Contents

Introduction:

Our aim for this project consists in few points, to start, we need to build a mathematical model applicable on the real process and verify the parameters according to the data results, the next step is to test and measure and collect the maximum data possible in order to identify a virtual model relatively to those measurements, then finally build a solid robust controller using different methods, and apply it on the real process and proceed to any modification possible in order to make It work as smooth fast as possible

Chapter I

Description of the task:

our task is to control the horizontal movement of a spherical ball through a cylindrical tube, the ball moves longitudinally by means of a powerful jet of water, pushing the ball up or leaving a height desired falling relatively to the power of the jet exerts on the latter, an ultrasonic high frequency sensor is setup on the top of the cylinder to record the longitudinal movement of the ball, sending the data collected to the computer.

Using Matlab and Simulink we are able to set the desired voltage sent to the pump, and transmit the position of the moving body at any moment to the workspace.

Mechanical design:

Figure 1: Pump inline 24V/10l/min

Tableau 1: Inline pump ELEGANT – Inline 24V

Spannung / voltage:	24 Volt = / 24 volt DC
Stromstärke / current:	max. 1 A
Fördermenge / throughput:	max. 10 l/min
Förderhöhe / delivery height:	max. 5 m
Förderdruck / pressure:	max. 0,5 bar / 7,2 psi
Verbrauch / power:	15 – 25 Watt
Ø / Höhe / diam. / height:	38 / 120 mm
Kabellänge / wire length:	1 m
Anschluss / connection:	(+) braun / brown (-) blau / blue

Characteristic of the pump:

- Do not start to suck.
- Run Dry up to 2hours without any damage.
- Suitable for continuous operations.
- Running time 1000hours when operating in 12V, otherwise 500h.
- For hoses with 10mm inner diameter.

Here is collected from the pump characteristic data sheet, the most important features. In the figure bellow the dependencies respectively, between the pressure/throughput, and the throughput/current.

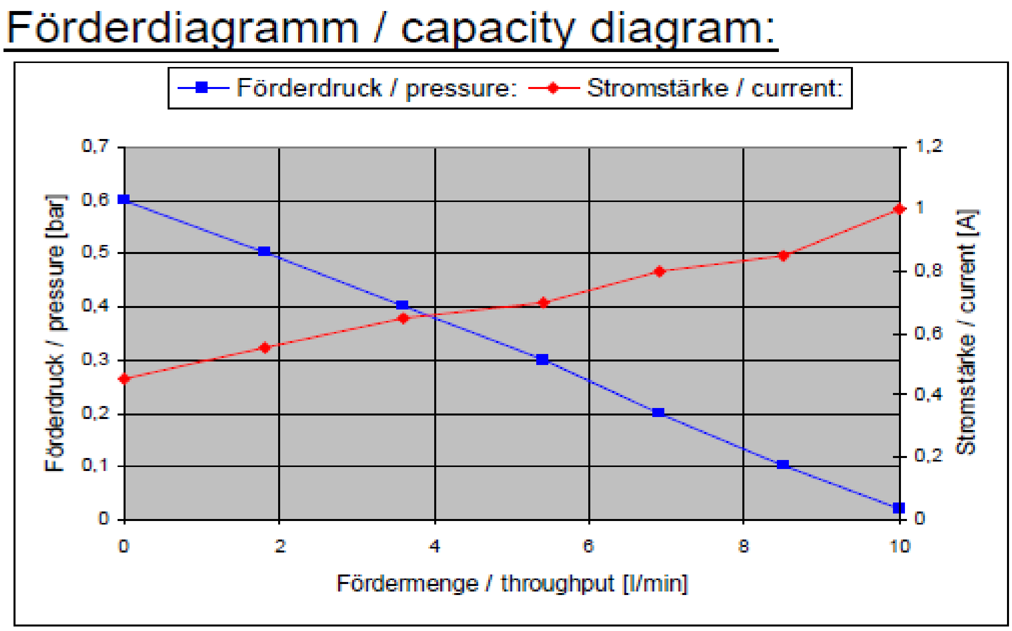

Figure 2:capacity diagram

Figure 3: Ultrasonic Sensor: UC 2000-30GM-IUR2-V15

General specifications

Sensing range	80 ... 2000 mm
Adjustment range	120 ... 2000 mm
Dead band	0 ... 80 mm
Standard target plate	100 mm x 100 mm
Transducer frequency approx.	180 kHz
Response delay	65 ms minimum, 95 ms (factory setting)

Electrical specifications

Operating voltage	UB 10 ... 30 V DC, ripple 10 %SS
Power consumption	P0 ≤ 900 mW

Factory settings

Output evaluation limit	A1: 200 mm
Evaluation limit	A2: 2000 mm/rising ramp

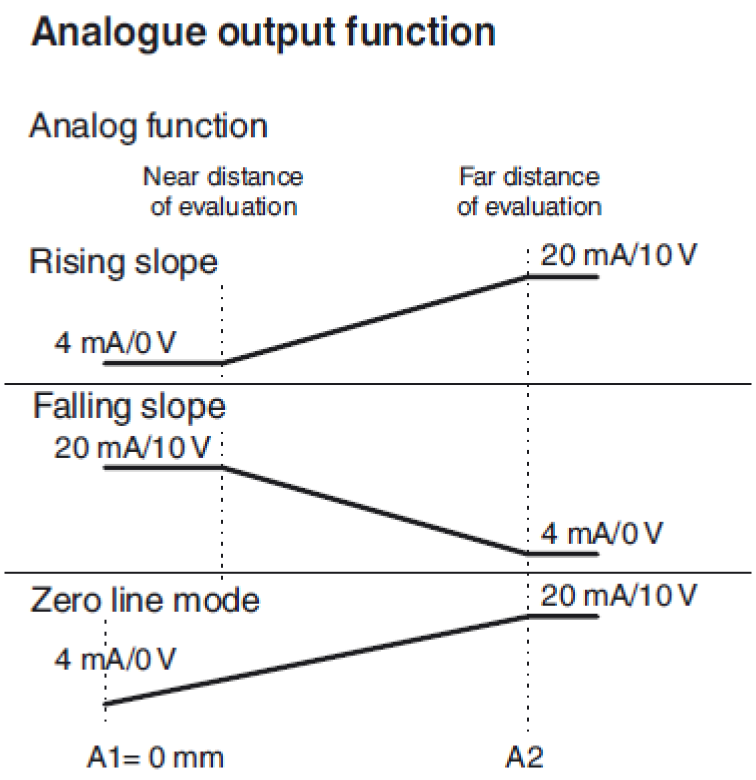

Figure 4: Slope of the Analogue Output according to the current input/Voltage

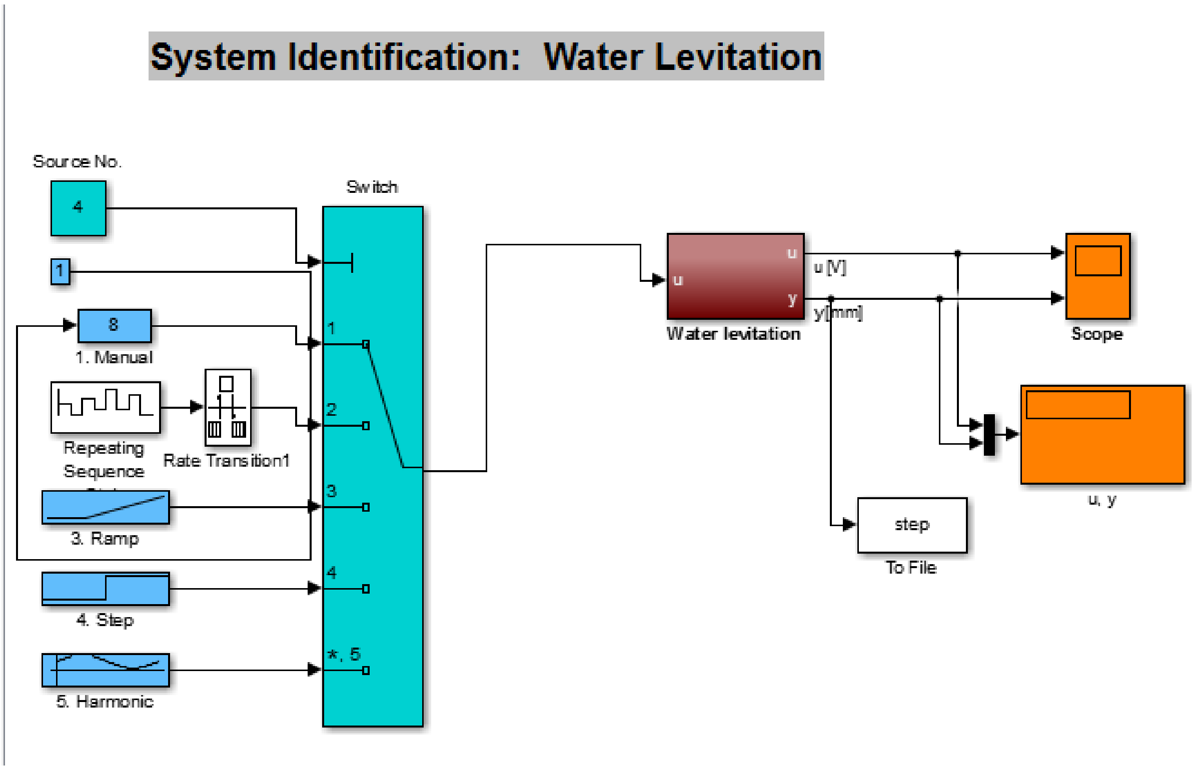

Figure 5: Levitation real Simulink process

Here is our Simulink setup for the real process, on the left side we are going to mainly use the step input, which controls the desired voltage input to the pump, the scope is used to plot the results on real time, the step block send the saves the data directly to the workspace, and the screen is used to show at every single moment the ball height while the process is in use.

The red Water Levitation Block is the real process, and consists of the adapter card in charge of the sensor, coupled with some filters, and a block to linearize and control the desired input voltage as shown on the Figure bellow:

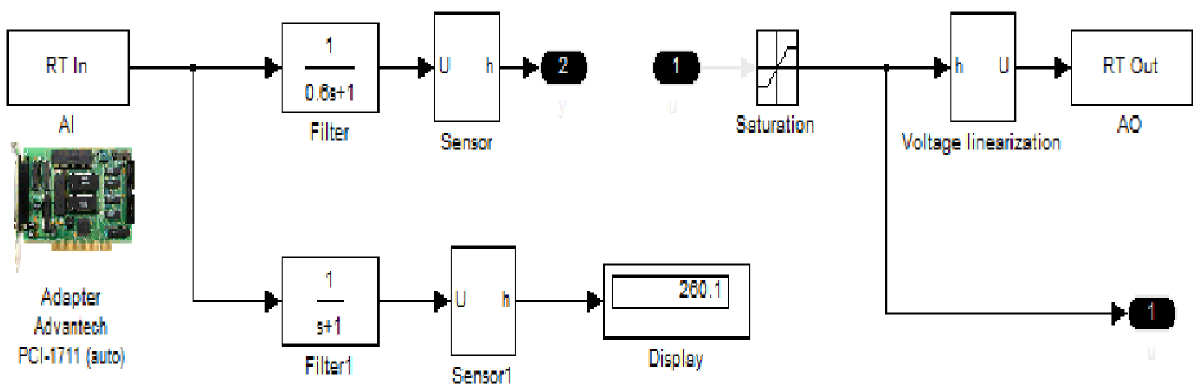

Figure 6: Water levitation laboratory setup of the real process

Chapter II

Formulation of a mathematical model of the process:

In order to build a solid mathematical model for our analysis, it is necessary to define and determine the variables involved in the physical experiment.

The main characteristic of flow around sphere is the existence of turbulent wake with recirculation, which has a dominant effect on the drag and lift of the sphere. The extent of this region is dependent on the shape, orientation, and size of the body, the velocity and viscosity of the fluid, and may be influenced by a wide variety of small flow disturbances, which may be generated in various ways.**[VISUALIZATION OF FLOW AROUND SPHERE FOR REYNOLDS NUMBERS BETWEEN 22 000 AND 400 000 V. BAKIĆ[1] and M. PERIĆ[2]]**

The model is related to the water flow rate around a spherical object, many research have been achieved in this domain, wither it is water air or some other form of fluid, it has determined somehow the almost equivalence between a fluid flowing around a stationary sphere, or a sphere moving through fluid, and the dependency of the force acting on the object in question to the Reynolds number.

Let's name the exerted force on the spherical object by the fluid FD drag force. FD for a given fluid in the current case water, and a given sphere, constrained us to develop some a set of certain other variables involved in the motion process, in many reviews and articles FD is not in the current case the dependent variable in the problem, for a spherical object settled under its own weight, it has been preferably established that the velocity of the water flow as the dependent variable.

The flow acts differently under certain circumstances, large studies have been conducted in order to describe this phenomenon.

It is important to take into account the conditions around which our flow is subjected, thus leading to a model adequate to our spot.

The phenomenon has been studied by Osbourne Reynolds that in 1880 in an experience which then became a classic in the mechanics fluid helped demonstrate that the flow is in its dependence (mean velocity, viscosity, diameter, and density of the fluid).The main parameter to describe Laminar flow, Turbulent flow and the transition from laminar to turbulent flow, is a Reynolds number.

