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ENGINEERING**



DIPLOMA THESIS

**TIME DELAY COMPENSATION
ALGORITHMS
FOR ROLLING MILS**

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MASTER THESIS ASSIGNMENT

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- Perform state of the art in rolling mills modelling and control with the focus on delay and eccentricity compensation
- Derive the dynamical model of a rolling mill and apply internal model control scheme to compensate the transportation delay
- Design repetitive controller to compensate eccentricity and validate it on a simulation model.
- Propose the final control scheme for implementation on a real plant
- Summarize results

Literature resources:

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The student is aware that the thesis has to be accomplished through an independent and unassisted student's work, supported only by recognized consultations. Literature and other information resources as well as consultants' names have to be acknowledged in the thesis.

27. April 2017

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Declaration of Own Work

I, Mert Okar, hereby declare that this work, which is submitted on 16.06.2017 to Czech Technical University in Prague, Faculty of Mechanical Engineering is my own work, supported only by recognized consultations. Literature and other information resources as well as consultants' names have been acknowledged in the thesis.

Date: 16.06.2017

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A handwritten signature in black ink, appearing to read 'Mert Okar', with a horizontal line drawn through the middle of the signature.

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Nomenclature

Δh_{GM}	Variation of gauge meter thickness
ΔS	Variation of work roll gap
ΔP	Roll force variation between top/bottom work roll received from the strip
M	Mill modulus of rolling mill
T_L	Value of time delay
\tilde{T}_L	Nominal value of delay
$r(t)$	Reference signal
$\eta(s)$	Sensitivity Function
$\epsilon(s)$	Complementary Sensitivity Function
$Q(s)$	Internal Model Controller
$C(s)$	Classical Feedback Controller
$P(s)$	Plant
$\tilde{P}(s)$	Plant Model
J	Cost Function
m_m	Mass of material
m_r	Mass of roll
m_b	Mass of back-up roll
c_m	Damping coefficient of material
c_r	Damping coefficient of roll
c_b	Damping coefficient of back-up roll
k_m	Spring coefficient of material
k_r	Spring coefficient of roll
k_b	Spring coefficient of back-up roll
τ	Time delay of model

1. Abstract

In metalworking, rolling is a metal forming process in which metal stock is passed through one or more pairs of rolls to reduce the thickness and to make the thickness uniform. The final thickness of the strip is measured at a little downstream from the roll gap where the deflection of strip happens. In other words, there is an unavoidable non-negligible transport delay in the measurement of steel strip thickness. Furthermore, there exists a periodic disturbance signal in rolling mills. In this master thesis two control approaches, namely internal model control and repetitive control are designed and numerically simulated in MATLAB Simulink to compensate this delay and eccentricity in rolling mills.

2. Introduction

2.1 Overview

In the flat rolling process metal is passed through a pair of rolls and the final shape of product is classified as plate or strip. If the rolling process occurs above recrystallization temperature it is called hot rolling and if it occurs below, it is called cold rolling [1], [2]. The starting material is usually large pieces of metal such as slabs [1].

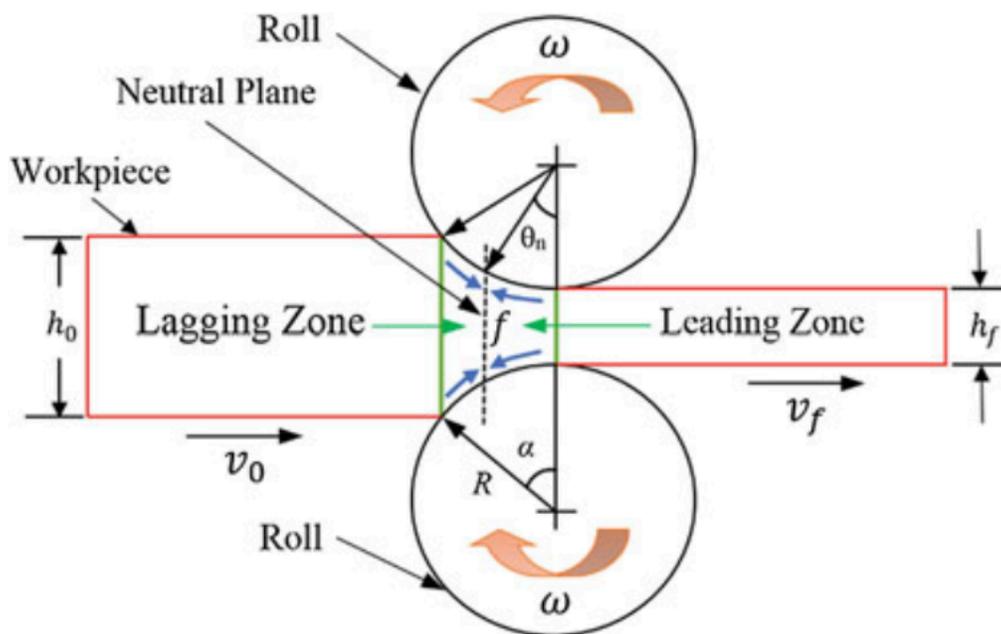


Figure 1 Rolling process [3]

Rolling, in steel strip manufacturing industry has a key role because this process is directly affecting the quality of final strip [4]. This rolling machine should be controlled precisely and appropriately to guarantee the uniformity on the final product which is a key property indicating the quality of final product. For this purpose, the roll force or gap is controlled based on the measurements of the thickness of the final product steel strip. [4]

During the rolling process, very high amount of forces act on the mill which causes the deformation of the mills [4]. As for the quality expectations of final product to be met the mill must be set up in a way that the roll gap between the two work-rolls is as flat as possible to ensure a uniform exit thickness of the plate in lateral direction. However, the measurement can't be done close to the gap because of unsafe environment which is consisting of immense heat and steam. The measurement therefore is being made some distance away from the gap which is causing a time delay in the system [4]. It is not useful for real time thickness control due to the time lag. Hence the exit thickness must be calculated with a mathematical model. The roll used in strip rolling process systems normally has eccentricities, which affects the precision of the thickness as a periodic disturbance with respect to the angle of the roll [4]. This means this periodic disturbance should be compensated under existence of a transport delay. [5]

2.2 Control System of Rolling Mills: Automatic Gauge Control (AGC)

The most common and well proven control strategy for thickness control in industrial rolling mills is the Automatic Gauge Control [5]. Automatic gauge control is a control loop which is designed to control the thickness of the output plate. From Figure 2 one can see, an example of AGC scheme. One should note that there are many kinds of Automatic Gauge Control types. For example, M-AGC and GM-AGC are two types of AGC [4].

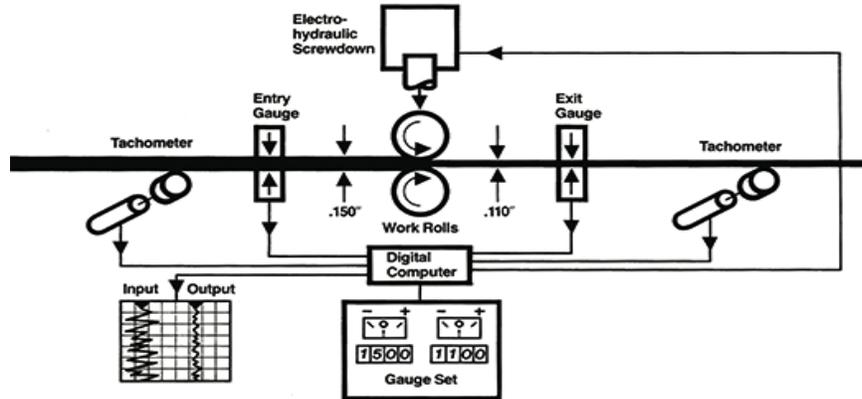


Figure 2 Automatic gauge control scheme [6]

It is basically, measuring inlet & outlet speeds and thicknesses and adjusting the gap accordingly. However, this control system is not able to control thickness accurate enough because of the time delay and periodic disturbance. Also, it is important to note that the dynamics of the process is very fast compared to the delay.

2.3 Problem Statement

The thickness of the final product is a very important quality measure. The rolls are normally composed of working rolls and back up rolls as shown Figure 3 [1]. A usual problem is that these rolls are not completely circular [4]. In other words, they are eccentric. This will result in unwanted thickness deviations in final product. So, the goal of this study is to compensate this thickness deviations in rolling mills with time delays. Two control methods namely, repetitive control and internal model control will be applied to the derived model of rolling mills and then results of simulations will be presented.

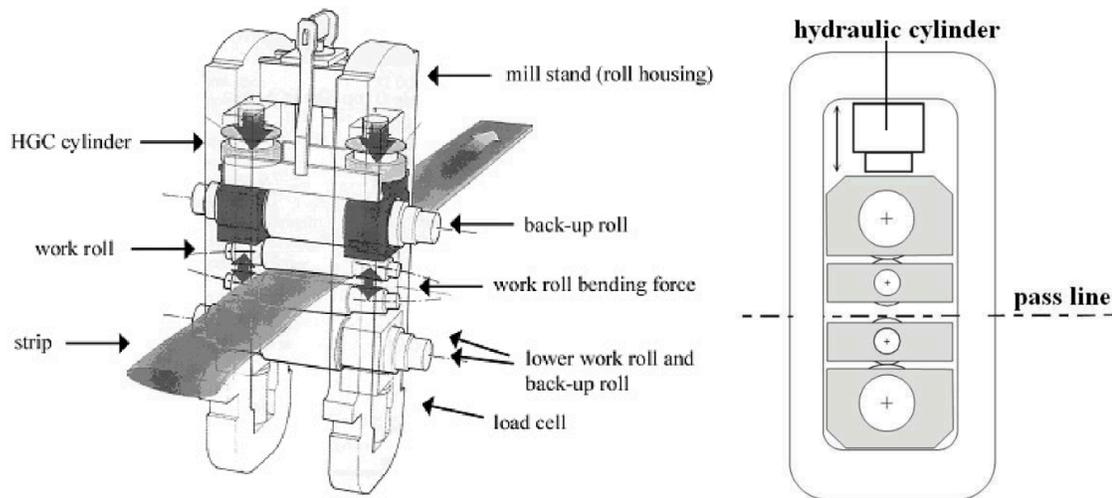


Figure 3 Rolling mill machine stand scheme [1]

3. State of Art Analysis

In this section, the paper called '**Attenuation of roll eccentric disturbance by modified repetitive controllers for steel strip process with transport time delay**' by Kazuki, Hiroto, Osamu, Yuki, Shigeru, Hiroyoku, Tokujiro is reviewed. This paper is published in 2015 by IFAC (International Federation of Automatic Control). In this paper, they have proposed a new modified repetitive controller to attenuate the effect of roll eccentric disturbance in a rolling process with time delay. They claim that a conventional repetitive controller cannot address the transport delay adequately.

In Section 2 of their work, they explain the strip mill and its involved control system. In Section 3, they explain why there exists a roll eccentric disturbance in rolling mills and related problems caused by this. In section 4, they present how new modified repetitive controller is changed to compensate the effect of the

input delay. In next sections, they perform a μ synthesis to cancel the effect uncertainty. Finally, they conclude with numerical simulations and results.

Overview of a typical conventional hot strip mill is shown in Figure 4 from the paper.

