Czech Technical University in Prague

Faculty of Electrical Engineering

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Pavel FEXA

Czech Technical University in Prague

Faculty of Electrical Engineering
Department of Measurement

Non-traditional ADC and DAC testing with poly-harmonic signals

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Pavel Fexa

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Supervisor: ***Assoc. Prof. Josef Vedral, CSc.***
Supervisor-Specialist: ***Assoc. Prof. Jan Holub, Ph.D.***

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Declaration

I hereby declare I have written this doctoral thesis independently and quoted all the sources of information used in accordance with methodological instructions on ethical principles for writing an academic thesis. Moreover, I state that this thesis has neither been submitted nor accepted for any other degree.

Prague, August 2015

Pavel Fexa

Abstrakt

Tato disertační práce shrnuje výsledky výzkumu testování dynamických vlastností analogově-číslicových a číslicově-analogových převodníků s využitím polyharmonických signálů.

Cílem práce je analyzovat vlastnosti netradičních metod testování dynamických vlastností převodníků za účelem rychlého a efektivního určení jejich vlastností v širším frekvenčním spektru. Při hledání těchto metod se práce zaměřuje na testování převodníků amplitudově a frekvenčně modulovanými signály, vícetónovými signály s rovnoměrně rozloženým amplitudovým spektrem a impulsní signály, jímž jsou tlumená sinusoida a signál typu sinc (sinx/x).

Tyto signály byly zvoleny tak aby reflektovaly měření či generování signálu ve skutečných podmínkách. Modulované a multitónové signály jsou typickými signály v radiotechnice, tlumená sinusovka odpovídá odezvě systému 2. řádu a signál sinc (sinx/x) obsahuje rovnoměrně rozložené diskrétní amplitudové frekvenční spektrum se shodnými amplitudami všech jeho složek.

Všechny tyto signály mají jedno pojítko a to je, že jejich amplitudová spektra obsahují více než jednu frekvenci. Tím získáme během jednoho měření komplexní informaci o dynamických vlasnostech testovaných převodníků, což výrazně redukuje dobu potřebnou k určení jejich parametrů. Tento fakt je významný zejména při mezioperačním měření parametrů převodníků při jejich hromadné výrobě.

Vlastnosti metod testování převodníků multiharmonickými signály byly nejprve teoreticky zdůvodněny, pak simulovány a poté ověřeny při testování reálných převodníků, či digitalizátorů spojitých signálů.

Výsledky testů převodníků multitónovými signály jsou vyhodnoceny jak v časové, tak frekvenční oblasti. Ze stěžejních parametrů, určujících dynamické vlastnosti převodníků jsou zvoleny *odstup signál šum včetně zkreslení vyššími harmonickými* (*SINAD*) a *efektívní počet bitů* (*ENOB*).

Výsledky těchto testů jsou porovnány s výsledky testů získanými klasickými metodami jak v časové oblasti - metodou nejlépe proložené sinusovky SWFT, tak ve frekvenční oblasti - metodou diskrétní Fourierovy transformace DFT v souladu se světovými standardy IEEE 1241 a IEEE 1658.

Problém s měřením je v jeho někdy zdánlivé jednoduchosti...

Autor neznámý

Abstract

This thesis summarizes the results from research in the field of dynamic testing Analog to Digital and Digital to Analog Converters using poly-harmonic signals.

The aim of the work is to analyze and verify properties of non-traditional methods for dynamic converters testing. The goal is to measure these parameters effectively and quickly in broader frequency range at the same time. When searching for new methods, this work was inspired with Amplitude Modulated, Frequency Modulated signals, Multi-Tone signals with Uniformly Distributed Amplitude Spectral Lines, and impulse signals such as Damped Sine Wave and (sinx/x) signal.

Previously mentioned signals were chosen to reflect measurement or generation in the real conditions. Modulated and Multi-Tone signals are common signals in radio engineering. A Damped Sine Wave signal corresponds with a step response of the second order system. Sinc (sinx/x) signal contains uniformly distributed amplitude lines in its frequency spectrum.

Signals which are stated here have one common link. Their amplitude spectra contain more than one frequency. This feature gives us complex information in one measurement and significantly reduces the time for estimation of converter’s dynamic parameters. This fact is significant in end of line testing during manufacturing of converters.

Attributes of new proposed methods using multi-harmonic signals were analyzes theoretically then simulated and finally verified by measurement on the real devices like digitizers or generators.

Results from tests using Multi-Tone signals are evaluated both in time domain and frequency domain. The dynamic parameters *Effective Number of Bits* (*ENOB*) and *Signal to Noise and Distortion* (*SINAD*) were chosen as the main parameters for methods evaluation.

Non-traditional test’s results are compared with results from standard methods like Best Sine Wave Fit Test evaluating converters in time domain or Sine Wave DFT Test testing converters in frequency domain according to a standards IEEE 1241 and IEEE1658.

The trouble with measurement is its seeming simplicity . . .

Author Unknown

Contents

List of Tables

List of Figures

Chapter 1 State of the Art

1.1 The Best Sine Wave Fit Test

There are two reasons that this method is stated in the thesis: firstly the deeper mathematic background is missing in [A1], secondly knowledge of the fitting algorithm can help in future research [A14].

This method belongs to the standardized methods. The acquired sine wave (a series of M data u) is reconstructed using 3 or 4 parameters Minimum Square Error algorithm. *Effective Number of Bits* (*ENOB*) can be estimated from a *Root Mean Square* (*RMS*) value of residuals e_{RMS} (the series of differences between acquired and reconstructed data); see (1.1) and (1.2) [A1].

$$ENOB = N - \log_2 \frac{e_{\mathrm{RMS}}}{2^{-N}/\sqrt{12}} (\mathrm{bit}) \tag{1.1}$$

, where N is ADC's or DAC's nominal number of bits [A1]

$$e_{\mathrm{RMS}} = \sqrt{\frac{1}{M}\sum_{n=1}^{M}\left(u_{FIT}(n) - u_{REC}(n)\right)^2} \tag{1.2}$$

, where M is number of acquired samples (length of both series) [A1]

1.2 Three Parameters Sine Wave Fit Test

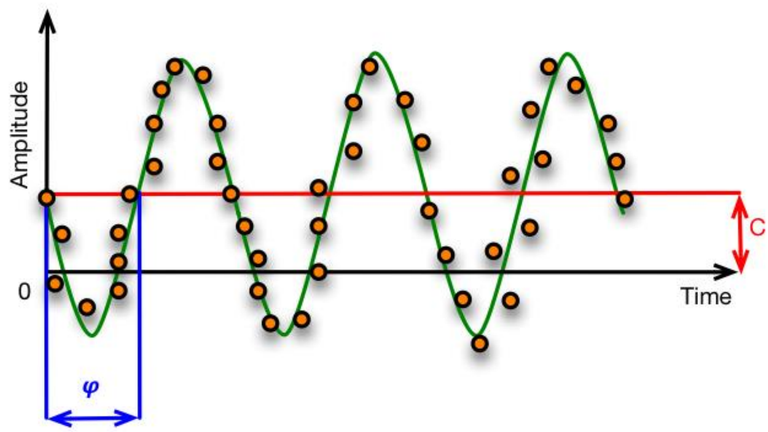

Figure 1.1 Three parameters sine wave fit test

This algorithm minimizes the error between acquired series of data $u_n=u_1, u_2,\ldots,u_n$, samples were acquired in times $t_n=t_1,t_2,\ldots,t_n$ and an ideal form of a sine wave signal described by the equation $X \cdot \cos(\omega_f t + \varphi)$. As you can see, the phase is in the argument of the non-linear function (cosine), but it is possible to find an elegant solution to avoid the non-linear fitting. The cosine function is split into two orthogonal components $A \cdot \cos(\omega_f t_n) + B \cdot \sin(\omega_f t_n)$ without the phase shift. Therefore there is linear problem to solve. The fitting algorithm tries to find coefficients A, B and offset C such that the (1.3) reaches its minimum [A1].

$$\sum_{n=1}^{M}\left(u_n - A\cdot\cos\left(\omega_f\ t_n\right)+B\cdot\sin\left(\omega_f\ t_n\right)-C\right)^2 \tag{1.3}$$

Using matrix notation, this equation can be rewritten as

$$(\mathbf{u} - \mathbf{Dx})^{\mathrm{T}}(\mathbf{u} - \mathbf{Dx}) \tag{1.4}$$

Where matrix **D,** vectors **u** (measured data) and **x** (vector of fitted parameters according to matrix **D**) are explained in equations (1.5) - (1.7)

$$\mathbf{D} = \begin{pmatrix} \cos(\omega_f\ t_1) & \sin(\omega_f\ t_1) & 1 \\ \cos(\omega_f\ t_2) & \sin(\omega_f\ t_2) & 1 \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cos(\omega_f\ t_n) & \sin(\omega_f\ t_n) & 1 \end{pmatrix} \tag{1.5}$$

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{pmatrix} \tag{1.6}$$

$$\mathbf{x} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \tag{1.7}$$

In real case there is no such a vector **x** that satisfied condition **u** = **Dx**. So there is a non-trivial error vector **e = u - Dx**, where **e** is defined by the next equation [A14]:

$$\mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{pmatrix} \tag{1.8}$$

Minimum of (1.3) can be found using matrix calculus [A15], [A17]. The local minimum is in this case also a global minimum (see the definition of the Euclidean Norm [A16]).

$$\begin{aligned} \left(\mathbf{e}^{\mathrm{T}}\mathbf{e}\right)' &= \left[(\mathbf{u}-\mathbf{Dx})^{\mathrm{T}}(\mathbf{u}-\mathbf{Dx})\right]' = \left[\left(\mathbf{u}^{\mathrm{T}} - \mathbf{x}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\right)(\mathbf{u}-\mathbf{Dx})\right]' \\ &= \left[\mathbf{u}^{\mathrm{T}}\mathbf{u} - \mathbf{u}^{\mathrm{T}}\mathbf{Dx} - \mathbf{x}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{u} + \mathbf{x}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{Dx}\right]' = -2\mathbf{D}^{\mathrm{T}}\mathbf{u} + 2\mathbf{D}^{\mathrm{T}}\mathbf{Dx} = 0 \end{aligned} \tag{1.9}$$

Now it is possible to solve the equation and compute desired vector **x**; see (1.10)

$$\mathbf{x} = \left(\mathbf{D}^{\mathrm{T}}\mathbf{D}\right)^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{u} \tag{1.10}$$

Fitted sine wave is written as:

$$u_{fit} = A\cdot\cos(\omega_f\ t_n) + B\cdot\sin(\omega_f\ t_n) - C = X\cos(\omega_f\ t_n + \varphi) + C \tag{1.11}$$

Where

$$\begin{aligned} X &= \sqrt{A^2+B^2} \\ \varphi &= \mathrm{arctg}\left(\frac{B}{A}\right) \text{ if } A \geq 0 \\ \varphi &= \mathrm{arctg}\left(\frac{B}{A}\right) \pm \pi \text{ if } A < 0 \end{aligned} \tag{1.12}$$

The *ENOB* is computed from the effective value of residuals e_{rms} see (1.1) and (1.2). The three parameters algorithm is used if the fundamental frequency is known.

1.3 Four Parameters Sine Wave Fit Test

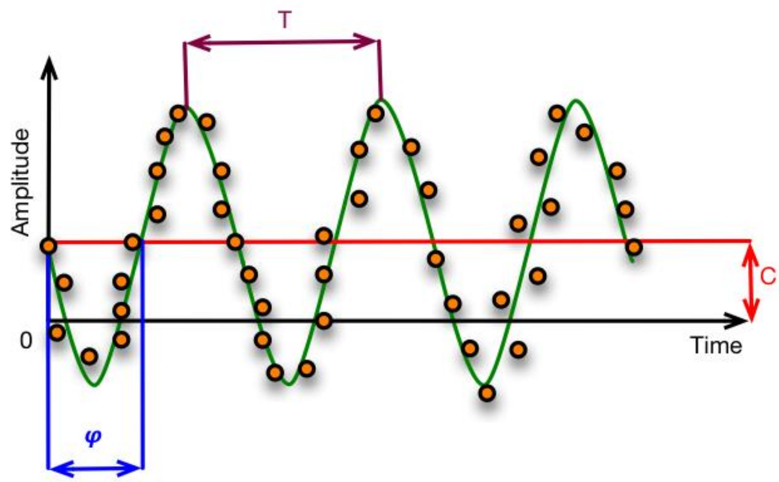

Figure 1.2 Four parameters sine wave fit test

Usually a fundamental frequency is not precisely known. The procedure is necessary in order to estimate a fundamental frequency using a frequency analysis or measure frequency with i.e. counter. Then the three parameters test is performed to get A_0, B_0, and C_0 (parameters corresponds to *A*, *B*, *C* (1.3) index 0 represents an initial condition in the iterative algorithm). The frequency is unknown therefore the (1.3) has to be modified, but it is in the argument of the non-linear function. So in each step *i* sine and cosine are expanded using Taylor series around estimated frequency $\hat{\omega}_i$; see (1.13) and (1.14). Thus additional information about frequency update is available [A18].

$$\cos(\omega_f\, t_n) \approx \cos(\hat{\omega}_i\, t_n) - \Delta\hat{\omega}_i t_n \sin(\hat{\omega}_i\, t_n) \tag{1.13}$$

$$\sin(\omega_f\, t_n) \approx \sin(\hat{\omega}_i\, t_n) + \Delta\hat{\omega}_i t_n \cos(\hat{\omega}_i\, t_n) \tag{1.14}$$

Where $\Delta\hat{\omega}_i$ is a difference between fundamental and estimated frequency

$$\Delta\hat{\omega}_i = \omega - \hat{\omega}_i \tag{1.15}$$

The fitting problem according to (1.16) is still nonlinear. When $\Delta\hat{\omega}_i \cong 0$, the problem as a pseudo linear one.

$$\sum_{n=1}^{M}(u_n - A\cos(\hat{\omega}_i\, t_n) - A\Delta\hat{\omega}_i t_n \sin(\hat{\omega}_i\, t_n) + \\ + B\sin(\hat{\omega}_i\, t_n) + B\Delta\hat{\omega}_i t_n \cos(\hat{\omega}_i\, t_n) - C)^2 \tag{1.16}$$

The Equation (1.16) says how to build the matrix $\mathbf{D}_i$, and vectors $\underline{\mathbf{u}}$ and $\mathbf{x}$.

$$\mathbf{D}_i = \begin{pmatrix} \cos(\hat{\omega}_i\, t_1) & \sin(\hat{\omega}_i\, t_1) & 1 & -\hat{A}_{i-1}\Delta\hat{\omega}_i t_1 \sin(\hat{\omega}_i\, t_1) + \hat{B}_{i-1}\Delta\hat{\omega}_i t_1 \cos(\hat{\omega}_i\, t_1) \\ \cos(\hat{\omega}_i\, t_2) & \sin(\hat{\omega}_i\, t_2) & 1 & -\hat{A}_{i-1}\Delta\hat{\omega}_i t_2 \sin(\hat{\omega}_i\, t_2) + \hat{B}_{i-1}\Delta\hat{\omega}_i t_2 \cos(\hat{\omega}_i\, t_2) \\ & \cdots & \cdots & \\ & \cdots & \cdots & \\ \cos(\hat{\omega}_i\, t_n) & \sin(\hat{\omega}_i\, t_n) & 1 & -\hat{A}_{i-1}\Delta\hat{\omega}_i t_n \sin(\hat{\omega}_i\, t_n) + \hat{B}_{i-1}\Delta\hat{\omega}_i t_n \cos(\hat{\omega}_i\, t_n) \end{pmatrix} \tag{1.17}$$

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{pmatrix} \tag{1.18}$$

$$\mathbf{x}_i = \begin{pmatrix} A \\ B \\ C \\ \Delta\hat{\omega}_i \end{pmatrix} \tag{1.19}$$

Iterative procedure according to a IEEE 1241 is shown in the next list, point 1) to 7) [A1].

1) Set i= 0 and compute 3 parameter test
2) Set i= i+ 1
3) Update $\hat{\omega}_i = \hat{\omega}_{i-1} + \Delta\hat{\omega}_{i-1}$, $\Delta\hat{\omega}_{i-1}$=0 for i=1
4) Create $\mathbf{D}_i$
5) Compute $\mathbf{x}_i = \left(\mathbf{D}_i^{\mathrm{T}}\mathbf{D}_i\right)^{-1}\mathbf{D}_i^{\mathrm{T}}\mathbf{u}$
6) Compute e_{RMS} and ENOB
7) Repeat 2) - 6) until convergence (practically $\Delta ENOB$ < 0.1 LSB) [A1]

Another point of view is stated in the IEEE1057. The 3 parameter test is shown in appendix A and the derivation of the 4 parameter test is presented appendix B in IEEE Std. 1057 [A3].

1.4 Step Gauss Test

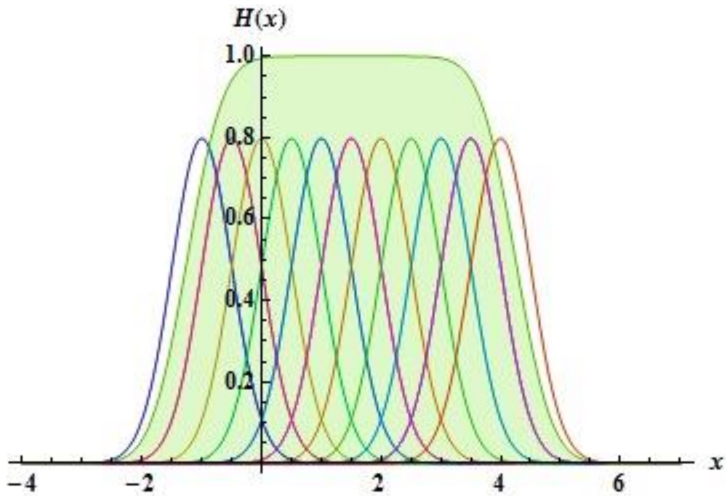

Figure 1.3 Histogram – Sum of Gaussian noises with equidistantly spaced DC shifts

A random or pseudorandom signal with uniform distribution is created using noise with normal distribution as a source. This signal is offset by another DC source. The result is slightly ripped uniform histogram, but it's possible to reduce this ripple in order to achieve conditions suitable for testing of high resolution DUTs. Benefit of this method is that we can test the full scale of a DUT. Two chosen parameters (standard deviation of the random signal and DC offset) also limit the signal which is out of range of the DUT. Disadvantage of this method lies in a huge amount of samples are needed for testing (approximately 3.10^8 Samples) [A5], [A6].

1.5 Discrete Fourier Transform Test

This method applies *Discrete Fourier Transform* on the set of acquired data according to (1.20) and a frequency spectrum of the signal is computed [A8].

$$X(k) = \sum_{i=0}^{M-1} u(i) e^{-j2\pi ik/M} \tag{1.20}$$

In (1.20) $X(k)$ is a sequence of M complex frequency components, M is a number of samples.
The frequency resolution (*frequency bin*) is given by a sampling frequency and the number of samples according to the sampling theorem [A8]

$$\Delta f = \frac{f_s}{2M} \text{(Hz)} \tag{1.21}$$

To reduce noise and improve accuracy the averaged spectrum is computed (RMS averaging). IEEE 1658 uses magnitude averaging; see (1.23) [A4]. Measurements in Chapter 5 and Chapter 6 were programmed in LabVIEW. Function used in LabVIEW's code use RMS averaging which was applied on acquired data; see the (1.22). [A4]

$$X_{RMS}(k) = \sqrt{\frac{1}{M}\sum_{i=0}^{M-1} X_i^2(k)}, i = 0,1,2,\ldots,M-1 \tag{1.22}$$

$$X_{avg}(k) = \frac{1}{M}\sum_{i=0}^{M-1} \left|X_i\ (k)\right|, i = 0,1,2,\ldots,M-1 \tag{1.23}$$

A condition for *coherent sampling* is satisfied if the integer number of the periodic signal's cycles is acquired; see (1.24) [A21].

$$\frac{f_{SIGNAL}}{f_{SAMPLING}} = \frac{k_{CYCLES}}{N_{RECORD}} \tag{1.24}$$

Where f_{SIGNAL} is the frequency of the signal, f_{SAMPLING} is the sampling frequency, k_{CYCLES} is integer number of the signal's cycles, N_{RECORD} is number of acquired samples [A21].

