-]	4.19

Table 42 CHP, 1B, Case I result table

1B Case II

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

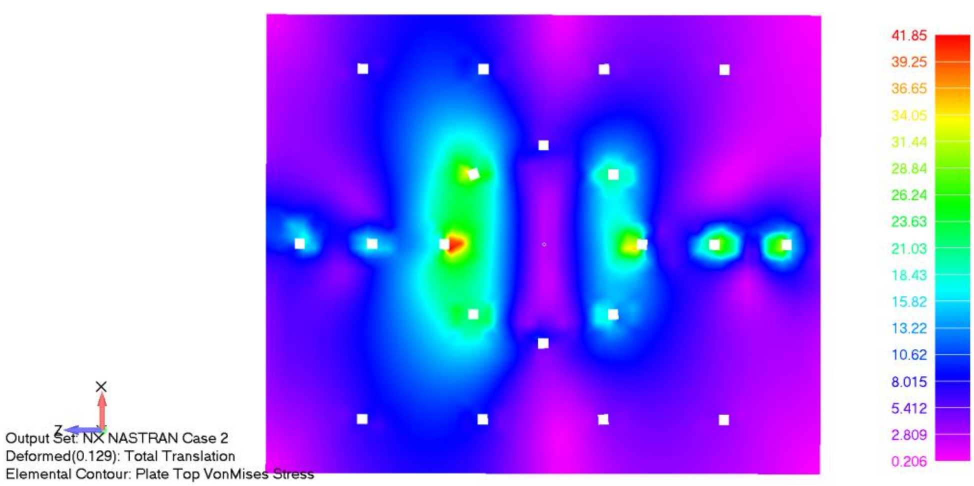


Figure 4-53 CHP, 1B, Case II, steel made

Max. stress [MPa]	42 MPa
Max. deformation [mm]	0.13
Stress reserve factor, j [-]	3.1

Table 43 CHP, 1B, Case II result table

2B Case I

All subcases have similar results, from all frame variants was chosen the one that has the biggest results from them all.

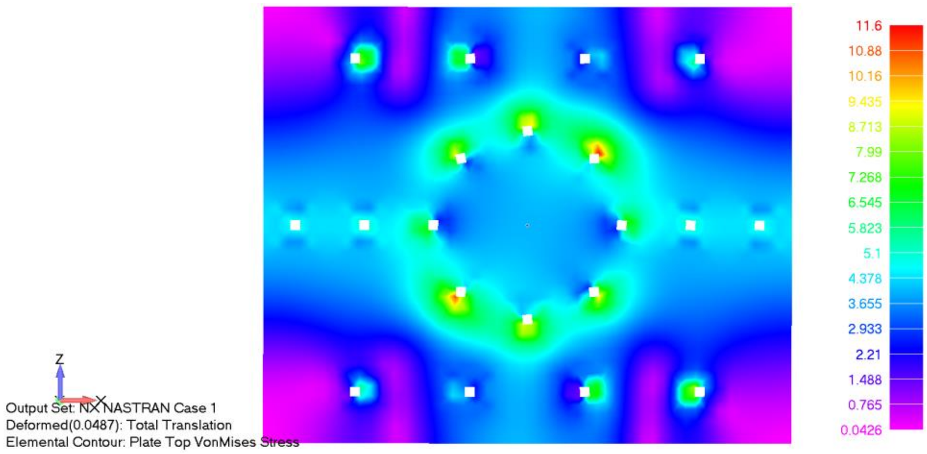


Figure 4-54 CHP, 2B, Case I, steel made

Max. stress [MPa]	12 MPa
Max. deformation [mm]	0.05
Stress reserve factor, j [-]	10.8