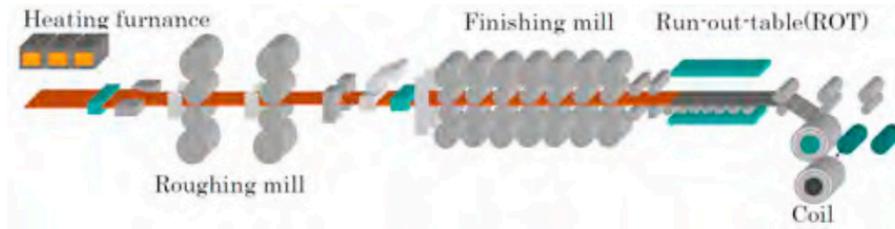


Figure 4 Overview of conventional hot strip mill [4]

The control system of the finishing mill almost determines the final strip quality. Thickness is the main strip quality to control. Automatic Gauge control, which is explained in above section is applied to all stands. They explain two types of AGC's, namely gauge meter AGC (GM-AGC) and monitor AGC (M-AGC). In GM-AGC, there is no time delay because it uses mill stand as a thickness sensor [4]. It uses the thickness approximation shown Eq.1 [4].

$$\Delta h_{GM} = \Delta S + \frac{\Delta P}{M} \quad (1)$$

Where Δh_{GM} is variation of gauge meter thickness, ΔS is the variation of work roll gap, ΔP is the roll force variation between top/bottom work roll received from the strip, M is mill modulus of rolling mill. M-AGC uses the measured value of the thickness, with a sensor placed on the exit side of the mill. However, since the sensor is placed at some distance from the gap there exists a transport time delay, which should be compensated when designing controller [4].

In section 3, they describe the cause effect of roll eccentric disturbance in hot strip mill. According to that paper, it is caused by the existence of keyway on the Back-up Roll [4]. Because the part of keyway is relatively weaker than other parts

of the machine, key deforms when a very large roll force exists on the system [4]. It causes 0.03-0.05 mm deviation of final products thickness [4]. For better product, it is very important to attenuate the effect of this disturbance.

In section 4, firstly they describe the conventional modified repetitive control system, and then describe their proposed modified repetitive controller for systems with time delay. More detailed explanations can be found in following sections, from Figure 5 Figure 6 one can see the conventional modified repetitive controller (MRC from now on) and proposed MRC.

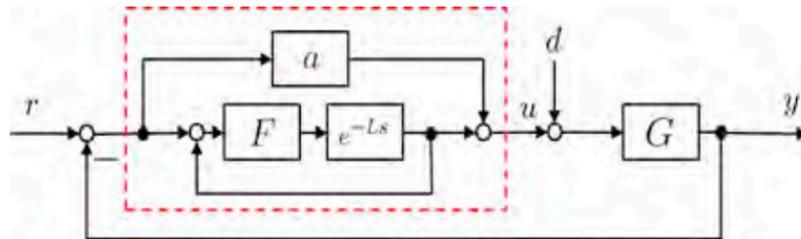


Figure 5 Conventional modified repetitive controller scheme [4]

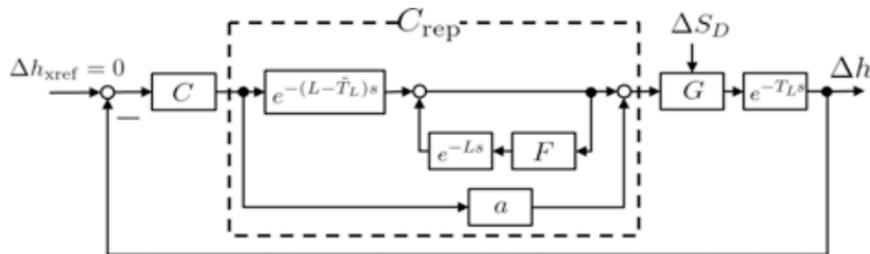


Figure 6 Proposed MRC scheme [4]

Here, L is the period of disturbance signal, $F(s)$ is low pass filter, T_L is transport delay and \tilde{T}_L is the nominal value of delay. Here it should be noted that they have introduced a new controller $C(s)$ which is used to guarantee the stability [4].

In final section, they have introduced a numerical example to illustrate the effects of proposed MRC. For details of this numerical example, please refer to the paper itself [4].

4. Theoretical Background

4.1 Internal Model principle

4.1.1 Overview & Basic Idea

In this section, I will explain with a brief example the main principle of internal model control. Suppose we have a control system shown in Figure 7.

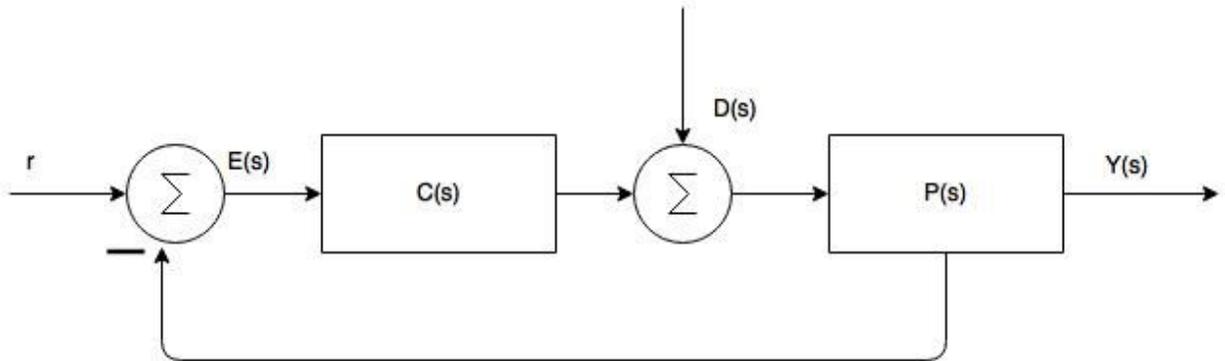


Figure 7 Simple block diagram of a control system

Assume a constant disturbance d_0 . In Laplace domain, it is equal to

$$D(s) = \frac{d_0}{s} \quad (2)$$

In Figure 7, $C(s)$ is the controller, $P(s)$ is the plant, $E(s)$ is control error and $D(s)$ is the disturbance in Laplace domain.

The transfer function between $E(s)$ & $D(s)$ is then;

$$E(s) = \frac{-P(s)}{1 + P(s) * C(s)} * D(s) \quad (3)$$

Assuming first order system with transfer function $P(s)$ and PID controller, the resulting transfer function would be as follows;

$$P(s) = \frac{1}{m * s + b} \quad (4)$$

$$E(s) = \frac{\frac{-1}{ms+b}}{1 + \frac{1}{ms+b} * (K_p + K_i/s + K_d * s)} * D(s) \quad (5)$$

Multiplying both denominator and numerator with $(ms + b) * s$ the equation yields;

$$E(s) = \frac{-s}{(ms+b)s + sK_p + K_i + K_d s^2} * \frac{d_0}{s} \quad (6)$$

so as one can see since the s ' are cancelled in the Eq.6, error will go to zero in time domain because of final transfer function we obtain.

$$E(s) = \frac{d_0}{(ms+b)s + sK_p + K_i + K_d s^2} \quad (7)$$

So, another approach to this conclusion is, as one can see the integral part of the controller is contains the "structure" of the disturbance. [7]

$$D(s) = \frac{d_0}{s} \text{ and integral part of controller is } C(s) = \frac{K_i}{s}$$

This is not only true for where disturbance is constant. Internal Model Controls basic principle states that "If the controller contains the structure of disturbance it can asymptotically reject the effect of disturbance" [8].

4.1.2 IMC structure

Figure 9 is the Internal Model Control structure where $Q(s)$ is controller $P(s)$ is plant and $\widetilde{P}(s)$ is the model of the plant. Given the response of the system as follows [7],

$$Y(s) = \frac{P(s)Q(s)}{1 + Q(s)(P(s) - \widetilde{P}(s))} * R(s) + \frac{1 - \widetilde{P}(s)Q(s)}{1 + Q(s)(P(s) - \widetilde{P}(s))} D(s) \quad (8)$$

$$Y(s) = \eta(s)R(s) + \epsilon(s)D(s) \quad (9)$$

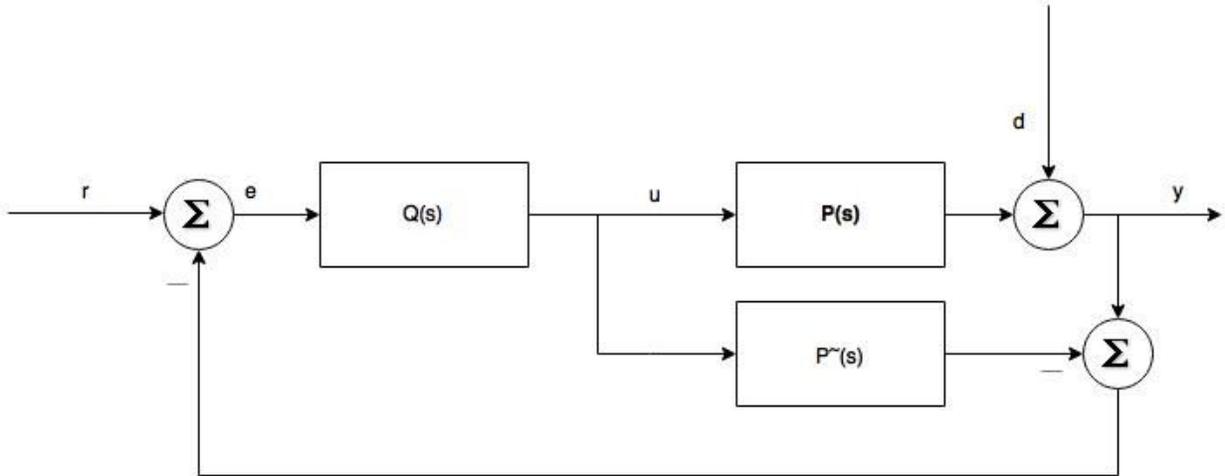


Figure 8 Internal model control structure

In the absence of plant/model mismatch, i.e. $P(s) = \widetilde{P}(s)$ sensitivity and complementary sensitivity functions as follows

$$\eta(s) = \widetilde{P}(s)Q(s) \quad (10)$$

$$\epsilon(s) = 1 - \eta(s) = 1 - \widetilde{P}(s)Q(s) \quad (11)$$

yielding the following relation,

$$Y(s) = \overline{P(s)}Q(s)R(s) + \left(1 - \overline{P(s)}Q(s)\right)D(s) \quad (12)$$

These, Eq.10, Eq.11, Eq.12 will be referred at following sections of this work.

4.1.3 Requirements for Physical Replicability on IMC

For $Q(s)$, the internal model controller, to produce meaningful manipulated variable responses, it must satisfy the following criteria [7];

- **Stability**

The controller $Q(s)$ must be stable [7], which means it must have all poles in Left Hand Complex Plane.

- **Properness**

$Q(s)$ must be proper [7]. For a transfer function to be proper, the degree of the numerator should be equal or lower than the degree of the denominator. Recall that we call the transfer function to be strictly proper if the degree of numerator is smaller than the degree of denominator, and we call the transfer function to be semi-proper if the degree of the denominator is equal to the degree of the numerator. Both are allowable for internal model controller $Q(s)$.

- **Causality**

Recall that we call a system causal if and only if it doesn't require a future value, in other words, it must rely on current and previous plant measurements and inputs [7]. For example, if a system has an inverse of a time delay i.e. the system below, then it is not a casual transfer function.

$$K_c e^{+\theta s}$$

4.1.4 Design Procedure

The internal model control design step is a two-step approach that provides a feasible tradeoff between performance and robustness [7]. One obvious benefit of internal model control is that its directly indication of sensitivity and complementary sensitivity functions ϵ, η which directly specifies the output characteristics of a closed loop [7]. The first step guarantees that q is stable and casual, second one guarantees that it is proper [7].

Step 1: Factorization

It is needed to factor p in to two parts, first part is the part containing the minimum phase elements (will be represented by p_-) and second part is the part containing non-minimum phase elements (will be represented by p_+) which are Right Hand Plane (RHP) zeros and time delays [7]

$$p = p_- * p_+ \tag{13}$$

Internal model controller at this stage is specified as follows:

$$q = p_-^{-1} \tag{14}$$

It is simply inverse of the minimum phase elements of the plant model. This step guarantees that $Q(s)$ is stable and casual.