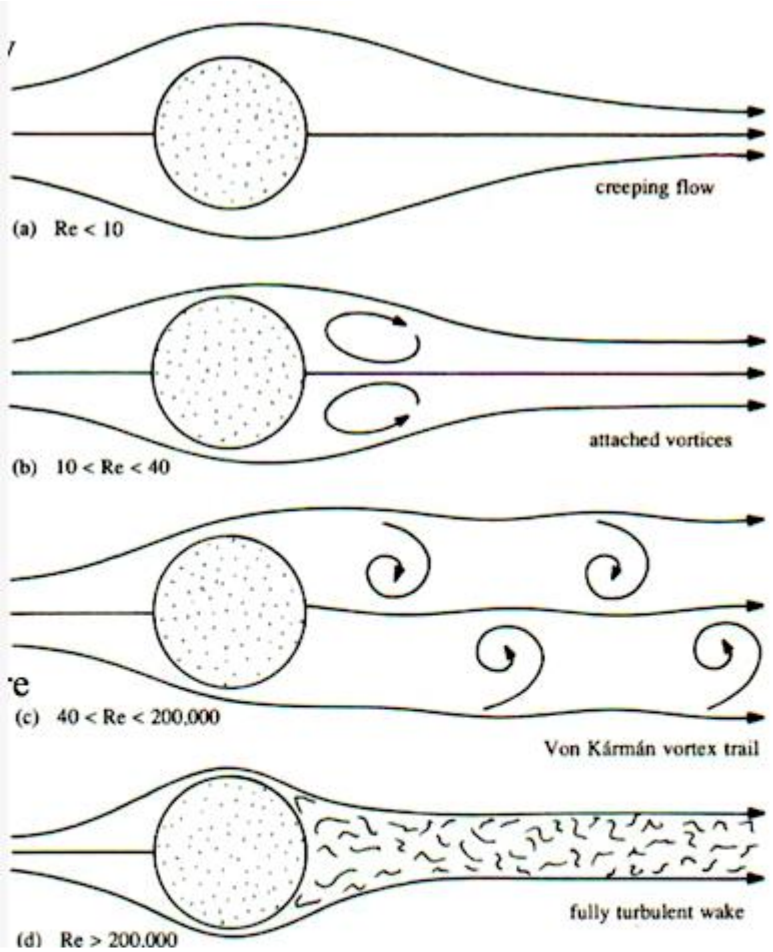

Figure 7: flow behavior at different Reynolds number

[Figure from S. Vogel, Life in Moving Fluids, Princeton University Press, 1994]

Therefore;

$$Re = \frac{\rho u d}{\mu}$$

Where:

ρ: Water density in the Lab condition *(kg/m^3)*

u: Velocity of the water flow *(m/s)*

d: diameter of the ball (m)

μ: Dynamic viscosity at Lab condition *(kg/ms)*

A characteristic feature of the turbulent flow relatively to Reynold's number, is a change in the behavior of a drag coefficient, knowing that the Drag, called sometimes air or fluid resistance in some case ,is a type of friction acting on the opposite to the relative motion of any object moving with respect to a surrounding fluid

$$C_D = \frac{2F_D}{\rho u^2 A}$$

Where;

F_D: Drag force (by definition the force component in the direction of the flow velocity) (N)

C_D: Drag coefficient (related to the object's geometry) (dimensionless)

A: Area of the orthographic projection of the object on the plane perpendicular to the direction of motion. (m^2)

u: Velocity of the object relative to the fluid, (m/s)

ρ: Mass density of the fluid (kg/m3)

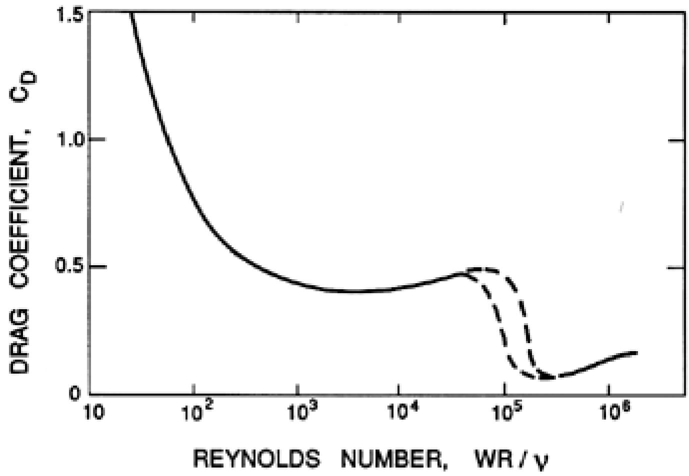

Figure 8: Drag coefficient on a sphere as a function of Reynolds number [CAVITATION AND BUBBLE DYANMICS by Christopher earls Brennen]

Now that we know the Drag force for a flow around a spherical object, we can search for the minimum speed to make this object move, we obtain using Newton's first Law, rotational motions and rugosity have not been considered;

$$F_D = m.g$$

$$v_i = \sqrt{\frac{2mg}{\rho A C_D}}$$

$A = \frac{\pi d^2}{4}$= 0.05024m^2

$C_D = 0.47$

ρ =998.2071 (kg/m^3)

m =4g

v_i: is the minimal velocity to start the motion, at a null voltage

After substitution of the parameters;

v_i= 3.29m/s

Let's express the dynamic velocity;

$$v_i = v_{set}(h) - v$$

Where;

h: is a desired height

v : The additional velocity from starting point to reach v_{set}

v_{set}: is the flow velocity at desired height *h*

According to Stokes theorem for rigid spheres, we have; $C_D = 24/Re$

$$F_D = \frac{24}{Re}\rho\frac{\pi d^2}{4}\frac{\left(v_{set}(h)\right)^2}{2}$$

And simplification of this equation could be done by approximation according a Taylor series around the steady state operating system

$$F_D = \frac{24}{Re}\rho\frac{\pi d^2}{4}\frac{\Delta v}{2} = F_{Coef}(v_i + \Delta v)$$

$\Delta\dot{h}$: The difference of velocity added to initial one, to obtain the desired height

After substitution we obtain: $F_{Coef} = 32$

Due to the fact that the system is being non-linear non-stationary

$$\sum F = m.a$$

$$F_D - m.g = m.a$$

Rearranging the equation we obtain;

$$F_{Coef}\left(v + \Delta\dot{h}\right) - m.g - m.a = 0$$

Using the balance of kinetic and potential energies, and assuming that initial height h=0m we have:

$$m.g.h = \frac{1}{2}.m.(v_{set}^2 - v_i{}^2)$$

The final height is obtained by the following equation:

$$h = \frac{(v_{set}^2 - v_i{}^2)}{2g}$$

Using the results obtained in the measurements on the next chapter we achieve the following results detailed in the tab;

Tableau 2: dependencies of voltage, velocity and Drag force according to Math. Model and measurements

Voltage input(V)	Steady state (m)	Velocity (m/s)	Drag Coeff. sphere	Drag Force Fd
0	20	3.29	0.47	346.37
1	50	3.43	0.47	376.5
2	115	3.61	0.47	417
3	130	3.657	0.47	427
4	160	3.73	0.47	445.2
5	185	3.8	0.47	462
6	215	3.87	0.47	479.26
7	240	3.94	0.47	496.75
8	265	4.003	0.47	512

Using this table we can conclude the outbuildings, as well as references to choose virtually apply our model, and in order describe a certain linearity between the different parameter of our spot.

Measurements and identification of the process:

In this chapter, a set of measurement will be executed by setting a range of voltage as an input in our valve, and noting the level of water for each voltage using the high precision sensor and at a sampling period of *dt=0.01s*, we will use those results to locate the linear part of our experiments results dataset, in order to choose our working point as a reference for our future estimation and control purposes.