Table 44 CHP, 2B, Case I, result table

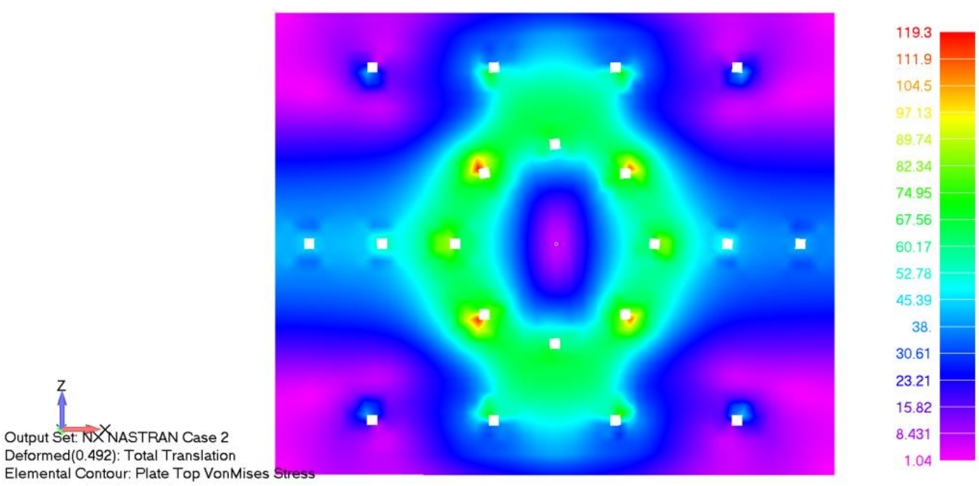


Figure 4-55 CHP, 2B, Case II, steel made

Max. stress [MPa]	120 MPa
Max. deformation [mm]	0.5
Stress reserve factor, j [-]	1.1

Table 45 CHP, 2B, Case II, result table

Summary

Again, we can see that for the symmetric case it is better to use a 2B variant, however when the load is completely asymmetric 1B variant is better for the stress and for the deformation.

Very big value of the stress of the 2B Case II can be explained. In the FEM model holes are modulated as squares, which due to presence of right angle increase the stress peak much higher than it should be in reality, this is so called local notch. In addition the force transmitted from the Lug-out part to the Lug (female lug) in 2B variant is higher than for the 1B variant, see Figure 4-11.

4.9.2. Hub

The most critical load case is the asymmetrical load, Case II for 1B variant. Its stress and deformation are shown on the Figure 4-56.

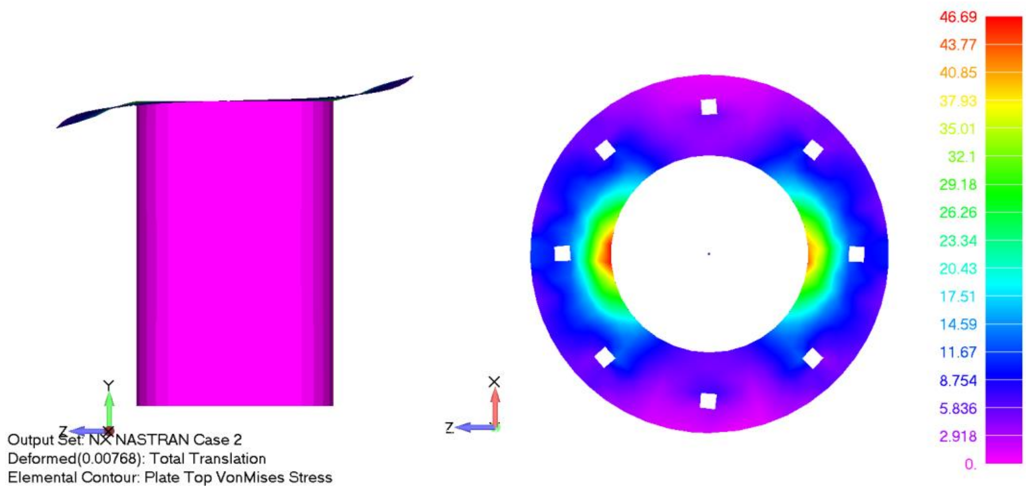


Figure 4-56 Hub, 1B, Case II, steel made

Max. stress [MPa]	47 MPa
Max. deformation [mm]	0.008
Stress reserve factor, j [-]	2.76

Table 46 Hub, 1B, Case II, table result

All results satisfies all necessary condition.

4.10. Shaft's critical speed

It was already provided a hand calculation for this problem in the paragraph §3.3.2, p. 32. There was made an assumption that the shaft is the massless, which is not true, it was done to simplify the problem. The advantage of the FEA will help to find the critical shaft speed with proper mass model of the shaft. For this purpose was used MSC Patran software as pre-processor and MSC Nastran software as a calculator.

Shaft was designed as 1D element, namely CBEAM. Shaft was divided into 5 sections. Additional masses, as a centrifuge's head of 60 kg mass and driven pulley of 4 kg mass were modeled as a shaft section with relatable cross-section. Driven pulley was modeled as a beam with circular cross-section of radius R_{M2} = 84 mm and density $\rho_{M_2} = 6.445 * 10^{-6}\ kg/mm^3$ in order to have a proper mass character in this section. Centrifuge's head was modeled as a thin rectangle of thickness t = 5mm, height H = 230 mm and width W = 190 mm, density of this section is $\rho_{M_1} = 2.746 * 10^{-4} kg/m^3$. FEM shaft model is shown on the Figure 4-57.