Dynamic range is defined in (1.25) when condition of coherent sampling is fulfilled [A22].

$$DR_{DFT} = 6.02n + 1.76 + 20\log\frac{M}{2} \text{(dB)} \tag{1.25}$$

The window function has to be applied in case of non-coherent sampling. A windowed DFT is stated in (1.26) [A4]

$$X_w(k) = \sum_{i=0}^{M-1} w(i)u(i)e^{-j2\pi ik/M} \tag{1.26}$$

The brief table of commonly used windowing function is stated in Table 1. Dynamic Range is modified with the *Equivalent Noise Bandwidth ENBW* [A19].

Table 1 Windowing functions

Type of window	Side lobe level (dB)	*ENBW*
Hann	- 32 dB	1.5
Hamming	- 43 dB	1.37
4 Term Blackman-Harris	- 58 dB	2.01
Flat top	-96 dB	3.77

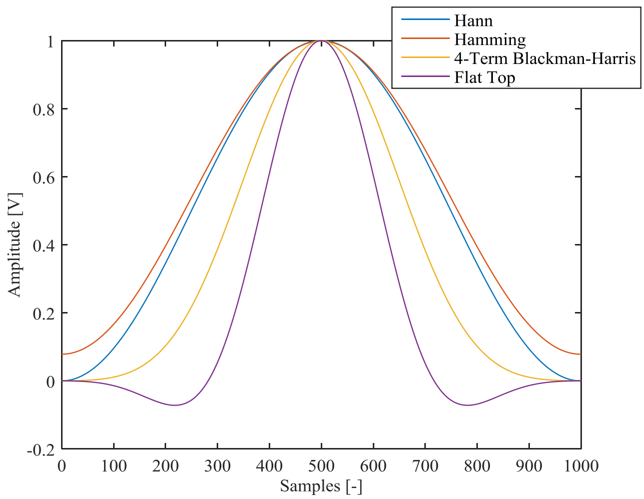

Figure 1.4 Normalized windowing function in the time domain plotted in Matlab [A23]

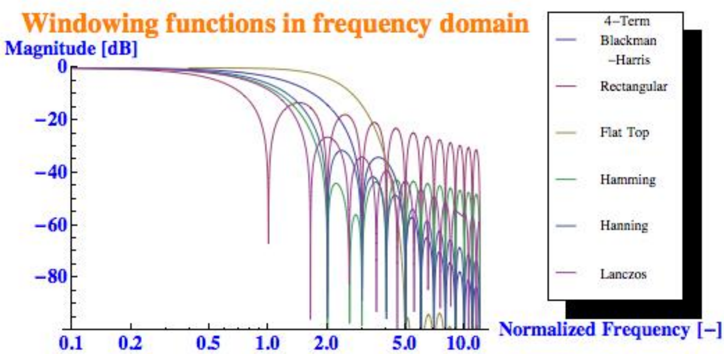

Figure 1.5 Normalized windowing function in the frequency domain plotted in Mathematica [A20]

Figure 1.5 shows chosen types of the windowing functions. It is necessary to choose the main lobe gain -10 dB bellow expected noise of the DUT. Standard window for testing is a 4th order *Blackman-Harris window* [A20].

From the spectrum one can compute following parameters: *Total Harmonic Distortion* (*THD*), *Signal Noise and Distortion* (*SINAD*), *Signal to Non Harmonic Distortion* (*SNHR*), *Spurious Free Dynamic Range* (*SFDR*) [A13].

THD is defined as the total power of a specified set of harmonics divided by the power of fundamental. It is often expressed as a percent or in dBc [A1].

$$THD_{1T} = \frac{X_{f_1}}{\sqrt{\sum_{i=2}^{M/2} X_{f_i}^{2}}} \tag{1.27}$$

SINAD is defined according to a standard IEEE 1658 as:

For a pure sine-wave input of specified amplitude and frequency, the ratio of the root-mean-square (*RMS*) amplitude of the digital-to-analog converter (DAC) filtered reconstructed output sine wave to the *RMS* amplitude of the output noise and distortion [A4].

$$SINAD_{1T} = \frac{X_{f_1}}{\sqrt{X_n^2 - \sum_{i=2}^{M/2} X_{f_i}^2}} \tag{1.28}$$

SNHR is defined only for ADC

$$SNHR = \frac{X_1}{\sqrt{\sum_{i=1}^{l} X_{ni}^2 - RMS_{NOISE}^2}} \tag{1.29}$$

SFDR specifies the available signal range as the difference in magnitude between the amplitude of the fundamental and the amplitude of the largest spurious component in the frequency band of interest [A4].

$$SFDR = \frac{X_1}{X_m} \tag{1.30}$$

ENOB is related to *SINAD* according to the next formula (1.31) [A2]:

$$ENOB_{1T} = \frac{SINAD_{1T}(dB) - 1.76}{6.02}(\text{bit}) \tag{1.31}$$

1.6 Summary

There are several disadvantages when ADC or DAC is tested using the fitting sine wave method and DFT test.

Parameters of a reference testing system have to be much better than expected parameters of a DUT (expensive low noise and distortion generators, high resolution digitizers and or precise filters are needed to satisfy this condition).

When digitizing a sine wave, it is necessary to acquire *M* samples (1.32) to test every code word; (1.32) is applicable in ideal case when no random noise is presented [A1].

$$M = \pi 2^{n} \tag{1.32}$$

One disadvantage when evaluating DUT using a DFT test is that a condition of coherent sampling (1.24) has to be satisfied or windows has to be applied.

Another disadvantage of DFT test is that the number of acquired samples has to satisfy recommendation that noise floor (1.33) of the computed spectrum including used window is 10 dB lower than noise floor of the tested device [A1]. This fact is described using a parameter *Noise Floor NFL.*

$$NFL = 6.02n + 1.76 + 10\log_{10}\left(\frac{M}{2ENBW}\right)(dBFs) \tag{1.33}$$

where *M* is acquired number of samples, *ENBW* is an effective noise bandwidth of a chosen window and *n* is a tested device’s nominal number of bits.

The Step Gauss test is one of the most demanding method as far as number of acquired samples is concerned. For estimation of *ENOB* it is necessary to acquire *M* samples; see (1.34) [A6].

$$M \approx 2^{2n} \tag{1.34}$$

Chapter 2 Objectives and thesis's arrangement

The aim of the thesis was to find new non-traditional signals for ADC or DAC testing. The work came from standartized method such as sine wave fit test or sine wave DFT test.

The thesis is motivated from the previous research in my master degree thesis which was focused on minimizing the number of samples required. The main contribution of that thesis was the formula which shows you how many samples are necessary for chosen uncertainty of the *ENOB* for the case the ADC is tested by a *Step Gauss Method* [A5]. The previous research led to one important result. Histogram methods are well defined. It is possible to calculate how many samples are needed for chosen uncertainty. But the number of samples is enormous. Those methods could be used in future for very fast ADCs [A6].

My work comes out from several well described standard methods for testing of ADCs and DACs. These methods such as a Single tone Best Sine wave Fit Test or a FFT (resp. DFT) test are using high spectrum purity input signal [A1]. *Effective Number of Bits ENOB*, *Total Harmonic Distortion THD*, *Signal Nonharmonic Distortion SNHR*, *Spurious Free Dynamic Range SFDR* and *Intermodulation Distortion IMD* can be determined with high accuracy using these methods. However, tests are relatively time-consuming especially, when it is necessary to analyze frequency characteristic of previously mentioned parameters. Some papers deal with an application of a frequency swept or an exponential signal suitable for economic tests. The methods for ADC testing are described in [A7], [A9], [A10], [A11], [A12]. The thesis will be linked to this research, and will extend it into usage of multi-tone band signals.

The aim of this thesis is to find new non-traditional methods for fast testing of *Digital to Analogue Converters* (DAC) or *Analogue to Digital Converters* (ADC). Most of dynamic methods can be applied on a ADC as well as on a DAC. It is possible to combine them into one group: Devices Under Test (DUT). This abbreviation will be used in the rest of the text unless there is explicitly stated an ADC or a DAC [A1], [A4]. The major of the thesis deals with testing in frequency domain using Discrete Fourier Transform Test (DFT) [A8]. Methods other than DFT methods will be explicitly stated in the text, if not, FFT method is meant. For demonstration and comparison purposes with FFT methods, I will show a fitting algorithm for testing in time domain.

The main goal was to find such a signal which was suitable for fast ADC's or DAC's testing. What exactly do words like "*fast*" "*effective*" or *"economic"* (testing of DUTs) mean? They mean testing a DUT in reduced amount of time in comparison with standardized FFT methods. How to achieve this goal? It is possible to increase sampling or generating rate, but there is always limit for a chosen DUT. The second option is to reduce the number of samples, which are acquired during a test. This way is also sporadic especially when evaluating a DUT in frequency domain. Frequency resolution of computed spectrum is reduced by lower number of samples.

The alternative way is to use a poly-harmonic signal. DUT's performance is checked at multiple frequencies at once in this way. The results are expected to be worse than results from a single tone test, but they should reflect the real performance of the DUT. The thesis will focus on methods using band signals (signals with wide frequency range). Signals had to fullfil requirements for fast and effective testing. It means a testing process with a signal has to be fast. This condition was satisfied when there was less signal processing or the number of samples were reduced, or a test result with a chosen signal showed more information in one measurement in comparison with the standardized method. To achieve this several points had to be fulfiled:

1. Theoretical analysis (see Analysis in Chapter 4 - Chapter 6).
2. Finding out the set of suitable signals (Chapter 4 - Chapter 6).
3. Designing a test system to get consistent results from proposed methods (Chapter 3).

Analyzing and verifying non-traditional tests using:

4. Fitting algorithm using exponential and chirp signals (Chapter 4).
5. Modulated signals in frequency domain (Chapter 5).
6. Impulse signals in frequency domain (Chapter 6)

Each chapter is also focused on:

7. Each chaper is also focused on evaluation methods, finding out their benefits and disadvantages (see Summary in Chapter 4 - Chapter 6).
8. Each chaper also includes comparison of new methods with standardized method (see Summary in Chapter 4 - Chapter 6).

The thesis was published as a commented set of published paper in journals or conference proceedings. There are 5 papers inserted in Chapter 3 to Chapter 6. These ones are enriched with introduction, analysis and broadening comments to the article in summary. The list of publication is divided into sections related to the chapter and paper or papers inserted in the chapter. Citations related to the inserted paper are one to one copy from the article. This makes citations easier to follow in the text. Thesis contains 39 pages from papers and 40 pages with extension text. Thesis contains a DVD with an electronic form of inserted articles.

Chapter 3 Developing Automated Data Acquisition System for ADC and DAC Testing

3.1 Introduction

The paper *Developing Automated Data Acquisition System for ADC and DAC Testing* is stated in this chapter in order to introduce different test setups which were used in this work. Before finishing development of the testing system, two tasks have to be finished. A right hardware has to be chosen. The second job was to build an application with chosen SW. Selection of hardware affects a software parts and vice versa. Solution of such a task is presented in the inserted paper.

This chapter is divided into four subparts.

1. Introduction,
2. Analysis,
3. Summary,
4. Inserted article:
   Developing Automated Data Acquisition System for ADC and DAC Testing.

3.2 Analysis

There were several devices suitable for developing an automated system. List of some solutions is stated in Table 2

Table 2 Possible HW, SW solutions

DUT	Reference Device	Platform/Interface	SW
Oscilloscope Tektronix MSO 4000	14-bit generator Tektronix AFG3102	GPIB [B17]	LabVIEW, LabWindows [B16], C#
Altera II kit	Oscilloscope Tektronix MSO 4000	FPGA/GPIB	VHDL/LabVIEW, LabWindows, C#
Soundcard	24-bit digitizer NI PXI 5922	USB/PXI	LabVIEW, LabWindows, C#
16-bit multi-function card NI PXI 6251	24-bit digitizer NI PXI 5922	PXI	LabVIEW, LabWindows, C#

Some interesting information follows from the Table 2. The first and second rows present homogenous system based on the same bus (GPIB) or the same platform (PXI) and the same software environment. Other possible solutions build on heterogeneous platforms offers an interesting programming challenge; so PXI platform in combination with LabVIEW was a perfect match when minimizing effort during developing system.

3.3 Summary

Preliminary measurements were done manually using an arbitrary generator Tektronix AFG3102 [B1] which has 14 bit DAC. The DUT was an oscilloscope Tektronix MSO 4000 [B2]. Signal were collected and processed in Matlab [B3]. This procedure was quite time consuming so it was necessary to create a testing system using PXI chassis [B4] with 24 bit reference digitized NI PXI 5922 [B5].

A testing of several devices like a sound card, a PXI generator NI PXI 6251 [B6] or a generator SMIQ 03B [B7], [B8], [B9] and [B10] (see Chapter 5, paper *FM and QAM signals for ADC Testing*) was allowed by PC driven PXI system. PXI system was programmed in a graphical environment LabVIEW [B11], [B12], [B13], [B14] from National Instruments [B15].

System could generate signals such as

Chirp (see Chapter 4) [C3],

AM, FM, QAM (see Chapter 5) [C4], [C6]

Sinc, DSW, DSW with symmetrical repetition, single tone dual tone, triple tone, quad tone signals (see Chapter 6), [C5].

The final version was presented as the authorized software. Decentralized modification of the system was prepared for cases when the performance of the PC is not enough. It was interesting job to do. Measurement task and Generating task were split into two separate programs with client server architecture.

The system can operate the generator NI PXI 6251 and the digitizer NI PXI 5922 at the same time. Generation and acquisition more than 4Msamples was achieved with distributed (decentralized) solution. Post-processing of the signal was made in Matlab and has not been implemented into a testing system.

The 6th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications
15-17 September 2011, Prague, Czech Republic

Developing Automated Data Acquisition System for ADC and DAC Testing

Note! Please DO NOT write author(s) name(s) and other contact information in the extended abstract as it should be reviewed anonymously by the IDAACS'2011 International Program Committee.

Abstract— **This paper shows implementation of Data Acquisition System for Analog-to-Digital-Converters ADC and Digital-to-Analog-Converters DAC in LabView. This system was developed for purpose of evaluation of ADC's and DAC's dynamic parameters using single and multi-frequency signal. Virtual Instrument was created in LabView to control data generation and acquisition. Paper briefly shows you methods, which were used for ADC and DAC testing.**

Keywords*— *ADC, DAC, dynamic testing, SINAD, ENOB, LabView, FFT test, single and multi-tones, AM, FM, SINC, sweep signals, damped sine wave

I. Introduction

This paper shows a development automated system for multi-frequency dynamic test of ADC and DAC. In Fig. 1 is shown primordial arrangement for ADC Testing System.

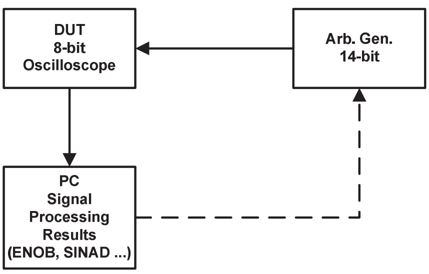

Figure 1. Simple measuring arrangement

The generator TEKTRONIX AFG3102 with 14 bit DAC and 250 MSa/s was used as a reference. A device under test was internal ADC in Mix Signal Oscilloscope TEKTRONIX MSO 4000 with sample rate 5 GSa/s and memory 10 MByte. This arrangement was used for AM and FM FFT test verification [3]. This arrangement was suitable for testing Device Under Test DUT with 8-bits nominal resolution. To achieve better resolution it was necessary to find reference device with higher resolution. A solution was to use PXI system. Reference device was Chosen 24-bit digitizer (NI PXI 5922). Tested device was a 16 bit generator (NI PXI 6251 multifunction card) .

II. Realisation

PXI system from National Instruments offer modularity in chosen device. Together with hardware and graphical development environment LabView a Virtual Instrument is created.

We can change measuring device to change target application. In this case we can easily set up test of the NI PXI 6251 Fig. 2.

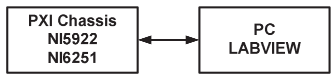

Figure 2. Arrangement for NI PXI 6251 Testing

Or we can easily change measuring system to test i.e. Soundcard, see Fig. 3.

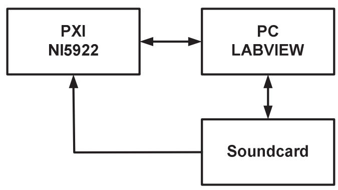

Figure 3. Arrangement for Audio Codec Testing

PC with multi-core processor enable us to generate and acquire data at the same time in two parallel tasks.

Decentralized arrangement was established for the case if it is necessary to divide measuring and data acquisition task (if the performance of the PC is not enough), see Fig. 4. Trick is to use PXI Chassis with controller inside. Then requirements for the performance are divided into PC and PXI controller. Again rearranging system was not time-demanding. A server client application, using TCP protocol, is used for synchronization measuring and generating tasks. Both tasks start to run independently through initialization phase. When measuring task is ready sent start signal to generating task via TCP and waits for trigger from signal. When generating task was prepared before measuring task, it waits for start signal. After receiving signal generating task can generate chosen signal. After both tasks were completed, they continue again to init phase.

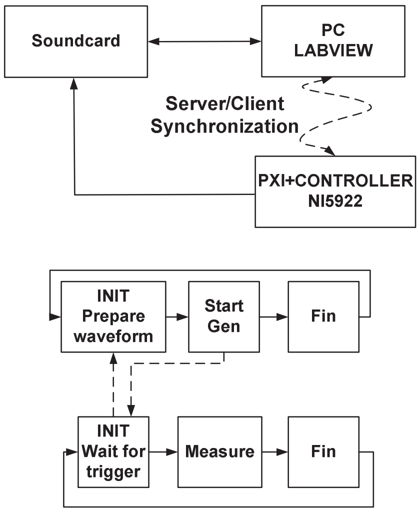

Figure 4. Decentralized solution

User can change parameters of the signal (frequencies, modulation index, damping ratio). Acquired data and spectra are saved automatically to chosen destination folder, to maximize saving time for user. For correction method the Crest Factor is computed from generated waveform and from measured waveform (for comparison).

III. EXAMPLES OF GENERATED SIGNALS AND METHODS

A. Single tone test

Signal Noise and Distortion SINAD is the ratio of the *RMS* value of the carrier frequency to the mean value of the root-sum-square of all other spectral components, including harmonics, but excluding dc component. [1], [2]

$$SINAD = n - \log_2 \frac{RMS_{\mathrm{NOISE}}}{2^{-n}/\sqrt{12}} \qquad (1)$$

where n is number of nominal bits of converters under test.

For single tone signal Signal Noise and Distortion and Effective Number of Bits is given by following equation.

$$SINAD = 6.02\,ENOB + 1.76 \;\; (\mathrm{dB}) \qquad (2)$$

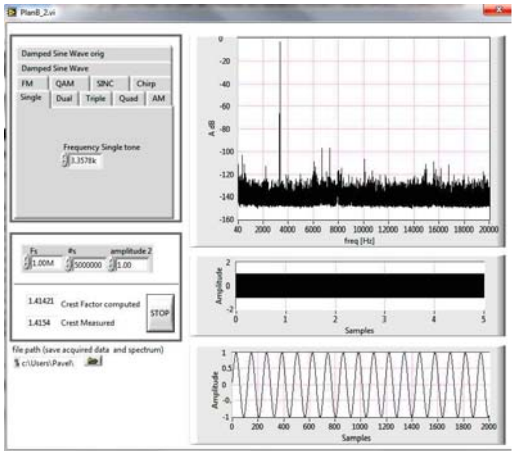

Figure 5. GUI of the Automated Data Acquisition System for Testing - sine wave test

B. Dual tone test

Crest Factor $CF = X_{pp} / (2\sigma_x)$ can be computed for every signal. Formulas (1), (2) are derived for the case the single sine wave signal is used in the test. For other case it is necessary to compensate the influence of different *CF* and correct *SINAD* and *ENOB* using $\Delta SINAD$ (3) and $\Delta ENOB$ (4) correction.

$$\Delta SINAD_{\mathrm{AM}} = 20 \log CF_{\mathrm{AM}} \; (\mathrm{dB}) \qquad (3)$$

$$\Delta ENOB_{\mathrm{AM}} = \frac{\Delta SINAD_{\mathrm{AM}}}{6.02} \qquad (4)$$

CF for multi-tone signal with uniform amplitudes (6) is:

$$CT_{\mathrm{MT}} = \sqrt{2m} \qquad (5)$$

$$u_{\mathrm{MT}} = \sum_{i=1}^{m} U_i \sin(\omega_i t) \qquad (6)$$

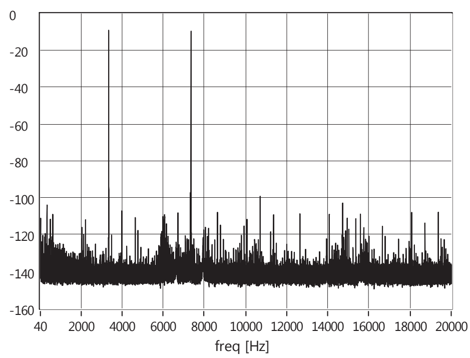

Figure 6. Dual tone test spectrum

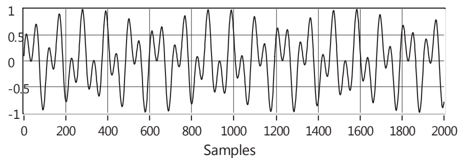

Figure 7. Dual tone test - 2k Samples cut from the waveform

C. *Quad tone test*

In Fig. 8 is presented test using quadruple tone signal.