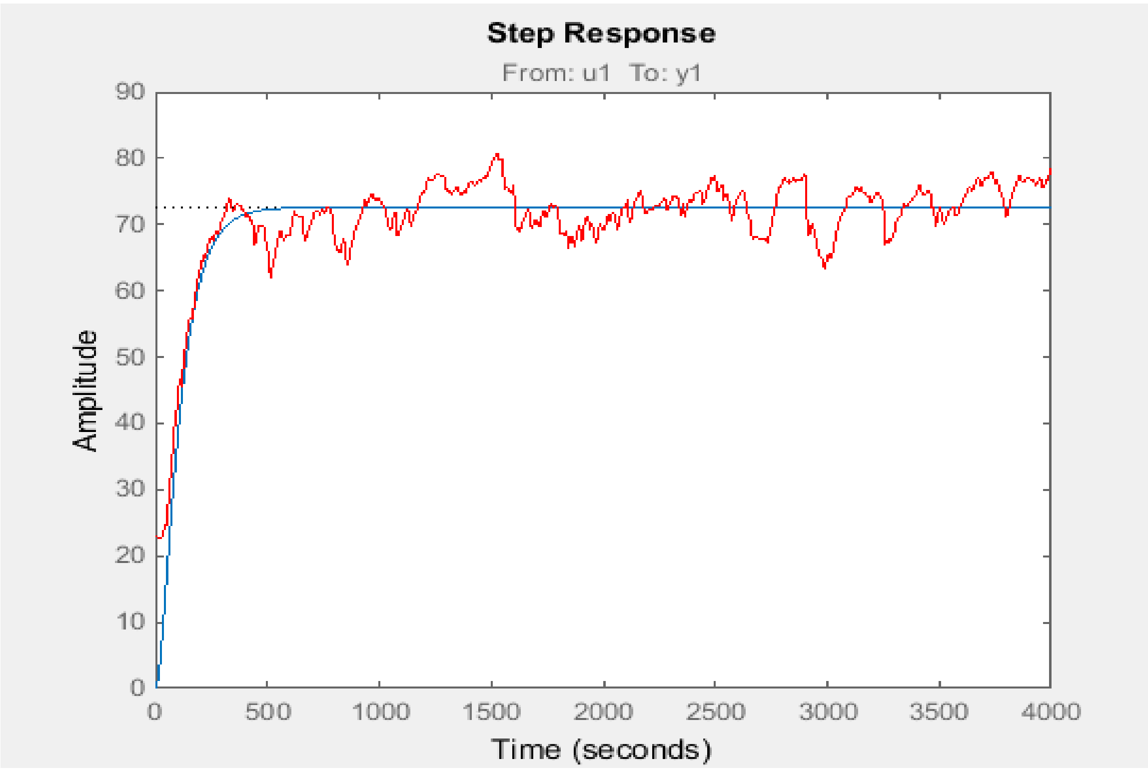

Figure 9 : Step response approximated to a second order TF for V=1v

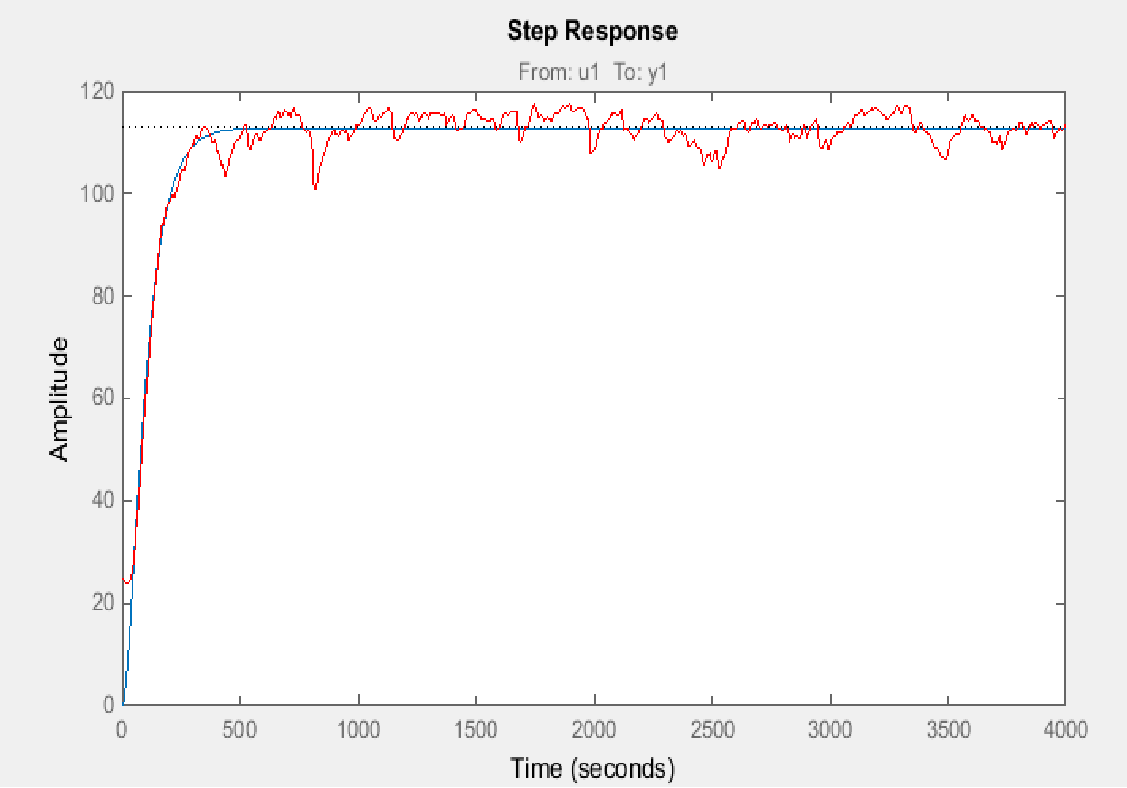

Figure 10 : Step response approximated to a second order TF for input V=2v

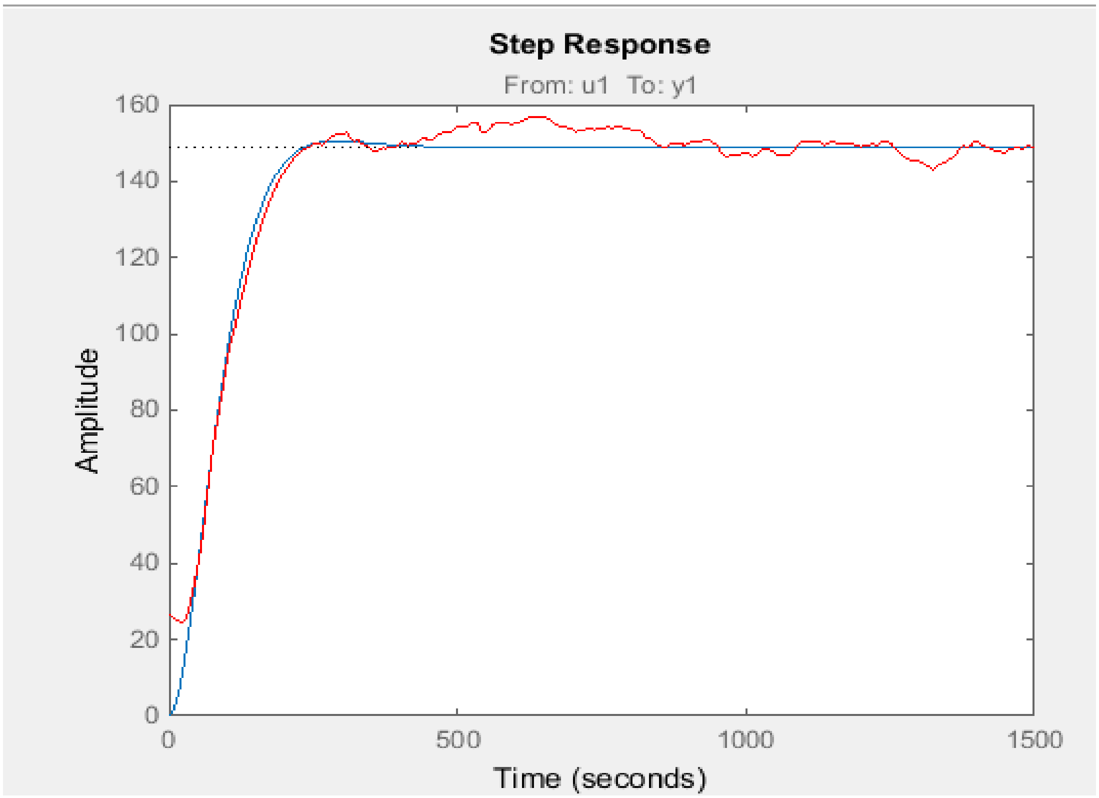

Figure 11: Step response approximated to a second order TF for input V=3v

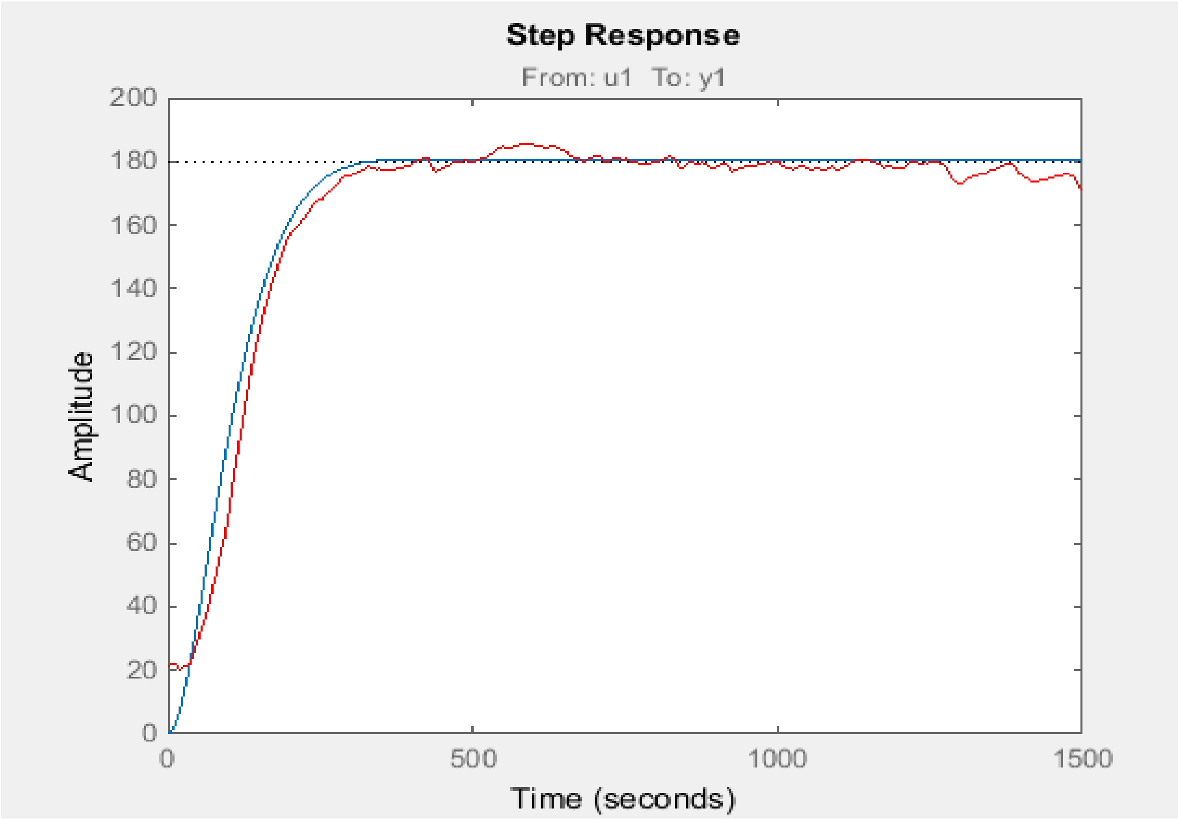

Figure 12: Step response approximated to a second order TF for input V=4v

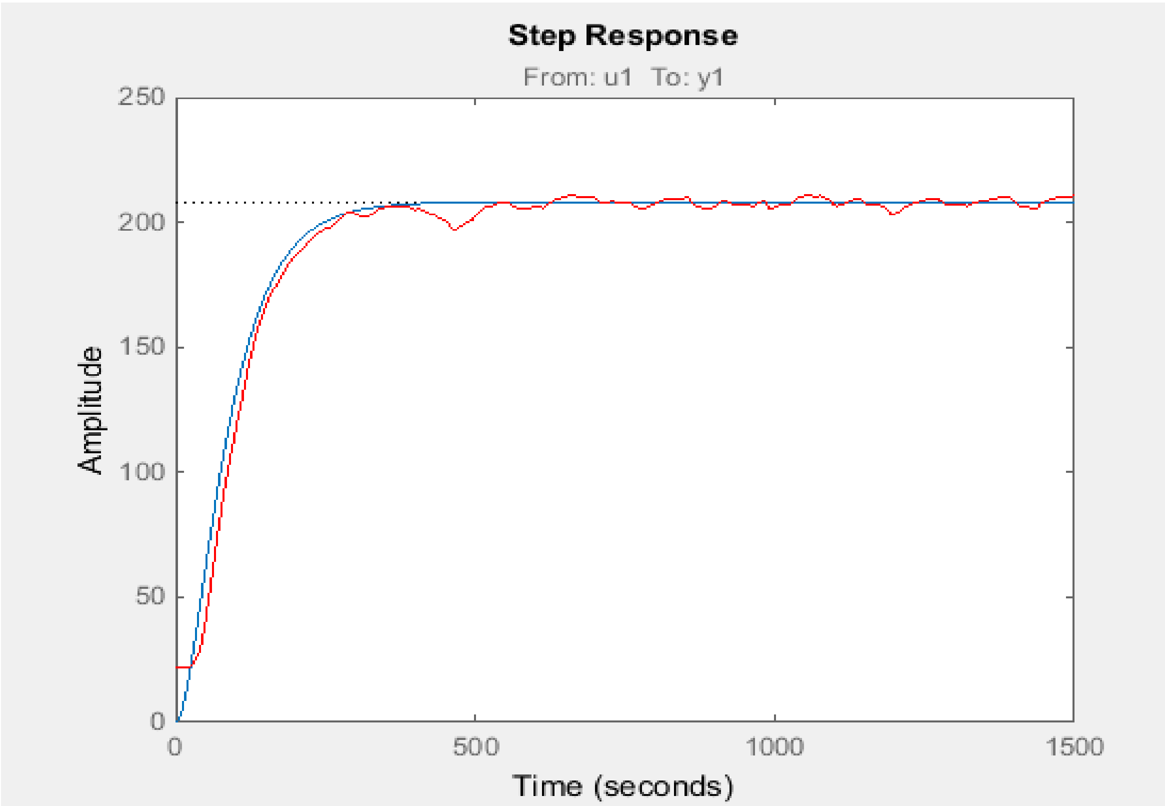

Figure 13 : Step response approximated to a second order TF for input V=5v

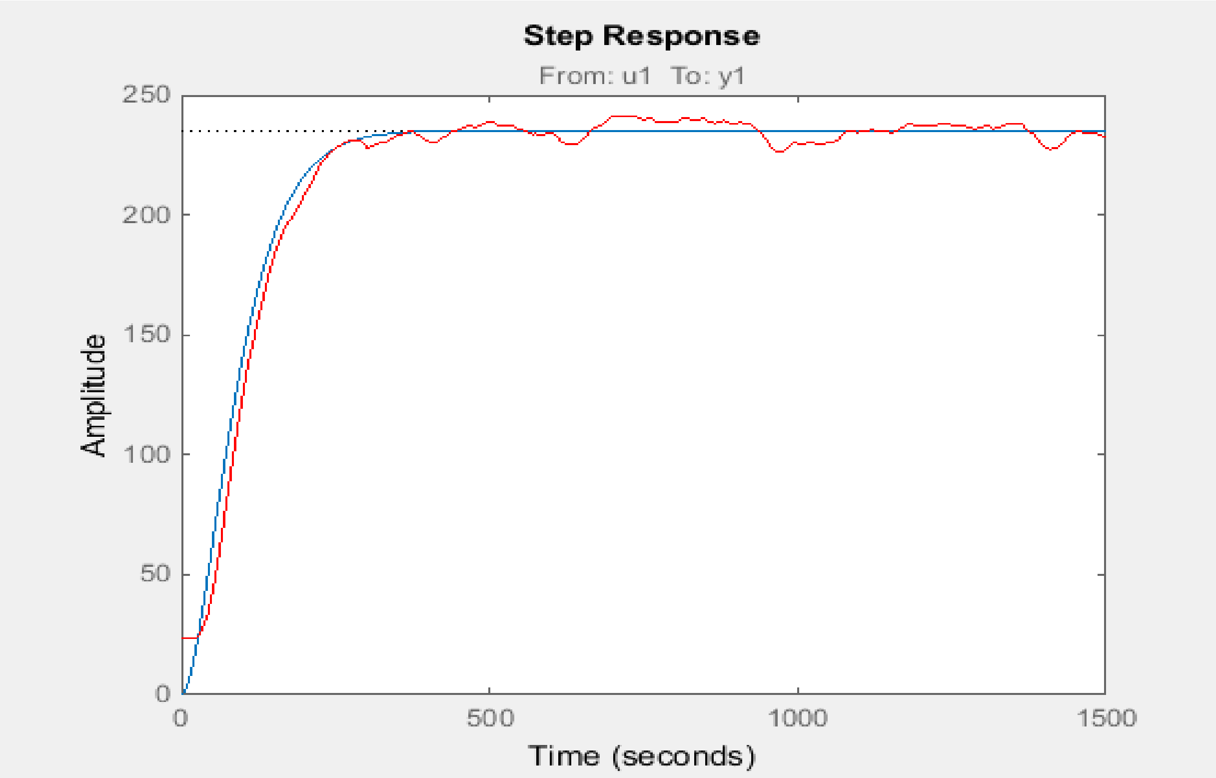

Figure 14 : Step response approximated to a second order TF for input V=6v

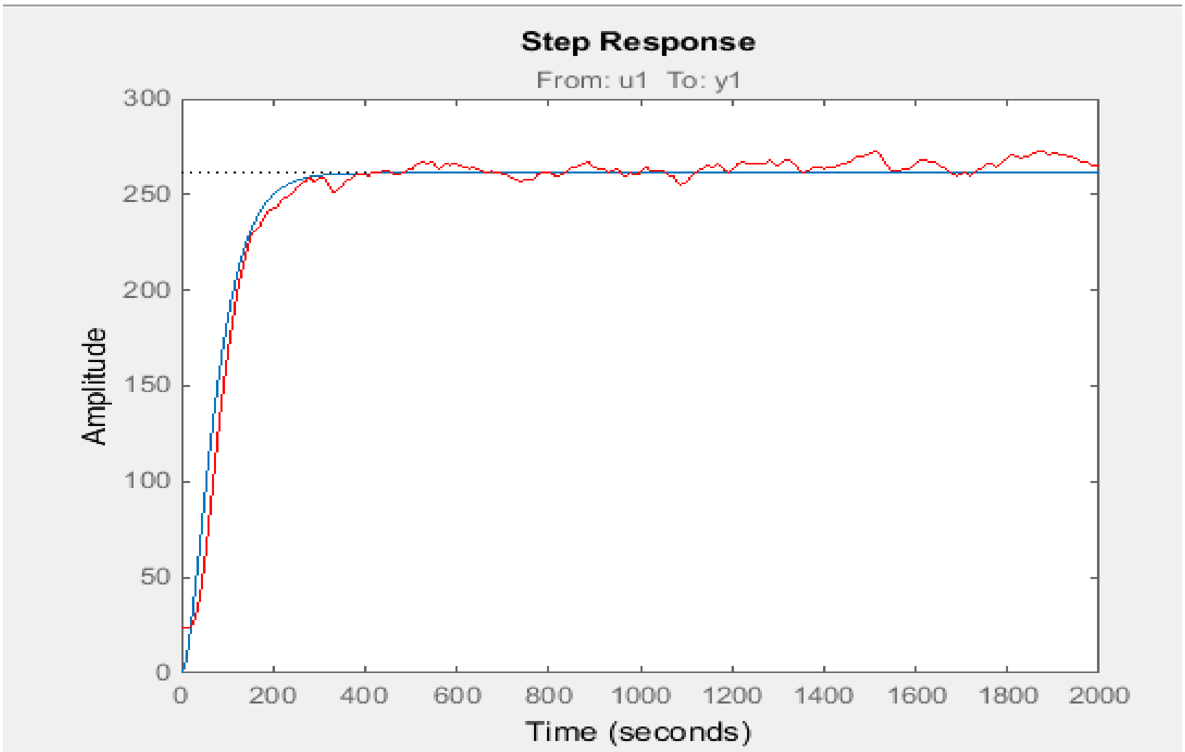

Figure 15: Step response approximated to a second order TF for input V=7v

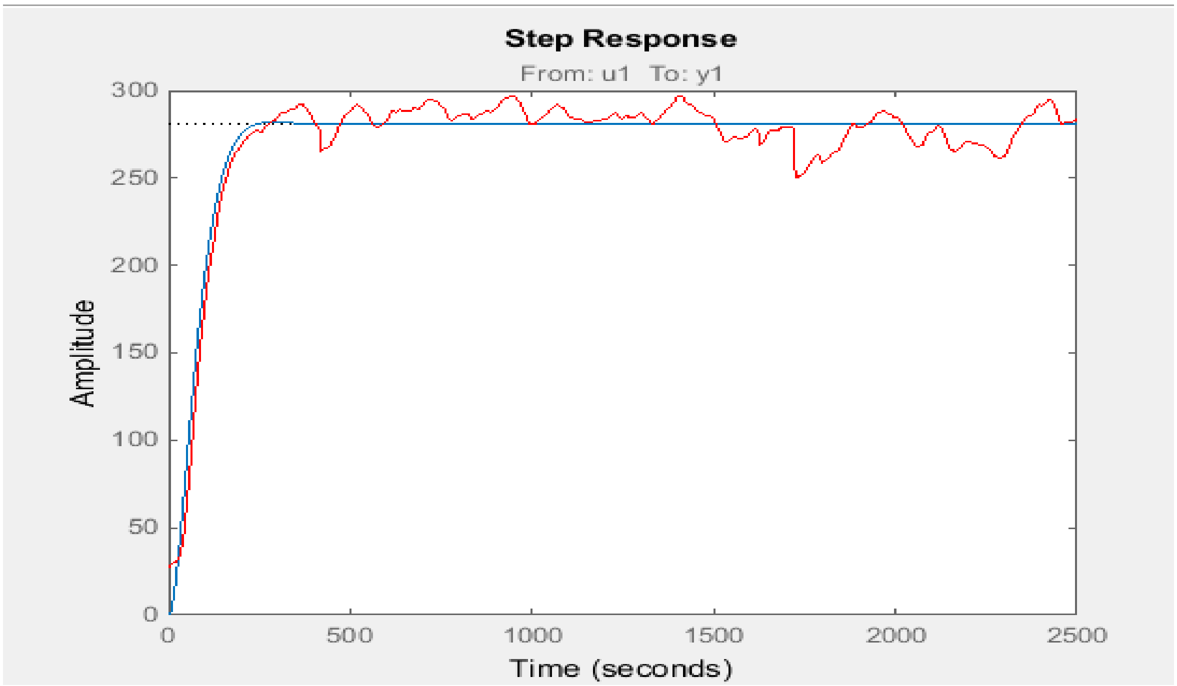

Figure 16 : Step response approximated to a second order TF for input V=8v

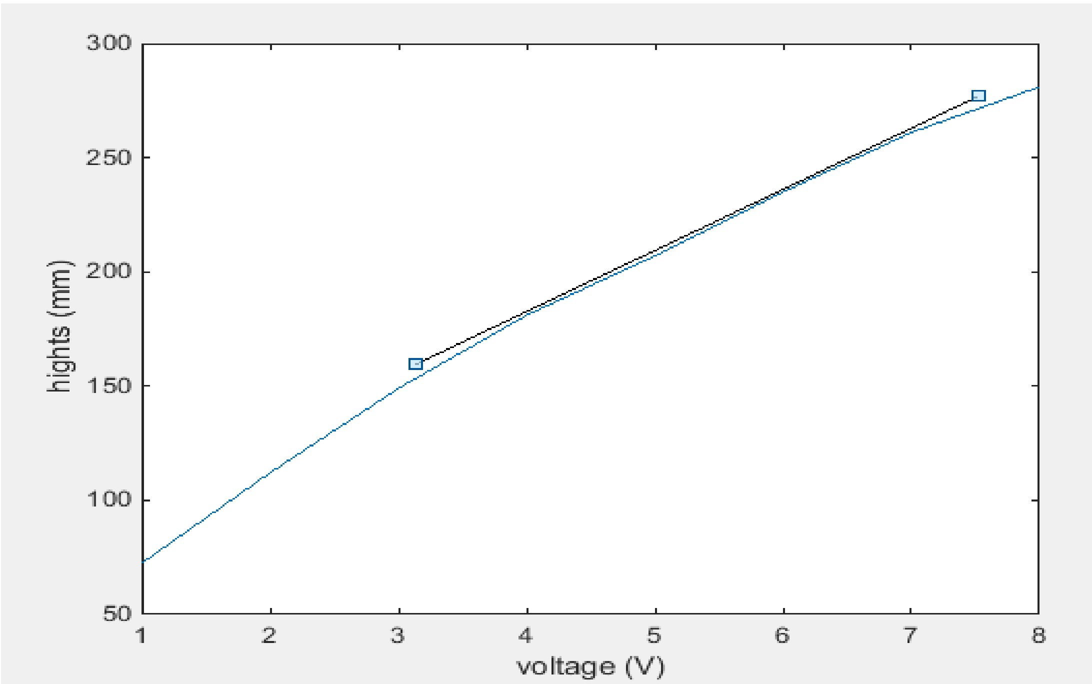

Figure 17: defining the Linearity of the system H=f(v), linear in the interval 4 to 7 volts

This graphic gives us the dependency of the height relatively to the voltage input, knowing that the voltage Is also directly related to pump velocity, and the clear almost linearity of the relations ,we can now set the desired set of the height changing the step input range from 1v to 8v using this curve.