One should note that factorization of p_+ is dependent upon the objective function chosen [7]. Recall that the objective function is the function that we want to minimize.

For example:

$$p_+ = e^{-\theta s} \prod (-\beta s + 1) \quad Re(\beta) > 0 \tag{15}$$

is IAE-optimal (Integral of Absolute Error) factorization [7]. In Eq.16 you can see ISE-optimal factorization. One should consider these factorization options after deciding the cost function [7].

$$p_+ = e^{-\theta s} \prod \frac{(-\beta s + 1)}{(\beta s + 1)} \quad \text{Re}(\beta) > 0 \quad (16)$$

Please note that: For ramp, exponential or other factorizations would require different factorizations [7].

Step 2: Adding a filter to Internal Model Controller Q(s)

In this step, in addition for Q(s) to be causal and stable, we will guarantee that it is also proper. This will be done by adding a filter which usually has a form of,

$$F(s) = \frac{1}{\lambda s + 1} \quad (17)$$

If it is needed to increase order of Q(s) more than one, one can simply augment a filter in a form shown in Eq.18 where n represents the order required to make Q(s) proper [7].

$$F(s) = \frac{1}{(\lambda s + 1)^n} \quad (18)$$

λ is a parameter which effects the systems speed of response [7]. Smaller λ 's will decrease the time constant and larger λ 's will increase the time constant.

4.1.5 Why Factorization?

For perfect control, i.e. controlled variable y , tracking the reference input, recall the response of the system and sensitivity and complementary sensitivity functions [7],

$$Y(s) = \eta(s)R(s) + \epsilon(s)D(s) \quad (19)$$

as can be seen here, we need $\eta = 1$ and $\epsilon = 0$. Also for physical replicability, we need Q to be proper, stable and casual [7]. So, to achieve this (for $\eta = 0$) it is necessary that $q = p^{-1}$. However, the nonminimum phase elements in p (RHP zeros & transport delays) cause $Q(s)$ to be unstable. Taking inverse of $P(s)$ RHP zeros of $P(s)$ would be RHP poles of $Q(s)$. Also, if $P(s)$ is strictly proper this will also cause $Q(s)$ to be improper degree of denominator would be less than that of numerator for $Q(s)$. This is the reason why we need factorization.

4.1.6 Simple Example Regarding to IMC design

In this section, a simple example regarding to IMC design is shown. Second order plant with transport delay is shown in Eq.20.

$$G(s) = \frac{9e^{-9s}}{s^2 + 18s + 9} \quad (20)$$

Comparison between responses with PID and IMC controllers are shown in Figure 9 of the plant is as follows.

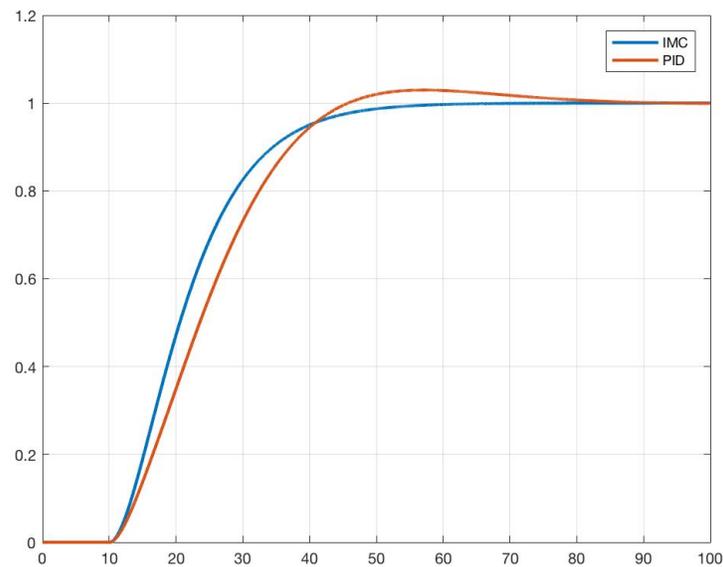


Figure 9 IMC vs PID

The internal model control structure and design procedure is applied (assuming no model mismatch) to plant and Figure 10 and Figure 11 are scheme and response of a system.

IMC of Second order system with time delay

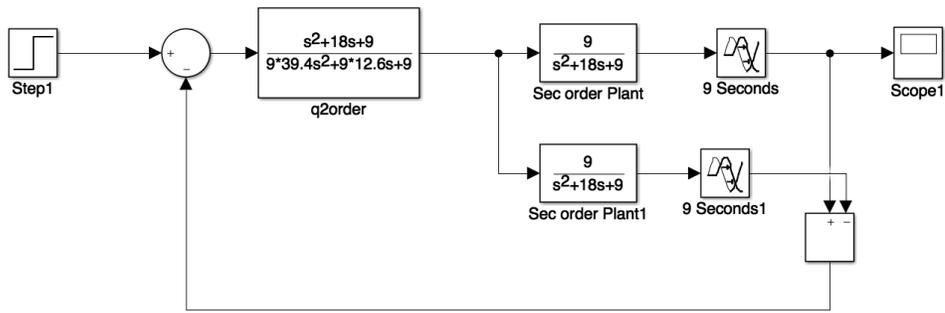


Figure 10 IMC scheme for second order system with time delay

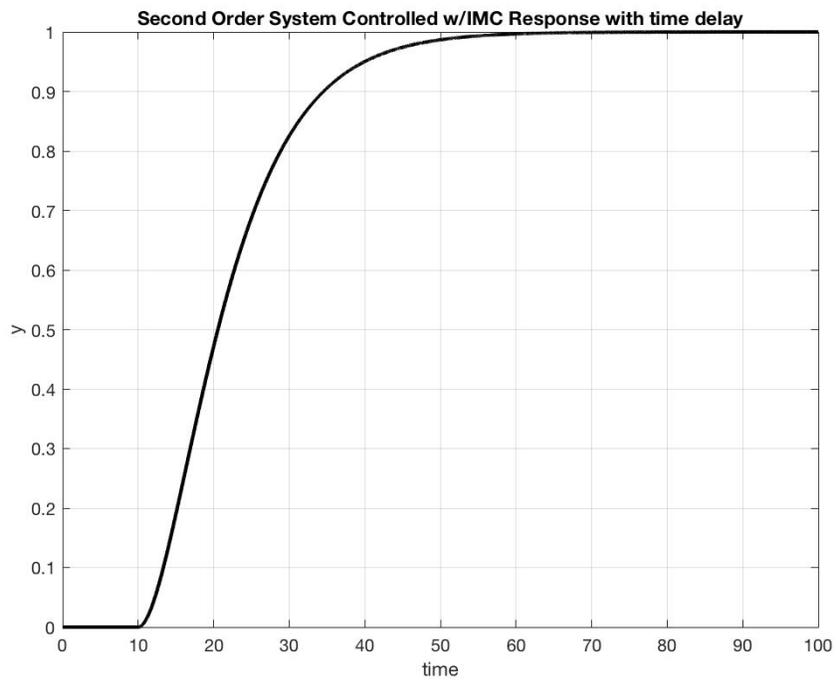


Figure 11 IMC loop response for second order system with time delay

4.2 Repetitive Control

4.2.1 Overview of Repetitive Control

Repetitive control is a very popular way to compensate periodic disturbances. As an extension to the internal model principle, repetitive control employs internal model principle, consists of a periodic signal generator, enabling asymptotic disturbance rejection of periodic disturbances. [9] One of the drawbacks of repetitive control is the requirement of knowing the exact form of periodic external signals [9]. As explained above, internal model principle states that, for asymptotically rejection of disturbance and tracking reference, the model of this disturbance should be included in the controller. An example, considering this disturbance as a constant is explained previously. Constant disturbances can be modeled as a constant times an integrator (Laplace of constant), so including integral action to controller (i.e. integral part of the controller) can prevent the steady state error for constant disturbances.

Similarly, this principle is valid for periodic disturbances as well. In next section principle and structure of repetitive controllers are presented.

4.2.2 Principle and Structure of Repetitive Controllers

Any periodic signal with period L can be generated by the block diagram shown Figure 12 with an appropriate initial function [10]. So as stated previously, according to internal model principle, asymptotic periodic disturbance rejection is expected. Any controller including this model is said to be a repetitive controller and a system which this controller is used is called repetitive control system. [10] Repetitive control is a popular technique for tracking periodic signals and rejecting periodic disturbances [11]. Repetitive control achieves accurate tracking

of periodic trajectories by incorporating a periodic signal generator within the feedback loop shown in Eq.21 [11].

$$\frac{1}{1 - e^{-Ls}} \quad (21)$$

In this equation L, the time delay amount is equal to the period.

The result is a controller with an infinite number of marginally-stable poles with infinite gain at the harmonics of the periodic reference [11].

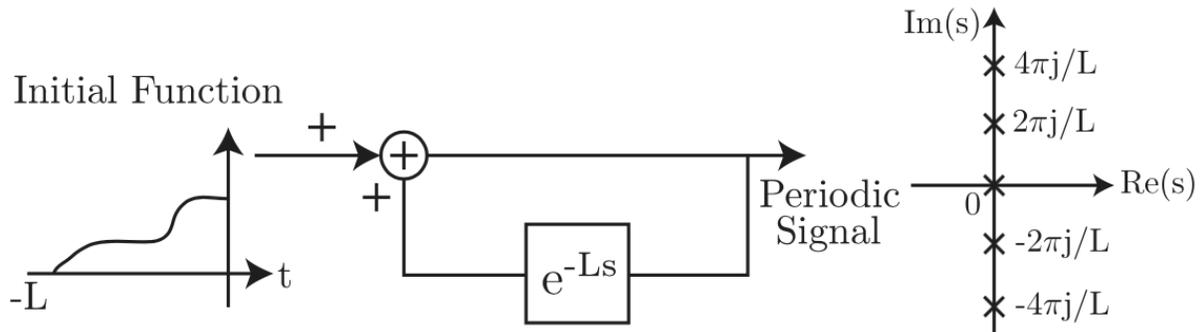


Figure 12 An infinite number of imaginary roots is required to replicate an arbitrary periodic input. [11]

However, the biggest challenge in Repetitive control is the stability. This problem can be avoided using appropriate filters in the positive feedback repetitive control loop.

In Figure 13, one can see the conventional modified repetitive control system scheme [10]. The dotted part represents Repetitive controller in Figure 13, where alpha is the tradeoff between stability and performance [4].

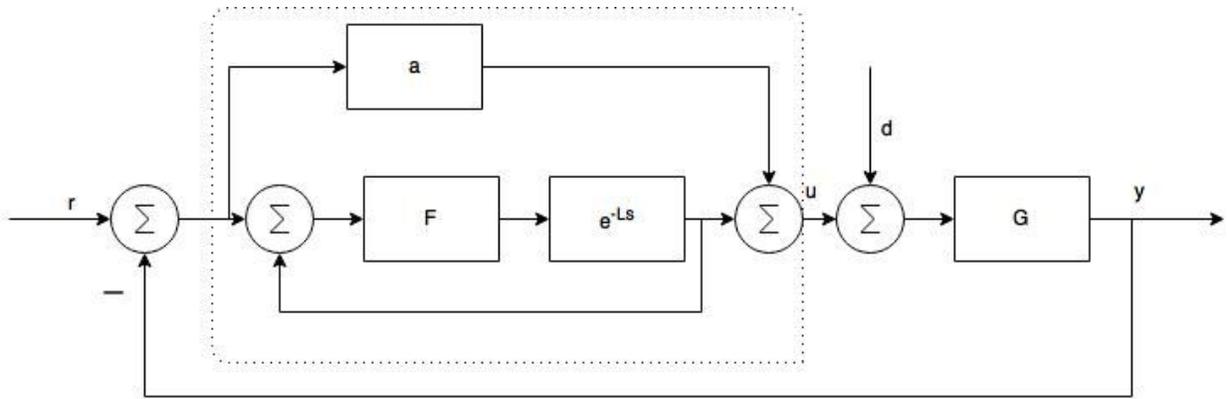


Figure 13 Conventional modified repetitive control system structure [4]

4.2.3 New Modified Repetitive controller for systems with time delay.

For repetitive control system to work effectively, we need to guarantee that the system has no time delay [4]. However, the model (rolling mills, will be discussed next section) has a transport time delay. That is the reason why we need to modify conventional modified repetitive controllers to compensate time delays. Omura et al. proposed a new modified repetitive controller to compensate these time delays. Figure 14 you can see the structure of this new modified repetitive controller [4].