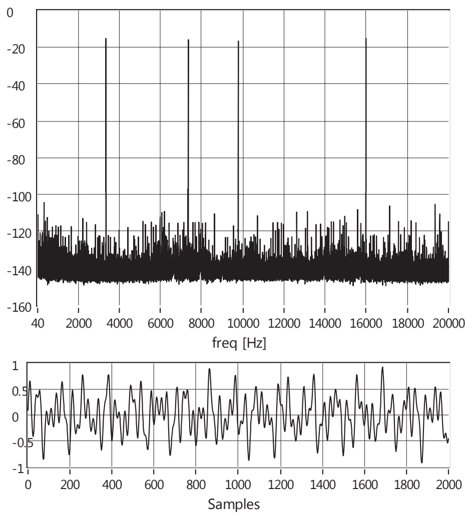

Figure 8. Quadruple tone test spectrum and 2k Samples cut from the waveform

D. *AM test*

The amplitude modulated signal is defined by formula:

$$u_{\mathrm{AM}} = \left(U_{\mathrm{c}} + U_{\mathrm{m}} \cos \omega_{\mathrm{m}} t\right) \sin \omega_{\mathrm{c}} t =$$
$$= U_{\mathrm{c}} \sin \omega_{\mathrm{c}} t + \frac{U_{\mathrm{m}}}{2} \left[\sin\left(\omega_{\mathrm{c}} - \omega_{\mathrm{m}}\right) \cdot t\right] \quad (5)$$
$$+ \frac{U_{\mathrm{m}}}{2} \left[\sin\left(\omega_{\mathrm{c}} + \omega_{\mathrm{m}}\right) \cdot t\right]$$

Crest factor of AM signal could be expressed as:

$$CF_{\mathrm{AM}} = \frac{2(1 + m_{\mathrm{AM}})}{\sqrt{2 + m_{\mathrm{AM}}^2}} \quad (6)$$

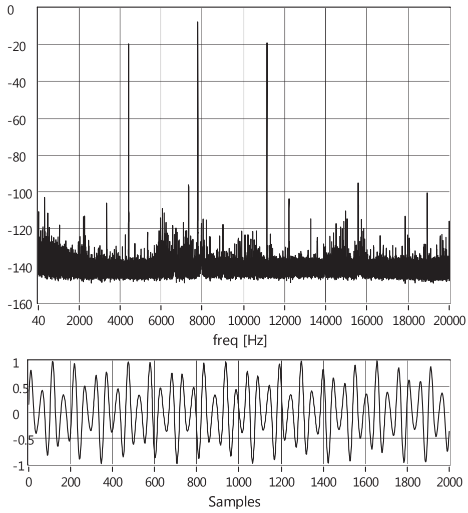

Figure 9. AM tone test spectrum and 2k Samples cut from the waveform

E. *FM test*

Waveform of the frequency modulated signal is:

$$u_{\mathrm{FM}} = U_{\mathrm{c}} \sin\left(\omega_{\mathrm{c}} t + \frac{\Delta\omega}{\omega_{\mathrm{m}}} \cos \omega_{\mathrm{m}} t\right) \quad (7)$$

FM signal has same Crest Factor. It is possible to use standard formula (1), (2).

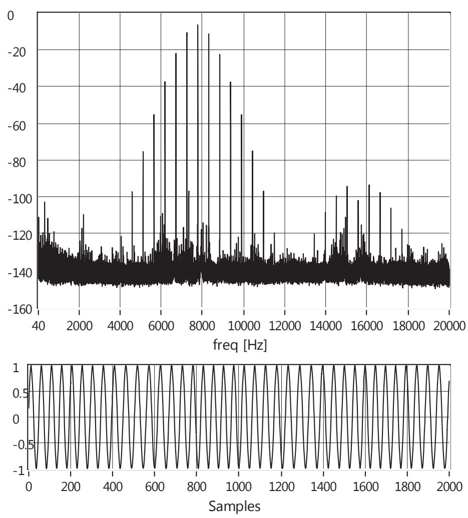

Figure 10. AM tone test spectrum and 2k Samples cut from the waveform

F. Sinc test

Waveform of the sinc signal is:

$$u(t) = \frac{\sin\left(\frac{2\cdot\pi\cdot t}{T_2}\right)}{\frac{2\cdot\pi\cdot t}{T_2}}$$

And the crest factor can be computed by this equation or measured directly using our measuring systems. Then correction could be applied.

$$CF = \frac{Max[u(t)]}{RMS} = \frac{1}{\sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} u^2(\mathrm{t})dt}}$$

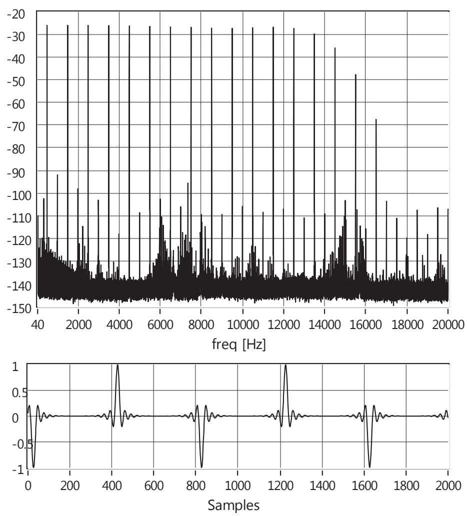

Figure 11. Sinc tone test spectrum and 2k Samples cut from the waveform

IV. Results

The testing system was created for a purpose of a method evaluation. A 16 bit multi-function card NI PXI 6251 was chosen as DUT. 24-bit digitizer NI PXI 5922 provides a function of reference device. Whole system was controlled by PC with LabView. Acquired data were processed in Matlab.

Application of the automated measurement system was successfully proven for DAC testing. Table I. shows comparison multi-tone tests (like 2-Tone, 3,4-Tone, AM and FM) with single tone sine wave FFT Test. All results was evaluated in frequency domain. Hanning window was used. Crest Factor *CF* is computed form original signal, CF_c respect influence of the Hanning window. $SINAD_C$ and $ENOB_C$ are corrected using CF_c. Details about methods correction are shown in [4],[5].

TABLE I. METHODS COMPARISON USING *ENOB* AND *SINAD* AS QUALITY INDICATOR

Methods:	*CF* (-)	CF_C (-)	*ENOB* (bit)	*SINAD* (dB)	$ENOB_C$ (bit)	$SINAD_C$ (dB)
1 Tone	1.41	2.31	13.4	82.5	14.1	86.7
2 Tone	2.00	3.26	12.9	79.4	14.1	86.7
4 Tone	2.83	4.61	12.4	76.7	14.2	86.9
AM (*m*=0.25)	1.74	2.84	12.9	79.9	14.0	86.0
AM (*m*=0.50)	2.00	3.26	12.8	79.0	14.0	86.2
AM(*m*=1.00)	2.31	3.77	12.6	77.8	14.1	86.3
FM (*m*=0.25)	1.41	2.31	13.2	81.4	13.9	85.7
FM (*m*=0.50)	1.41	2.31	13.2	81.4	13.9	85.6
FM (*m*=1.00)	1.41	2.31	13.2	81.3	13.9	85.5
Sinc 17 Tones	5.6	9.1	69.9	11.3	14.2	86.9

Acknowledgment

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Chapter 4 Methods for economical test of dynamic parameters ADCs

4.1 Introduction

The paper *Methods for economical test of dynamic parameters ADCs* brought my the research into the area of band signals. The idea came from measuring the frequency dependency of an *ENOB*. Measurement shows that function *ENOB*(*f*) is not a flat function. An *ENOB* reduction was observed close to the bandwith of the ADC. The *ENOB* drop at higher frequencies was interesting with combination of using a band signal in this case Exponential Fit Test and Wobbler Test.

This chapter is divided into four subparts.

1. Introduction,
2. Analysis,
3. Summary,
4. Inserted article:
   Methods for economical test of dynamic parameters ADCs.

4.2 Analysis

Definitions of an exponential (chapter 4.2.1) and linear chirp signal (4.2.2) are presented in this chapter. Theoretical equations are enriched with experiences which came from evaluation of test's results.

4.2.1 Exponential Fit Test

The digitized signal in Exponential Fit Test is fitted and *ENOB* and *SINAD* is evaluated from an *RMS* of residuals between fitted and measured signals. The function which represented testing signal is stated in (4.1)

$$u = Ae^{-Bt} + C \tag{4.1}$$

Fitting such a signal means to estimate parameters *A*, *B*, *C*. This task thus end with non-linear fitting problem. Measured data were fitted using Matlabs' curve fitting toolbox which offer also non-linear fitting [D1]. During fitting of the (4.1) this toolbox failed to converge.

Practical experience showed that previously stated equation does not reflect the phase shift of the signal. I have choosed two ways how to overcome this problem. The first solution is to add phase shift φ into (4.1), see (4.2)

$$u = Ae^{-B(t+\varphi)} + C \tag{4.2}$$

This leads there is another parameter inside exponential function to fit. My experience with a non-linear fitting is that estimation of fitted parameter‘s initial value close to the real value is required; otherwise the algorithm fails.

The other method is synchronous measurement or post processing of acquired signal in order to eliminate the phase shift. Then it is possible to use (4.1) or (4.2) but with known φ. The second aproach led to the result comparable with the sine wave fit test.

Result of exponential fit test should be closer to the *ENOB* measured at lower frequencies because spectrum of the exponential signal drops with a slope 20dB/decade. In this case major deviation causes the drop of the fitting process quality.

4.2.2 Wobbler Test

More promising signal for measuring *ENOB* in whole bandwith during one measurement is frequency swept sine wave signal in Wobbler Test. Linear chirp uses a *linear chirp signal* [D2].

$$u = \sin\left(2\pi\left(f_0 t + \frac{k^2}{2}t^2\right)\right) \tag{4.3}$$

Spectrum of such signal is shown in the Figure 4.1. The figure was plotted in Matlab [D1].

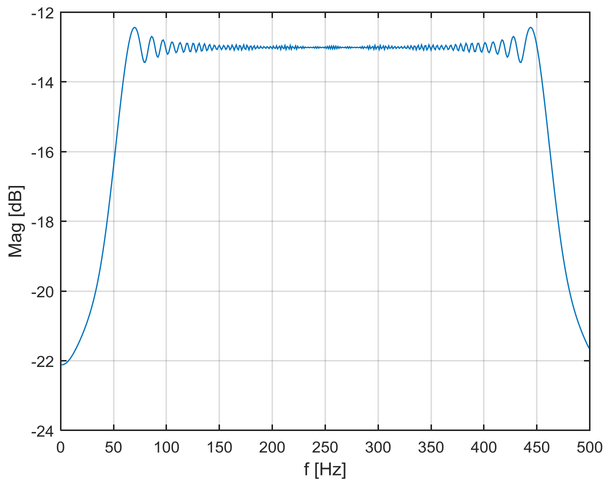

Figure 4.1 An example of chirp signal using linear sweeping function

The inserted paper shows that non linear fitting of a lineary swept signal could be solved by splitting signal into series quasi-periods and fitting them one by one.

Again a fitting of a parameter inside non linear function leads to troubles. Transformation of a nonlinear task into a linear one is possible by using dicreete series of sweeping frequencies instead of using continuous linear sweep function. Thanks to this procedure signal is divided into series of sine wave signal step-sweep like in vector analyzers or ground penetration radars [D3], [D4] which are possible to fit with three parameters sine wave fit test. [E2], [D5].

The results of Wobbler Test should represent average *ENOB* in chosen bandwith.

4.3 Summary

There are two methods shown in the paper *Methods for economical test of dynamic parameters ADCs*. The first one uses an exponential signal. General application of this signal however leads to non-linear fitting problem. The same situation concerns Wobbler Test. But solving problems with non-linear fitting will open new possibilities when it comes to dynamic testing of DUTs.

The following table presents test results from measurement using *Sine Wave Fit Test* (SWFT), *Discrete Fourier Transform* Test (DFT), *Wobbler Test* and *Exponential Fit Test*. In this case an internal ADC in FPGA Altera II kit was tested. The reduction of necessary samples is evident in the last row of the table.

Table 3 Test Results from Economic Tests

Sine Wave Fit Test and Discrete Fourier Transform Test			Wobbler Test	Exponential Fit Test
Frequency of input signal	1 kHz	10 kHz	20 Hz – 20 kHz	
ENOB	14.7 bit	13.8 bit	14.1 dB	13.9 dB
SINAD	90.2 dB	84.8 dB	-	-
THD	103 dB	95 dB	-	-
SFDR	102 dB	92 dB	-	-
SNHR	94 dB	86 dB	-	-
ER	15.4 bit		-	-
Number of samples	64 k in 10 steps of frequency range 20 Hz – 20 kHz		5120	1024

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METHODS FOR ECONOMICAL TEST OF DYNAMIC PARAMETERS ADCS

Josef Vedral, Pavel Fexa, Jan Svatoš

Czech Technical University in Prague, Faculty of Electrical Engineering, Department of Measurements, Technicka 2, CZ 16627, Prague 6 Czech Republic, (☒ *vedral@fel.cvut.cz,*, +420 2 2435 2182, *fexap1@fel.cvut.cz, svatoj1@fel.cvut.cz*)

Abstract

This paper describes two methods for economical test of dynamic parameters ADCs. First method is Exponential Fit Test, second method is Wobbler Test. Common testing methods are mentioned as far the accuracy and time necessary for the complete test are concerned. The tests for fast evaluation of the dependence of an effective number of bits on frequency of input signal are described and the comparison of proposed method with the standard methods is given. The suitable area of proposed method application is "each-piece" factory testing requiring extremely short time testing.

Keywords: ADC Test, Exponentional Fit Test, Wobbler Test, Effective Number of Bits.

1. Introduction

There are several common well-described indirect methods of ADC testing that are suitable for an evaluation of the reduction of the Effective Number of Bits (*ENOB*) on the frequency of input signal [1], [2]. For example, the if one has 16-bits AD converter, it is necessary requires to take 64 kilo samples for 0,1 LSB error of estimation of *ENOB* with Sine Wave Fit Test and equivalent number of samples for 0.1dB error of estimation of Signal Noise and Distortion SINAD with Discrete Fourier Transform Test. These series of samples must be taken for each frequency point individually. Similarly to the previous case, the series of harmonic signals must be sampled. It is necessary to avoid leakage error by using coherent sampling.

2. Principles of the exponentional fit test

The Exponentional Fit Test is based on best fitting of an exponential signal to the tested digitizer output signal. If samples are acquired from one period of the output signal, it is possible to reconstruct the exponential signal by means of the least-square

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$$x_n = Ae^{-Bt} + C, \tag{1}$$

where A is the reconstructed signal amplitude, $1B$ is its time constant and C is its DC value. The RMS error of this fit ε denotes the tested digitizer Average Effective Number of Bits is defined

$$\overline{ENOB} = n - \log_2\left(\frac{\varepsilon}{RMS_q}\right), \tag{2}$$

where n is the nominal number of digitizer bits and $RMS_q = 2^{-n}/\sqrt{12}$ is the RMS value of its quantizing error.

The exponentional signal is easily generated out of a rectangular signal by means of passive RC element filtration, where the time constant $\tau = RC$, see Fig.1.
The exponentional signal is defined as

$$u_C(t) = U_m\left(1 - e^{-\frac{t}{\tau}}\right), \tag{3}$$

where U_m is the amplitude of the input rectangular signal.

To achieve the final output steady-state signal differing from the theoretical value less than the resolution of the tested digitizer with nominally n bits, the minimum ratio between T_1 (T_2) and time constant τ is given by

$$T_{1(2)} \geq \tau\,(n+1)\,ln2 \tag{4}$$

For example, a 16-bit digitizer requires a period of input rectangle $T \geq 11\tau$.

If the duty factor is $T_1 = T_2 = T$ the frequency spectra of both exponential curves are identical and given by the expression

$$A(\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\tau}\right)^2}} \tag{5}$$

For $\omega >> \tau A(\omega) = \tau/\omega$ and amplitude decreases with a slope – 20 dB/decade, (see Fig. 2.)

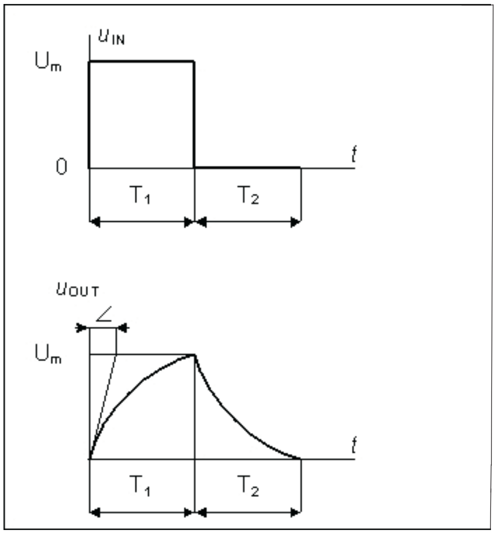

Fig. 1. Exponential signal generation.

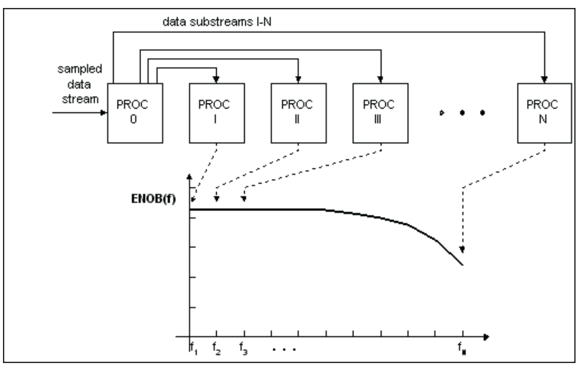

Fig. 2. Frequency spectra of exponentional signal.

3. Principles of the wobbler test

The basic idea is to apply the full-scale wobbler signal to the input of the tested ADC located on the chip of the microprocessor. The frequency sweep of the wobbler signal should cover the desired range of dynamic test and the length of the wobbler depends on available memory space for the output series of samples as well as on desired accuracy of the test and on the availability of synchronisation of signal sampling. An estimation of the reduction of *ENOB* due to an increase of the input signal frequency is calculated from measured series of samples. The Fig. 3 shows signal generated in an ADC test.

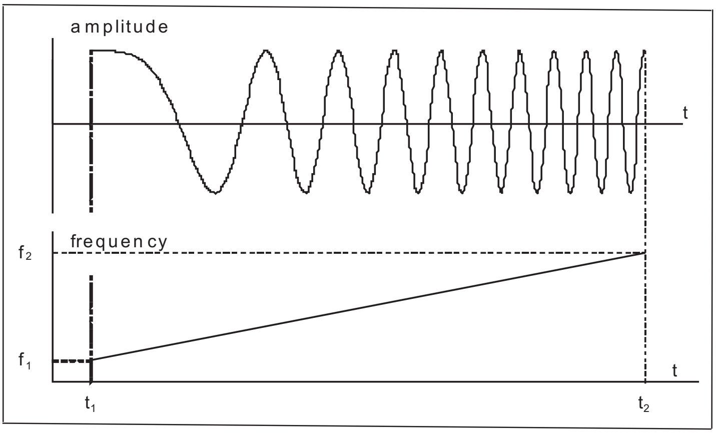

Fig. 3. Wobbler signals generated in ADC test.