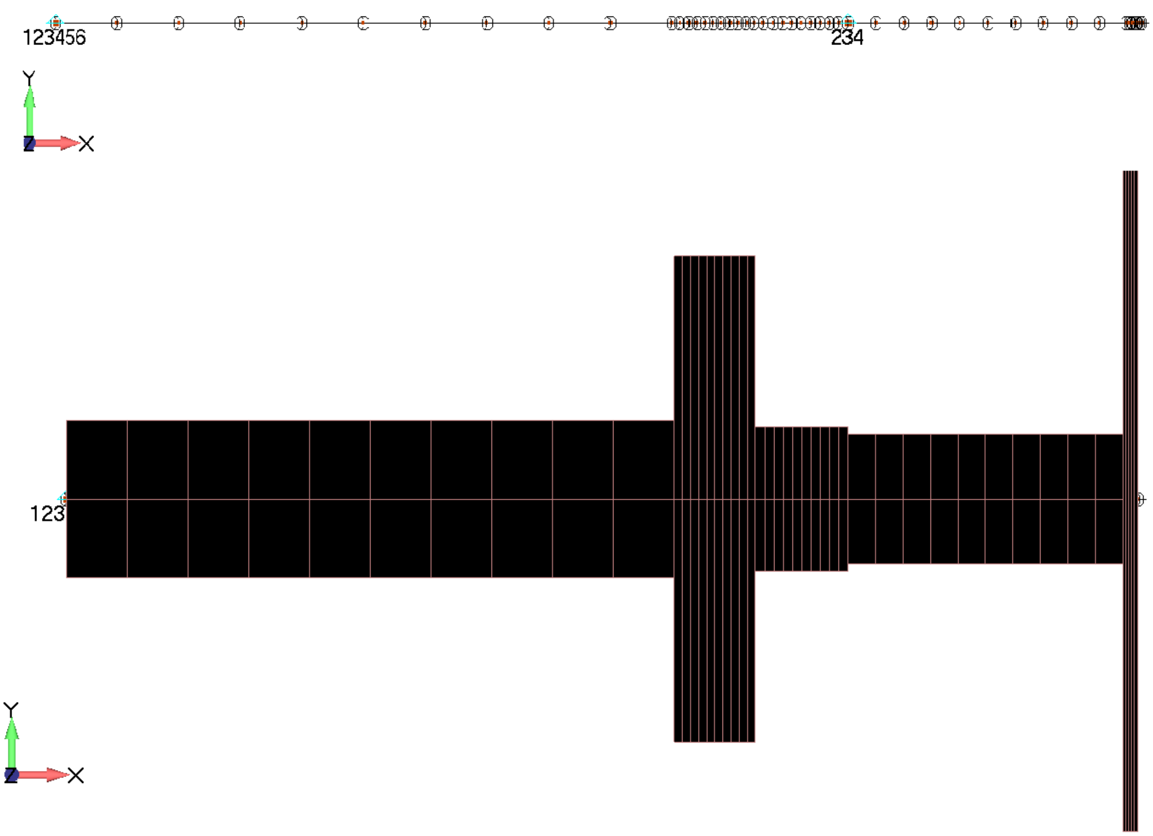


Figure 4-57 Shaft FEM model (CBEAM)

Using rotor dynamic tool in MSC Patran, we have defined rotational orientation and speed unit. The solution method was chosen to be Complex Eigenvalue solver with direct formulation, SOL 107. The solution result is chosen to be given as a Campbell diagram (plot represents a system's response spectrum as a function of its oscillation regime; Eigen frequencies as a function of the shaft's rotation speed. This case is also called "whirl speed map" [17] [18]). In addition, FEM will help to show what shape of each mode will be.

From the Campbell diagram, Figure 4-58, it is seen that first critical speed is 2.52 rps (very first intersection between first speed and Eigen frequency of Mode 1 = Mode 2), which makes 151.2 rpm.

