

## Robust Focal Length Computation

by Oleh Rybkin

The thesis of Oleh Rybkin deals with the problem of recovering focal lengths of two perspective cameras from a set of pairwise correspondences. This allows for the decomposition of the fundamental matrix into (partly known) calibration matrices and the essential matrix. Such computation is an important component of structure-from-motion pipelines, where the same goal can be achieved by theoretically equivalent or similar but in practice differently performing methods. Thence, this belongs to a group of geometric and computational problems that is still studied in computer vision.

The main part of the thesis contains six sections. These are written in excellent English, their content it is a well-structured, lucid, and easy to follow text that I read with joy. The introductory Section 2 presents standard material about epipolar geometry and its parameter estimation algorithms in a self-contained, exhaustive, and precise way. The author has shown a good knowledge of the state of the art methods and their implementations. Sections 4 and 5 that describe new contributions are then more compact, in the usual style of a scientific/engineering paper that is usually not fully self-contained. This I see as a positive.

There are two experimental sections, one dealing with performance evaluation of the seven-point algorithm and the Bougnoux formula for estimating focal lengths of the camera pair. The experiments are synthetic, but convincing, with a careful design and a good documentation of the results. The other experimental section then discusses modifications of two algorithms: (1) computing fundamental matrix from six correspondences given focal length ratio (assuming known principal point and square pixels), using a solver for the six-point problem obtained from an automatic generator, and (2) Hartley's local optimization for estimating fundamental matrix given initial focal length estimates. Both modifications were successful in achieving good (better) performance. I found the results and the discussion quite interesting.

The main contribution of the thesis is contained in Section 4 where the author describes three algebraically independent, automatically derived solvers for computing focal lengths, one of which coincides with the Bougnoux formula and the remaining two are novel. Degenerate configurations are discussed and it is shown that the new formulae have a common set of degenerate configurations which is smaller than the set of degeneracies for any of the single formula. In addition a formula for computing focal length ratio is derived. The author found that there is only one algebraically independent formula for this ratio. The author also notes an interesting observation: Independent algebraic constraints on the set of fundamental matrices for partially calibrated cameras with unknown focal lengths are the same as for the fully uncalibrated cameras. As discussed in the thesis, this means that the trace constraints for the partially calibrated case are superfluous and that every seven-tuple of correspondences can be explained by totally uncalibrated cameras as well as partly calibrated cameras with unknown focal lengths. Undoubtedly, all these are very interesting results.

My goal here is not to give an exhaustive review of the thesis. The interested reader is encouraged to read it. The thesis is easy to follow, each major section starts with a description of its goals and concludes with a brief and accurate summary.

I have two questions:

1. On p. 11 it is written that the eight-point algorithm satisfies epipolar constraint better, unlike Algorithm 1 (which is seven-point). Could the statement be made more precise? In what sense does it satisfy the constraint better? Was the statement confirmed by the experiments? (I think not...)
2. As far as I understood, Algorithm 6 (p. 37) uses the ratio of focal lengths  $r$  from the Bougnoux formula. Why was the ratio from (4.4) not used?

I have several minor comments regarding scientific/engineering writing style:

1. Abstract should contain no numbered references if their list is not a part of the abstract.
2. If all numbers in a sentence are zero to nine, then all of them are written in words.
3. References like Figure 2.1, Algorithm 1, Chapter 9, Example 4.2.1, etc. are written without an article, hence we do not write “the Figure 2.1,” etc.
4. When we have to include a footnote reference at the end of a sentence, we do it after the period. A nice example of what happens otherwise is shown on p. 10, where we can see “3<sup>3</sup>.” (which is ambiguous) instead of “3.<sup>3</sup>” (which is not).
5. It is customary to make references to numbered equations or formulas parenthesized. For instance “(2.6)” means “Equation 2.6.”
6. Problems with font size in exported Matlab figures can be fixed by setting an appropriate paper size in the Matlab figure.
7. Bibliographic reference [4] is incomplete, it should contain the name of the publishing institution.

Some additional comments:

1. The  $\mathbf{p}_1$  and  $\mathbf{p}_2$  in (2.6) and (2.7) are left unexplained.
2. Some results or statements that are not generally known should be accompanied by a bibliographic reference, eg. the Sampson error (3.2) on p. 17.
3. A ratio of positive random values cannot have a Gaussian distribution, as suggested on p. 18. Perhaps a log-ratio could.

In summary, considering that this is a bachelor’s thesis, it must be rated as outstanding. The student is clearly able to carry out good research work in the area of computer vision, including novel modifications of existing methods. There must have been a non-trivial effort on the side of the student to understand and use algebraic geometry techniques properly. The notes and remarks in this review are meant as a feedback and should not be a basis for downgrading the thesis. I recommend it for presentation and suggest the **A-grade (excellent)**. I suggest the defense committee considers this thesis a candidate for nomination for the dean’s prize.

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