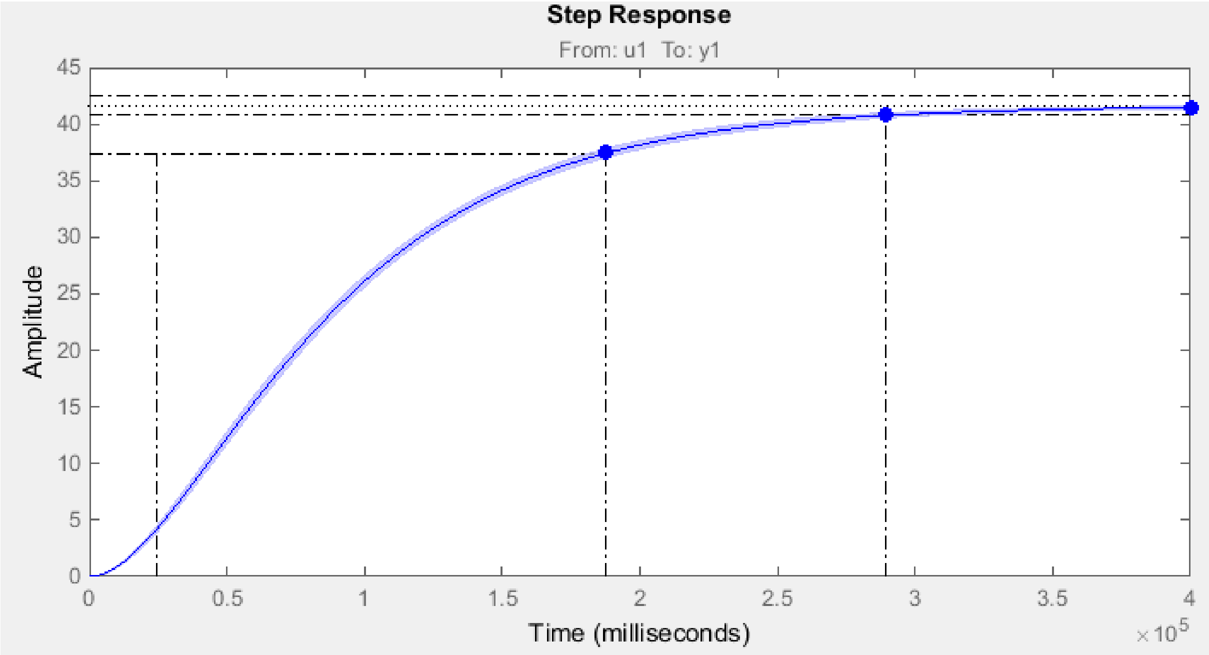

Figure 18 : Step response approximated around 5volts (working point, rising time ….)

Estimated step response to input 5Volts, including the in green brushed the confidence region, steady state, rising time.

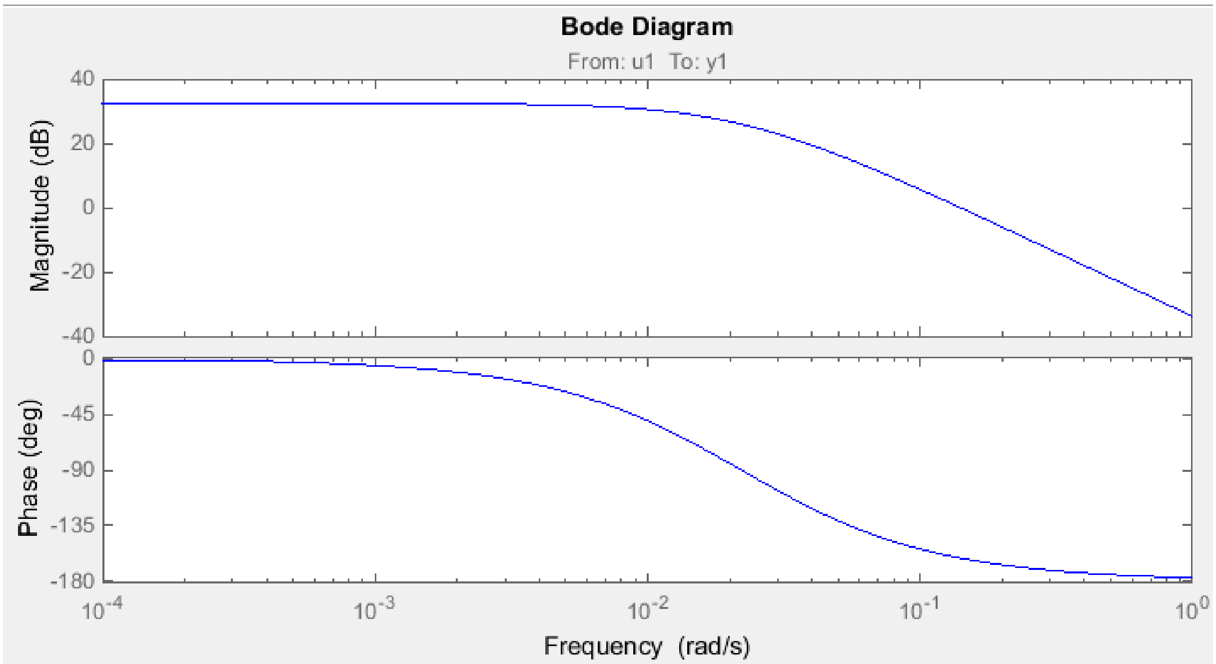

Figure 19: Bode plot of the tf5 v=5volt

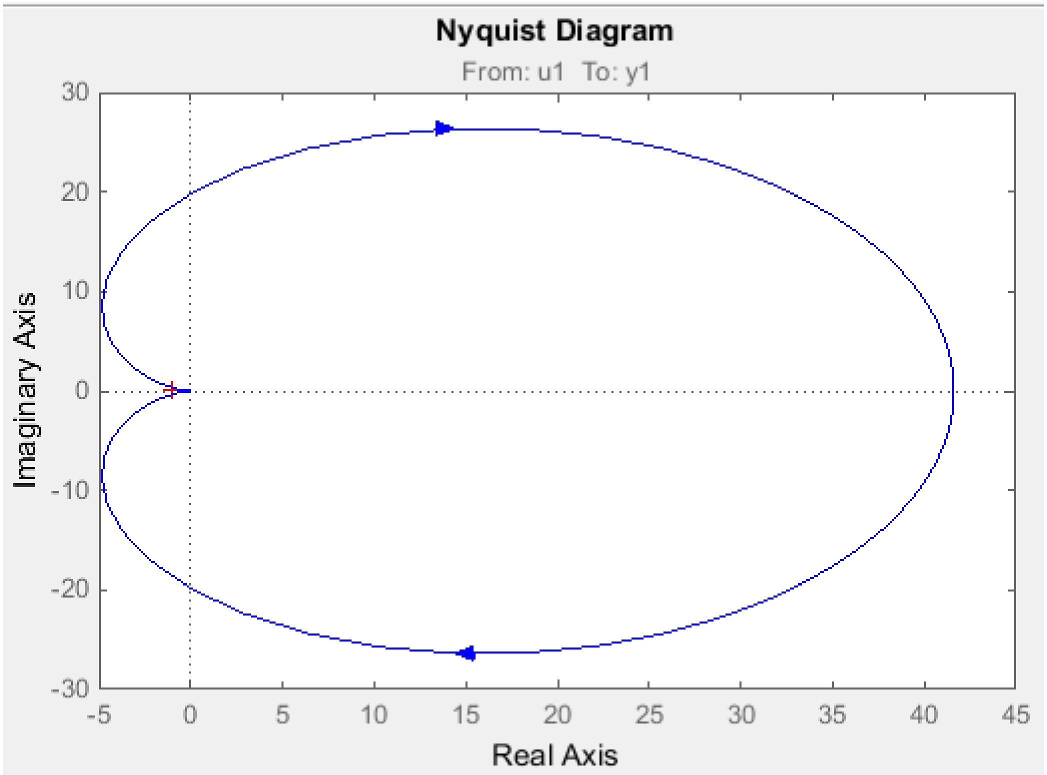

Figure 20: Nyquist plot at working point V=5v

Identification of the model

No matter the order number of poles and zeros used to estimate the transfer function of the working point at V=5volts ,the estimation is all the time close to 90 percent due to the fluctuations in the steady state cause by a random movement (the spinning motion) of the ball around the steady height set -+ 5mm

The Transfer function is set by the identification toolbox on Matlab as represented in the working point figure in blue:

$$G(s) = \frac{10880}{s^2 + 33.63s + 298}$$

Here is a set and orders tried before choosing the previous TF as best fit for our model:

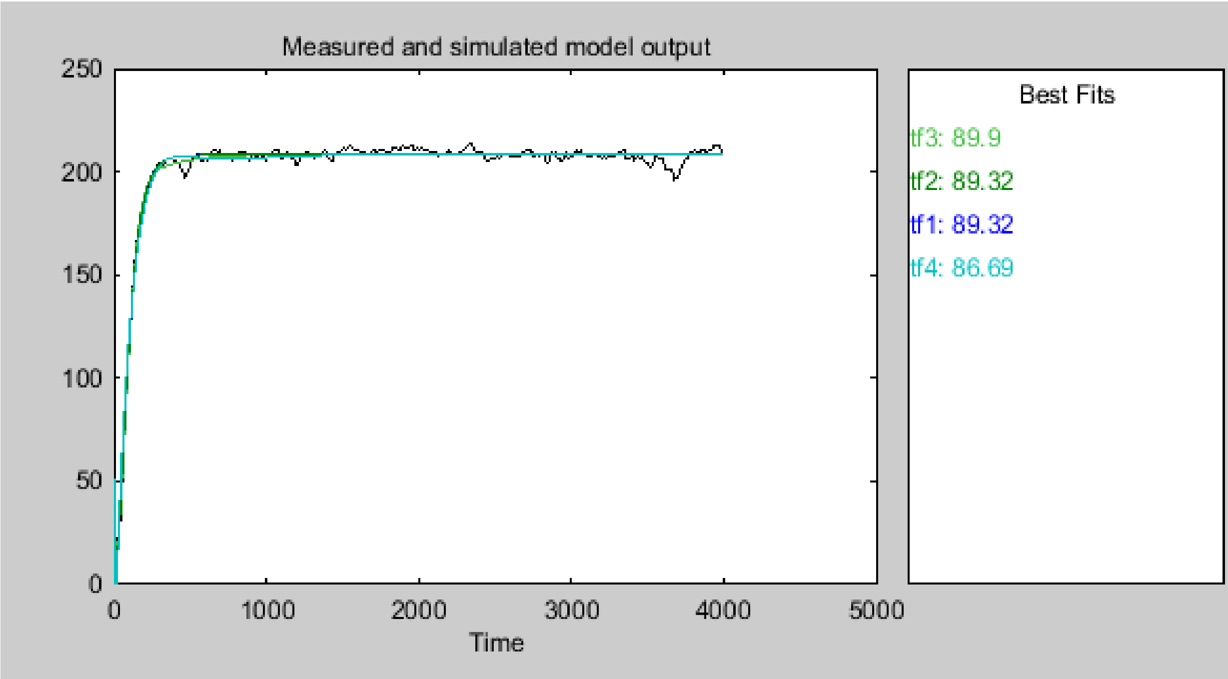

Figure 21: best fits estimated responses to our input signal 5V

Where tf1 and tf2 are second order transfer functions with different zeros and poles, tf3 is third order and tf4 is 4^{th} order estimated T.F.