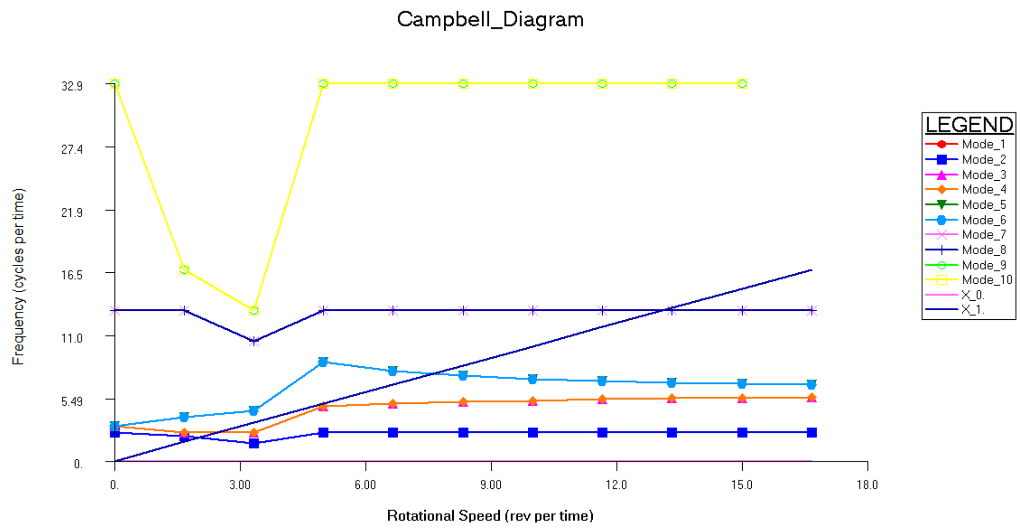
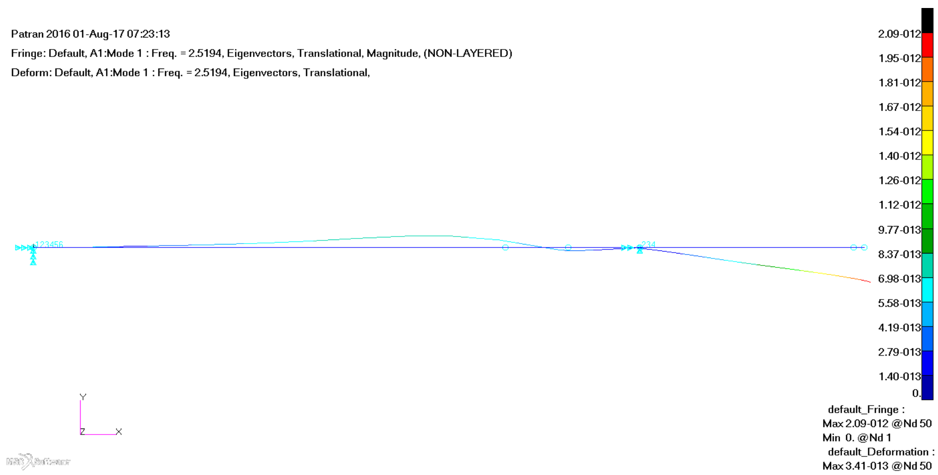


Figure 4-58 Campbell Diagram

Comparing results from FEM calculation with hand calculation (see eq. (3.83)) where the the frist critical speed was 17.5 rad/s = 2.7 rps, we can see that the result given by numerical solution is relatively smaller than the one we calculated by hand. It is because the FEM solution was solving with another method and with proper mass model of the shaft.

Even though the FEM result is less, still it is bigger than the chosen minimum required speed, which is 2 rps or 120 rpm. The shaft design satisfies requirements.