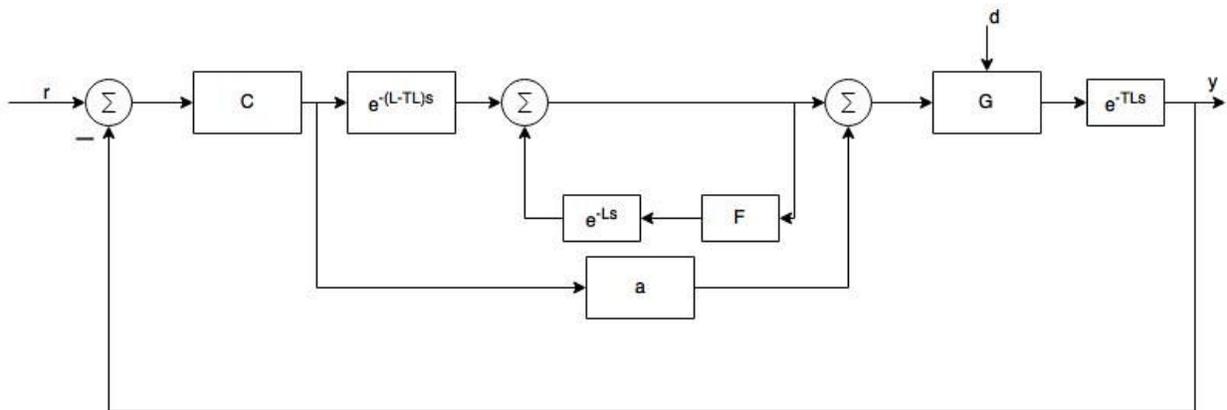


Figure 14 New modified repetitive controller for systems with time delay [4]

In this scheme, T_L is the nominal value of time delay and L is the period of the external periodic disturbance. And to do this Omura et. Al. has modified the controller to delay the input of the controller $L - T_L$ [4]. The idea here is to synchronize the periodic signals so that they cancel each other out. Next sections you will see that this idea is implemented in our design of controller and numerical simulation as well. Because Omura et. Al.'s motivation for proposing such a controller was to extend the capability of repetitive controller to the systems which are having time delays. Since rolling mills has non-negligible and non-escapable transport delays, we will use the same trick as well.

4.2.4 Comparison of traditional and modern modified repetitive controllers for a system with time delay

A simple first order plant is selected to compare traditional and modified repetitive controllers in this section. Time delay T_L is equal to 0.75 seconds, and period of external periodic disturbance L is equal to 1 seconds. A simple Simulink scheme is constructed to do this comparison. From Figure 15 Figure 16 Figure 17 and Figure 18 one can see the schemes and results.

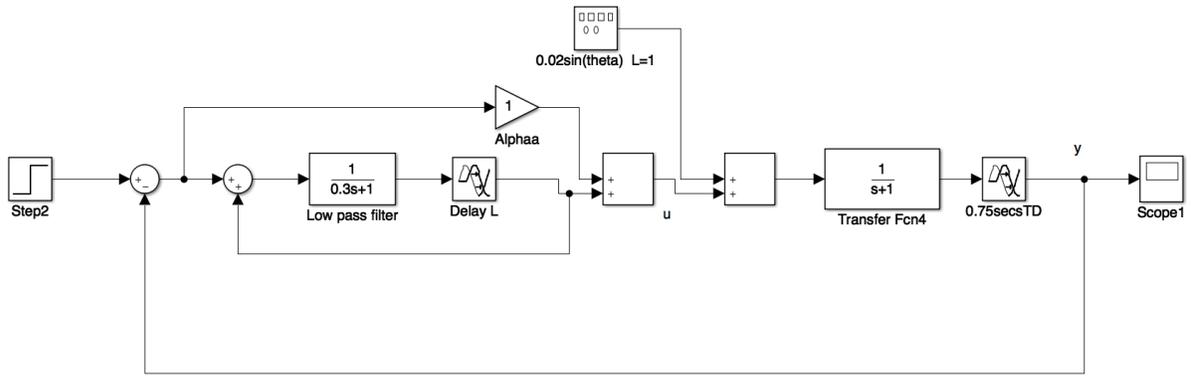


Figure 15 Conventional Modified Repetitive control scheme

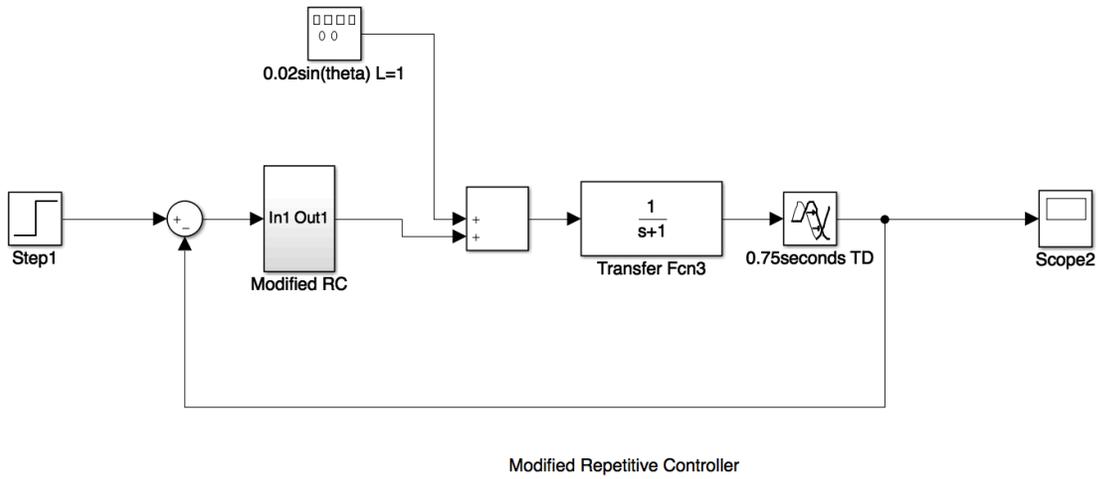


Figure 16 Simulink scheme of modern modified repetitive control system with time delay

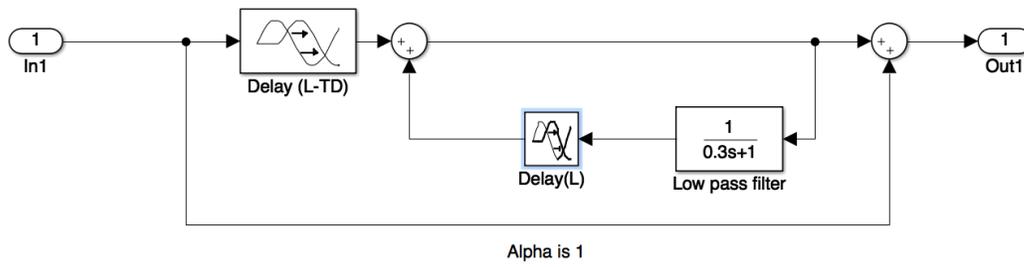


Figure 17 Simulink scheme of modern modified RC

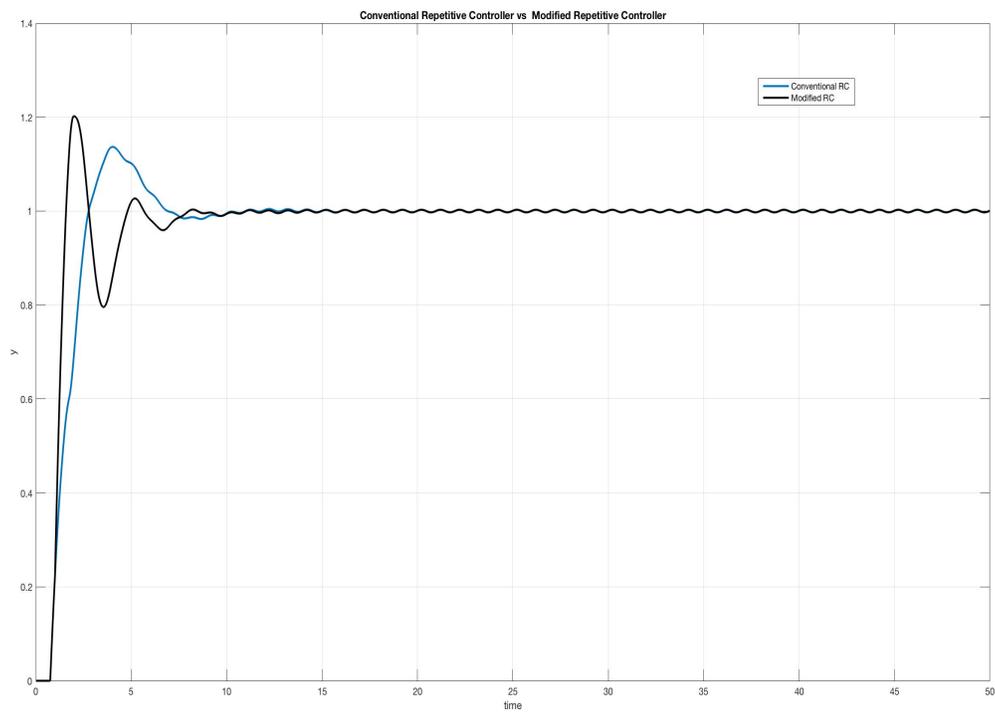


Figure 18 Comparison of responses of conventional vs new modified repetitive control systems

As one can see from above figure, the periodic disturbances are only attenuated.

5. Modelling Rolling Mill Machine

5.1 Modelling of Rolling Machine as Three Mass, Spring, Damper System

As a starting point the model of the rolling mill is created. Several approaches are tried in modelling procedure. Firstly, the state space representation of model is created assuming the behavior of the rolling mill is like mass spring damper system. In literature, Atilla et.al. has approached this problem in a similar way [1]. Figure 19 you can see the mass spring damper system used to model the rolling mill machine.

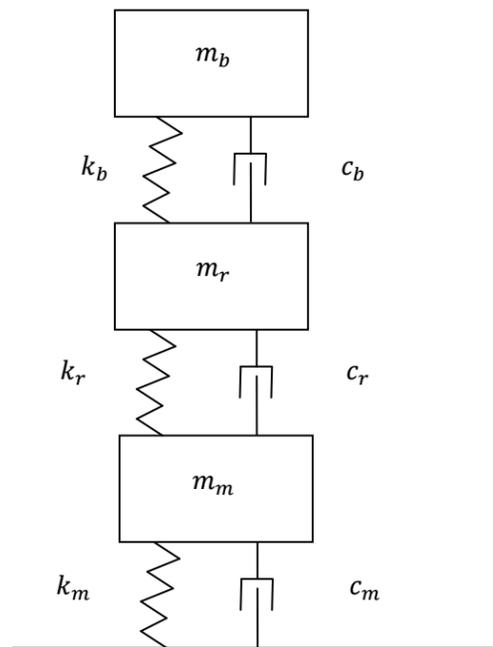


Figure 19 Mass spring damper model of the system.