We notice that the 3^{rd} one is probably the one matching the most percentage of compatibility to the real data of the experiment, however for control purposes and calculation purposes, and also due to the weak percentage of difference between the two models we choose to work with TF2.

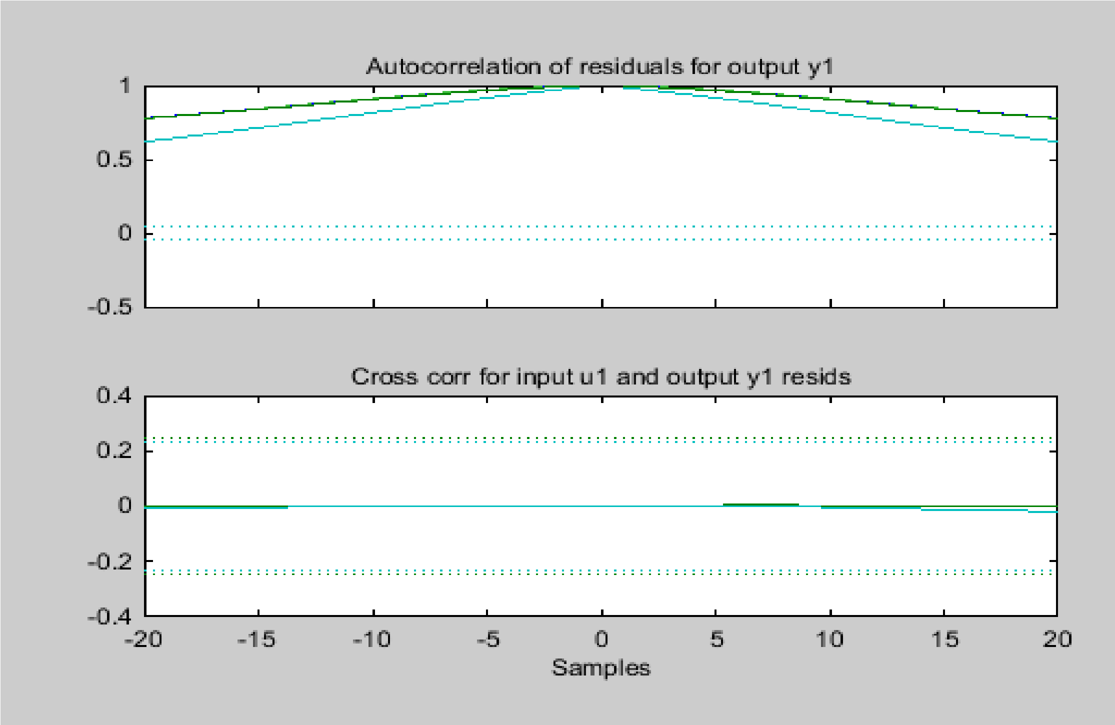

Figure 22: Residual analysis

Residuals are differences between the one-step-predicted output from the model and the measured output from the validation data set. Thus, residuals represent the portion of the validation data not explained by the model.

Residual analysis consists of two tests: the whiteness test and the independence test.

According to the *whiteness test* criteria, a good model has the residual autocorrelation function inside the confidence interval of the corresponding estimates, indicating that the residuals are uncorrelated.[http://fr.mathworks.com/help/ident/ug/what-is-residual-analysis.html]

For the *continuous state space model* identification of the variables, the model fitting the best our data, on a range of 1 to 10th order is the 2^{nd} order one:

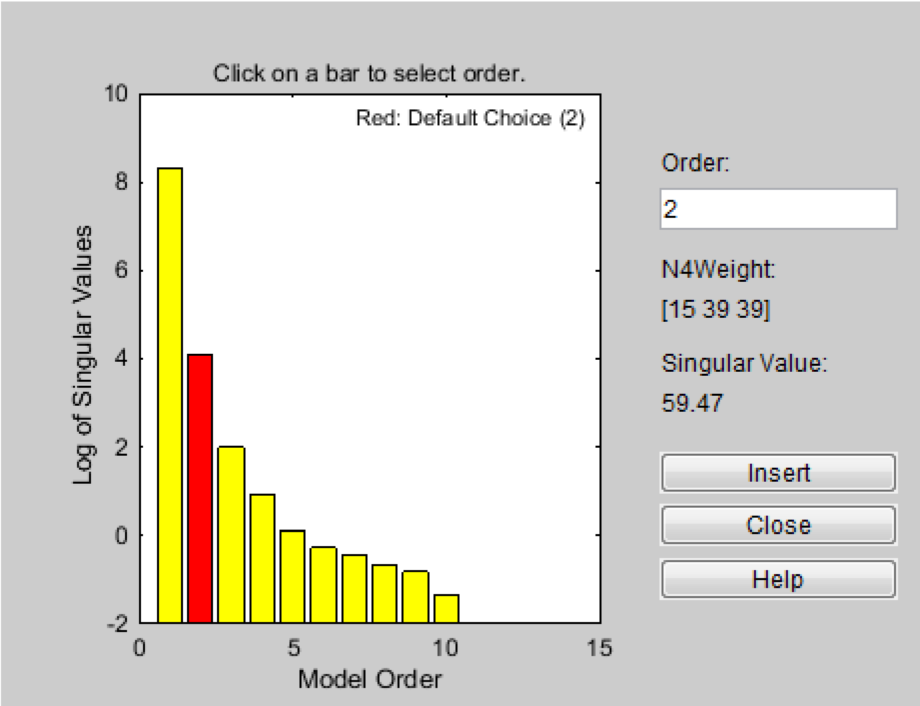

Figure 23 : Model Order Selection

Continuous-time identified state-space model:

The continuous state space approximated by the Identification Matlab toolbox is the following one, it reaches more than 90% of fits to our real laboratory data, and as mentioned previously the order chosen has been based on a model order selection, and different approximation methods.

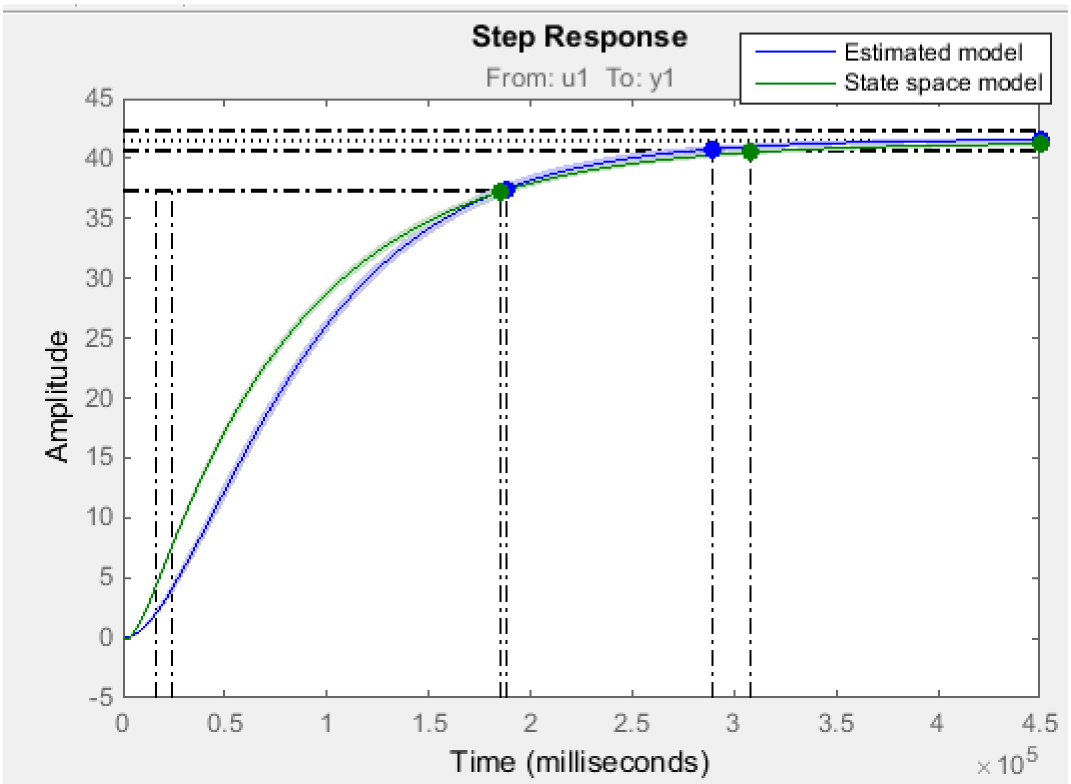

Figure 24 : Continuous state space model in compare to the estimated model

Mathematical estimation of the Continuous-time identified state-space model:

$$dx/_{dt} = A\,x(t) + B\,u(t) + K\,e(t)$$

$$y(t) = C\,x(t) + D\,u(t) + \ e(t)$$

Where ;

$$A = \begin{matrix} -0.0108 & -0.008665 \\ 0.03332 & -0.1405 \end{matrix}$$

$$B = \begin{matrix} -0.0001167 \\ -0.009386 \end{matrix}$$

$$K = \begin{matrix} 0.0005917 \\ -0.04601 \end{matrix}$$

And ;

$$C = \begin{matrix} 1143 \\ -40114 \end{matrix}$$

$$D = \begin{matrix} 0 \\ 0 \end{matrix}$$

Chapter IV

PID Controller:

Using the previous data results, and having found the best fitting identified model to our task, almost 90% (fluctuations around the steady state with a low range of fluctuations) according to our analysis, and fitting graphic, also the model setup is only a second order, which simplifies hugely the design of the Controller.

In the next part we will be to design a robust controller for the whole range of the ball position, and experimentally verify the results on Laboratory real plant.

IMC tuning control method

IMC Background

In process control, model based control systems are mainly used to get the desired set points and reject small external disturbances. The internal model control (IMC) design is based on the fact that control system contains some representation of the process to be controlled then a perfect control can be achieved. So, if the control architecture has been developed based on the exact model of the process then perfect control is mathematically possible. **[INTERNAL MODEL CONTROL (IMC) AND IMC BASED PID CONTROLLER GARIMA BANSAL ABHIPSA PANDA SANYAM GUPTA]**

IMC design procedure

Consider a process model Gp*(s) for an actual process or plant Gp(s). The controller Qc(s) is used to control the process in which the disturbances d(s) enter into the system. The various steps in the Internal Model Control (IMC) system design procedure are: **[INTERNAL MODEL CONTROL (IMC) AND IMC BASED PID CONTROLLER GARIMA BANSAL ABHIPSA PANDA SANYAM GUPTA]**

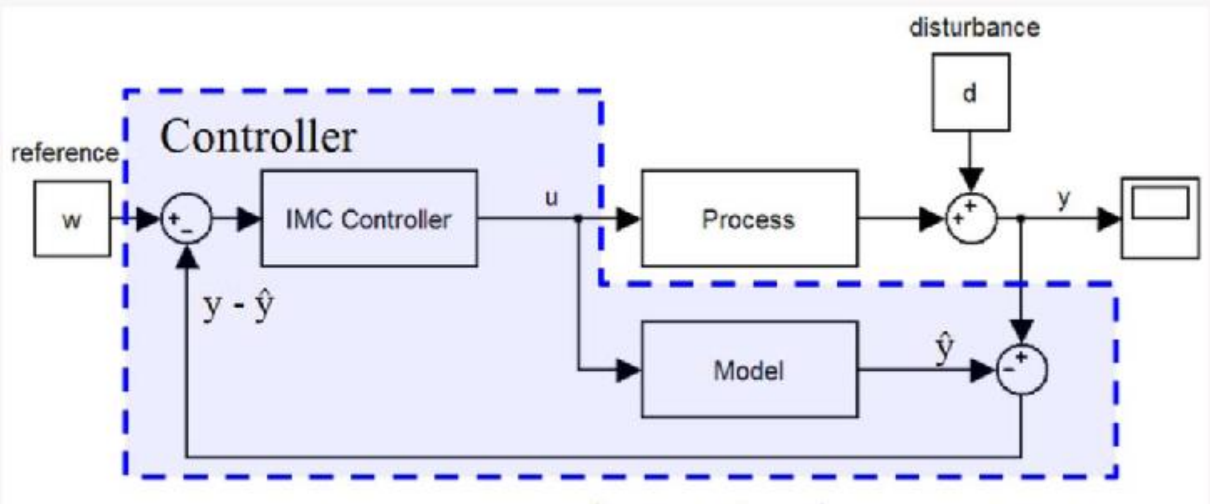

Figure 25: Classic IMC control [Modelling and Internal Fuzzy Model Power Control of a Francis Water TurbineKlemen Nagode 1, and Igor Škrjanc 2]*

After rearrangement of the controller we obtain;