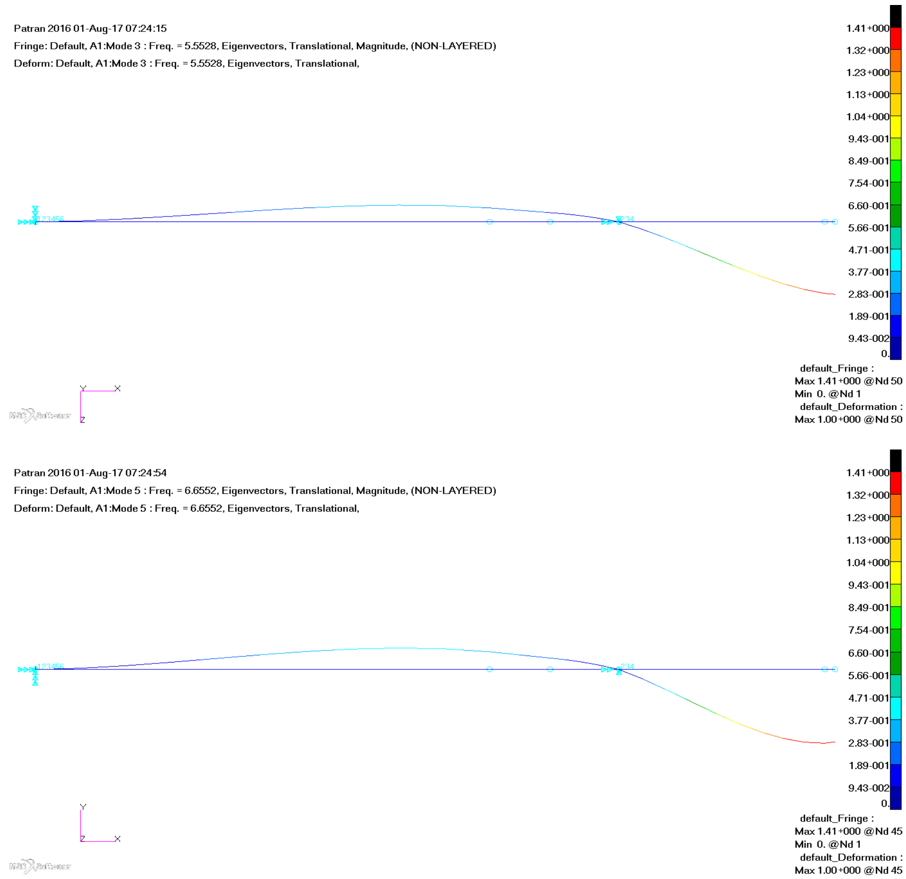


Figure 4-59 Shaft rotational shape, Mod 1 = Mod 2 (f=2.52 Hz), Mod 3 = Mod 4 (f = 5.55 Hz) and Mod 5 = Mod 6 (f = 6.65 Hz)

4.11. Shaft's force (Buckling)

In order to find a critical force of the shaft in buckling, we have to use non-linear Nastran solver SOL106. We apply on the free end of the shaft FEM model that is shown on the Figure 4-57 a nodal force 1 000 000 N. When a deformation of the shaft (of specific node), under the percentage of the applied load, will start to be non-linear, then this load will be counted as a critical load.

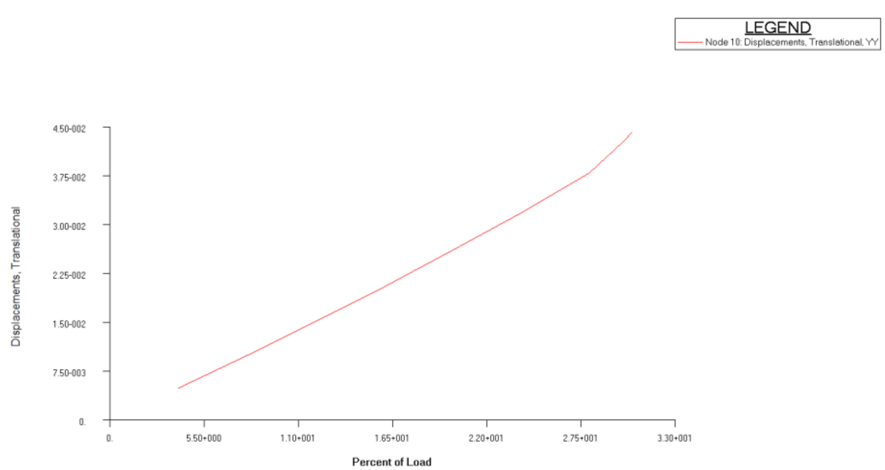


Figure 4-60 Shaft's node deformation vs Force

From the Figure 4-60 it is seen that the last point when deformation has a linear dependency on the force is approximately 27.5% of 1 000 000 N, which is 275 000 N.

$$F_{crit} = 275\,000\ [N]$$

$$F_{all} = \frac{F_{crit}}{f} = \frac{275000}{2} = 137\,500\ [N] \approx 135\,000[N] \tag{4.13}$$

Allowable compression forcce of the shaft is then 135 000 N, which is ≈ 13 500 kg. Which is far beyond of needs, and shaft seems to be oversized for the buckling. However, due to bending and critical speed, we can not reduce the diameter in order to remain sufficient margin safety.

4.12. Fasteners

All bolts, nuts, washers and threaded rods are chosen from the standards and can easily be purchased on the market. From the global FEM model, I chose the most critical stress that appeared on the fastener. All fasteners were modeled as a beam element with circular cross-section. The result is written in the table, material of standard fasteners are taken from the NX material library.