The rolling machine is modelled as three mass-spring-damper systems as shown above where m_b is mass of back-up roll, m_r is the mass of working roll and m_m is the mass of the material. Similarly, stiffness's and dampers are named with same indexes. Equations of motions of the system is shown are given in Eq.22, Eq.23, Eq.24 .

$$m_m h'' + c_m h' + k_m h = c_r * (x_r - h') + k_r (x_r - h) \quad (22)$$

$$m_m x_r'' = c_r (h' - x_r) + k_r (h - x_r) + c_b (x_b' - x_r') + k_b (x_b - x_r) \quad (23)$$

$$m_b x_b'' = c_b (x_r' - x_b') + k_b (x_r - x_b) - F \quad (24)$$

and corresponding state equations are as follows,

$$x_1' = x_2 \quad (25)$$

$$x_2' = \frac{c_r}{m_m} (x_4 - x_2) + \frac{k_r}{m_m} (x_3 - x_1) - \frac{c_m}{m_m} x_2 - \frac{k_m}{m_m} x_1 \quad (26)$$

$$x_3' = x_4 \quad (27)$$

$$x_4' = \frac{c_r}{m_r} (x_2 - x_4) + \frac{k_r}{m} (x_1 - x_3) - \frac{c_b}{m_r} (x_6 - x_4) - \frac{k_b}{m_r} (x_5 - x_3) \quad (28)$$

$$x_5' = x_6 \quad (29)$$

$$x_6' = \frac{c_b}{m_b} (x_4 - x_6) + \frac{k_b}{m_b} (x_3 - x_5) - F \quad (30)$$

In Appendix A; one can see the m-file and Simulink model of the system. Parameters of this model is obtained from Mr. Jaromir Fišer. Figure 20 are the results of this model with PI control for set point 4.323mm. Please note that this is only the model of the rolling machine itself. So, in this model there is no periodic disturbance and no delay.

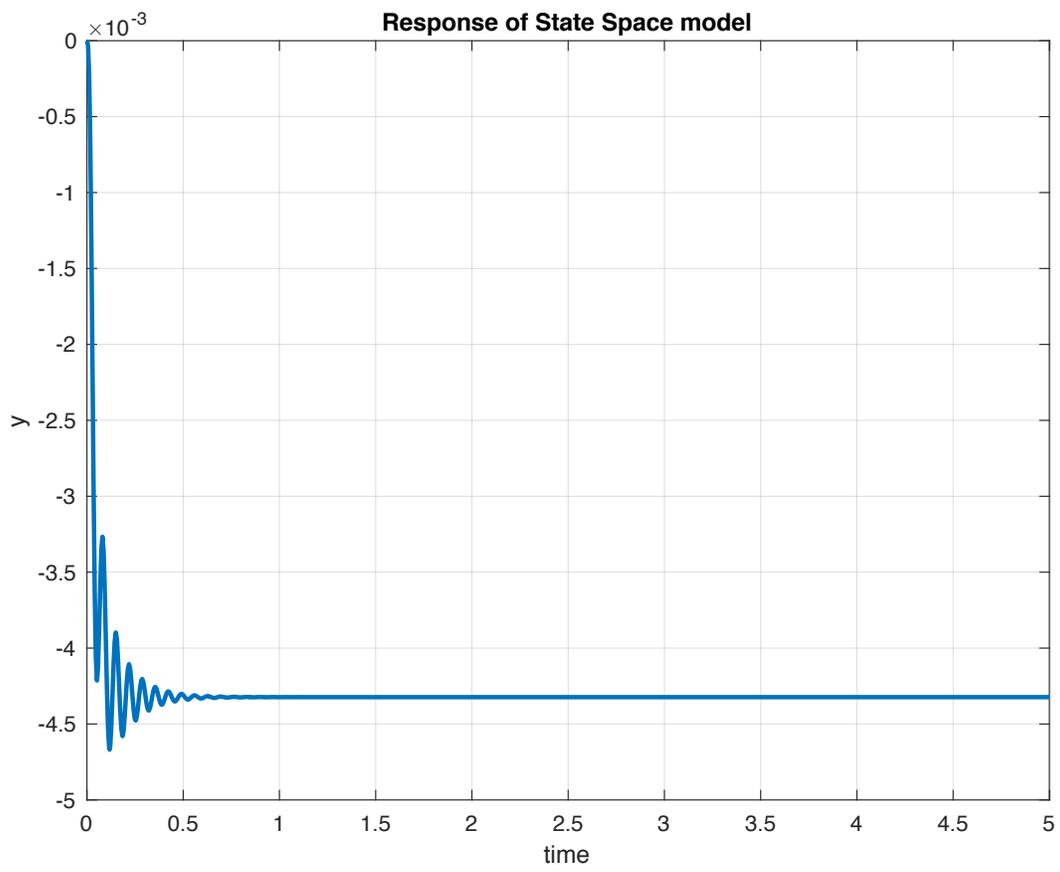


Figure 20 Response of milling machine model with PI control, modelled as 3 mass-damper-spring system.

5.2 Data-based model

As a second attempt, a data-based model is derived with use of real, industrial data from a rolling mill machine. Doing so, the delay, periodic disturbances and machine (including its AGC to the model) could be modelled more realistically. There is a slave controller already in use to adjust the gap according to the set point, but it is inefficient to obtain required steady thickness. This data-based model is from gap to thickness.

The goal of this approach is to obtain a transfer function from gap to thickness. Figure 21 you can see the data obtained from the industrial machine. The MATLAB Code for this can be seen in Appendix C. Figure 21 is the plot of this data.

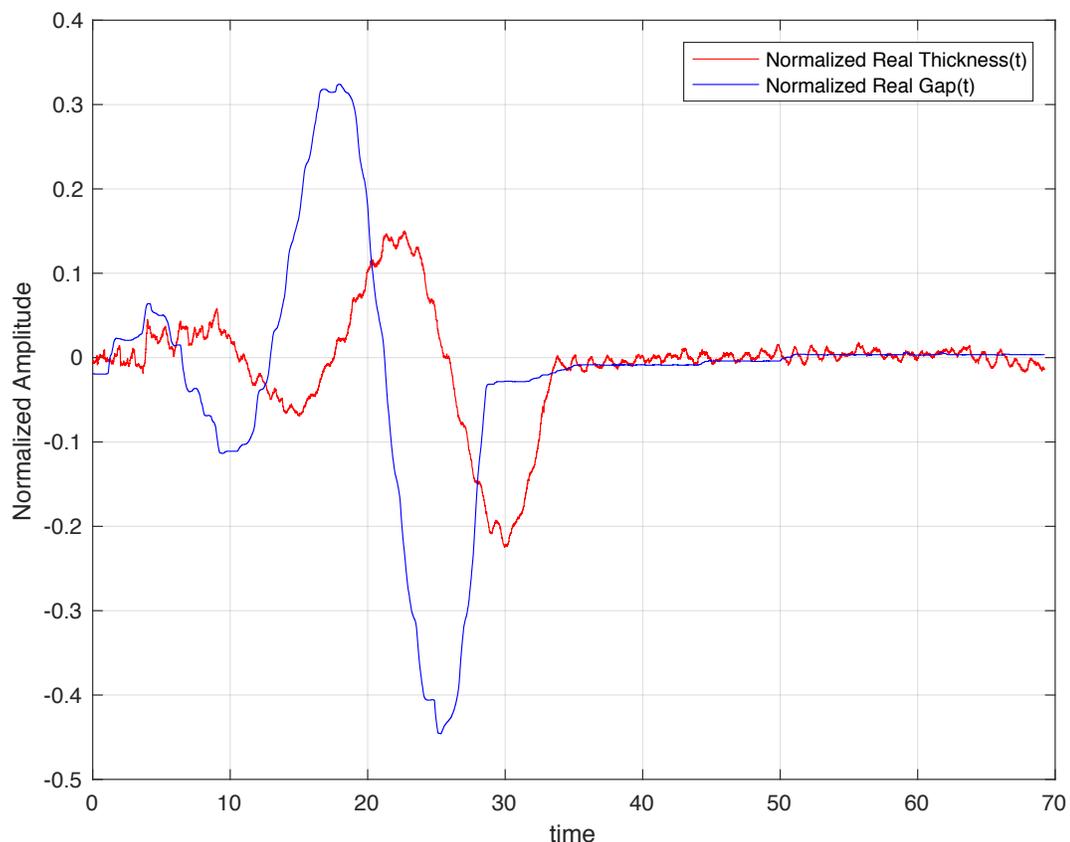


Figure 21 The data from industrial rolling mill machine

As one can see above clearly, there is a time delay and periodic disturbances in the thickness. The amount of time delay according to this data is identified as 4 seconds.

Also, the frequency of the disturbance is modelled according to this data. Figure 22 you can see the frequencies in output signal. 0.86 hertz is the frequency of the disturbance. Disturbance is modelled as a sine wave with the frequency of 5.4 rad/s.

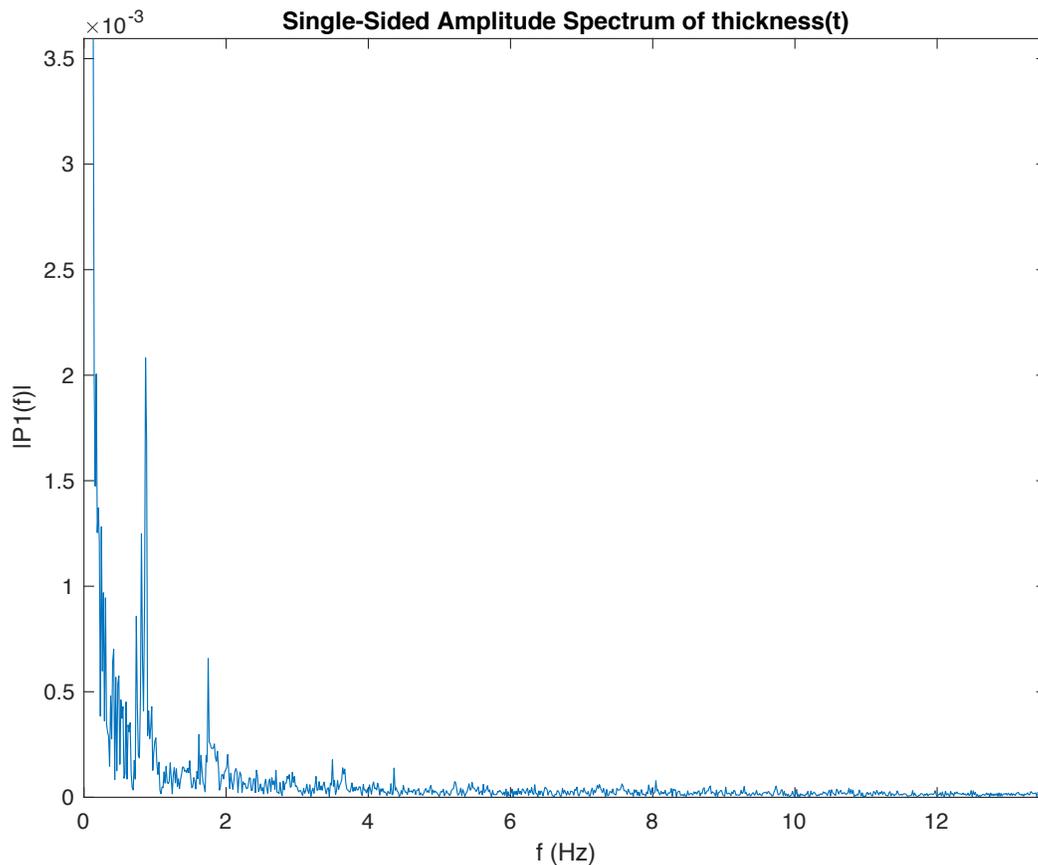


Figure 22 Frequencies exist in real data of thickness

In Appendix D, one can see the MATLAB code for this analysis. From real data which is represented in Fig.23, the disturbance signal is modelled as;

$$d(t) = 0.011 \sin(2 * pi * 0.85 * t) \tag{31}$$

After this a, first order system with delay of 4 seconds is derived with gain of 0.5 and time constant of 0.5 seconds. Model is plotted on the same plot in Figure 23, to verify the results with the input data of dy (normalized gap data). In Eq.32 you can see the model and its fitting to real data from Figure 23.

$$G_{data\ model}(s) = \frac{0.5}{0.5s + 1} e^{-\tau s} \quad (32)$$

where $\tau = 4$.