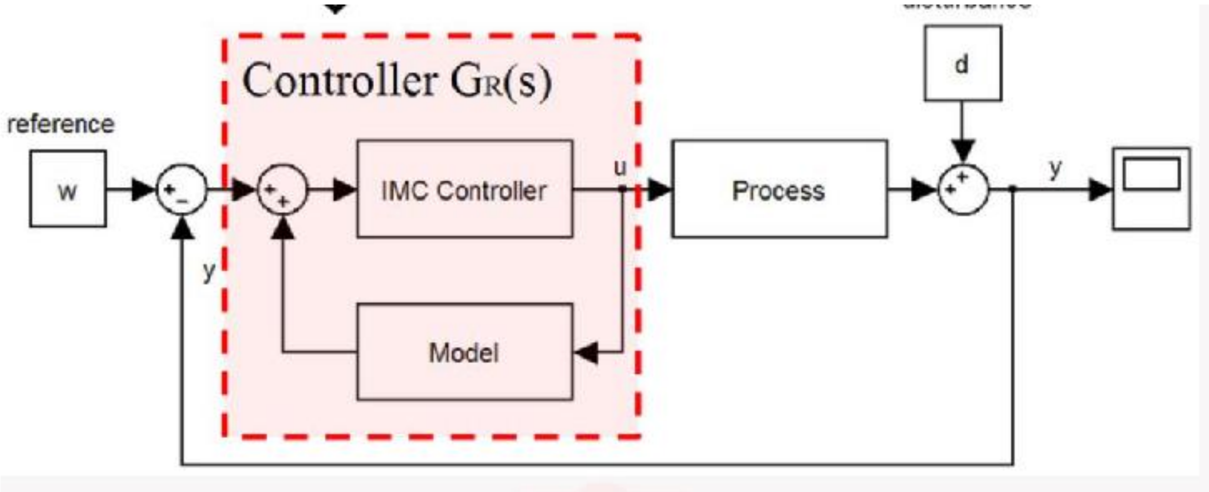

Figure 26: rearranged model [Modelling and Internal Fuzzy Model Power Control of a Francis Water Turbine]

Where the model is *G(s)* estimated previously, at V=5volts

$$G(s) = \frac{10880}{s^2 + 33.63s + 298}$$

For our IMC controller we will use the following filter

$$F(s) = \frac{1}{(Ts+1)^2}$$

Our controller *C(s)* will set as :

$$C(s) = G^{-1}(s) * F(s)$$

By substituting the values we obtain ;

$$C(s) = \frac{s^2 + 33.63s + 298}{10880 * (Ts+1)^2}$$

Now let rearrange the structure to get a little bit more classical one;

$$R(s) = \frac{C(s)}{1 - G(s)C(s)}$$

Knowing that;

$$\frac{Y}{W} = \frac{R(s)G(s)}{1 + R(s)G(s)}$$

After substitution we obtain;

$$R(s) = \frac{s^2 + 33.63s + 298}{10880 * (T^2\, s^2 + 2Ts)}$$

Which finally give us;

$$F(s) = \frac{1}{(Ts + 1)^2}$$

We conclude finally that the IMC control stability and speed, is based only on the value of *T*.

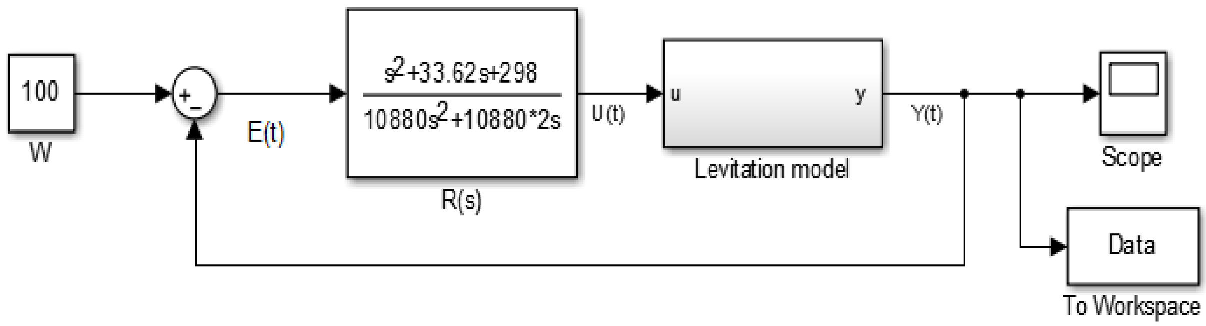

Figure 27 : Final IMC controller Simulink model applied on the model

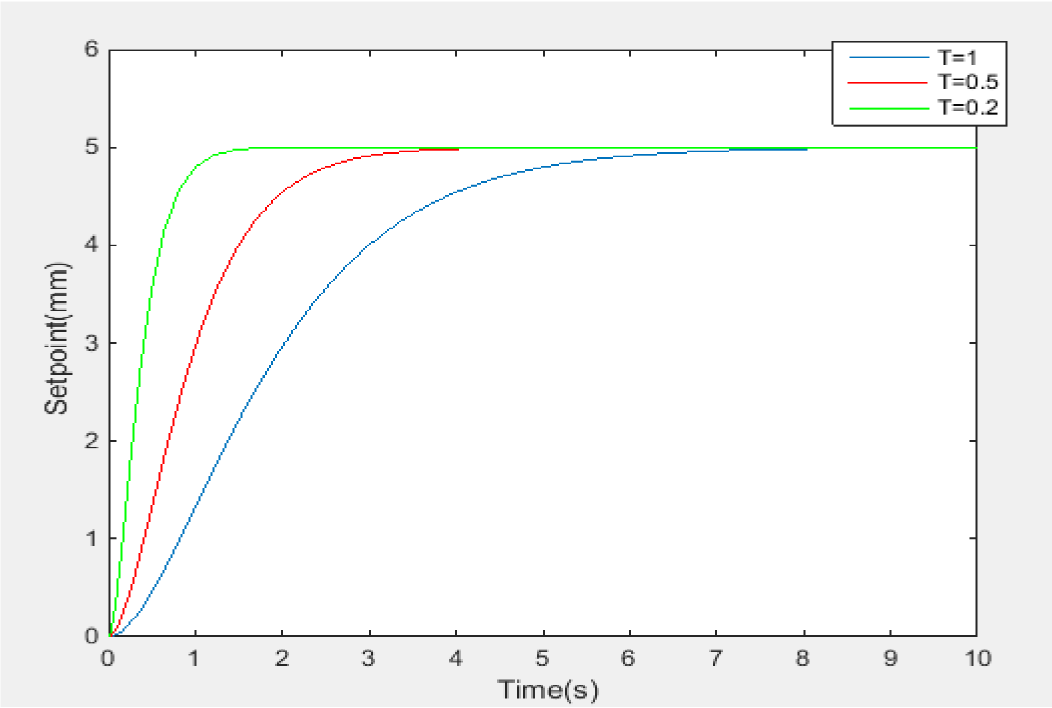

Figure 28: response to 1 as a Set point for our model G(s), T=0.2s, T=0.5s and T=0.05s

We notice that the smaller the value of T is, the more stable the system become and reach the Set point with no overshoot quite fast enough.

T is set with respect to the sampling theorem;

$$\Delta t = 0.1 \qquad then \quad T > 2 * \Delta t$$

Yet the optimal value obtained is T=0.2s.

Application and results on the process:

Applying the previous results on the real Levitation task gave us functional PID, we are now able to set the desired height of the ball as shown on the figures bellow for different desired set points, managing T as a unique and very simple tuning parameter for our controller.