Designation	Parts connected	σ_U [Mpa]	σ_{all} [Mpa]	$\sigma_{\max}$ [Mpa]	j [-]
rod M5 x 80	Lug - 45x45 profile	275	137.5	10	13.75
DIN M6 x 35	CHP - Hub	275	137.5	45	3.06
M6 x 80	45x45 - CHP	275	137.5	65	2.12
DIN M8 x 55	BHP - 45x45	275	137.5	25	5.50
321345.1	45x45 - 45x45 (angle connection)	100	50	2	25.00
DIN M12 x 40	BHP - Bearing House	137	68.5	5	13.70
DIN M12	UCF 209 - 45x45	137	68.5	5	13.70
M8	Connection between Frame members	137	68.5	4	17.13

Table 47 Fasteners FEM result table

All reserve factors are above than one, which means all designed fasteners satisfies necessary conditions. Some of the fasteners are oversized, however from the design point of view it is not advantageous to redesign these connections.

5. Remarks

In the remarks, I would like to recalculate the time required to reach 10g and 5g with the complete knowledge of the mass moment of inertia and torque on that will be applied on the Centrifuge's head.

From the CAD software Siemens NX 11® we can read the weight characteristics of the Centrifuge's head with all lugs, pins, bearings, fastener and attached capsule with 15 kg mass and size of 340 x 650 (D x H in mm). Final result for the mass moment of inertia for 2B centrifuge's head is:

$$I_{\max} = 38135230\ [kg.mm^2] = 38.14\ [kg.m^2] \approx 40\ [kg.m^2] \tag{5.1}$$

Using method described in the paragraph §2.3, p. 8 we can recalculate the time that will be need to reach such a rotation that will create a 10 g loading.

Using gear ratio of i = 1.6, there will be next values:

rps	T [N.m]	ω[rad/s]	v [m/s]	Re[-]	$2 * M_d$	T_2 [N.m]	α' [rad/s2]	t'[s]
0	20	0.00	0.00	0.00	0.00	30.40	0.76	**0.00**
0.37	20.5	2.32	2.32	54138.68	1.61	29.55	0.74	**3.15**
0.95	21.2	5.97	5.97	139004.72	10.61	21.61	0.54	**11.05**
1.5	20.44	9.42	9.42	219481.13	26.45	4.62	0.12	**81.66**
1.576	20.35	9.90	9.90	230653.77	29.21	1.72	0.04	**230.60**
2	20.05	12.57	12.57	292641.51	47.03	-16.55	-0.41	**-30.37**

Table 48

Where $2 * M_D$ is the moment from the drag force from the both capsules of 340 x 650 mm size. T_2 is the torque on the shaft. Time that is required to reach the tested sample of 10g acceleration is 230.6 [s]. In order to reach 5g load we need as rotational speed to be 1.15 rps, which makes approximate 30 seconds.

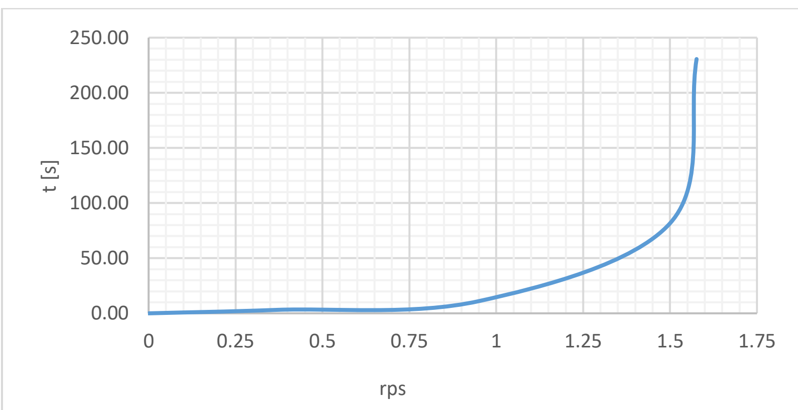


Figure 5-1 Time demand for specific rotational speed, for I=40kg.m² and a drag from both sides of arms.

Please note that, the drag force, respectively moment is not necessary always equal to the value that is written in the Table 48, since it is the function of the shape of the tested sample by manipulating with it we can decrease drag force, hence a resisting drag moment. The less drag moment is then less time is needed to reach the desired rotational speed. In addition, drag force has not to be always symmetrical, since the second arm can be counterweighted by a mass of different shape.

If we would like to reduce the time that is needed to reach specific rotational speed, also, we can use a motor with better torque characteristics. For example, if we use a motor with constant torque value T =35 N.m up to 2 rps and transmission with ratio i = 1.25, the time to reach speed that will load tested sample for 10g will be t = 41.25 s.