Also, Figure 23 one can see the modelled disturbance, with real data to see its fitting to real thickness deviations. This model is used for testing time delay compensation algorithms from now on.

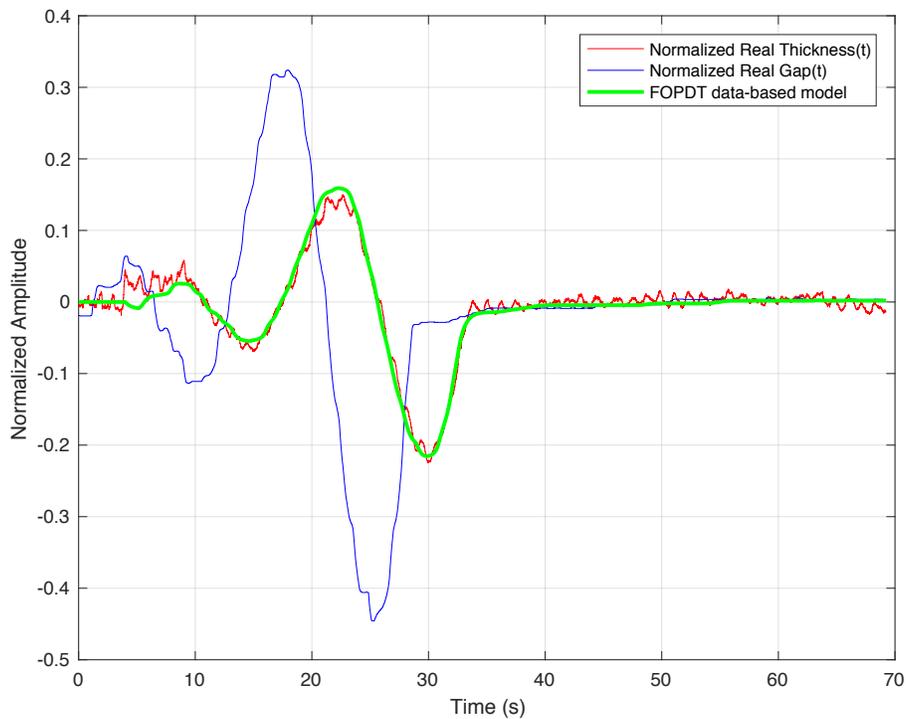


Figure 23 FOPDT data based model and its fitting to real data

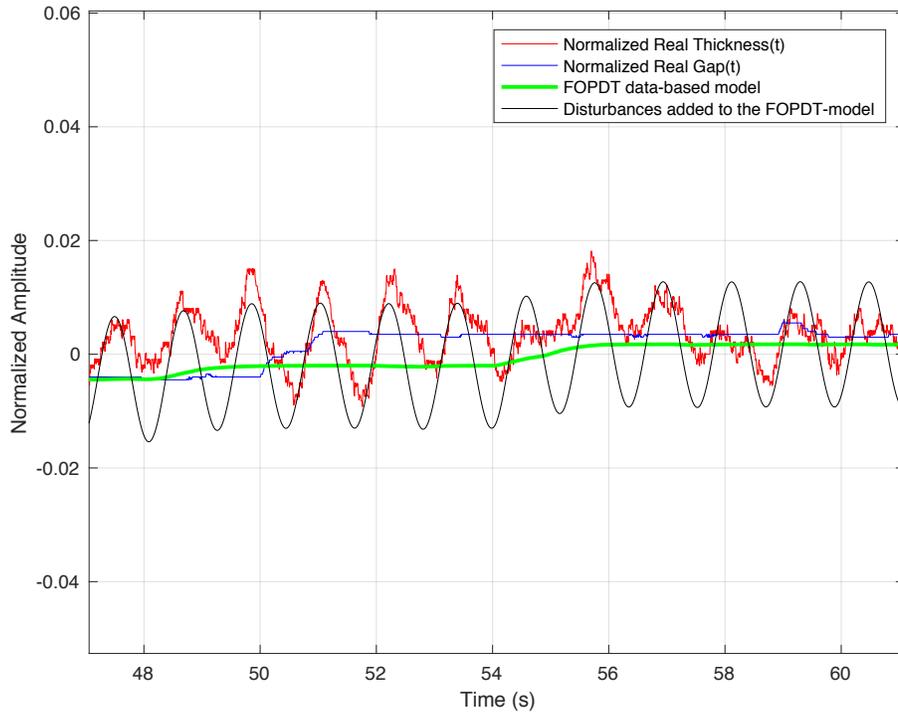


Figure 24 Fit of modelled disturbances to the real thickness deviations.

6. Internal Model Control of model without disturbance

As an initial task system without disturbance is controlled using Internal Model Control. Note that there is no disturbance yet. This part is only for compensating the time delay with IMC. The model in Eq.33 is used with very fast dynamics and very large time delay compared to system dynamics,

$$G(s) = \frac{0.5}{0.5s + 1} e^{-4s} \quad (33)$$

The design procedure explained above is implemented. Firstly, non-minimum phase and minimum phase elements are factorized. Giving us,

$$q_- = \frac{0.5}{0.5s + 1} \quad (34)$$

And the non-minimum part which is having non-minimum phase elements like time delay and RHP zeros is,

$$q_+ = e^{-4s} \quad (35)$$

Then, as design procedure suggests, inverse of minimum phase elements of plant has been taken,

$$Q(s) = q_+^{-1} = \frac{0.5s + 1}{0.5} \quad (36)$$

For $Q(s)$ to be proper a first order filter is augmented to the system.

$$F(s) = \frac{1}{\lambda s + 1} \quad (37)$$

After augmenting the filter, the final controller Q is shown in Eq.38,

$$Q(s) = \frac{0.5s + 1}{3.25s + 0.5} \quad (38)$$

The response and Simulink scheme of the system may be seen in Figure 25.

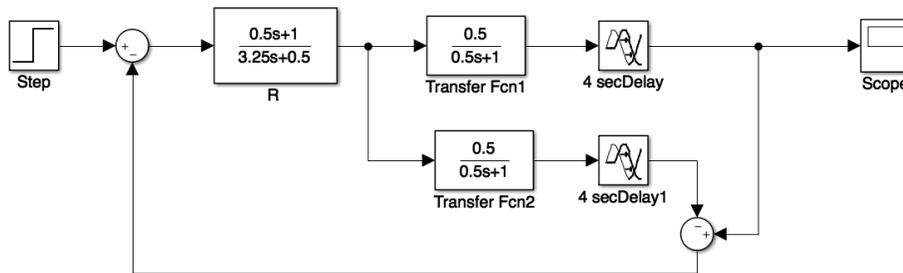


Figure 25 Simulink scheme for IMC control of time delay system

The response of the IMC controlled system and step response of plant can be seen Figure 26. Red line, as one can predict, system without IMC control and blue line is the response of the system with IMC control. As one can see from Figure 26 the steady state error is eliminated.

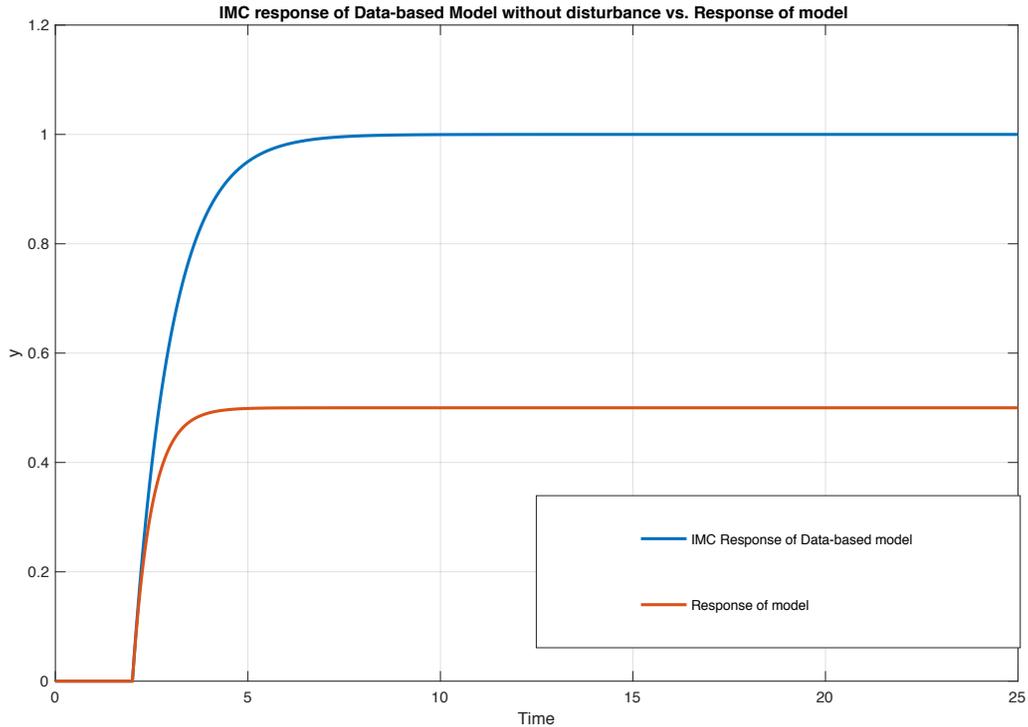


Figure 26 Data-based model response vs IMC loop response free of disturbance

7. Filter design for IMC and Repetitive control in MATLAB

Simulink

Consider the equivalent classical feedback controller $C(s)$, can be obtained with some basic algebraic equations [7]. In Eq.39 one can see the algebraic equation equal to IMC controller.

$$C(s) = \frac{Q(s)}{1 - \overline{P(s)}Q(s)} \quad (39)$$

Now substituting controller and plant model to Eq.39 where plant model is as derived previously,

$$\overline{P(s)} = \frac{0.5}{0.5s + 1} e^{-s\tau} \quad (40)$$

and the controller

$$Q(s) = \frac{0.5s + 1}{0.5 F(s)} \quad (41)$$

substituting Eq.40 and Eq.41 to Eq.39,

$$C(s) = \frac{\frac{0.5s + 1}{0.5 F(s)}}{1 - \frac{0.5e^{-s\tau}}{0.5s + 1} \frac{0.5s + 1}{0.5 F(s)}} \quad (42)$$

and multiplying with P(s),

$$C(s)P(s) = \frac{\frac{0.5s + 1}{0.5 F(s)}}{1 - \frac{e^{-s\tau}}{F(s)}} * \frac{0.5}{0.5s + 1} e^{-s\tau} \quad (43)$$

simplifying Eq.43,

$$C(s)P(s) = \frac{e^{-s\tau}}{F(s) - e^{-s\tau}} \quad (44)$$

and finally, disturbance to controlled variable transfer function is shown in Eq.45,

$$G_{dy}(s) = \frac{1}{1 + C(s)P(s)} = \frac{1}{1 + \frac{e^{-s\tau}}{F - e^{-s\tau}}} \quad (45)$$

Simplified version of Eq.45 is shown in Eq.46 & Eq.47.

$$G_{dy}(s) = \frac{1}{1 + \frac{e^{-s\tau}}{F - e^{-s\tau}}} = \frac{F - e^{-s\tau}}{F} \quad (46)$$

$$G_{dy}(s) = \frac{F - e^{-s\tau}}{F} = 1 - \frac{e^{-s\tau}}{F} \quad (47)$$

To make Eq.47 zero, $\left|\frac{1}{F}\right|$ should be equal to one and we should add an extra delay.