The sampling process and the process of chirp generation are controlled by microprocessors. In this case, only one chirp is generated and sampled. The advantage is the reduction of acquisition time (to 50%) but a more complicated arrangement and software are required.

4. Algorithm of data processing in the wobbler test

The first step of the algorithm is a rough analysis of sampled data. The measured data stream is divided into substreams. Each substream contains an integer number (one or more to reach the necessary number of samples for the next steps of the algorithm) of quasi-periods of the sampled chir*RMS* calculation). The behavior of the analog input part of the ADC is estimated during this step. The input chirp can be described by the following formula

$$u(t) = A.\sin\left[2\pi\, t\left(\frac{t-t_0}{\Delta t}(f_1-f_2)+f_2\right)+\varphi\right], \tag{6}$$

where A is the amplitude of the chirp, t_0 is the start time of the chirp, Δt is the duration of the chirp, f_0 is the start frequency of the chirp, f_1 is the stop frequency and φ is the start phase of the chirp. In (1), a linear frequency sweep is considered.

$$\frac{df}{dt} = \frac{f_1 - f_2}{\Delta t}. \tag{7}$$

The least squares algorithm is applied to each substream to fit the ideal quasi period (1) described above to the measured one.To reduce the necessary time of solving the non-linear system equation, not every parameter is optimized during the fitting. In the concrete case f_0, f_1 are the optimized parameters while other parameters are found by other ways before. Parameter t_0 (start time of quasi-period defined as the zero crossing of the measured signal) is calculated using linear interpolation from the two nearest samples (one negative and one positive) samples. In the case of a noisy signal, more complicated higher-order interpolation using more samples should be used. Parameter φ is automatically equal to zero when the above described definition of t_0 is considered.

Parameter Δt (the duration of the current quasi-periods of the chirp) is calculated as the difference of the current t0 and the value of t_0 of the next substream.
The effective value of the current quasiperiod – A – is calculated using the following formula:

$$A_{est} = \sqrt{\frac{2}{\Delta t} \sum_{t_i} u^2(t_i)}, \tag{8}$$

where i includes all indices of samples taken between t_0 and $t_0 + \Delta t$.

The last step of the algorithm is the calculation of the reduction of *ENOB* on the instant frequency of the input wobbler. The calculation of the Average Effective Number of Bits is done using formula

$$\overline{ENOB} = \log_2 \frac{FS}{\sigma_f \sqrt{12}} \tag{9}$$

where σ_f is the standard deviation obtained as a final result of chirp fitting of the substream in which

$$f = (f_1 + f_2)/2 \tag{10}$$

Minimum of σ_f is the criterion of the best fitting by the least square method.

5. Accuracy and speed

The accuracy of the proposed method can be estimated by comparison with the values of sine-fit test for he 16-bit AD converter. The 256 samples must be taken to achieve 0,1 bit accuracy in t estimation of *ENOB*. If each substream contains more than 256 samples, one can say that the accuracy is equal or better than 0.1 bit (each substream is considered as part of a harmonic signal with the frequency $(f_1 + f_0)/2$). If 20 measured points are necessary to plot the graph showing the dependence of *ENOB* on the frequency, at least 5120 (=20×256) samples must be taken for such accuracy.

A superior chirp generator is required. Most of DDS-based generators are not suitable because the frequency sweep is synthesised from discrete frequency steps and these degrade the results of test.

The proposed test seems to be the fastest way of *ENOB* estimation. The speed of processing may be even enhanced by introducing the multiprocessor approach, see Fig 4.

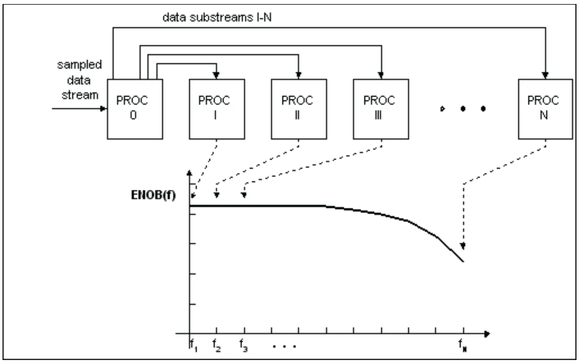

Fig. 4. Multiprocessor approach to test algorithm.

6. Comparison and results

Each method described here has been implemented in the internal ADC of the FPGA Start Development Kit Cyclone II by Altera. The internal controlled DDS generator with 24-bit DAC has been used to generate the wobbler signal in a frequency range from 20 Hz to 20 kHz.

The average *ENOB* = 13,9 bits by Exponential Fit Test and average *ENOB* = 14,1 bits by Wobbler Test in the frequency range up 20 Hz to 20 kHz is relevant to *ENOB* plot, determined by the classical Sine Wave Fit Test method. In Fig. 5 is presents the *ENOB* plot for internal ADC by Sine Wave Fit Test. The average *ENOB* by this method is 14,2. The difference of 0.3 bit between *ENOB* and $ENOB_M$ is practically insignificant.

In Fig. 6, Fig. 7 and Fig. 8 the FFT plot of Noise Histogram Test with grounded input, Code Words Histogram Test and FFT plot of sinus signal 1 kHz are presented. The Effective Resolution of internal converters under test defined by formula

$$ER = \log_2 \frac{FS}{RMS_{NOISE}} \tag{11}$$

is 15,4 bits. In the FFT plot in Fig.6, evident parasitic spectral components with USB supply noise under 100 dB are evident.

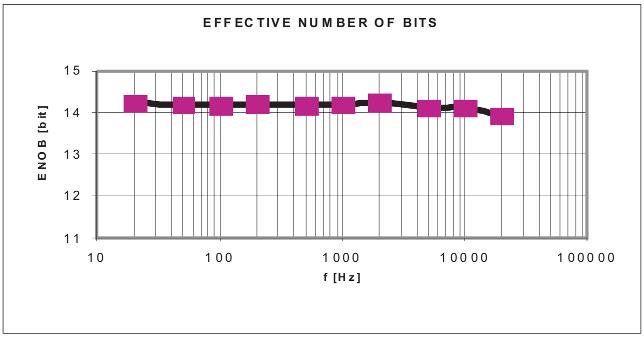

Fig. 5. ENOB plot for Sin Wave Fit Test by FPGA from Altera.

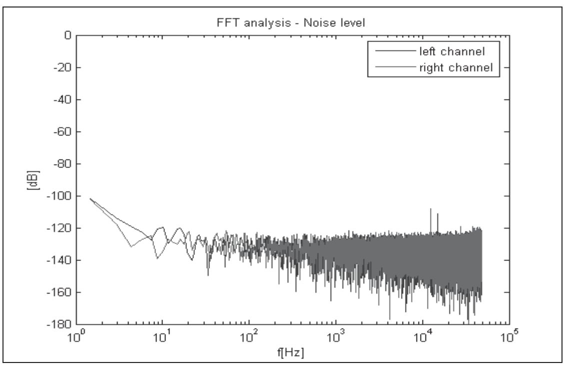

Fig. 6. FFT plot of Noise Histogram Test.

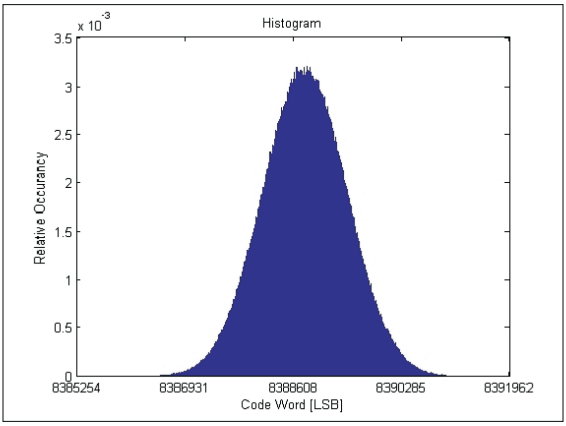

Fig. 7. Code words histogram test.

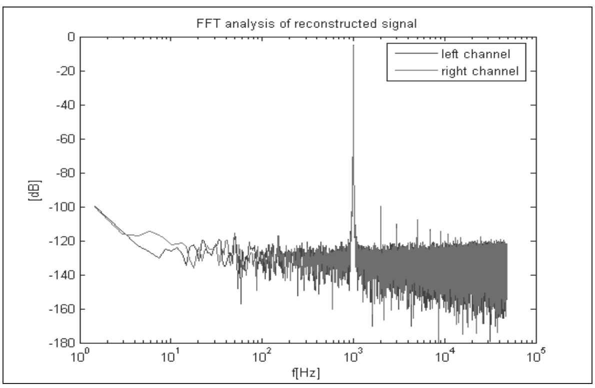

Fig. 8. FFT plot of sinus signal 1 kHz.

Metrol. Meas. Syst. Vol XVI (2009), No. 1, pp. 161-170

In Table 1 is a comparison of results of all tests applied to the internal ADC in FPGA Altera II.

Other parameters in Table 1 are: THD (Total Harmonic Distortion), SFDR (Spurious Free Dynamic Range) and SNHR (Signal to Non-Harmonic Ratio).

From measured examples in Table 1 a very large reduction of the number of samples in application of the Wobbler Test and Exponentionel Fit Test is evident. The reduction of samples is very significant for economical aspects of dynamic testing of internal ADC in embedded Data Acquisition Systems.

Table 1. Comparison results of ADC test.

Sine Wave Fit Test and Discrete Fourier Transform Test			Wobbler Test	Exponential Test
Frequency of input signal	1 kHz	10 kHz	20Hz – 20kHz	
ENOB	14,7 bit	13,8 bit	14,1 dB	13,9 dB
SINAD	90,2 dB	84,8 dB		
THD	103 dB	95 dB		
SFDR	102 dB	92 dB		
SNHR	94 dB	86 dB		
ER	15,4 bit			
Number of samples	64 k In 10 steps of frequency range 20Hz – 20kHz		5120	1024

7. Conclusions

In the article is two high-speed test methods are proposed. Due to the high speed of the test, the method described above can be suitable when the necessity of testing of many pieces of ADC is required, *e.g.* at the end of a manufacturing process.

Acknowledgements

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Chapter 5 ADC and DAC testing using modulated signals in frequency domain

5.1 Introduction

The AM, FM and QAM signals, which are frequently used in communications and radio communication, can be used for the ADC testing too. It is possible to use similar approach to test of the Digital to Analogue Converters DACs. If we realize that the DACs are widely used in signal generators, it is very important to test them by inartificial signals. Another aspect is saving time, while using test signal with wider frequency range.

This chapter is divided into five subparts.

1. Introduction,
2. Analysis,
3. Summary,

Inserted articles:

4. *DAC Testing Using Modulated Signals*,
5. *FM and QAM Signals for ADC Testing*.

5.2 Analysis

There is presented a *Crest Factor* (*CF*) correction in the chapter 5.2.1. The corrections of *ENOB* and *SINAD* are stated there. Theoretical analyses of modulated signals including analytically derived formulas modulated signal's *CFs* are stated in chapter 5.2.2. One disadvantage of using modulated signal for testing is presented in chapter 5.2.3. Intermodulation Distortion products are masked with a tested signal.

5.2.1 Crest Factor correction

Standard signal for testing is sine wave [G2]. When signal with different *CF* then sine wave is used a *CF* correction has to be applied. Examples of such a situation are:

When a coherent sampling condition is not satisfied, it is necessary to apply a window function on an input signal. Every operation, including windowing, which changes shape of the signal, changes also a *CF*.

Signal with different ratio between peak-to-peak value of the signal and signal's *RMS* than sine wave is the candidate for *CF* correction.

This correction fix also problem with signal which does not fully cover the Full-Scale (*FS*) of the DUT.

When one of previously stated condition occurs then another formula (5.4) for an *ENOB* computation has to be applied. This phenomenon is stated in [F1], [G7]. Its principle will be briefly shown. For estimating *ENOB*, it is necessary to compensate *CF* influence. There is known formula for *SNR*:

$$SNR = 10\log\frac{\sigma_{\mathrm{x}}^2}{\sigma_{\mathrm{n}}^2} = 10\log\frac{12\sigma_{\mathrm{x}}^2}{2^{-2n}X_{\mathrm{PP}}^2}\ \text{(dB)} \tag{5.1}$$

$$SNR = 10.8 + 6.02n - 20\log\frac{X_{\mathrm{PP}}}{\sigma_n} = 4.77 + 6.02 - 20\log CF\ \text{(dB)} \tag{5.2}$$

where σ_n is standard deviation of noise, σ_x is standard deviation of the input signal, X_{pp} is peak-to-peak value of the input signal, n is the nominal number of bits and and $CF = X_{pp} / (2\sigma_x)$.

For sine wave signal or FM signal, is the amplitude $X_m = X_{pp}/2 = FS/2$, the $\sigma_x = X_{pp}/(2\sqrt{2})$, then n is given by (5.3) [F1]

$$n = \frac{SNR - 1.76}{6.02}\ \text{(bit)} \tag{5.3}$$

which is the standard formula.

It is possible to express general formula from (5.2) for *ENOB* using a *CF* correction:

$$ENOB_{\mathrm{cor}} = \frac{SINAD - 4.77 + 20\log CF}{6.02}\ \text{(bit)} \tag{5.4}$$

The formula (1.31) is used for the *SINAD* correction; only *ENOB* is replaced by $ENOB_{\mathrm{cor}}$. This approach is equivalent to the correction of window influence [G11].

5.2.2 Harmonized equations

In order to harmonize differences in used symbols in different papers important formulas are stated here:

The amplitude-modulated signal is defined by formula [G10]

$$u_{\mathrm{AM}} = \left(U_{\mathrm{c}} + U_{\mathrm{m}} \cos \omega_{\mathrm{m}} t\right) \sin \omega_{\mathrm{c}} t = \\ = U_{\mathrm{c}} \sin \omega_{\mathrm{c}} t + \frac{U_{\mathrm{m}}}{2} \left[\sin\left(\omega_{\mathrm{c}} - \omega_{\mathrm{m}}\right) \cdot t + \sin\left(\omega_{\mathrm{c}} + \omega_{\mathrm{m}}\right) \cdot t\right] \tag{5.5}$$

A *SINAD* is defined as:

$$SINAD_{\mathrm{AM}} = \sqrt{\frac{\frac{X_{\mathrm{c}}^2}{2} + \frac{X_{\mathrm{m}}^2}{4}}{\sum_{f \neq 0, fc, fm} \frac{X_f^2}{2} - \frac{X_{\mathrm{c}}^2}{2} - \frac{X_{\mathrm{m}}^2}{4}}} \tag{5.6}$$

Here U_{c} is carrier frequency amplitude in time domain and and X_{c} is corresponding spectral line in frequency domain .The same relationship is between amplitude of modulated signal U_{m} and it's spectral partner X_{m}.

A *FS* condition for an AM signal has to be satisfied:

$$X_{\mathrm{FS}} = X_{\mathrm{c}} + X_{\mathrm{m}} \tag{5.7}$$

CF of the AM signal:

$$CF_{\mathrm{AM}} = \frac{2(1+m_{\mathrm{AM}})}{\sqrt{2+m_{\mathrm{AM}}^2}} \tag{5.8}$$

Crest factor corrections for *ENOB* and *SINAD*:

$$\Delta SINAD_{\mathrm{AM}} = 20\log\left(CF_{\mathrm{AM}}/\sqrt{2}\right)(\mathrm{dB}) \tag{5.9}$$

$$\Delta ENOB_{\mathrm{AM}} = \frac{\Delta SINAD_{\mathrm{AM}}}{6.02}\,(\mathrm{bit}) \tag{5.10}$$

Frequency modulated signal is defined by formula:

$$u_{\mathrm{FM}} = U_{\mathrm{c}} \sin\left(\omega_{\mathrm{c}} t + \frac{\Delta\omega}{\omega_{\mathrm{m}}}\cos\omega_{\mathrm{m}} t\right) \tag{5.11}$$

$$SINAD_{\mathrm{FM}} = \sqrt{\frac{X_{\mathrm{c}}^2 + \sum_{k=1,2,..} X_{k\mathrm{m}}^2}{\sum_{f\neq 0,\, f\mathrm{c},\, k\cdot f\mathrm{m}} X_f^2 - X_{\mathrm{c}}^2 - \sum_{k=1,2,..} X_{k\mathrm{m}}^2}} \tag{5.12}$$

where X_{c} is spectral component at the carrier frequency, X_{km} are modulated spectral products and X_{f} are other spectral components. Hence the signal has the same *CF* as a sine wave signal (from the principle of FM modulation); therefore *CF* correction is not necessary.

The QAM signal is defined by (5.13), where E_{min} is the energy of the signal with the lowest amplitude, a_i b_i are pairs of independent integers chosen according the location of the particular signal point, f_c is a carrier frequency, T_s is a symbol period **Error! Reference source not found.**:

$$u_{\mathrm{QAM}} = \sqrt{\frac{2E_{\mathrm{min}}}{T_S}} a_i \cos 2\pi f_c t + \sqrt{\frac{2E_{\mathrm{min}}}{T_S}} b_i \sin 2\pi f_c t, 0 \le t \le T_s, i = 0..M \tag{5.13}$$

A *SINAD* is defined as:

$$SINAD_{\mathrm{QAM}} = \sqrt{\frac{X_{\mathrm{c}}^2 + \sum X_{\mathrm{m}}^2}{\sum X_{\mathrm{SD}}^2 - X_{\mathrm{c}}^2 - \sum X_{\mathrm{m}}^2}} (\mathrm{dB}) \tag{5.14}$$

Where X_{c} is carrier spectral component X_{m} represents component of the side lobes of a QAM signal and X_{SD} belongs to the rest of the spectral components which are considered as noise or distortion

The Crest Factor of the QAM signal is defined by formula:

$$CF_{\mathrm{QAM}} = \frac{u_{ppQAM}}{RMS_{QAM}} \tag{5.15}$$

Corrected *SINAD* and *ENOB* for AM and QAM signals:

$$SINAD_{cor} = SINAD + \Delta SINAD = SINAD + 20\log CF \ (\mathrm{dB}) \tag{5.16}$$

$$ENOB_{cor} = ENOB + \Delta ENOB = ENOB + \frac{\Delta SINAD}{6.02} (\mathrm{bit}) \tag{5.17}$$

5.2.3 Inter Modulation Products Overlapping

It is necessary to think about a chosen signal's spectrum. There is one issue when using signals with uniformly distributed frequencies of tones in their spectra. Then some inter-modulation products have the same frequencies as signal components see Figure 5.1 and Figure 5.2. This phenomenon affects the results of the test. Signals with uniformly or symmetrically displaced frequencies of tones are not suitable if the goal of the test is to measure *IMD* [G2].

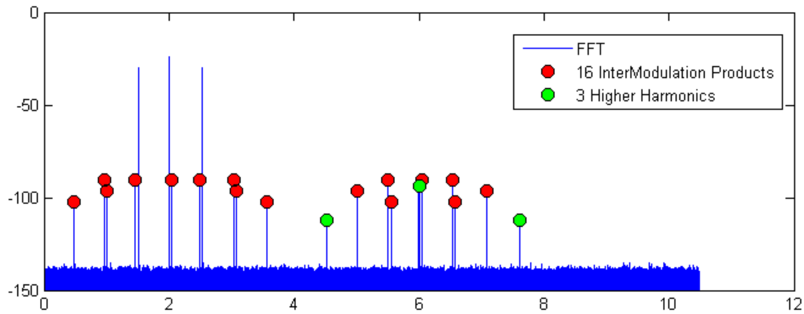

Figure 5.1 A three tone signals 150.760, 200, 253.892 kHz,

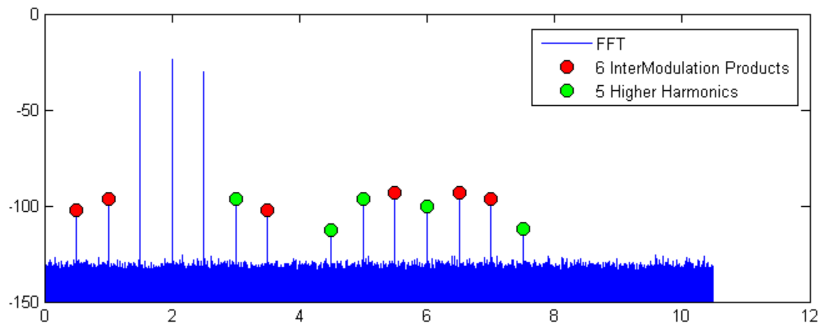

Figure 5.2 A three tone signals 150, 200, 250 kHz

5.3 Summary

It is rational to test digitizers or generator with signals using signals natural to their application. Thus modulated signal were chosen for non-traditional DUT testing.