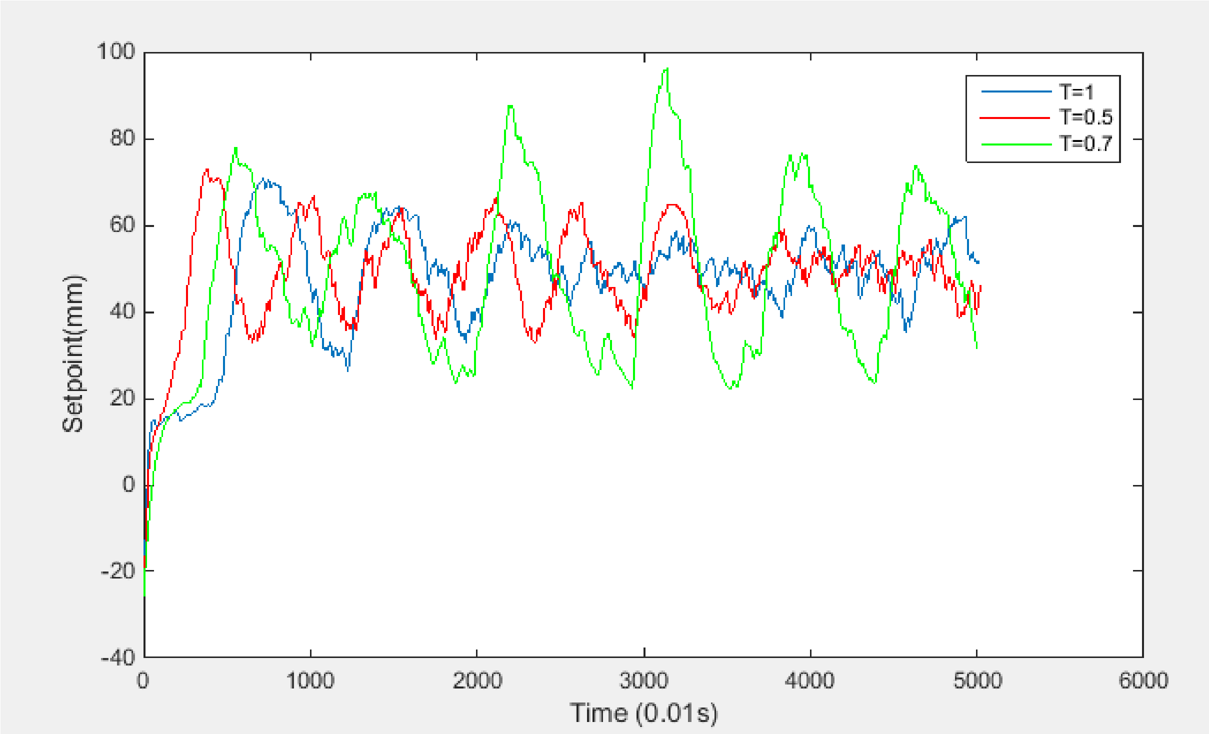

Figure 29: Controller IMC system for Set point 50mm

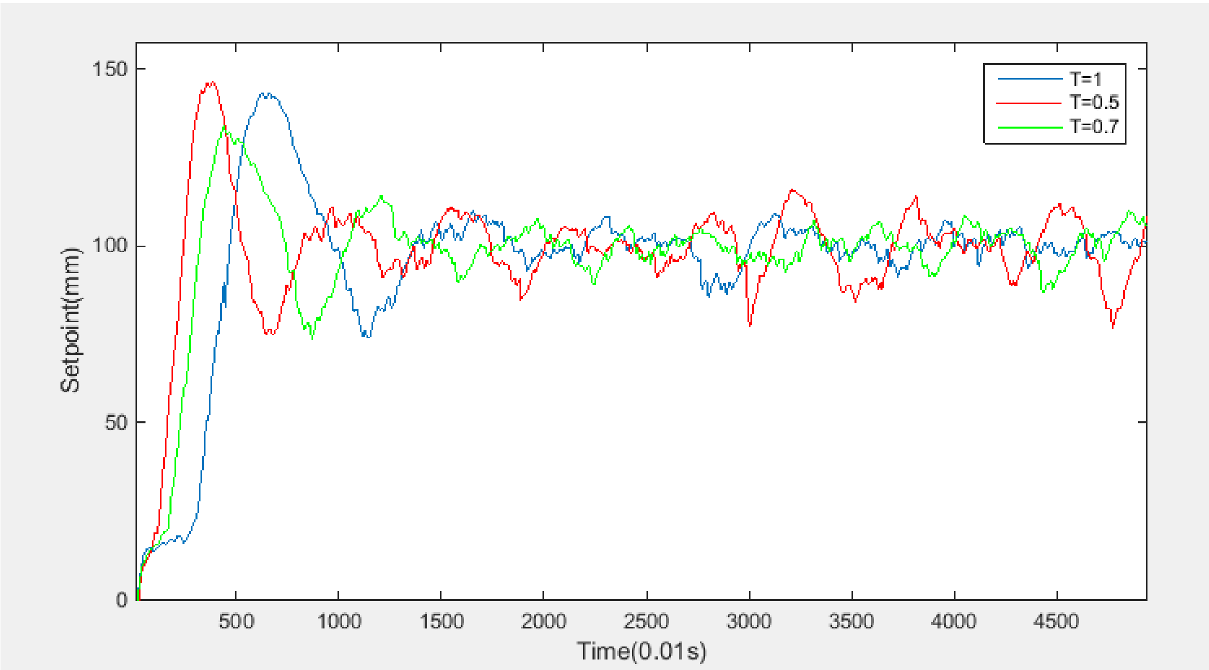

Figure 30: Controller IMC system for Set point 100mm

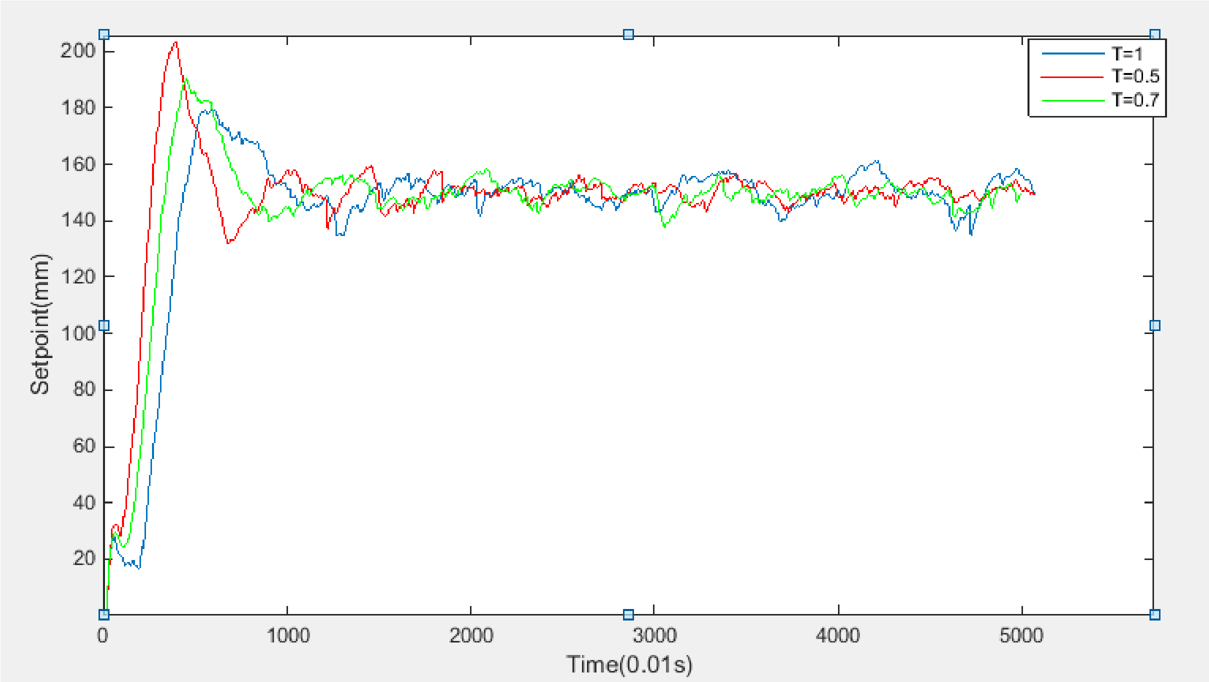

Figure 31: Controller IMC system for Set point 150mm

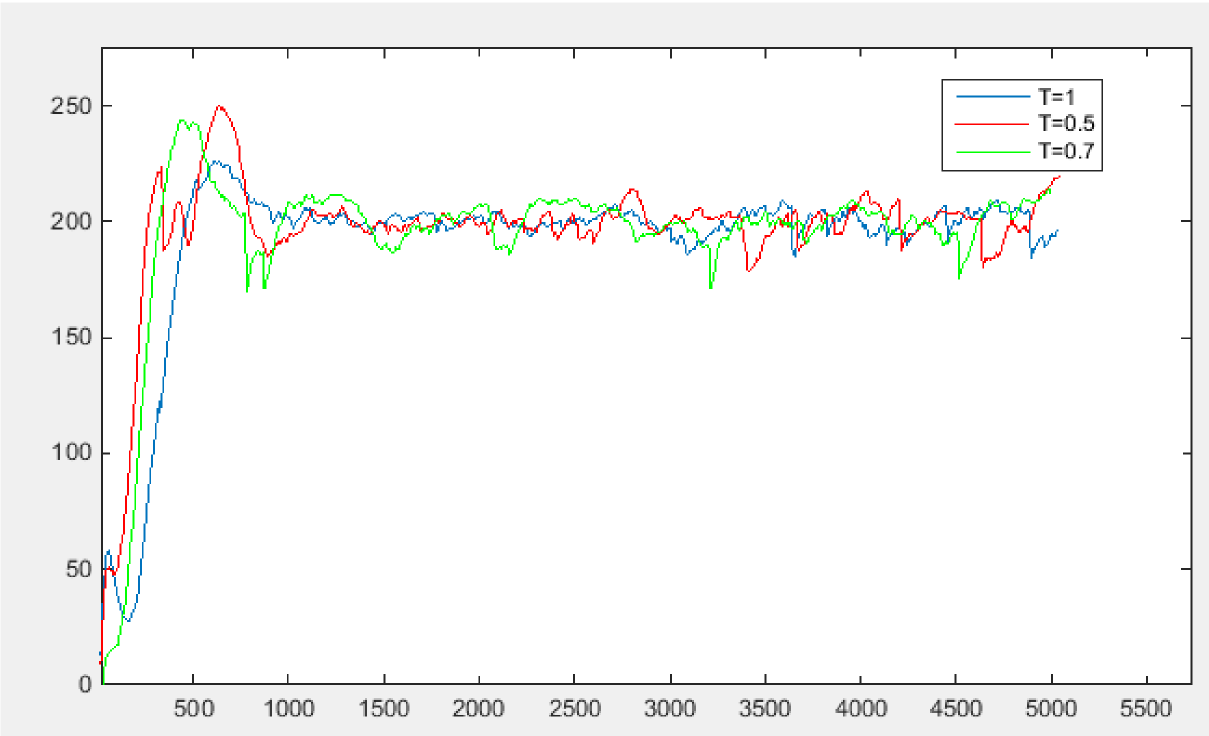

Figure 32: Controller IMC system for Set point 200mm

Ziegler Nichols Controller:

The Ziegler Nichols method for tuning PID controller, with the parameters Kp, Ki and Kd, are mainly based on measured parameters ,the ultimate gain Ku and the ultimate period Pu, where Ku is the proportional parameter of the control that allow the process to be marginally stable

Setting a Proportional controller only, and from some low values of Kp (the integrative Ki and derivative Kd are set to 0 value for this step) ,we will test the system increasing slowly Kp until it starts to oscillates continuously ,that means that at that value of Kp = Ku the system is now marginally stable.

If the oscillations decay, we need to increase Kp.

Is the oscillations increase in amplitude, this indicates an unstable system, then we will need to decrease Kp.

Using these indications to find out how the system reacts to Kp, and finding the desired value where the system is stable marginally we can now according to the following table find the parameters of P, PI and PID if necessary to tune our system ;

Note: the Kp,Ki and Kd found for the values of Ku Pu are not the most stable parameters for PID controller, but just a starting point to tune the controller ;

Tableau 3 : Ziegler Nichols parameters

Ziegler–Nichols method

Control Type	K_p	K_i	K_d
P	$0.50K_u$	-	-
PI	$0.45K_u$	$1.2K_p/P_u$	-
PID	$0.60K_u$	$2K_p/P_u$	$K_pP_u/8$

Using the previous Nyquist plot of our estimated model, we will try to define the Ku the ultimate gain, and period

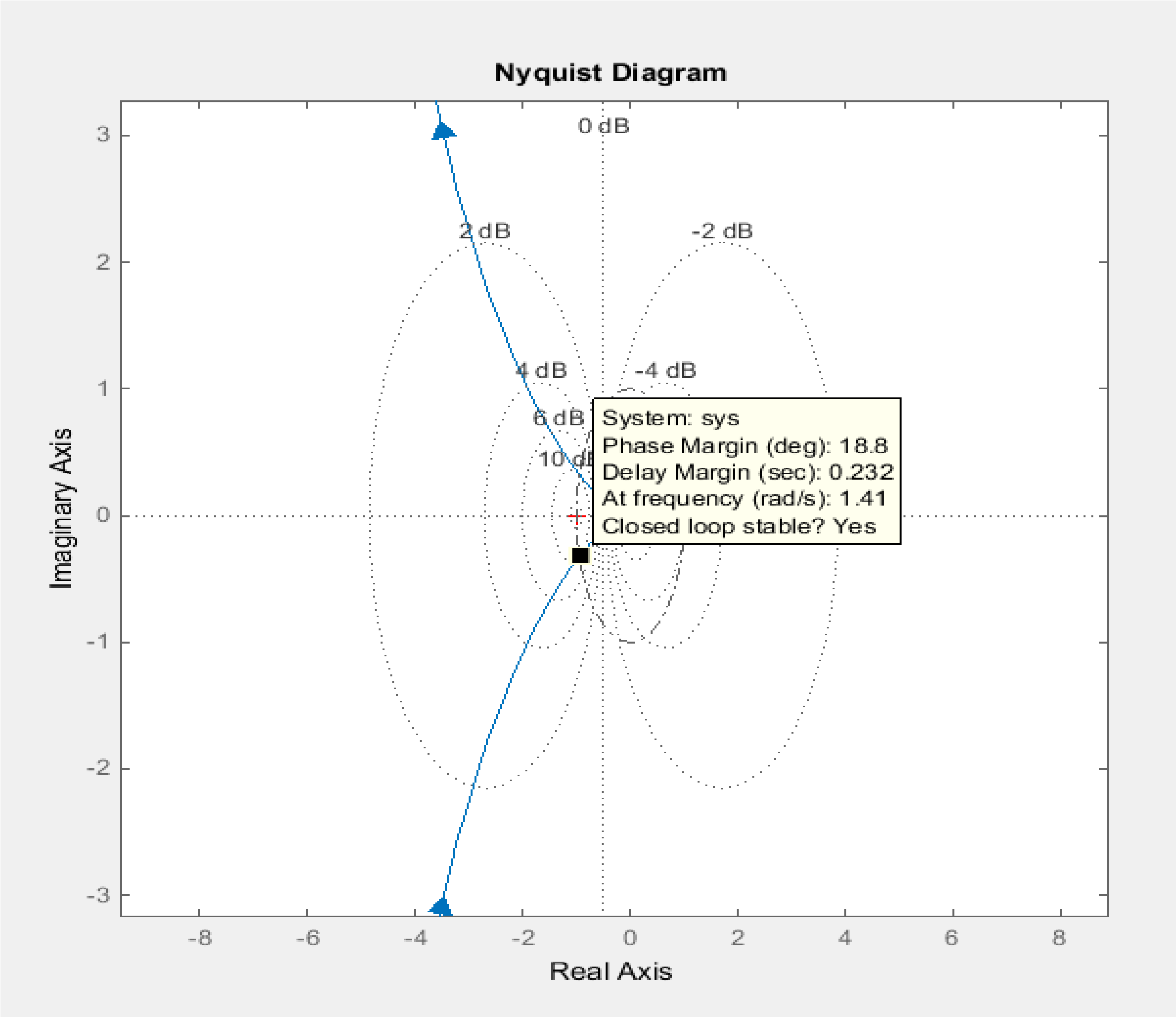

Figure 33 : minimum Phase and Stability margin of the model from Nyquist plot

From this figure we can establish the Critical amplitude and period;

Gm=10 db, Converted to amplitude Kc=3.1623

Frequency ω_c=1.41 rad/s, knowing that $Pu = \frac{2*\pi}{\omega_c}$ i.e.: Pu=4.453 sec

For those parameters of Ku and Pu the system is not marginally stable

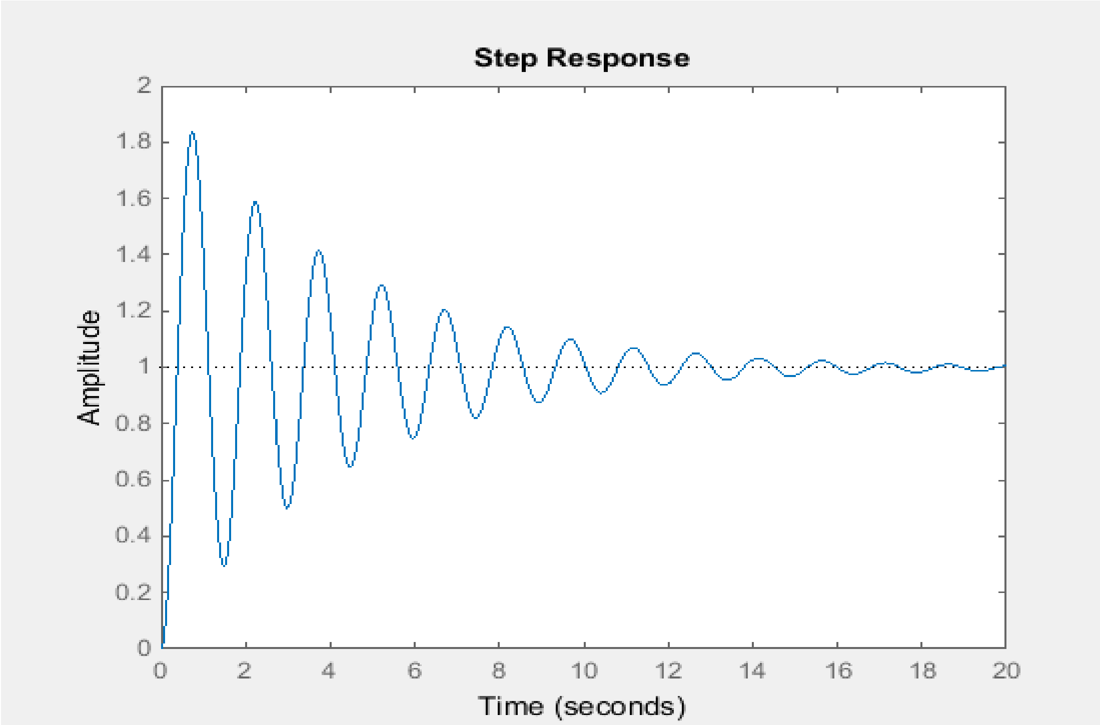

Figure 34: System response to the previous parameters of Ku Pu

Now let's try another method to determine the marginal stability of the model, the *Astrom Hagglund* method:

Ideal Relay Test:

In 1984, Astrom and Hagglund (Astrom et al., 1984) suggest the relay feedback test to generate sustained oscillation as an alternative to the conventional continuous cycling technique. This relay feedback test was soon (Luyben, 1987) referred as autotune variation (abbr. ATV) test.

Controller tuning using ATV test is attractive, because it is operated under closed loop and no a priori knowledge of system is need. The test provides ultimate gain and ultimate period for applying Z-N rules to tune a PID controller. **[Model-based Auto-tuning System Using Relay Feedback Hsioa-Ping Huang, Kuo-Yaun Luo]**

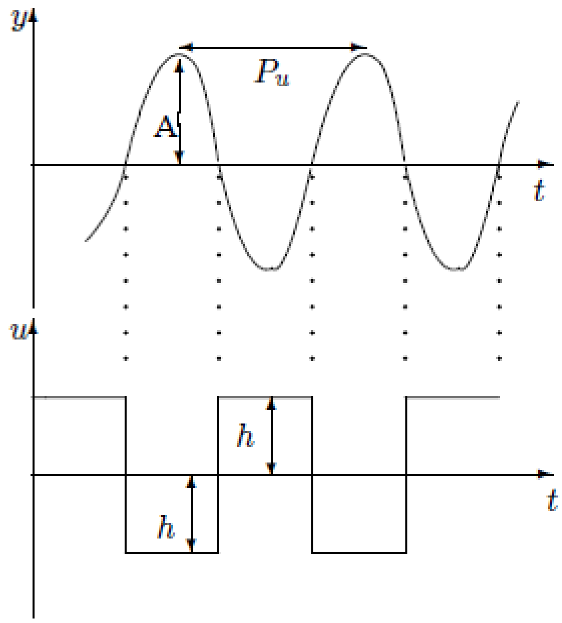

Figure 35: Auto tune with ATV test, determination of Ku,Pu

[Model-based Auto-tuning System Using Relay Feedback
Hsioa-Ping Huang, Kuo-Yaun Luo]