Another way to improve this machine is to add a rope that will be pinned into the centrifuge's arms and connected to the Holder that. For example, preliminary, as it shown on the Figure 5-2. This will help ro reduce a deformation of the centrifuge's arms.

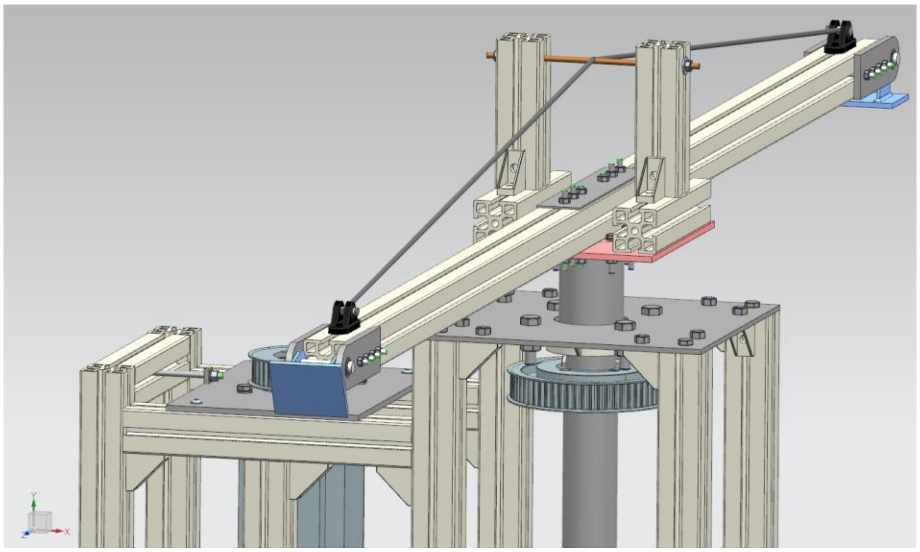


Figure 5-2 1B centrifuge with rope (or rod) support

Mainly, space rockets accelerates approximately with 5g value. For the acceleration the designed centrifuge machine fits perfectly with accomplishing all necessary requirements, however the maximum acceleration, for which the machine is designed is 10g.

Technological remarks

Lug-out

Centrifuge's part Lug-out was meant to be manufactured by welding connection of two aluminum elements, however aluminum welding is the complicated task and requires from the person specific skills, which potentially can lead into the increasing of production cost, because will be needed skillful person who needs higher salary. Steel welding is easier compare to an aluminum one. Changing material from aluminum to steel if we remain geometry similar as it was before we will decrees cost by decreasing labor working time, but the mass of the part will be increased that will lead into the time of reaching the desired rotational speed (mass moment of inertia). Material swapping will simplify the task.

I don't find it necessary to recalculate the strength and rigidity of the part, since traditionally, steel is more strengthen and more rigid than aluminum. Young's modulus of the steel is 3 times higher than aluminum (210 GPa against 70 GPa). It would be required to recalculate the task if we decide to change geometry (primarily thickness) of the part, but it is cheaper and easier to use plate of the same thickness for all shell parts of the machine, which is 5 mm. The reason is that it is nearly impossible to by in the market raw material like a steel metal sheet of small size like 400x400mm; usually nominal size starts from 1000x1000 mm. So all plate look alike parts of the centrifuge machine will be produced form 5mm steel sheet.

Shaft

At the first place shaft had to be manufactured from the aluminum material (AlMgSi0.5F25), however after consultation with the technology staff of Aerospace Department of Czech Technical University, it was stated that it is complicated task to manufacture by turning aluminum bar of 60mm cross-section with necessary tolerances. It was recommended, again, to change material to the designed part from aluminum to steel, so it will be easier for the university staff to manufacture this part. Changing shaft material will increase critical speed, which is advantageous for us, but it will increase the weight of the rotating part, so the time required reaching the desired rotational speed will increased too. The critical speed at the first mod (the first critical speed) is $\approx$ 180 rps, which is more than enough.

By changing the shaft material to steel, which has tensile modulus as 210 GPa and Rp02 is 275 MPa, the minimum diameter that can transfer loads that were mentioned in the paragraph §3.3.1 on the page 26 is then 35 mm. We will remain all geometry same as it was on aluminum made shaft except the section 4, the one that is connected to the thrust bearing. By reducing diameter from 45mm to 35mm we can change thrust bearing and its housing from UCF 209 to UCF 207, which will reduce the weight of the structure and cost.