A main benefit from FM Test is comparability of their results with Single Sine Wave FFT Test. Another advantage is that FM signal has wider amplitude frequency spectrum. AM test's amplitude spectrum has only three lines. Moreover *CF* correction has to be applied to get results comparable with FFT test using single sine wave.

The results from test using QAM signal as a testing signal are compared with sine wave and FM signal in Table 7. The measurement a multifunction card DAQ NI PXI 6251 was used for testing of a vector generator SMIQ 03B [F3]. 500kSa were acquired during measurement. Other parameters like *SINAD* and *ENOB* other spectral indicators such as *SNHR*, *SFDR*, *THD* were evaluated in the Table 7.

In the he first paper (*DAC Testing Using Modulated Signals*), it was said that a QAM signal is not suitable for a dynamic testing of DAC. The second paper (*FM and QAM Signals for ADC Testing*) surprisingly showed results from testing of a vector generator SMIQ 03B. In general QAM signals contains discontinuities such as phase steps amplitude steps which produces higher harmonics distortions and they might violate the condition that *Slew Rate* (*SR*) of a DUT has to be higher than maximum *SR* of the signal. Those are disadvantages of the QAM signal, but the second paper shows possibility of testing such DUT which was designed to generate QAM signal. In Non-traditional testing using a band signal; all its spectral components are considered as they belong to the signal itself and not to distortion. On the other hand when the generator SMIQ is tested using a QAM signal; higher harmonics which are caused by signal itself are here evaluated as distortion. In radio engineering parameter *ENOB* has no meaning. The factors like *SNR*, *SFDR* are more interesting items in this field [G2].

Following tables show summarized results of testing using modulated signal published in following papers [C4], [C6]. In Table 4, Table 5 and Table 6, there are stated results from test setup based on PXI system with DAQ NI PXI 6251 [B6] with sample frequency 400 kSa/s tested in the limited frequency bandwidth from 20 Hz to 20 kHz. 1 MSa/s were acquired during measurement.

There is shown a comparison of results from a testing using a QAM signal with sine wave and FM signal in Table 7. A multifunction card DAQ NI PXI 6251 was used as a reference device for a testing of a vector generator SMIQ 03B [B7]. The 500k samples were acquired during measurement. Parameters like *SINAD* and *ENOB* and spectral indicators such as *SNHR*, *SFDR*, and *THD* were evaluated in the Table 7.

Table 4 Test Results of Sine Wave Fit Test

Sine wave	
f (kHz)	*ENOB*(bits)
3.35780	14.3
7.3598	14.1
9.7845	13.9
15.9874	14.4

Table 5 Test Results with AM signals

AM , f_c = 7785		
m_{AM} (-)	$ENOB_{cor}$ (bits)	$ENOB_{cor}$ (bits)
0.25	7.8	14.2
0.50	7.8	14.2
1.00	7.79	14.2
0.25	12.4	14.4
0.50	12.4	14.4
1.00	12.4	14.4

Table 6 Test Results with FM signals

FM, f_c = 9785 Hz	
m_{FM} (-)	*ENOB* (bits)
0.25	13.9
0.50	14.0
1.00	14.1
2.00	14.1
4.00	14.2
8.00	14.2

Table 7 Test Results with Sine Wave, FM and QAM signals

	SINAD [dB]	*ENOB* [bit]	*SFDR* [dB]	*SNHR* [dB]	*THD* [dB]
Single tone	55,5	8,9	56,4	63,6	-56,3
FM (m_f = 0,25)	49,9	8,0	53,9	50,5	-58,6
FM (m_f = 0,5)	55,6	9,0	58,4	58,9	-58,4
FM (m_f = 1,0)	56,3	9,1	60,7	58,9	-59,7
16QAM (4CW)	48,9	9,0	50,1	49,0	-64,1

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DAC TESTING USING MODULATED SIGNALS

Pavel Fexa, Josef Vedral, Jakub Svatoš

CTU Prague, Faculty of Electrical Engineering Department of Measurement, Technicka 2, 16627, Prague 6, Czech Republic
(✉ fexap1@fel.cvut.cz , +420 22 435 2201, ✉ vedral@fel.cvut.cz , +420 22 435 2182, ✉ svatoja1@fel.cvut.cz, +420 22 435 2201)

Abstract

This document analyses qualities of methods used for testing dynamical parameters of Digital-to-Analog Converters (DAC) using a multi-frequency signal. As the source for these signals, Amplitude Modulated (AM) and Frequency Modulated (FM) signals are used. These signals are often used in radio engineering. Results of the tests, like *Effective Number of Bits* (*ENOB*), *Signal-to-Noise and Distortion* (*SINAD*), are evaluated in the frequency domain and they are compared with standard results of Sine Wave FFT test methods. The aim of this research is firstly to test whether it is possible to test a DAC using modulated signals, secondly to reduce testing time, while estimating band performance of DAC.

Keywords: Digital-to-analog converter, ENOB, Signal-to-noise and distortion - SINAD, FFT analysis, Crest Factor (CF).

1. Introduction

This work comes out from standards for Analog-to-Digital Converters (ADC) testing and uses this method for DAC testing. Almost all dynamic methods can be applied to ADC as well as to DAC. In the field of dynamic ADC testing there are several well-described methods used. Standardized methods like *Sine Wave Fit Test* or *FFT* (resp. *DFT*) *Test* are using high spectrum purity input signals. The *ENOB*, *SINAD*, *Total Harmonic Distortion* (*THD*), *Signal Non-harmonic Distortion SNHR*, *Spurious Free Dynamic Range SFDR*, and *Intermodulation Distortion IMD* can be determined with high accuracy by these methods [1, 2, 3]. However, these tests are relatively time-consuming, especially when it is necessary to analyze the frequency characteristic of these parameters. A possible way to shorten the test duration is to drive the input of an ADC by a multi-tone signal, or generate a multi-tone signal with a DAC.

For example, the exponential signal can be considered as a typical example of a multi-frequency signal, and it is to be generated with a passive RC circuit driven by a rectangular generator. Unfortunately, the spectrum amplitude frequency characteristic of the exponential signal falls with a slope of -20 dB per decade [4, 5, 6]. This fact causes variation of results reached by classic methods using a signal with constant amplitude and the exponential signal test method. Applications of a frequency-swept signal suitable for economic tests for ADC testing are described in [4].

The band signal is used to save the testing time. A test using an AM signal produces the average performance of the Devices Under Test (DUT) i.e. *SINAD* and *ENOB*. It means that 3 different tones are used for testing at the same time. The time-saving benefit is achieved using every band signal. However, AM and FM signals are frequently used in communications and they can be used for DAC testing too. It is possible to use a similar approach as one used for testing of the ADCs. The DACs are widely used in signal generators, thus it is very important to test them using inartificial signals. Still it cannot be

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clear why just modulated signals are used, a similar spectrum has generally a multi-tone signal, which is composed of sum of chosen sine-waves. In this case the answer is very simple: only 4 are needed to describe the modulated signal (amplitude, carrier frequency, modulation frequency, modulation index). It is significant when the number of tones is increasing. A FM signal which is composed of 33 spectral components is still defined by 4 parameters; a general multi-tone signal is defined by 33 amplitudes, 33 frequencies and optionally 33 phase shifts.

This paper shows only AM and FM signals for ADC or DAC testing, however the original idea is to test a general DUT by the signal which results from natural application of each DUT or it is similar to application. In radio engineering a DUT is for example a generator with chosen modulation and a receiver. The result from this test is the Bit Error Rate Ratio BERR, in this case it is a mixed signal test. The testing signal in this case is the chosen modulated signal. In this paper the DUT is not a transmitter, a channel with noise and a receiver, but the DAC and its performance is measured using *SINAD* and *ENOB*. Not every type of signal is suitable for DAC testing. It has to impeach limitation of DACs like bandwidth or slew rate etc. (digital modulation like QAM represents this type, which is not suitable for DAC testing)

2. FFT tests

The performance of the DUT is evaluated in the frequency domain in a FFT test. The data u_0, u_1... u_{M-1} are sent to the input of the DAC, generated, measured by an ADC and transformed by the DFT (FFT) algorithm.

In case of coherent sampling the following condition is valid: K × f_{SIGNAL} = m × f_{SAMPLING}, where K and m are integers. If this condition cannot be satisfied it is necessary to apply a window.

2.1 Standard sine wave FFT test of AD converters

Signal Noise and Distortion SINAD is the ratio of the *RMS* value of the carrier frequency to the mean value of the root-sum-square of all other spectral components, including harmonics, but excluding the DC component [1, 2 ,3].

$$SINAD = 20\log\frac{Signal_{\mathrm{RMS}}}{NAD_{\mathrm{RMS}}}, \tag{1}$$

where *n* is the number of nominal bits of the converter under test.

For a single-tone signal, *SINAD* and *ENOB* are given by the following equation [7],

$$SINAD = 6.02ENOB + 1.76 \ \ \text{(dB)}. \tag{2}$$

When the coherent sampling condition is not satisfied, it is necessary to apply the procedure of input signal windowing. Every operation, including windowing, which changes the shape of the signal, changes also the *CF*. Then another formula for an *ENOB* computation has to be applied (6). This phenomenon is stated in [8], its principle will be briefly shown. For estimating *ENOB*, it is necessary to compensate the *CF* influence. There is the known formula for *SNR*:

$$SNR = 10\log\frac{\sigma_{\mathrm{x}}^2}{\sigma_{\mathrm{n}}^2} = 10\log\frac{12\sigma_{\mathrm{x}}^2}{2^{-2n}X_{\mathrm{PP}}^2} \ \ \text{(dB)}; \tag{3}$$

$$SNR = 10.8 + 6.02n - 20\log\frac{X_{\mathrm{PP}}}{\sigma_n} = 4.77 + 6.02 - 20\log CF \ \ \text{(dB)}, \tag{4}$$

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 283–294

where σ_n is standard deviation of noise, σ_x is standard deviation of the input signal, X_{pp} is the peak-to-peak value of the input signal, n is the nominal number of bits and $CF = X_{pp} / (2\sigma_x)$. For a sine wave signal or FM signal, the amplitude $X_m = X_{pp}/2 = FS/2$, the $\sigma_x = X_{PP}/2/\sqrt{2}$, then n is given by the next equation [8],

$$n = \frac{SNR - 1.76}{6.02} \quad \text{(bit)}. \tag{5}$$

It is possible to express the general formula for *ENOB* using the *CF* correction [9]:

$$ENOB_{\text{cor}} = \frac{SINAD - 4.77 + 20\log CF}{6.02} \quad \text{(bit)}. \tag{6}$$

For the *SINAD* correction the formula (2) is used, only *ENOB* is replaced by $ENOB_{\text{cor}}$. This approach is equivalent to the correction of window influence [10].

2.2 Amplitude modulation FFT test

The amplitude modulated signal is defined by formula [11]

$$\begin{aligned} u_{\text{AM}} &= \left(U_{\text{c}} + U_{\text{m}} \cos \omega_{\text{m}} t\right) \sin \omega_{\text{c}} t = \\ &= U_{\text{c}} \sin \omega_{\text{c}} t + \frac{U_{\text{m}}}{2} \left[\sin\left(\omega_{\text{c}} - \omega_{\text{m}}\right) \cdot t + \sin\left(\omega_{\text{c}} + \omega_{\text{m}}\right) \cdot t\right]. \end{aligned} \tag{7}$$

The *modulation depth* $m_{\text{AM}} = U_{\text{m}}/U_{\text{c}}$ affects the character of the frequency spectrum of the signal. We consider Dual Side Band amplitude modulation with $m_{\text{AM}} \leq 1$. Its spectrum contains a carrier with frequency ω_c, amplitude U_c and two sideband components with frequencies $\omega_c \pm \omega_m$ and amplitude $U_m/2$. In the special case when $U_c = 0$, the carrier frequency is eliminated, but the sidebands remain. That is double-sideband suppressed-carrier transmission. In fact, we can use it as *Dual Tone Test* with symmetrically distributed spectral components and signal processing is the same as in the classic Dual Tone methods. It is possible to fit the AM signal by the *Multi-tone Fit Test* (*least square fit method*), which optimizes 3 amplitudes, 3 frequencies, 3 phase shifts and 1 offset. The easier way to obtain results is to apply spectral analysis of an AM signal. We can define $SINAD_{\text{AM}}$ similarly as in a classic FFT test without *CF* correction [9]:

$$SINAD_{\text{AM}} = \sqrt{\frac{\frac{U_{\text{c}}^2}{2} + \frac{U_{\text{m}}^2}{4}}{\sum_{f \neq 0, fc, fm} \frac{U_f^2}{2} - \frac{U_{\text{c}}^2}{2} - \frac{U_{\text{m}}^2}{4}}}, \tag{8}$$

where U_c a U_m are amplitudes of the carrier and the modulation, U_f represents amplitudes of other spectral components.

Input voltage in the time domain should be equal to full-scale of the ADC, i.e. it is necessary to satisfy the following condition:

$$U_{\text{FS}} = U_{\text{c}} + U_{\text{m}}. \tag{9}$$

The *CF* of an AM signal can be expressed by:

$$CF_{\mathrm{AM}} = \frac{2(1+m_{\mathrm{AM}})}{\sqrt{2+m_{\mathrm{AM}}^2}}. \tag{10}$$

The *ENOB* is computed from *SINAD* (6); the corrected *SINAD* has to be again recomputed by formula (2).

Another approach is to compute *SINAD* and *ENOB* using standard formulas and then apply a correction Δ*SINAD* (11) and Δ*ENOB* (12):

$$\Delta SINAD_{\mathrm{AM}} = 20\log\left(CF_{\mathrm{AM}}/\sqrt{2}\right)\ \text{(dB)}; \tag{11}$$

$$\Delta ENOB_{\mathrm{AM}} = \frac{\Delta SINAD_{\mathrm{AM}}}{6.02}. \tag{12}$$

Table 1. *SINAD* and *CF* reduction in case of using an AM signal

m_{AM} (-)	*CF* (-)	Δ*SINAD* (dB)	Δ*ENOB* (bits)
0.25	1.74	-1.8	-0.3
0.50	2.00	-3.0	-0.5
1.00	2.31	-4.3	-0.7

2.3 Frequency modulation FFT test

The FM modulated testing signal is defined as $u_{FM} = U_c \sin[\omega_c(t)\cdot t]$, modulation frequency $\omega_c(t) = \omega_{c0} + \Delta\omega\cos(\omega_m t)$, where Δω is the frequency deviation of the modulated signal, $m_{\mathrm{FM}} = \Delta\omega/\omega_{\mathrm{m}}$ is the modulation index. The waveform of the frequency modulated signal is [12] :

$$u_{\mathrm{FM}} = U_{\mathrm{c}} \sin\left(\omega_{\mathrm{c}} t + \frac{\Delta\omega}{\omega_{\mathrm{m}}}\cos\omega_{\mathrm{m}} t\right). \tag{13}$$

Spectrum of the FM consists of a carrier with frequency ω_{c} and symmetrically displaced spectral components around the carrier ω_{c} with multiples of frequency ω_{m}. Amplitudes of spectral components are given by first order Bessel functions with argument $\Delta\omega/\omega_{\mathrm{m}}$. In Tab. 2, the amplitudes of spectrum of the frequency modulated signal are shown for a modulation index $m_{\mathrm{FM}} = \Delta\omega/\omega_{\mathrm{m}}$ in the range from 0 to 2.

Table 2. FM signal amplitudes of the spectral component

m_{FM}	ω_{c}	$\omega_{\mathrm{c}} \pm \omega_{\mathrm{m}}$	$\omega_{\mathrm{c}} \pm 2\omega_{\mathrm{m}}$	$\omega_{\mathrm{c}} \pm 3\omega_{\mathrm{m}}$
0	1.00			
0.25	0.98	0.12		
0.50	0.94	0.24	0.03	
1.00	0.77	0.44	0.11	0.02
1.50	0.51	0.56	0.23	0.06
2.00	0.22	0.58	0.35	0.13

Fitting this signal is similar to the previous case, it is possible to use the *Least Square* method (Multi Tone Fit Test), but it is very dificult to optimize too many parameters. It is convenient to evaluate a signal in the spectral domain and determine the *Signal Noise and Distortion SINAD*$_{\mathrm{FM}}$, [9]

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 283–294

$$SINAD_{\mathrm{FM}} = \sqrt{\frac{U_{\mathrm{c}}^2 + \sum\limits_{k=1,2,..} U_{k\mathrm{m}}^2}{\sum_{f\neq 0,\, fc,\, k\cdot fm} U_f^2 - U_{\mathrm{c}}^2 - \sum\limits_{k=1,2,..} U_{k\mathrm{m}}^2}}, \tag{14}$$

where U_{c} is the *RMS* value of the carrier component ,$U_{k\mathrm{m}}$ corresponds to the *RMS* value of signal components around the carrier frequency, U_f are *RMS* values of other spectral components. The peak-to-peak value of an input voltage in the time domain has to cover the full scale of the ADC ($U_{\mathrm{FS}} = U_{\mathrm{c}}$), see (13).

The FM signal has the same *CF* as a sine wave signal, therefore equation (2) can be used for *ENOB* calculation.

3. Test setup and results

An application of modulated signals for DAC has to satisfy the following criteria: The DAC should be tested near its full scale range. The amplitude of the signal should respect this fact. The modulation index of the AM signal affects the amplitude of side-band spectral components. In this work the modulation index is chosen as 0.25 ÷ 1. A lower modulation index causes that the averaged *ENOB* is reassembling to the single tone test. The FM modulation index is essential for DAC testing, because it affects the signal bandwidth. The carrier frequency sets the central frequency of the signal in the spectral domain and the modulation frequency sets the spacing between spectral components.

For practical verification of the AM and FM methods, a PXI system was used. The first output channel of the DAQ NI PXI 6251 (2 analog outputs: 16-bit, 1.25 MSa/s or 1 analog output: 16-bit 1.8MSa/s, 16 analog inputs: 16-bit ADC, 1.25 MSa/s) was tested. The Digitizer (NI PXI 5922 24 bit, 500 kSa/s, or 16 bits, 15 MSa/s) was chosen as a reference device. The modulation method was compared with a sine wave DFT test. For all measurements the Hanning window was used. 1 MSa were acquired during measurement.

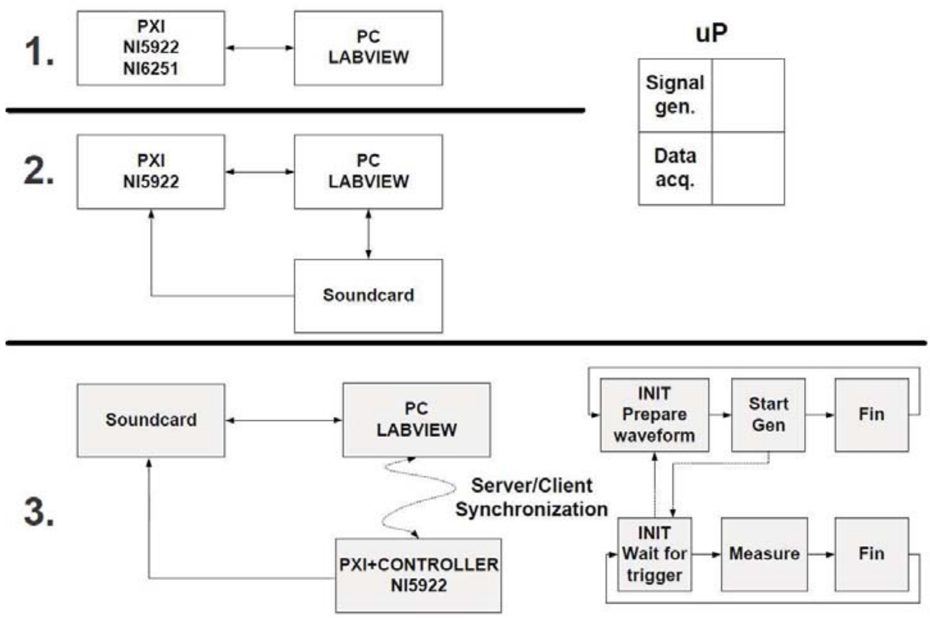

Fig. 1 Three arrangements of the testing system

The whole PXI testing system was programmed in LabView. For purpose of testing, 3 modifications of virtual instrument were developed, as shown in Fig. 1

The first two virtual instruments use one PC to control generating data and data acquisition. The generating part runs in one core of the processor and data acquisition runs in the second core.