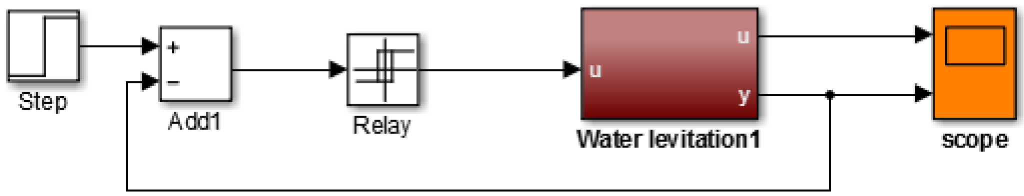

Figure 36: ATV tuning using relay feedback

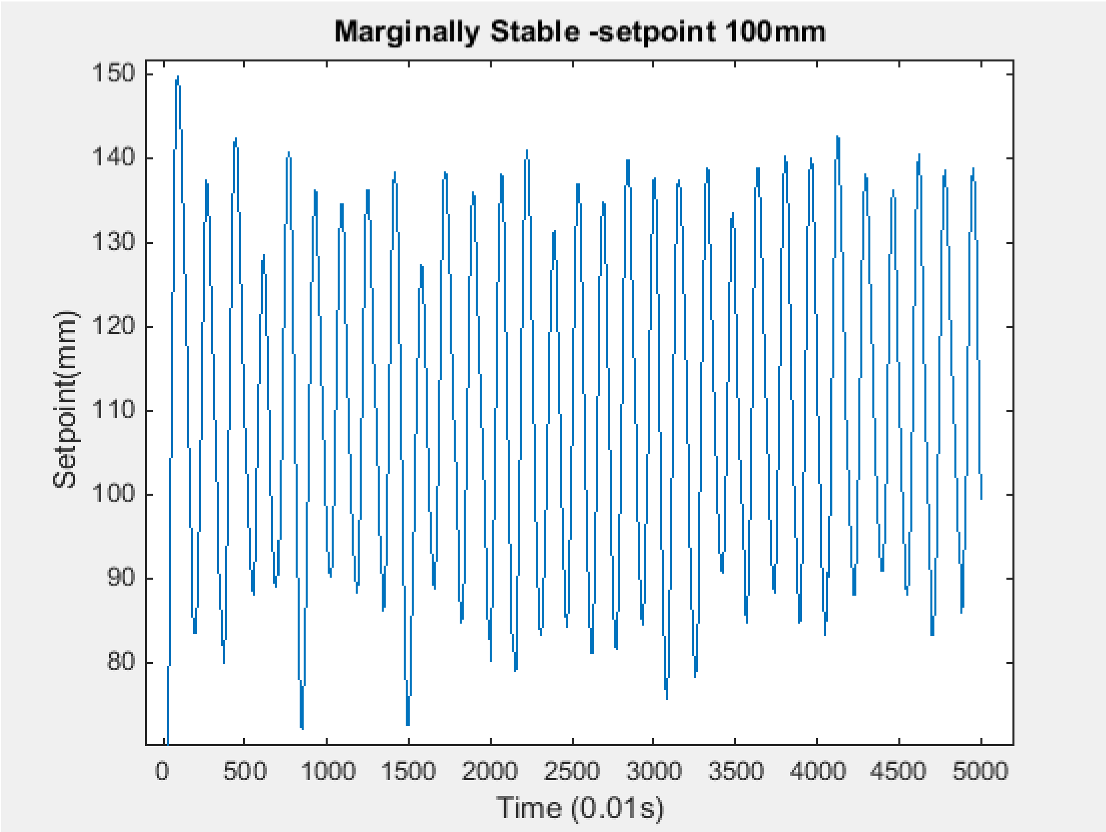

Figure 37: marginal stability for a set point of 100mm

The ultimate gain and ultimate frequency are estimated from the ATV test as:

$$K_{cu} = \frac{4h}{\pi A}$$

$$\omega_u = 2\pi / P_u$$

Where; *h=2 and A=26.91mm*

I.e.: $K_{cu} = 0.094677$

And P_u *=1.6213sec*

Using Z-N Tab for parameters of PID we obtain;

Kp=0.6Ku=0.0568062

Ki=2Kp/Pu=0.0700748

Kd=0.0006722

Application and results on the process

The following plots are the results of applying Ku and Pu from the feedback relay auto-tuning method to Ziegler Nichols PID parametrization for different set points, in this case they will be used as a starter to design a faster and more robust controller.

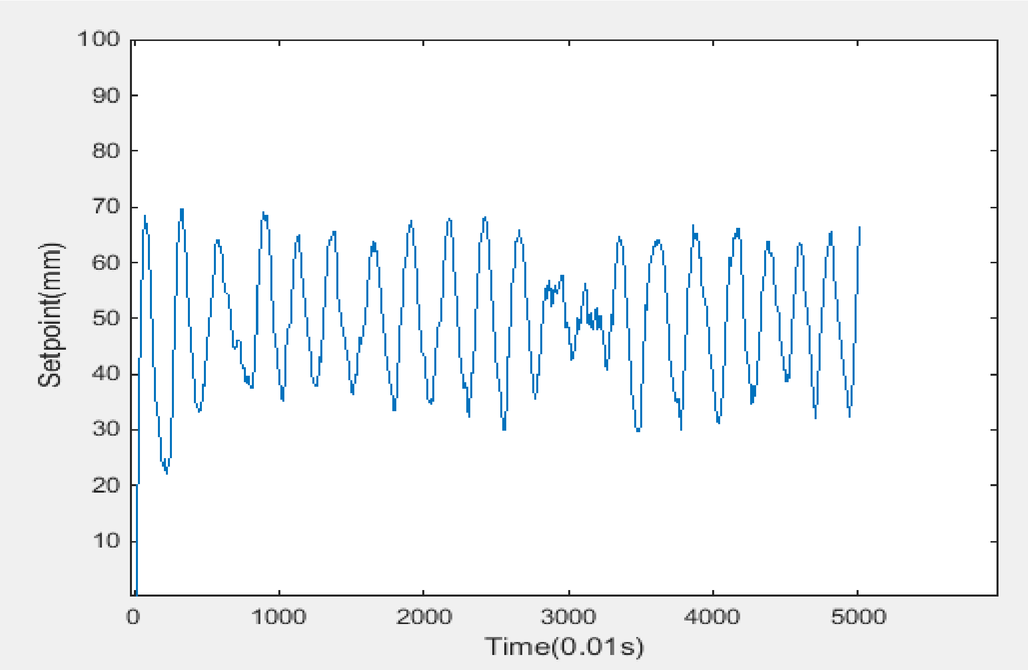

Figure 38: Set point 50mm controlled using ATV feedback relay method and Z-N parameters

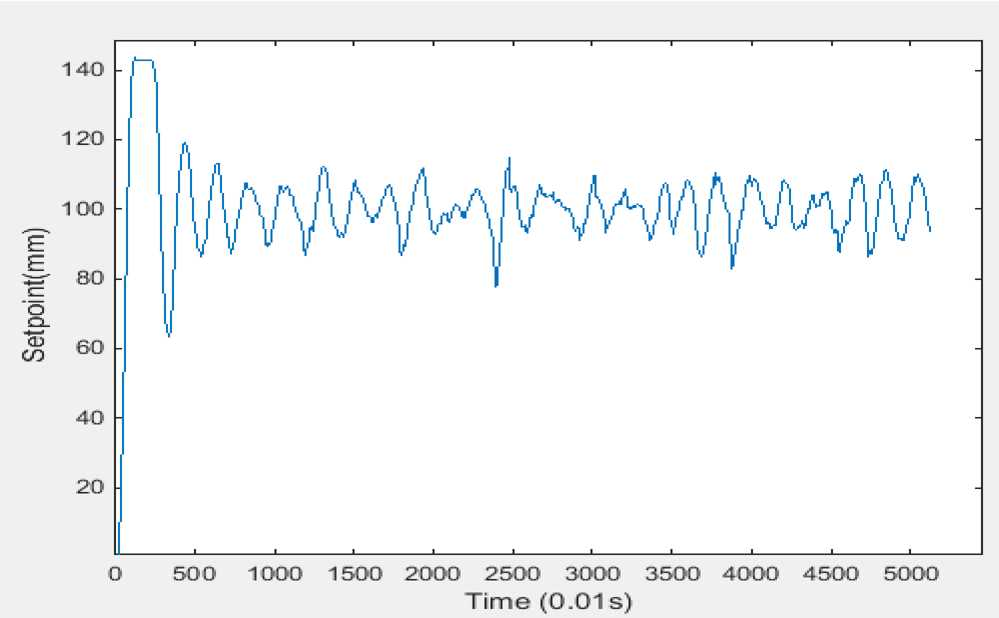

Figure 39: Set point 100mm controlled using ATV feedback relay method and Z-N parameters

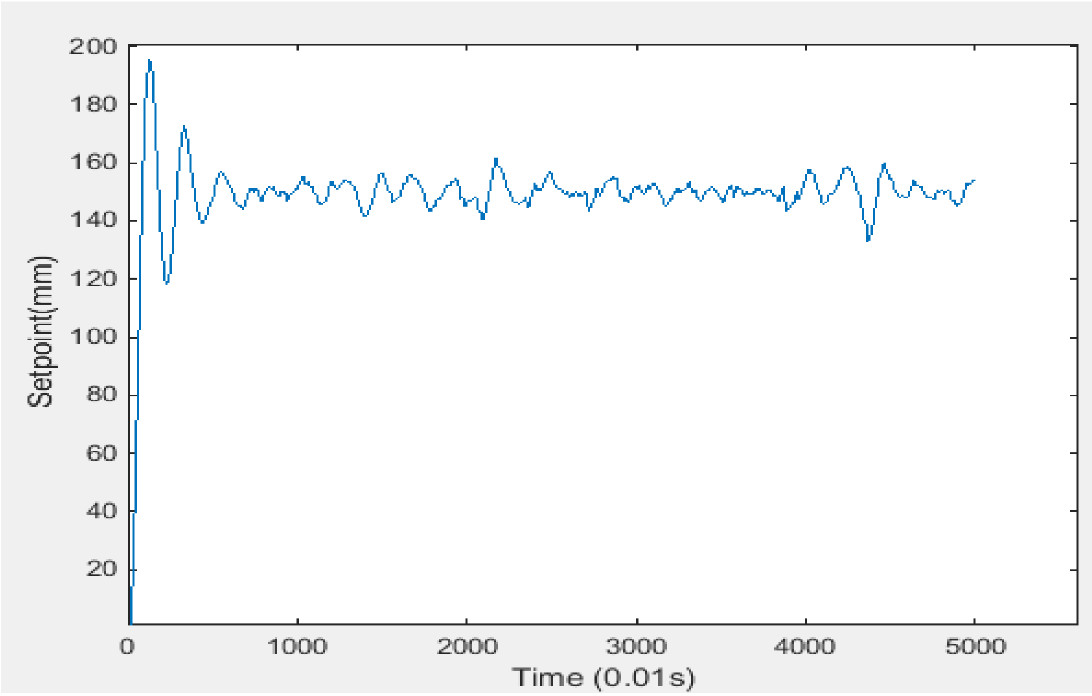

Figure 40: Set point 100mm controlled using ATV feedback relay method and Z-N parameters

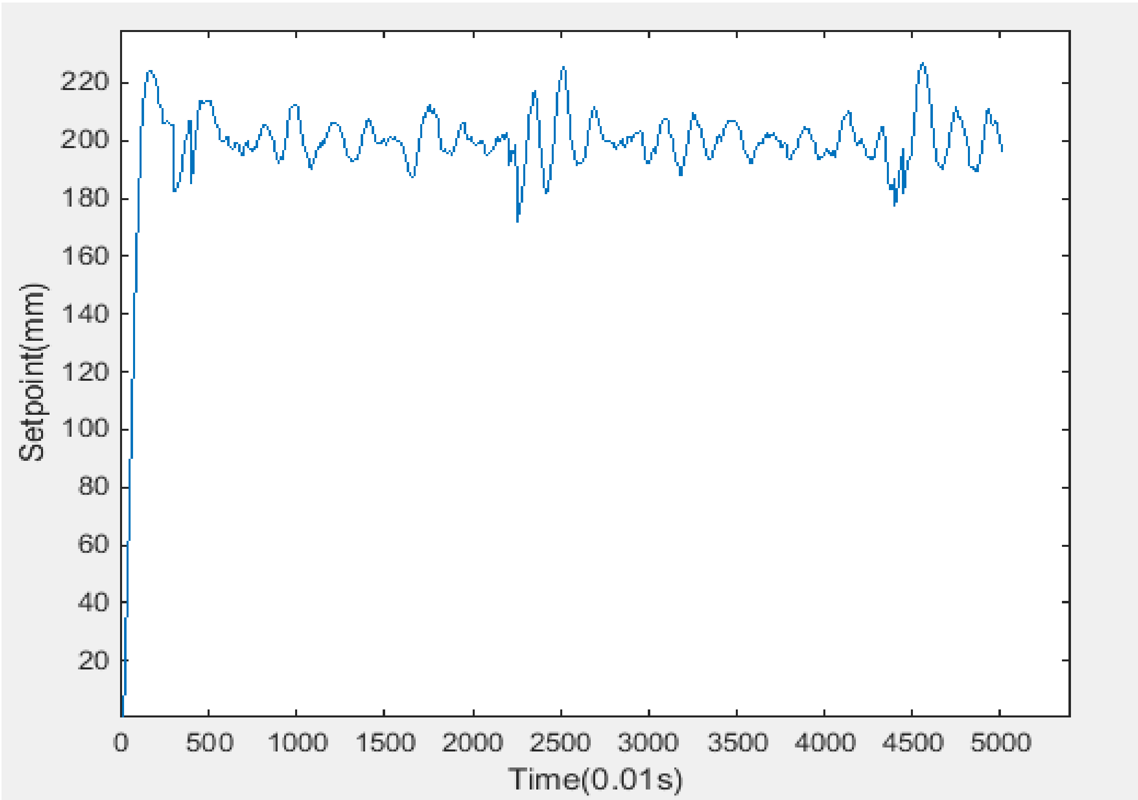

Figure 41: Set point 100mm controlled using ATV feedback relay method and Z-N parameters

We can obviously notice that using the starter parameter, the process respond positively with less oscillations to the higher heights than 100mm, which will make clearly and mainly based on lower heights, a new set of parameters of the PID mandatory.

Comparison of the controllers

The Z-N controller based on the feedback relay, appeared to be simple to achieve, however, tedious calculations are required in order to make it functional on the extent of voltage, it turns out to be much more complicated to achieve compared to the IMC controller, which of its simplicity and robustness, happens to be malleable, thanks to its single parameter T on the whole field tested.

Conclusion:

In this thesis we have successfully defined the levitation laboratory task, and presented the main parameters affecting the model identification, the estimated model has been implemented based on a large scale of measurements, led us to build two different perfectly working controller types applied to the real process.