Selected a second order filter, design of the second order filter is as follows.

$$F(s) = \frac{\Omega^2}{s^2 + 2\Omega\zeta s + \Omega^2} \quad (48)$$

$$\left| \frac{\Omega^2}{s^2 + 2\Omega\zeta s + \Omega^2} \right| = 1 \quad (49)$$

replace $s = j\omega$ in Eq.49,

$$\frac{|\Omega^2|}{|-\omega^2 + 2\Omega\zeta j\omega + \Omega^2|} = 1 \quad (50)$$

Real and imaginary components square,

$$|\Omega^2| = |-\omega^2 + 2\Omega\zeta j\omega + \Omega^2| \quad (51)$$

$$\Omega^4 = (-\omega^2 + \Omega^2)^2 + (2\Omega\zeta\omega)^2 \quad (52)$$

Substituting for Ω ,

$$\Omega = \sqrt{\frac{\omega^4}{2\omega^2 - 4\zeta^2\omega^2}} \quad (53)$$

And simplifying further,

$$\Omega = \frac{\omega}{\sqrt{2 - 4\zeta^2}} \quad (54)$$

$$\sqrt{2 - 4\zeta^2} > 0 \quad (55)$$

$$\zeta < 0.707 \quad (56)$$

It is not enough to make only the magnitude of filter equal to one. It is also necessary to achieve argument equal to $-\pi$ to achieve asymptotic rejection of the disturbance. That is the reason why we need to introduce an extra time delay.

Let this extra delay be theta. Then the equation for disturbance to output transfer function is as follows.

$$G_{dy}(s) = 1 - \frac{e^{-s(\tau+\vartheta)}\Omega^2}{s^2 + 2\Omega\zeta s + \Omega^2} \quad (57)$$

replace $s = j\omega$ again,

$$G_{dy}(s) = 1 - \frac{e^{-jw(\tau+\vartheta)}\Omega^2}{-\omega^2 + 2\Omega\zeta\omega j + \Omega^2} \quad (58)$$

Together with the argument of filter, equation to be satisfied is as follows where φ is the argument of the filter.

$$e^{j\varphi} * e^{-(\tau+\vartheta)wj} \quad (59)$$

$$e^{(\varphi - (\tau+\vartheta)w)} = e^{-\pi} \quad (60)$$

Argument of $\frac{\Omega^2}{-\omega^2+2\Omega z\omega j+\Omega^2}$ and this extra delay theta is calculated in MATLAB using angle method, and the code can be seen below. In the code, zeta is selected as 0.6, Ω is then calculated accordingly. Note that, wd in below MATLAB code is the disturbance frequency which is modelled in previous modelling section of this work. Q is the internal model control controller which is composed of inverse of non-minimum phase elements of plant and designed filter. The filter is then introduced with a symbolic variable z and then substituted to $j\omega_d$ where ω_d is the frequency of disturbance. Then extra delay theta is calculated to be used in previously described modified repetitive and internal model control structures.

```
Td=1.18;
wd=2*pi*0.85;
zeta=0.6;
W=wd/(sqrt(2-4*zeta^2))
Q=tf((T*s+1)/K/(1/W^2*s^2+2*zeta/W*s+1));
syms z
GRz=1/(1/W^2*z^2+2*zeta/W*z+1);
Tw=eval(angle(subs(GRz,z,j*wd)))/wd
Kw=eval(abs(subs(GRz,z,j*wd)))
theta=2*k*pi/wd-tau+Tw
```

In next section, you will see this filter and extra delay, together with the IMC controller Q applied to obtain the IMC structure.

8. Control and Disturbance Attenuation with IMC applied to plant model in Matlab-Simulink

In this section, all previously described concepts come together. IMC structure is used to compensate delay and the design procedure of controller is followed and previously described filter design and extra delay is used. We will see the disturbance and plant models applied with the IMC.

Firstly, the controller Q is constructed as the design procedure suggests

$$Q(s) = \frac{0.5s + 1}{0.5} F(s) \quad (61)$$

where F(s) is,

$$F(s) = \frac{\Omega^2}{s^2 + 2\Omega\zeta s + \Omega^2} = \frac{1}{\frac{s^2}{\Omega^2} + \frac{2\zeta s}{\Omega} + 1} \quad (62)$$

$$\zeta = 0.6 \quad (63)$$

$$\Omega = 7.1155 \text{ s}^{-1} \quad (64)$$

Resulting final internal model control is as follows,

$$Q(s) = \frac{0.5s + 1}{0.009876s^2 + 0.08432s + 0.5} \quad (65)$$

Assuming no model mismatch the plant and plant model is as follows.

$$G(s) = \frac{0.5}{0.5s + 1} * e^{-s\tau} \quad (66)$$

where $\tau = 4$ from the modeling section.

Disturbance model is also as follows,

$$d(t) = 0.011 \sin(2 * pi * 0.85 * t) \quad (67)$$

In Figure 27 one can see the Simulink model consisting of above information in form of IMC structure.

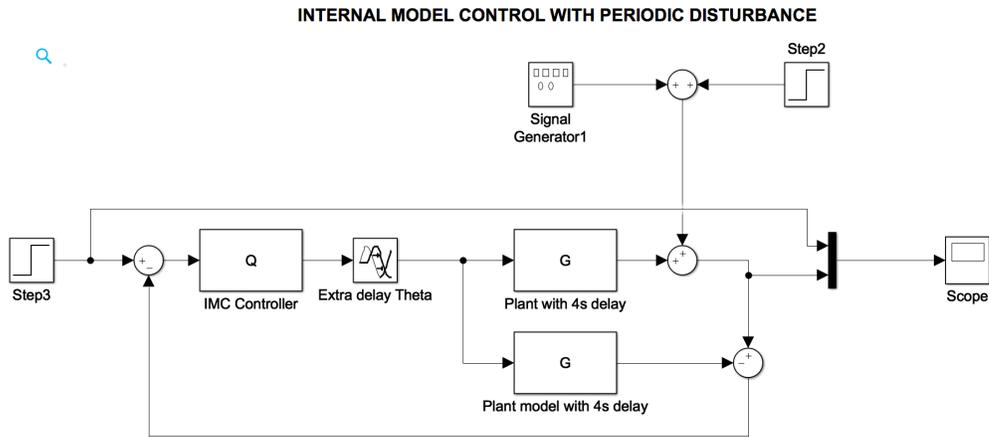


Figure 27 Internal model control with periodic disturbance

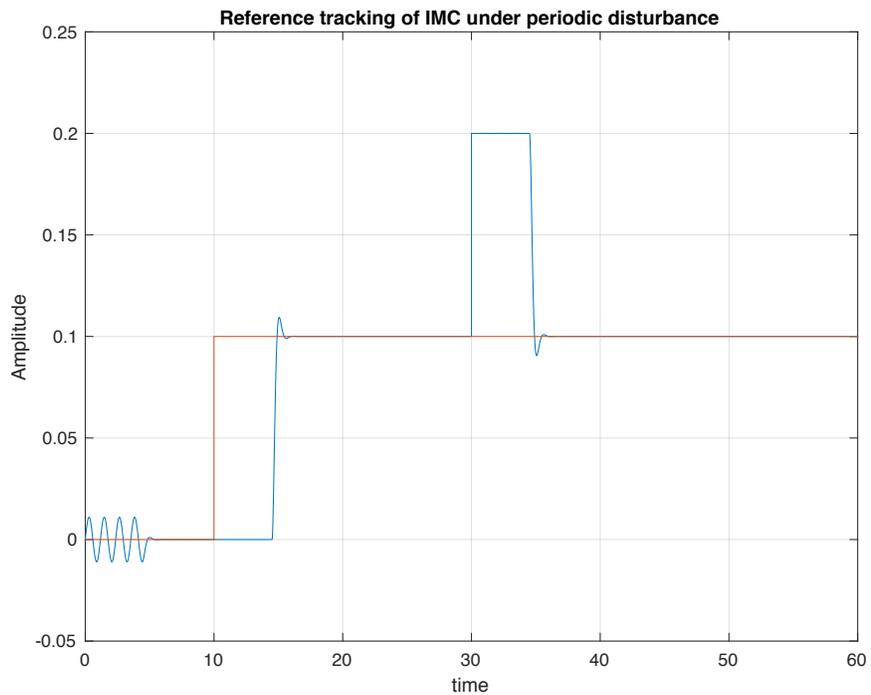


Figure 28 Response of the system with IMC control

9. Repetitive Control and disturbance attenuation applied to plant model in Matlab-Simulink

Internal Model Control scheme is modified to obtain the repetitive control scheme in Simulink. One can see the steps taken to achieve this.

First step is to move Q inside the feedforward loop. As the Plant model in Internal model control need to have the signal multiplied with Q , I multiply the plant model G with Q . Recall that controller Q is constructed with inverse of minimum-phase elements of plant, they cancel out and all we end up with is the filter with the time delay. In Figure 29 you can see this step.

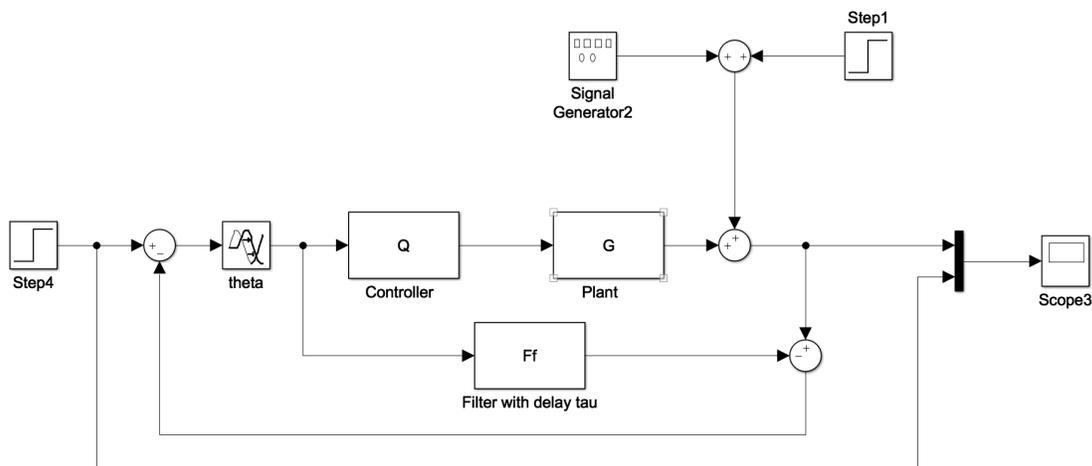


Figure 29 Q moved inside feedforward loop

Second and final step is to change negative feedforward loop to positive feedback loop. In Figure 30 one can see the obtained scheme after.