The third arrangement is suitable for those cases where the performance of the PC is not sufficient. A distributed measuring system was developed. The generating part runs in a PC and control soundcard. The data acquisition part runs in a PXI with controller. Synchronization is made via the TCP protocol.

3.1 Results of DAC testing using a sine wave DFT test

Firstly the DUT was tested by a single tone test. It was tested using four different frequencies (3357.8 Hz, 7359.87 Hz, 9784.52 Hz and 15987.41 Hz), see Fig. 2 and Fig. 3.

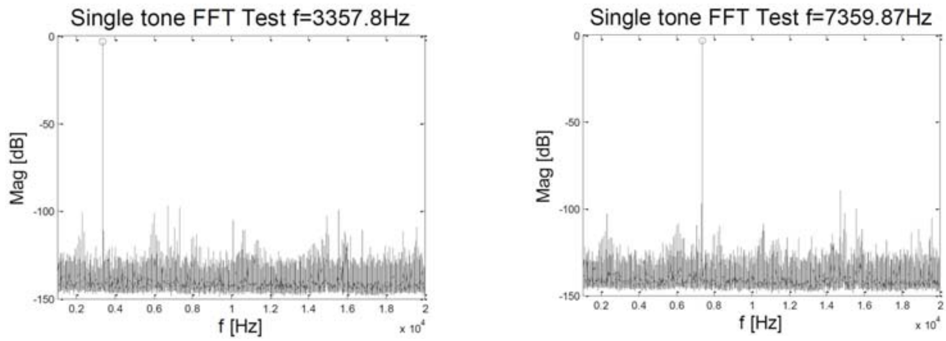

Fig. 2 Spectrum of sine wave signal f_1 = 3.3578 kHz (left) f_2 = 7.3599 kHz (right)

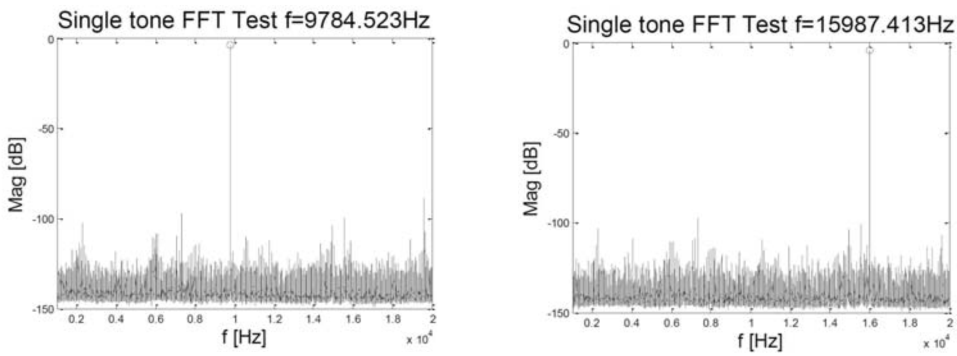

Fig. 3 Spectrum of sine wave signal f_3 = 9.785 kHz (left) f_4 = 15.987 kHz (right)

Table 3. Sine wave DFT test results

f (Hz)	CF (-)	CF_w (-)	$SINAD$ (dB)	$ENOB$ (bits)	$SINAD_{cor}$ (dB)	$ENOB_{cor}$ (bits)
3357.8	1.41	2.31	83.6	13.6	87.9	14.3
7359.8	1.41	2.31	82.2	13.4	86.5	14.1
9784.5	1.41	2.31	81.2	13.2	85.5	13.9
15987.4	1.41	2.31	84.2	13.7	88.5	14.4

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 283–294

Results are shown in Tab. 3. For incoherent sampling the Hanning window was applied. It is necessary to correct the influence of the Hanning window. The original sine wave has $CF = \sqrt{2}$. CF_w respecting the fact that the window changes the shape of the signal.

3.2 Results of DAC testing using an AM DFT test

Secondly the DUT was tested by an AM DFT test. Two different carrier frequencies f_c (7.785 kHz and 12.385 kHz) were used for testing, while the modulation frequency f_m is still the same (3.36 kHz). The modulation index was chosen as 0.25, 0.5 and 1, see Fig. 4 – 6.

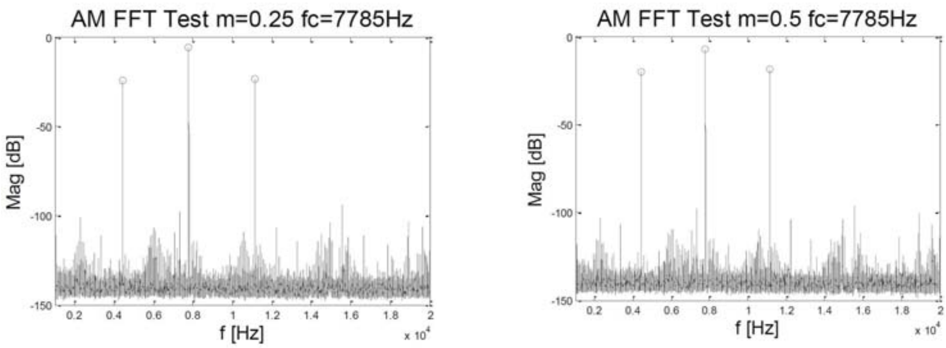

Fig. 4 Spectrum of AM signal f_c=7.785 kHz, f_m=3.36 kHz, m_{AM} =0.25 (left), m_{AM} =0.5 (right)

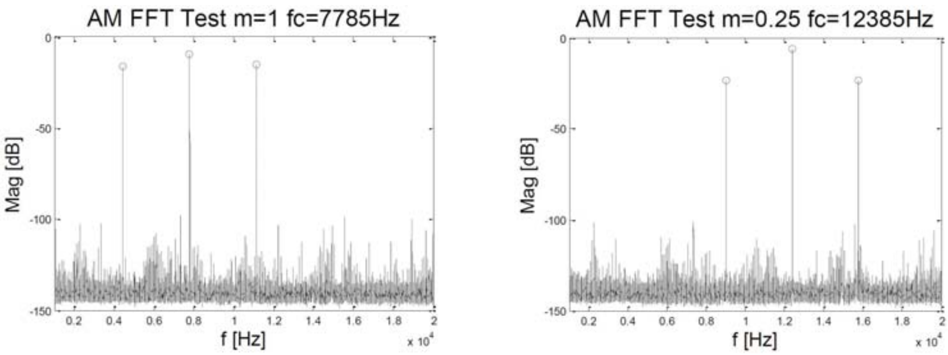

Fig. 5 Spectrum of AM signal f_c=7.785 kHz, f_m=3.36kHz, m_{AM} =1 (left),
f_c=12.385 kHz, f_m=3.36 kHz, m_{AM} =0.25 (right)

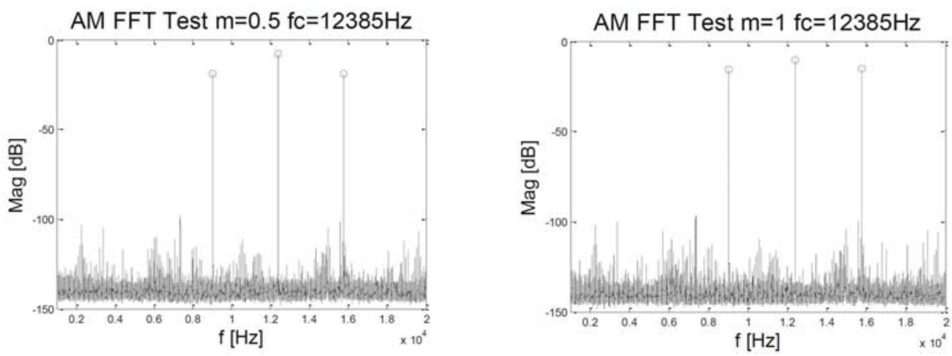

Fig. 6 Spectrum of AM signal f_c=12.385 kHz, f_m=3.36 kHz, m_{AM} =0.5 (left), m_{AM} =1 (right)

Results are shown in Tab. 4. In this case not only the window but also the modulation index affects the *CF*.

Table 4. AM DFT test results

f_m=3.36 kHz							
m_{AM} (-)	f_n (kHz)	*CF* (-)	CF_w (-)	*SINAD* (dB)	*ENOB* (bits)	$SINAD_{cor}$ (dB)	$ENOB_{cor}$ (bits)
0.25	7.79	1.74	2.84	81.2	13.2	87.2	14.2
0.50	7.79	2.00	3.26	80.1	13.0	87.4	14.2
1.00	7.79	2.31	3.77	79.0	12.8	87.5	14.3
0.25	12.39	1.74	2.84	82.7	13.4	88.7	14.4
0.50	12.39	2.00	3.26	81.3	13.2	88.6	14.4
1.00	12.39	2.31	3.77	79.8	13.0	88.3	14.4

3.3 Results of DAC testing using a FM DFT test

Thirdly the DUT was tested by a FM signal. Parameters of the signal were the following: carrier frequency f_c = 9785 Hz, modulation frequency f_m = 531.58 Hz. The modulation index was chosen as 0.25, 0.5, 1, 2, 4 and 8, see Fig. 7 - 9.

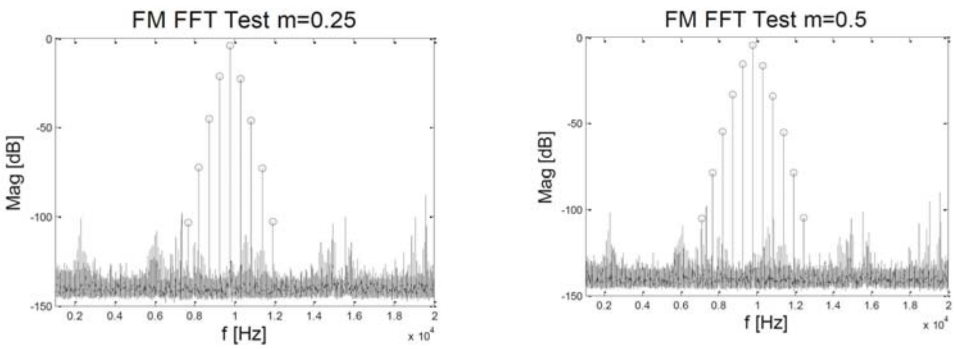

Fig. 7 Spectrum of FM signal f_c=9.785 kHz, f_m=531.58 Hz, m_{FM} =0.25 (left), m_{FM} =0.5 (right)

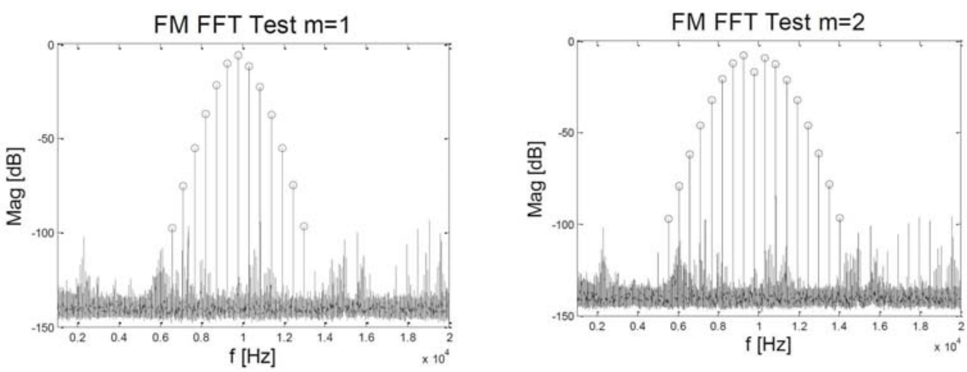

Fig. 8 Spectrum of FM signal f_c = 9.785 kHz, f_m = 531.58 Hz, m_{FM} = 1 (left), m_{FM} = 2 (right)

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 283–294

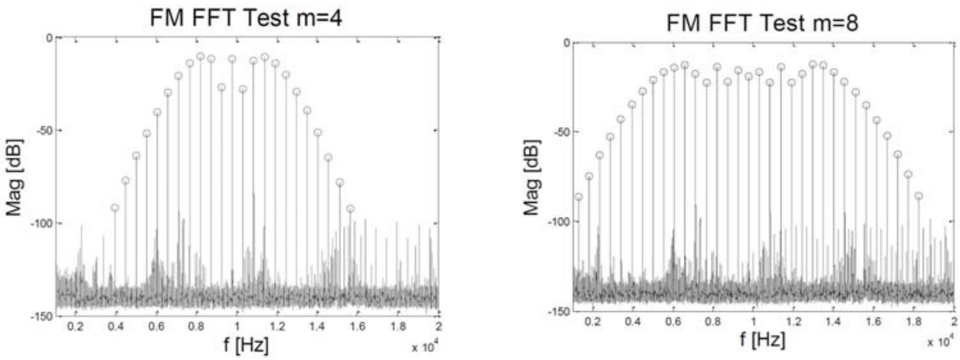

Fig. 9 Spectrum of FM signal f_c=9.785 kHz, f_m=531.58 Hz, m_{FM} =4 (left), m_{FM} =8 (right)

Results from the FM FFT Test are shown in Tab. 5. It is interesting that m_{FM} changes do not affect the *CF*.

Table 5. FM DFT test results

f_n=9785 Hz, f_m=531.58 Hz						
m_{FM} (-)	*CF* (-)	CF_w (-)	*SINAD* (dB)	*ENOB* (bits)	$SINAD_{cor}$ (dB)	$ENOB_{cor}$ (bits)
0.25	1.41	2.31	81.4	13.2	85.7	13.9
0.50	1.41	2.31	81.9	13.3	86.1	14.0
1.00	1.41	2.31	82.6	13.4	86.9	14.1
2.00	1.41	2.31	82.6	13.4	86.8	14.1
4.00	1.41	2.31	82.9	13.5	87.1	14.2
8.00	1.41	2.31	82.8	13.5	87.1	14.2

4. Summary

In Tab. 6. AM, FM methods with sine wave FFT test are compared. The comparison is made in such a way that the results from the test using a modulated signal with certain carrier frequency are compared with sine wave FFT test results using a similar signal frequency. For example a FM FFT test with 9.8 kHz carrier frequency is compared with a 9.8 kHz sine wave FFT test. The biggest value of the difference between the reference method and non-traditional method is equal to 0.3 bits. The reason why AM and FM methods show better results than a single-tone method, is that more signal spectral components cover a broader frequency band and potentially can mask distortion.

Table 6. Results of comparison between AM, FM methods and sine wave method

Sine wave		AM				FM, f_n = 9785 Hz		
f (kHz)	*ENOB* (bits)	m_{FM} (-)	$ENOB_{cor}$ (bits)	$\Delta ENOB_{cor}$ (bits)	m_{AM} (-)	f_n (kHz)	*ENOBcor* (bits)	$\Delta ENOB_{cor}$ (bits)
3.35780	14.3	0.25	7.8	14.2	**-0.1**	0.25	13.9	0.0
7.3598	**14.1**	0.50	7.8	14.2	**-0.2**	0.50	14.0	-0.1
9.7845	13.9	1.00	7.79	14.2	**-0.2**	1.00	14.1	-0.2
15.9874	**14.4**	0.25	12.4	14.4	***0.0***	2.00	14.1	-0.2
		0.50	12.4	14.4	***0.0***	4.00	14.2	-0.3
		1.00	12.4	14.4	***0.0***	8.00	14.2	-0.3

The aim of this article is to verify possibilities of DAC testing using multi-harmonic signals such as AM and FM signals. Signal processing of these signals in the time domain using fitting methods is quite complicated. An advantage of those methods is ascertaining DAC parameters in a wider frequency range without the need of measuring their frequency characteristic. Therefore it can be expected that the described methods will find application in industry for less-demanding and economical tests.

Another goal of this paper is to find a suitable testing signal which covers an evenly chosen bandwidth of the DUT and this signal is not completely artificial. This method is suitable for testing arbitrary generators (Agilent 33120A can generate an FM signal with *f*c from 10 mHz to 10 kHz and deviation from 15 mHz to 15 MHz). However this paper does not show results suitable for i.e. audio applications (the carrier frequency of the signal is much higher than the band of the modulated signal).

For example, if one cycle of data collection takes 2 seconds and 20 cycles are necessary for an averaged FFT, then measurement at 1 frequency lasts 40 seconds. If we want to test carefully the performance of the DAC in a range of 20 kHz in 33 steps, the test will take 1320 seconds. If we use an FM signal with $m = 8$, the duration of the test will be only 40 seconds. However the *ENOB* measured using an FM signal, represents the average performance of the tested DAC.

It is possible to shorten the test duration by reducing the number of samples. Tab. 7 shows how the number of samples affects the results of the test; a single tone test and a test using a FM signal with $m = 1$ was arranged for comparison. The single-tone FFT method is robust and works fine even if the number of samples is very limited (*RMS* of the signal and noise does not change significantly). A FM test for chosen parameters works properly, if up to 250 kSamples are acquired, otherwise the spectral lines are too close. This paper does not show results suitable for i.e. audio application (the carrier frequency of the signal is much higher than the band of the modulated signal), which needs special sampling methods.

Table 7. Results of comparison between AM, FM methods and the sine wave method

	Single-Tone	FM signal
Samples (MSa)	$ENOB_{cor}$ (bit)	$ENOB_{cor}$ (bit)
2.000	14.3	14.2
1.000	14.3	14.2
0.500	14.2	14.2
0.250	14.3	14.2
0.125	14.4	-
0.060	14.3	-
0.030	14.3	-

5. Acknowledgment

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Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 283–294

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FM and QAM Signals for ADC Testing

Josef Vedral, Pavel Fexa, Jakub Svatoš

CTU in Prague, Faculty of Electrical Engineering, Department of Measurement
Technická 2, 166 27 Prague 6, Czech Republic
Tel.: (+420) 22435 2182, Fax: (+420) 233 339 929
E-mail: vedral@fel.cvut.cz, fexapave@fel.cvut.cz, svatoja1@fel.cvut.cz

***Abstract* – In this paper the qualities of methods for testing dynamical parameters of A/D converters with analog frequency modulated and quadrature modulated signals are analyzed. Results of test are compared with standard Single-Tone Fourier Transform Test.**

Keywords – ADC testing, FM signal, QAM signal, Effective Number of bits, Signal Noise and Distortion

I. INTRODUCTION

Standardized methods for dynamic testing of ADC are Single Tone Fit Test and Discrete Fourier Transform Test [1], [2]. Typical dynamic parameters ADCs are *ENOB* (Effective Number of Bits), *THD* (Total Harmonic Distortion), *SNHR* (Signal to Nonharmonic Ratio), *SINAD* (Signal Noise and Distortion), *SFDR* (Spurious Free Dynamic Range), and *MTD* (Multi-tone Harmonic Distortion).

Effective Number of Bits is determined by standard deviation of difference between reconstructed and fitted signals ε

$$ENOB = n - \log_2 \frac{\varepsilon}{q} (bit) \tag{1}$$

where $q = 2^{-n} / \sqrt{12}$ is ideal quantization error of *n* bit ADC. Following ADC parameters can be identified from spectral analysis of digitized single sine wave signal.