Cover

It is good to cover the frame of the machine by plastic or wooden plate, so the dust will have limited access to the moving parts of a centrifuge, such as belt, shaft, bearings etc. There is no need to calculate the strength of it, because cover will not transfer any loadings and will execute only decorative function and protection from the dust or any other small particles. How preliminary wooden cover would look like you can see on the Figure 5-3.

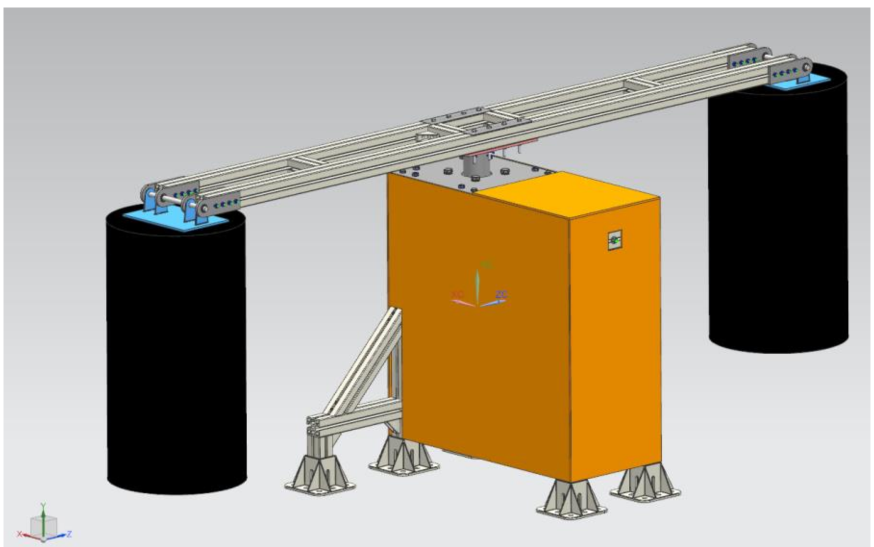


Figure 5-3 CAD model of centrifuge with wooden cover

6. Conclusion

The main goal of the work was to design a centrifuge machine that can load a tested sample up to 10g, which is 98.1 $m.s^{-2}$. Tested sample together with its capsule (box, cylinder or any other element that will be connected to the centrifuge arm and will carry a tested sample) should not exceed the mass of 15 kg. In this work, preliminary tested sample was chosen to be a cylindrical capsule of 650mm height and 350mm diameter. Centrifuge performance calculation was made according to chosen capsule that will call a drag force during the rotation. Bearing friction was neglected due to absence of precise date without experiment, as well as a drag force of the arms, due to no data of drag coefficient of a 45 x 45 profile and compare to the drag force and then drag moment to the rotation centre it very small, so it can be neglected.

The frame and centrifuge's arms were made from the aluminum profile 45 x 45 from Alutec KK supplier. Connection elements of the profile, again, were supplied by the Alutec KK company. Parts of the centrifuge machine that are not standardized, the one that are not selling in the market and requires manufacturing, were designed in the way that it will be cheap to manufacture them by the devices that are in the Aerospace Department workshop.

The centrifuge arm distance, the distance between rotation centre and pin centerline where a capsule with a tested sample is connected, is 1m. The centrifuge's span is then 2m. The centrifuge's head was designed in the way that it can be assembled as centrifuge with one-beam arm (1B) or two-beams arm (2B). Calculation showed that it is better to use 1B, due to less usage of a material and smaller deflection in asymmetric cases of load. However, for the symmetric loading cases the deflection of the arms is twice smaller for the same loading, but for 1B variant.

According to the calculation in paragraphs §4.7.1 and §4.8.1 on the page 94 and 96 respectively the optimum variant of the frame is the one reinforced window 2B-R or 1B-R, as it is shown on the Figure 4-37 p. 89.

Preliminary time demand to reach a desired rotation when centrifuge's arm tip reaches 10g acceleration is 4 minutes, for the gear ratio i = 1.6. The maximum speed of a rotation and torque can be manipulated by changing gear ratio that can be done by changing various pulleys. For details, see Figure 3-13 p.24. All centrifuge elements were designed to safely operate (safety factor f=2) for the rotational speed 95 rpm, which we can call as a maximum allowable speed of the centrifuge. However, due to high safety margins it can rotate maximum up to 120 rpm.

$$n_{all} = 95\ rpm \tag{6.1}$$

All necessary dimensions you can find in the attached assembly drawing for 2B-R, drawing number is 10-02-002.

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