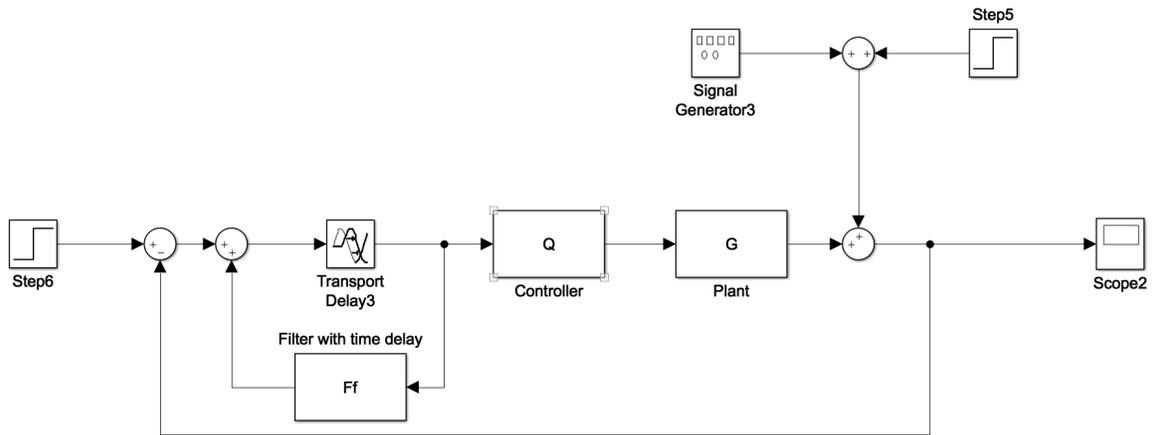


Figure 30 Repetitive control scheme obtained by modifying the IMC scheme

10. Results & Final Proposal

The response of the system depicted in Figure 30 is shown in Figure 31.

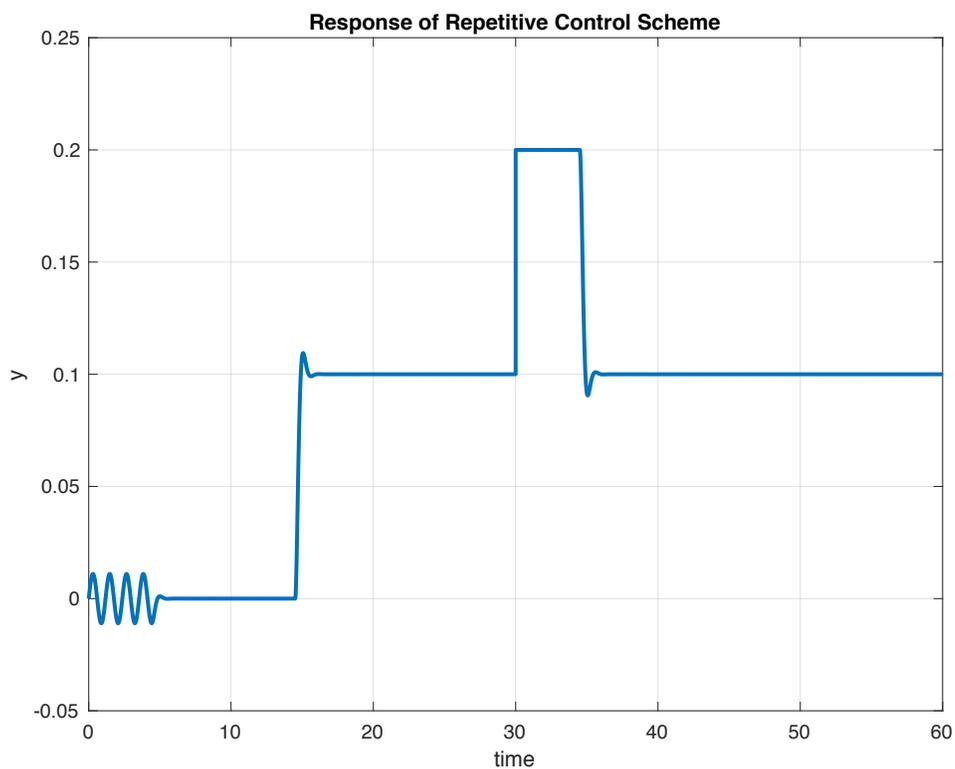


Figure 31 Response of repetitive control scheme

It is very clear from Figure 28 and Figure 31 which are the responses of the schemes with internal model control and repetitive control respectively, that

periodic disturbances which are acting on the Rolling Mill Machine data-based model are compensated ideally with large time delay compared to system dynamics. One should note that the responses of repetitive control scheme and internal model scheme are identical. The reason, obviously is that the schemes are identical. The problem with conventional repetitive controllers is stability, to guarantee stability filters are used, but then ideal response as in Figure 31 couldn't be obtained.

To make the response ideal, our contribution is to design filter and adding an extra delay as a part of industrial solution in [12]. Extra delay is added to adjust the phase so that the ideal response is to be obtained. Finally, our second contribution is to validate these results in simulations. The final controller to propose is the scheme depicted Figure 31 & Figure 28. Since they are identical schemes there wouldn't be any difference in response as this is validated in simulations.

11. Conclusion & Future Work

In steel strip manufacturing industry, thickness is an important quality measure. For this reason, it is needed to be controlled as precisely as possible. However, there exists a non-negligible and non-escapable transport time delay as well as periodic disturbances in these rolling machines. In this work, firstly a state of art analysis is made to see literature results. Then a modelling procedure was followed, first attempt was to model the system as three mass-spring damper systems, however for practical reasons it is decided to use a data based model using real industrial rolling mill machine data. Then internal model control and repetitive control concepts are applied and tested on this simulation model to obtain ideal response. A filter design and phase compensation approach is followed to obtain the ideal response. As one can see clearly from Figure 28 and Figure 31, two algorithms' results are identical and we have verified this with the simulation model. A future work should be testing this idea on an industrial rolling mill machine to verify the simulation.

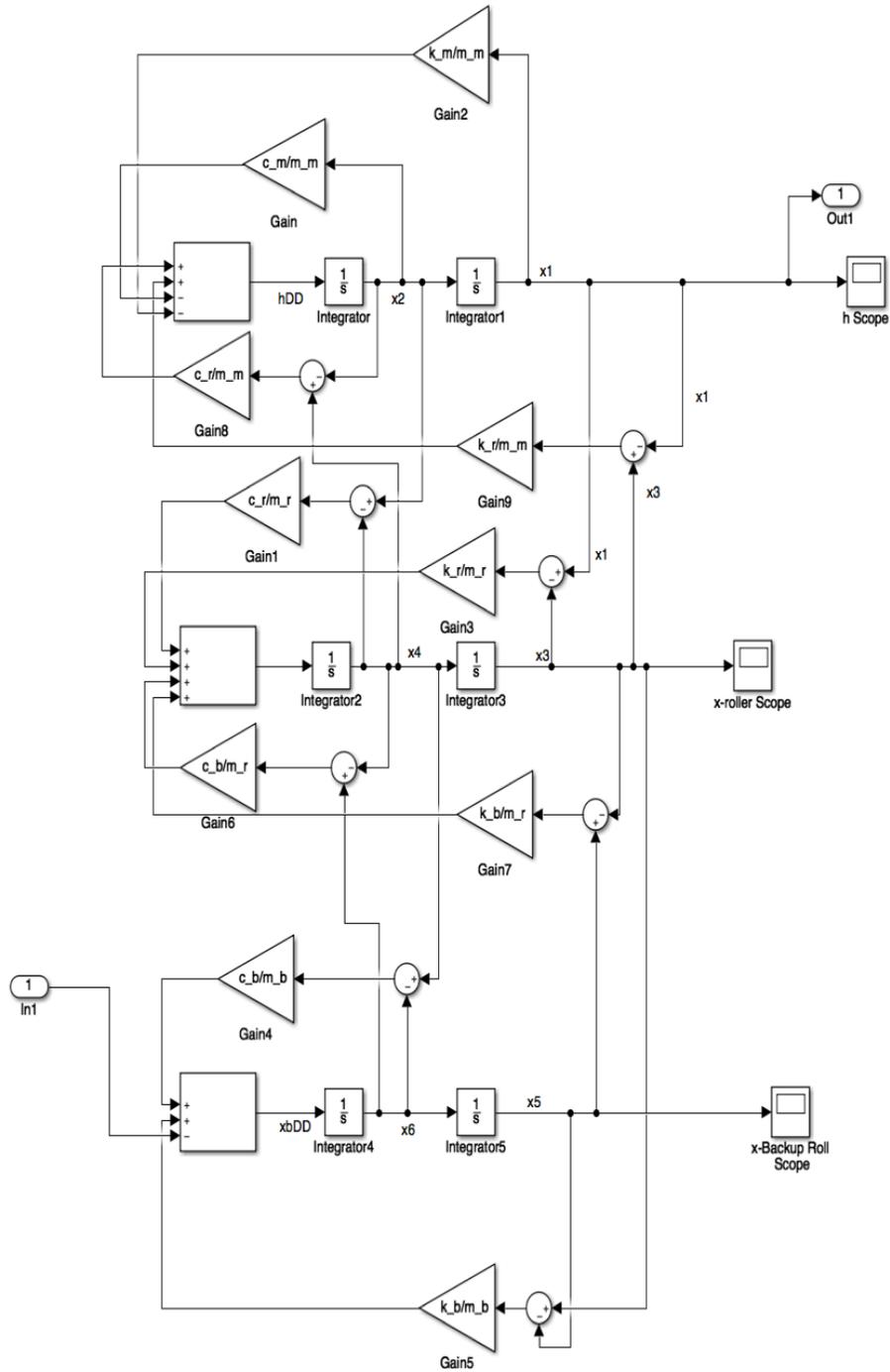
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13. Appendices

13.1 Appendix A

Simulink model of the rolling machine, modelled as three mass-spring-damper system.



13.2 Appendix B

m-file for parameters of state-space model.

```
% Mill stand data
L = 1270; % back-up and work roll length [mm]
w = 508; % Strip width [mm]
D1 = 254; % work roll diameter [mm]
D2 = 508; % back-up roll diameter [mm]
H = 25.4; % entry thickness [mm]
h = 21.077; % exit thickness [mm]
lc = sqrt((H-h)*D1/2); % Length of contact arc
E1 = 205; % work roll elastic modulus [GPa] in core
E1__ = 150; % work roll elastic modulus [GPa] in surface
E2 = 206.84; % back-up elastic modulus [GPa]
Es = 13.79; % strip elastic modulus [GPa = kN/mm^2]
ro = 8000; % steel density [kg/m^3]
F = 33949; % roll force [N]
% Stiffness computation k = E*A/L [kN/mm]
k__m = Es*w*(H-h)/L*10^6;
k__r = E1__*lc*w/L*10^6;
k__b = E2*(lc/2)*L/D2*10^6;
% Complete stand modulus < 10^4 [kN/mm]
k = (1/k__m + 1/k__r + 1/k__b)^(-1)
% Masses (stand weight including frame ~ 25 tons) kg
m__m = lc*w*(H+h)/2*ro*10^(-9);
m__r = pi*(D1/2)^2*L*ro*10^(-9);
m__b = pi*(D2/2)^2*L*ro*10^(-9);
% Damping factors c__m,c__r,c__b
c__m=k__m/200;
c__r=k__r/200;
c__b=k__b/200;
```

13.3 Appendix C

Plotting of experimental data for data-based modelling.

```
close all
clear
load Tomas_data_1

dy=y-4;
du=u-3.58;
figure
plot(t,dy,'r')
hold on
plot(t,du,'b')
grid
```

13.4 Appendix D

Frequency of disturbance analysis.

```
L=13848; % Length of the signal
Y = fft(dy);

Ts=t(2)-t(1); %% Sampling Period
Fs=1/Ts; %% Sampling frequency

f = Fs*(0:(L/2))/L;
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
figure(2)
plot(f,P1)
title('Single-Sided Amplitude Spectrum of thickness(t)')
xlabel('f (Hz)')
ylabel('|P1(f)|')
```