Total Harmonic Distortion is the ratio of the mean value of the root-mean-square (RMS) harmonics U_i to the RMS value of the first harmonic frequency U_1

$$THD = \frac{\sqrt{\sum_{i=2}^{n} U_i^2}}{U_1} \tag{2}$$

Signal Non-harmonic Distortion is the ratio of the RMS value of the first harmonic frequency U_1 to the RMS amplitude of the output noise U_{NH} (*Noise Floor*)

$$SNHR = \frac{U_1}{U_{NH}} \tag{3}$$

Signal Noise and Distortion is the ratio of the RMS value of the basic frequency to the mean value of the root-sum-square of all other spectral components, but excluding dc component.

$$SINAD = \frac{1}{\sqrt{\frac{1}{SNHR^2} + THD^2}} \tag{4}$$

Interdependency between *Signal Noise and Distortion* and *Effective Number of Bits* is given by following equation

$$SINAD = 6{,}02 ENOB + 1{,}76 (dB) \tag{5}$$

Dynamic range of tested ADCs can be characterized by parameter *Spurious Free Dynamic Range*, which is ratio of the RMS value of the basic frequency U_1 to RMS value of next higher harmonic or non-harmonic component U_m.

$$SFDR = \frac{U_1}{U_m} \tag{6}$$

Multi-tone Distortion MTD is the ratio of the mean value of the root-mean-square (RMS) poly-harmonics U_i to the RMS value of the major non-harmonic frequency.

When it's necessary to analyze frequency characteristic of mentioned parameters, these tests are relatively time-consuming. One possible way how to decrease duration of the test is to drive input of ADC by poly-harmonic signal [3]. Application of frequency swept signal which is suitable for economic tests of ADC testing are described in [4], [5].

Another possible signals for ADC testing is discrete multi-harmonic signal with discrete frequency components or AM, FM, QAM signals. Last three signals are used in radio communication techniques.

II. TESTING WITH FREQUENCY MODULATED SIGNAL

Principle of frequency modulation *FM* consists in modulation of carrier frequency according the expression $\omega_n(t) = \omega_{n0} + \Delta\omega \cos \omega_m t$, where $\Delta\omega$ is frequency deviation of modulated signal and $m_{FM} = \Delta\omega / \omega_m$ is modulation index. Waveform of the frequency modulated signal is

$$u_{FM} = U_n \sin\left(\omega_n t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right) \qquad (7)$$

Spectrum of the *FM* consists of carrier frequency ω_n and symmetrically displaced spectral component around carrier ω_n with multiples of frequency ω_m. Amplitudes of spectral components are given by Bessel's first order functions with argument $\Delta\omega/\omega_m$.
In Table 1 is amplitudes of the spectrum of frequency modulated signal are shown for *modulation index* $m_{FM} = \Delta\omega/\omega_m$ in range from 0 to 2.

Table 1. Spectral magnitudes of frequency modulated signal

m_{FM}	n	$n \pm m$	$n \pm 2m$	$n \pm 3m$
0	1.00			
0.25	0.98	0.12		
0.50	0.94	0.24	0.03	
1.00	0.77	0.44	0.11	0.02
1.50	0.51	0.56	0.23	0.06
2.00	0.22	0.58	0.35	0.13

Modulation indexes 0.25, 0.5 and 1 are suitable for dynamic ADC testing. Hence, the amplitudes of all other spectral components are smaller than the amplitude of carrier frequency.
It is possible to use *LMS* error method *Multi-Tone Fit Test*, but for this case is very hard to optimize too much parameters ($m_{FM} > 0.25$). Another way is to evaluate a signal in spectral domain and determine *Signal Noise and Distortion SINAD$_{FM}$*

$$SINAD_{FM} = \sqrt{\frac{U_n^2 + \sum_{k=1,2,..} U_{km}^2}{\sum_{i=kM/2,k=1,2,..}^{m} U_{f_i}^2 - U_n^2 - \sum_{k=1,2,..} U_{km}^2}} \qquad (8)$$

And other parameters (*THD*, *SFDR*, *SNHR*) can be similarly derived from single tone such as $SINAD_{FM}$ definition by extension of number of tones.

$$THD_{FM} = \sqrt{\frac{\sum_{l=2,3,..} U_n^2 + \sum_{l=2,3,..} \sum_{k=1,2,..} U_{km}^2}{U_n^2 + \sum_{k=1,2,..} U_{km}^2}} \qquad (9)$$

It is necessary to satisfy this condition that input voltage is equal to full-scale of the ADC.

III. TESTING WITH QUADRATURE AMPLITUDE MODULATED SIGNAL

Quadrate amplitude modulation (QAM) has been widely used in adaptive modulation because of its efficiency in power and bandwidth.
Modern modulation techniques exploit the fact that digital baseband data may be sent by varying both envelope and phase/frequency of a carrier wave. Because the envelope and phase offer two degrees of freedom, such modulation techniques map baseband data into four or more possible carrier signals. Such modulation techniques are called M-ary modulation, since they can represent more signals than if just the amplitude or phase were varied alone. In an M-ary signaling scheme, two or more bits are grouped together to form symbols and one of M possible signals is transmitted during each symbol period.
Usually, the number of possible signals is $M = 2^n$, where n is an integer. Depending on whether the amplitude, phase, or frequency is varied, the modulation technique is called M-ary ASK, M-ary PSK, or M-ary FSK. Modulation which alters both amplitude and phase is M-ary QAM. As with many digital modulation techniques, the constellation diagram is a useful representation. It provides a graphical representation of the complex envelop of each possible symbol state.
The constellation diagram of 16-QAM is shown in Figure 1. The constellation consists of a square lattice of signal points. The general form of an M-ary signal can be defined as (9)

$$u_{QAM}(t) = \sqrt{\frac{2E_{min}}{T_S}} a_i \cos 2\pi f_0 t + \sqrt{\frac{2E_{min}}{T_S}} b_i \sin 2\pi f_0 t \qquad (10)$$
$$0 \le t \le T_S \quad i = 0..M$$

Where E_{min} is the energy of the signal with the lowest amplitude a_i and b_i are a pair of independent integers chosen according to the location of the particular signal point; f_0 is the carrier frequency; T_s is the symbol period.

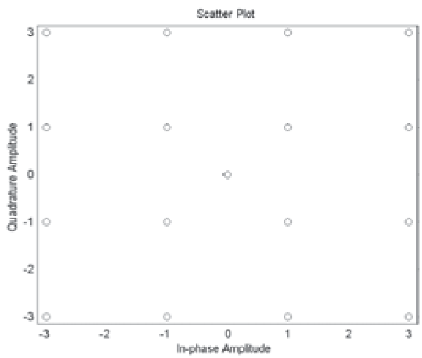

Fig. 1 Scatter plot – 16 rectangular QAM

QAM signal is evaluated in spectral domain and. It is possible to determine *Signal Noise and Distortion SINAD$_{QAM}$*

$$SINAD_{QAM} = \sqrt{\frac{U_n^2 + \sum_{k=1,2,..} U_{km}^2}{\sum_{i=kM/2,k=1,2,..}^{m} U_{f_i}^2 - U_n^2 - \sum_{k=1,2,..} U_{km}^2}} \qquad (10)$$

It is also necessary to satisfy this condition that input voltage is equal to full-scale of the ADC.

IV. EXPERIMENTAL TEST RESULTS

Generator SMIQ 03B - Rhode Schwarz was used for testing. A device under test was used 16 bit *Data Acquisition System* PXI 5922. Measured data were sampled with sample rate 15 MSa/s in memory depth 1 Mbyte. For DFT signal processing has been used windowing with Blackman-Harris 4. order. 500k samples were acquired.

Firstly *Single-Tone* and *FM* spectrum was measured for signals at frequency: 1MHz, modulation frequency of the *FM* signal 100kHz, modulation index 0.25, 0.5 and 1 was chosen.

Spectra are shown in Fig.2 to Fig. 5.

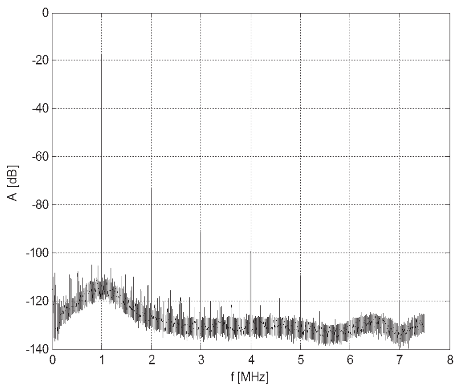

Fig. 2 FFT Single-Tone Test
f_1 = 1 MHz

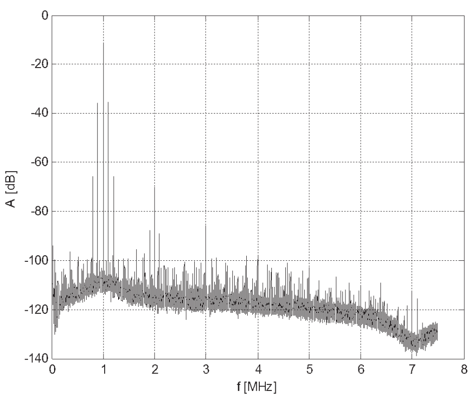

Figure 3 FFT test with FM signal
fn = 1 MHz, fm = 0.1 MHz, m_{FM} = 0.25

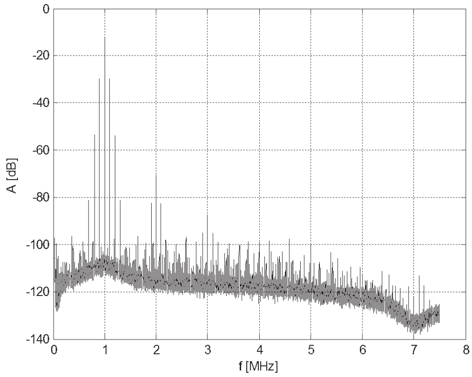

Figure 4 FFT test with FM signal
fn = 1 MHz, fm = 0.1 MHz, m_{FM} = 0.5

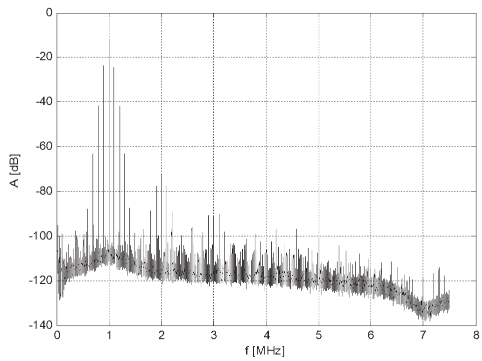

Figure 5 FFT test with FM signal
fn = 1 MHz, fm = 0.1 MHz, m_{FM} = 1

Secondly the system has been tested with 16 QAM. 4 code words were generated to create suitable testing signal with 1 carrier and 4 modulated components. Examples of frequency spectra's are shown in Fig. 6.

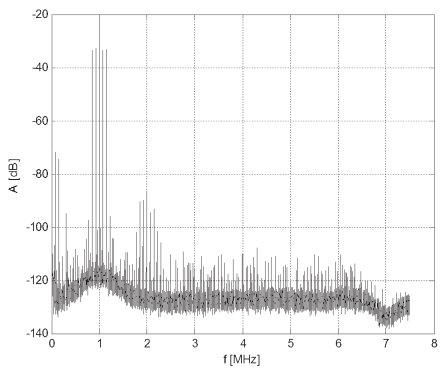

Figure 6 FFT test with 16QAM (4CW) signal fn = 1MHz,
f_m = 75k Hz

The examples from these methods were compared. Experimental results of these methods are presented in Table 2. Generally its confirmed by all obtained results that multi-tone signals can be used for economical testing of ADCs in wider frequency range.

Table 2. Experimental results multi-tone test signals

	SINAD [dB]	*ENOB* [bit]	*SFDR* [dB]	*SNHR* [dB]	*THD* [dB]
Single tone	55,5	8,9	56,4	63,6	-56,3
FM (mf = 0,25)	49,9	8,0	53,9	50,5	-58,6
FM (mf = 0,5)	55,6	9,0	58,4	58,9	-58,4
FM (mf = 1,0)	56,3	9,1	60,7	58,9	-59,7
16QAM (4CW)	48,9	9,0	50,1	49,0	-64,1

VI. CONCLUSION

The aim of this work is to verify possibilities of ADC testing using poly-harmonics signals such as FM and QAM signals. Signal processing of these signals in time domain using fitting methods is quite complicated.

An advantage of this method is ascertaining ADC parameters in wider frequency range without need of measuring their frequency characteristic. Therefore it is possible to suppose, that stated methods finds application in industry in less demanding economical tests.

ACKNOWLEDGMENT

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Chapter 6 ADC and DAC testing using impulse signals in frequency domain

6.1 Introduction

The paper *DAC testing using impulse signals in frequency domain* is introduced in this chapter. This paper deals with:

1. A signal obtained as sum of several sine wave signals – *Uniformly Distributed Multi-Tone* (UDMT) signals. Idea comes from measuring *Inter Modulation Distortion* IMD [I2].
2. A Damped Sine Wave Signal (DSW)
3. A sinc (sinx/x) signal

Previous knowledge (from the Chapter 5) was used for the *CF* correction of *ENOB* and *SINAD*. It might be interesting question why MT signals are treated in this chapter. The answer is simple. Interferences between tones produce an impulse like signal.

Signals such as a DSW or sinc signals have continuous amplitude spectrum. The way how to discretize the amplitude spectrum is to use a cut of the signal and repeat it. These repetitions produce a spectrum with equally distributed tones with distance of tones corresponding to the frequency of the repetition. Thus generated signal i.e. in case of DSW has wide spectrum with slowly decreasing amplitudes of spectral lines. To prevent aliasing or for easier identification signal bandwidth has to be limited. Let’s have a practical application like e.g. testing of a generator using the DSW signal. DUT has some limitation as its final bandwidth. The signal has to fit to the chosen frequency band. Then the DSW signal which will be sent to generator has to be modified. The task could be achieved by generation DSW signal and using digital filter to remove the high frequency components. All major spectral lines are considered as signal and the rest is considered as noise or distortion with knowing the fact that the band of the signal is limited.

This chapter is divided into four subparts.

1. Introduction,
2. Analysis,
3. Summary,
4. Introduced article:
   ADC and DAC testing using impulse signals in frequency domain.

6.2 Analysis

Some problems occurred when DUT was tested using an impulse signal. Measurement could be devaluated if signal maximum change of the signal is faster than a *Slew Rate* (*SR*). This topic is covered in chapter 6.2.1. Theoretical equations including analytically or numerically derived signal's *CF* are shown in 6.2.2.

6.2.1 Slew Rate

A DUT was 16-bit digitizer in the introduced paper. There is another issue which is also related with bandwidth of the system. It is the *SR* of the DUT and also the *SR* of a testing device [I1]. If the signal changes are faster than generator SR the signal is distorted in the process of its creation. The next possible issue comes when the signal is digitized. The signal is chopped if analogue part of the digitizer has lower SR than the entering signal. *SR* problems occur when generating a repeated segment of signal. The discontinuities could be observed in the end of the period. To overcome the issue is to symmetrize the signal. Flipping a signal around x axis reverses polarity and or flip a signal around y axis connects descending part with an ascending part of flipped signal, see the Figure 6.1.

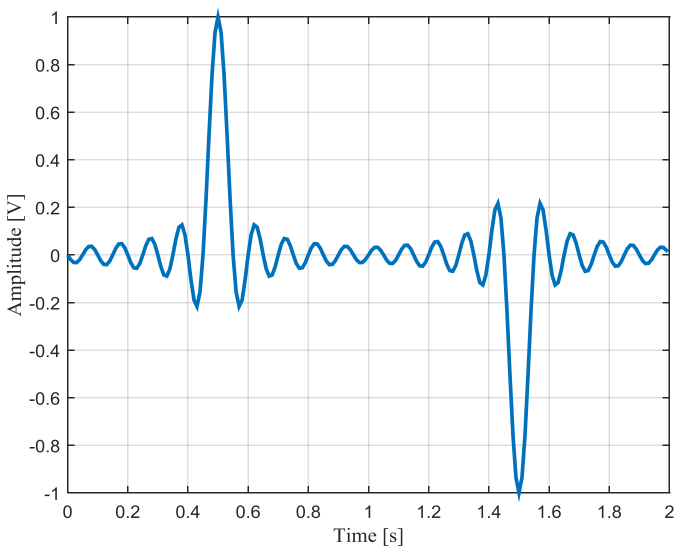

Figure 6.1 A symmetrized sinc function plotted in Matlab

6.2.2 Harmonized equations

To harmonize differences in used symbols in different papers important formulas are stated here:

A Multi Tone signal is defined:

$$u_{\mathrm{MT}} = \sum_{i=1}^{m} U_i \sin(2\pi f_i t) \tag{6.1}$$

Definition of the UDMT's *Crest Factor*:

$$CF = \sqrt{m} \tag{6.2}$$

where m is a number of tones for chosen frequencies which are coprime numbers.

A DSW signal

$$u(t) = e^{-2\cdot\pi\cdot f_2\cdot d\cdot t}\sin\left(2\cdot\pi\cdot f_2\cdot t\right) \tag{6.3}$$

A *CF* for the *DSW* for damping ratio d <<1

$$CF \cong \frac{2e^{-d\frac{\pi}{2}}}{\sqrt{\dfrac{\left(1-e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}}\right)f_1}{2\cdot\pi\cdot f_2\cdot d}}} \tag{6.4}$$

A sinc function is defined by (6.5) [I3].

$$\operatorname{sinc}(x) = \begin{cases} 1 & \text{for } x = 0 \\ \dfrac{\sin x}{x} & \text{otherwise} \end{cases} \tag{6.5}$$

A symmetrized testing signal derived from the sinc signal is expressed by (6.6), where *H* is the *Heaviside function*. The signal is composed of a sequence [*u*(*t*), -u(*t*), *u*(*t*), -u(*t*), ...].

$$u(t) = H\left(t + \frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right) - H\left(t - \frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right), t \in \left\langle -\frac{T_1}{2}, \frac{T_1}{2}\right) \tag{6.6}$$

A *Crest Factor* of the *sinc signal* could be solved numerically.

$$CF = \frac{Max(u(t))}{RMS(u(t))} = \frac{FS_{\mathrm{DAC}}}{\sqrt{\frac{1}{T}\int_{-T/2}^{T/2} u^2(t)\mathrm{d}t}} \tag{6.7}$$

A *SINAD* of the MT or the impulse signal is the ratio between signal and other spectral components i.e. *Noise and Distortion* (*ND*).

$$SINAD = \sqrt{\frac{\sum U_s^2}{\sum U_{ND}^2 - \sum U_s^2}} \tag{6.8}$$

Corrections of *ENOB* and *SINAD*:

$$ENOB_{\mathrm{cor}} = \frac{SINAD - 4.77 + 20\log CF}{6.02} \ (\mathrm{bit}) \tag{6.9}$$

$$SINAD_{\mathrm{cor}} = 6.02 ENOB_{cor} + 1.76 (\mathrm{dB}) \tag{6.10}$$

6.3 Summary

DSW is defined by a few parameters and it has wide amplitude frequency spectra. Sinc signal is defined even with less parameters and it has uniformly distributed amplitude spectral lines. As proved in this thesis, the sinc signal is one of the most promissing testing signal.

It it necessary to discretize spectra of Sinc‘s and DSW‘s signal by repetition of its segments, but steps on edges between them should be be removed. Another problem is that signal’s frequency range should be limited.

The main disadvantage of the UDMT signal was that many parameters has to be defined. At least three parameter for each tone has to be defined if phase shift is not considered. Advantage of this signal is its simplicity with no need for a filtering or a signal symmetrization.

The results from testing internal DAC in DAQ NI PXI 6251 from National Instruments using sample frequency 400 kSa/s in limited frequency bandwidth from 20 Hz to 20 kHz are shown in the Table 9. A sine wave, a MT, a DSW a sinc signal was used for testing.

