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Referee report on diploma thesis:

# Scheduling of energy-demanding operations with varying cost of electricity

In his diploma thesis, David Král deals with a problem of scheduling jobs on identical machines consuming electricity with time varying price. A schedule with a minimal overall cost of consumed electricity is sought, which is an NP-hard problem in the strong sense. While the scheduling is subject to release times, processing times and deadlines of jobs, it is not subject to any precedence constraints.

In the thesis, author proposes four exact methods for the problem. The first two methods are two different plain mixed-integer linear programming (MILP) formulations. Because not even with state-of-the-art MILP solvers, it is not possible to solve many real-world problem instances using plain MILP formulations, two other exact methods are proposed.

The third method is so-called Two-phase method based which divides the solution process in two phases – the first decides start times of jobs with the time granularity of one minute while the second one assigns the jobs to machines.

Finally, the last method is loosely based on Benders decomposition, resulting in an algorithm solving the problem in an iterative fashion – the master problem selects regions in which a majority of each job would be processed while the sub-problems fine-tune the start times of the jobs. If a sub-problem cannot find a feasible solution, a constraint is added into the master problem and the master problem is recomputed.

All the exact algorithms were tested on a set of artificially generated test cases up to the size of 60 jobs and 12 machines. The Two-phase method was reported to be the most efficient exact algorithm as it could solve all the problem instances within the time limit of 60s (the other three exact methods failed in many cases).

To solve larger problem instances (up to 200 jobs and 15 machines) two heuristics based on reducing the solution space were proposed. The first heuristics reduces the granularity of the Two-phase method from one minute to 15 minutes and thus extensively reduces the solution space. The second heuristics is iterative and reduces the solution space by tightening time-windows between job start times and deadlines.

The most efficient of the proposed methods is the Two-phase method with reduced granularity – it can solve the largest problem instances considered with 200 jobs and 15 machines within a second or two, which seems to be a very good result.

Nevertheless, I have several questions:

1. It seems to me, that the Two-phase approach is just a more efficient formulation of the time-indexed MILP formulation (the second proposed approach). I believe that if the machines are identical, it is sufficient to model the scheduling problem using only the resource constraint as used within the Two-phase method. I.e. it is not necessary to model each identical machine individually as this would only bring unnecessary symmetry into the MILP model, which may (and probably would) result in longer computation times. It should be also noted, that the second phase is straightforward in the case of identical machines. Could the author, please, comment on this?
2. In this thesis, no precedence constraints between the jobs are modelled. However, it seems to me that in most real-world applications some precedence constraints always take place. Could the author, please, comment, why it was not necessary to include precedence constraints into the model in this case?
3. Besides, would the proposed methods be usable if the precedence (or even general temporal) constraints were modelled?

From the formal point of view, the thesis is excellent. The language is clear and I especially appreciate the very thorough state-of-the-art analysis.

I believe that David Král deserves

**„A - excellent“**

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