Table 8 Test Results

Signal	Frequency	Number of tones	*CF*	*ENOB* (bit)	*SINAD* (dB)
SW	3358.87 Hz	1	1.4	14.3	87.8
SW	7359.87 Hz	1	1.4	14.1	86.5
SW	9784.52 Hz	1	1.4	13.9	85.6
SW	15987.41 Hz	1	1.4	14.3	87.9
Dual Sine	3357.87 Hz, 7359.87 Hz	2	2.0	14.2	87.4
Triple Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz	3	2.5	14.2	87.5
Quad Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz, 15987.41 Hz	4	2.8	14.3	87.6
DSW	170 Hz – 17 kHz	17	1.7	14.6	89.7
DSW	170 Hz – 17 kHz	17	2.6	14.5	89.1
DSW	170 Hz – 17 kHz	17	4.0	14.4	88.5
SINC	170 Hz – 17 kHz	17	5.6	14.2	86.9
SINC	170 Hz – 17 kHz	31	7.8	14.3	87.7
SINC	170 Hz – 17 kHz	141	17.3	14.3	87.6

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DAC TESTING USING IMPULSE SIGNALS

Josef Vedral, Pavel Fexa

CTU Prague, Faculty of Electrical Engineering Department of Measurement, Technicka 2, 16627, Prague 6, Czech Republic, (✉fexap1@fel.cvut.cz , +420 22 435 2201, vedral@fel.cvut.cz , +420 22 435 2182)

Abstract

The Multi-Tone (MT) signal with uniform amplitudes can be used for DAC testing. This paper shows an easier way to generate a MT signal using several impulse signals. The article also analyzes qualities of methods for testing the dynamic parameters of Digital to Analog Converters using an impulse signal. The MT, Damped Sine Wave (DSW) and Sinx/x (SINC) signals will be used as the source for these tests. The Effective Number of Bits (*ENOB*) and Signal to noise and distortion (*SINAD*) are evaluated in the frequency domain and they are modified using the *Crest Factor* (*CF*) correction and compared with the standard results of the Sine Wave FFT test. The first advantage of the test using an impulse signal is that you need fewer input parameters to create the band signal for testing the DAC. The second one is to reduce the testing time using a band signal in comparison with multiple tests using a single sine wave.

Keywords: DAC, ENOB, SINAD, FFT test, Crest Factor, Damped Sine Wave, SINC signal, Multi-Tone signal.

1. Introduction

Classical methods for testing the dynamic parameters of DACs are stated in a draft [1]. The non-standard methods for DAC testing are using multi-tone, AM and FM signals [2-4]. One of the possibilities how to create an impulse signal is to sum up the several tones which are equidistantly distributed and they have zero phase shifts. The contribution analyzes the possibilities how to use the DSW and SINC signals for DAC testing. These signals are generated by the DUT – a 16-bit-resolution multifunction card. The reference device is a 24 bit digitizer, see Fig. 1. There are two ways how to directly compare the impulse methods using a non-sinusoidal signal with the Sin Wave (SW) FFT test. The first approach is to use a general formula (2) and to recalculate the *SINAD* (from the corrected *ENOB*) using the standard formula for a SW signal [5].

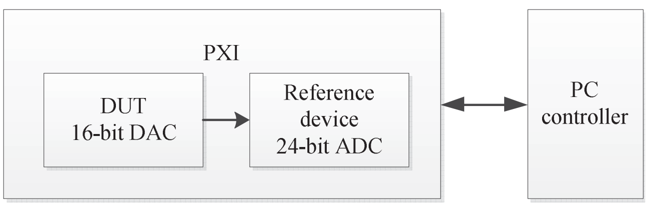

Fig. 1. The measurement setup.

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$$CF = \frac{Amplitude}{RMS}(-)\,, \tag{1}$$

$$ENOB_{\mathrm{GENERAL}} = \frac{SINAD - 4.77 + 20\log CF}{6.02}(bit)\,. \tag{2}$$

Second way is to use the standard formula and correct the *SINAD* and the *ENOB* using the Δ*SINAD* (3) and the Δ*ENOB* correction factors (4).

$$SINAD_{Corrected} = SINAD_{Measured} + \Delta SINAD = SINAD_{Measured} + 20\log CF\ (\mathrm{dB})\,, \tag{3}$$

$$ENOB_{Corrected} = ENOB_{Measured} + \Delta ENOB = ENOB_{Measured} + \frac{\Delta SINAD}{6.02}(\mathrm{bit}). \tag{4}$$

2. Testing with a Multi-Tone signal

There are two general but important presumptions for testing a DAC: [6-9]

- to test near *Full Scale*(*FS*) - it is necessary to ensure that the *Peak-to-Peak* value covers *FS* of DUT;
- the second condition is that the *Slew Rate SR* of the signal is slower than the *SR* of the DUT (a generator) and the digitizer as well (5). The *SR*'s maximum of the DSW has the same value as the *SR*'s maximum of the sine-wave so it is possible to use the standard formula for the *SR*.

$$SR_{MAX\_SIGNAL} < SR_{\mathrm{DUT}} < SR_{\mathrm{DIGITIZER}}. \tag{5}$$

The multi-tone signal with discrete frequency components (each tone has zero phase shift) is defined by the formula:

$$u_{\mathrm{MT}} = \sum_{i=1}^{m} U_i \sin(2\pi f_i t)\,, \tag{6}$$

where U_i, f_i, $i = 1,2,..m$ are amplitudes and frequencies of m spectral signal components. The *Effective Number of Bits* of the tested ADC and DAC is given by the equation (2). The CF_{MT} of this multi-tone signal is defined by the formula (7).

$$CF_{\mathrm{MT}} = \frac{\sum_{i=1}^{m} U_i}{\sqrt{\sum_{i=1}^{m} \frac{U_i^2}{2}}}\,. \tag{7}$$

CF is equal to $\sqrt{2m}$ for equal amplitudes of frequency components $U_i = 1/2m$. Table 1 shows how *CF* affects *SINAD* and *ENOB*. Δ*SINAD*, Δ*ENOB* are differences between the SW test and MT test. The signal contains only tones with the same amplitude. Frequencies are chosen such that the maximum (minimum) of the fundamental is supported by the maximum

Metrol. Meas. Syst., Vol. XIX (2012), No. 1, pp. 105-114.

of other tones. For example $f_2=4f_1$. It is possible to say that the maximum of the signal is equal to the sum of the amplitudes.

Table 1. The dependence of *SINAD* and *ENOB* on the number of tones.

Number of Tones	1	2	4
CF_{MT}	√2	2	2√2
Δ*SINAD* (dB)	0	- 3.01	- 6.02
Δ*ENOB* (bit)	0	- 0.5	- 1.0

A disadvantage of this method is that you need to set up the frequency of each tone. It means that it is necessary to pick up 20 parameters for a 20-tone test.

3. Damped Sine Wave Test

The DSW is a natural signal which is easy to generate. For ADC testing we can design a circuit with only one operational amplifier to generate the DSW from a square signal. In case of DAC testing, the signal is generated directly by the DAC.

This signal has one disadvantage. The *CF* differs from that of the sine-wave signal. It is necessary to compute the *CF* to correct the *ENOB* and *SINAD*. Let us do the analysis of the DSW. The solution of the harmonic motion of the under damping system can be expressed [10, 11]

$$u(t) = e^{-2\cdot\pi\cdot f_2\cdot d\cdot t}\sin(2\cdot\pi\cdot f_2\cdot t), \tag{8}$$

where d is the damping ratio, $f_1 = 1/T_2$ is the natural frequency, see Fig. 2.

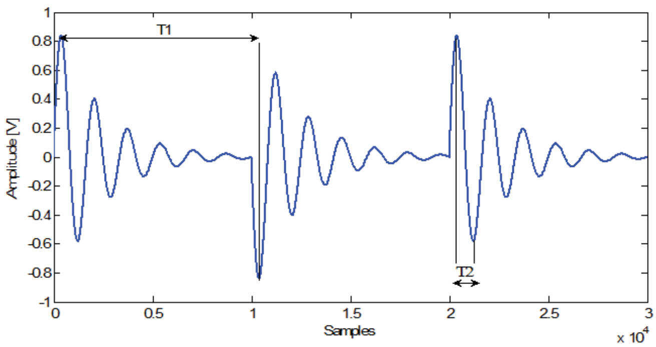

Fig. 2. Time plot of a DSW signal.

The signal's *RMS*, which is periodically generated with the repetitive frequency $f_1 = 1/T_1$, can be computed using (9):

$$RMS = \sqrt{\frac{1}{T_1}\int_0^{T_1} u^2(t)\,\mathrm{d}t}\,, \tag{9}$$

$$RMS = \sqrt{\frac{f_1 e^{\frac{-4\pi d f_2}{f_1}}\left(-1-d^2+e^{\frac{4\pi d f_2}{f_1}}+d^2\cos\left(\frac{4\pi f_2}{f_1}\right)-d\sin\left(\frac{4\pi f_2}{f_1}\right)\right)}{8\pi f_2 d(1+d^2)}}. \tag{10}$$

This formula can be simplified for the ratio of frequencies f_1 / f_2 from 0.5 to 2.

$$RMS \approx \sqrt{\frac{e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}} f_1\left\{-1+e^{\frac{4\cdot\pi\cdot d\cdot f_2}{f_1}}\right\}}{8\cdot\pi\cdot f_2\cdot d\cdot(1+d^2)}}. \tag{11}$$

The amplitude of this signal is the first maximum. It can be expressed as the following formula

$$Amplitude = \sqrt{\frac{f_2^2}{d^2+f_2^2}}\, e^{-2d\arccos\sqrt{\frac{1}{2}+\frac{d}{2\sqrt{1+d^2}}}}. \tag{12}$$

After some math the *CF* can be formulated as the equation:

$$CF = \frac{2\sqrt{\frac{1}{1+d^2}}\, e^{-2d\arccos\sqrt{\frac{1}{2}+\frac{d}{2\sqrt{1+d^2}}}}}{\sqrt{\frac{\left(1-e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}}\right) f_1}{2\cdot\pi\cdot f_2\cdot d\cdot(1+d^2)}}}. \tag{13}$$

This formula can be reduced if $d \ll 1$ (i.e. significantly or strongly under-damped systems). Then the *CF* is given by the formula:

$$CF \cong \frac{2e^{-d\frac{\pi}{2}}}{\sqrt{\frac{\left(1-e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}}\right) f_1}{2\cdot\pi\cdot f_2\cdot d}}}. \tag{14}$$

Just for illustration, there is the *CF* shown in Table 2 for the chosen parameters f_1, f_2 and d.

Table 2. The damped sinewave's crest factor for the chosen damping ratio and frequencies.

f_1 (kHz)	f_2 (kHz)	d (-)	CF (-)
5	1	0.016	1.74
5	1	0.032	2.05
5	1	0.064	2.59
5	1	0.127	3.33
5	1	0.255	4.04

Metrol. Meas. Syst., Vol. XIX (2012), No. 1, pp. 105-114.

4. Test with a Sinc signal

The SINC signal seems to be a very perspective signal for testing of dynamic parameters of DACs [12]. This signal is composed of two SINC signals with the same parameters, only the second part is inverted, see Fig. 3.

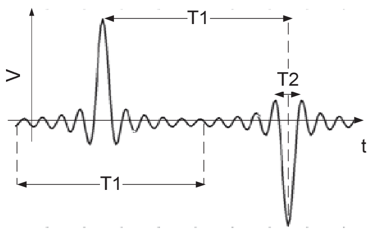

Fig. 3. Time plot of a *SINC* signal.

The first period of the signal is described by the next formula:

$$u(t) = \mathrm{H}\left(t+\frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right) - \mathrm{H}\left(t-\frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right),\ t \in \left\langle -\frac{T_1}{2}, \frac{T_1}{2}\right), \tag{15}$$

where H is the Heaviside function, which chops the SINC function in time, in the range $< -T_1/2, T_1/2)$.

In the frequency domain the signal contains equidistantly distributed components with uniform amplitudes in the ideal case. Because the function is chopped and inverted, the effect of the rectangular window causes leakage. The digital filter limits the frequency band and reduces this effect, but it also reduces the amplitude near the filter's cut-off frequency.

Thus the generated signal symmetrically covers the *FS* of the DAC. *CF* can be computed from one period of our signal (16). When the filter is applied, the amplitude of the signal must be normalized to cover the *FS* of the DAC; the signal *u*, which is stated in formula (16) is then the output signal behind the filter.

$$CF = \frac{Max[u(t)]}{RMS} = \frac{FS_{DAC}}{\sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} u^2(t)dt}}. \tag{16}$$

However this integral in equation (16) can be solved only numerically.

5. Test setup and results

A PXI system was used for the practical verification of the test methods with MT, DSW and SINC signals. The first output channel of the DAQ NI PXI 6251 (2 analog outputs: 16-bit, 1.25 MSa/s or 1 analog output: 16-bit 1.8MSa/s, 16 analog inputs: 16-bit ADC, 1.25 MSa/s) was tested. The Digitizer (NI PXI 5922 24 bit, 500 kSa/s, or 16 bits, 15 MSa/s) was chosen as the reference device, see Fig. 1. All impulse methods were compared with the sine wave DFT test. The Hanning 4th order window was used for all measurements. The frequency bandwidth is limited in range from 20 Hz to 20 kHz, for the reduction of the *Noise Floor* in all tests. The test system is programmed in NI LabView. The 2-MSamples were acquired during the test. The sampling frequency is 400 kHz.

Firstly the DUT was testing the internal DAC in DAQ NI PXI 6251 by single-, double-, triple- and quadruple-tone test. The results of this test are shown in Fig. 4 and Fig. 7.

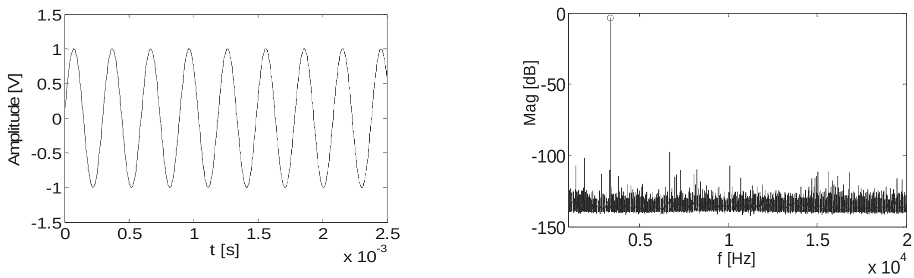

Fig. 4. The time plot and the spectrum of the sinewave signal (3357.87 Hz).

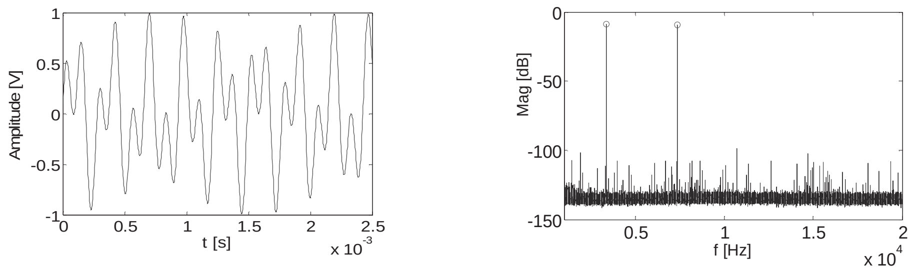

Fig. 5. The time plot and the spectrum of the 2-tone signal (3357.87 Hz, 7359.87 Hz).

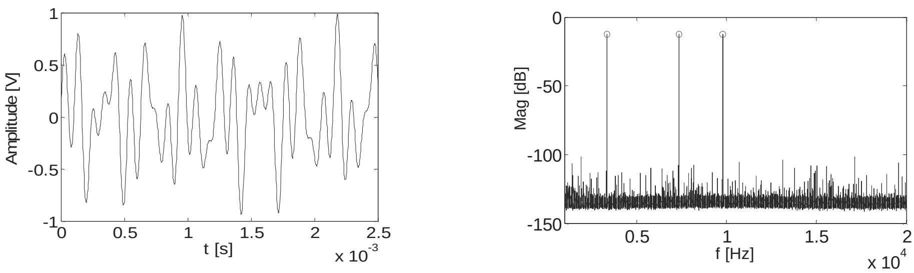

Fig. 6. The time plot and the spectrum of a 3-tone signal (3357.87 Hz, 7359.87 Hz, 9784.52 Hz).

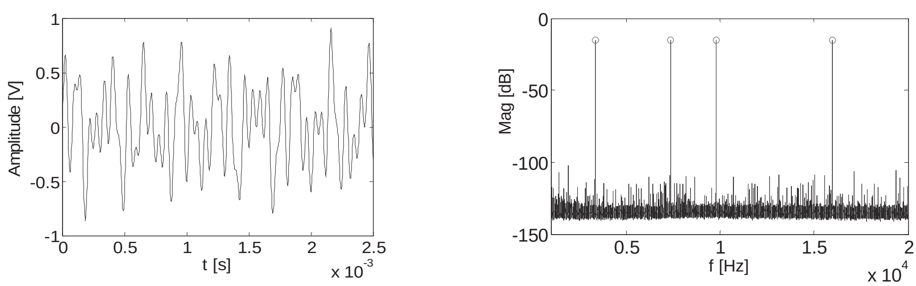

Fig. 7. The time plot and the spectrum of the 4-tone signal, (3357.87 Hz, 7359.87 Hz, 9784.52 Hz, 15987.41 Hz).

The spectral plots of the DSWs with the different damping ratios are shown in Fig. 8 and Fig. 11.

Metrol. Meas. Syst., Vol. XIX (2012), No. 1, pp. 105-114.

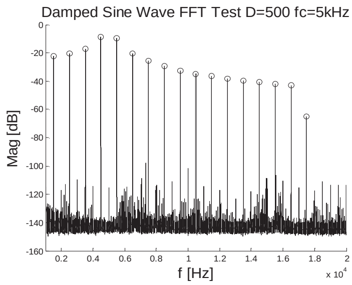

Fig. 8. The spectrum of the DSW, f_1 = 5 kHz, f_2 = 1kHz, d = 0.016.

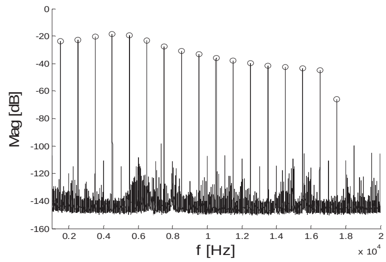

Fig. 9. The spectrum of the DSW , f_1 = 5kHz, f_2 = 1 kHz, d = 0.26.

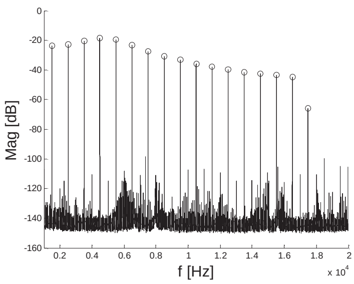

Fig. 10. The spectrum of the DSW d = 0.255, f_1 = 5 kHz, f_2 = 1 kHz.

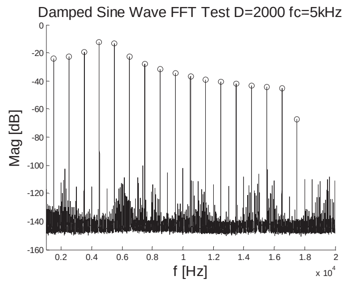

Fig. 11. The spectrum of the DSW d = 0.64, f_1 = 5 kHz, f_2 = 1 kHz.

The time and spectral plots of the SINC signals with the different CFs and number of tones are shown in Fig. 12 and Fig. 14.

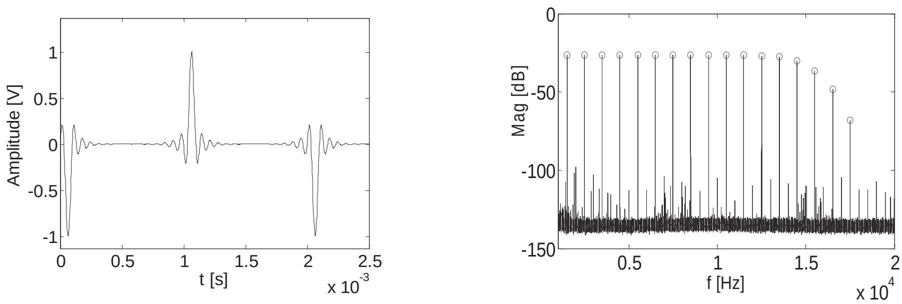

Fig. 12. The time plot of the SINC signal – 17, Tones CF = 5.5, BW = 15 kHz.

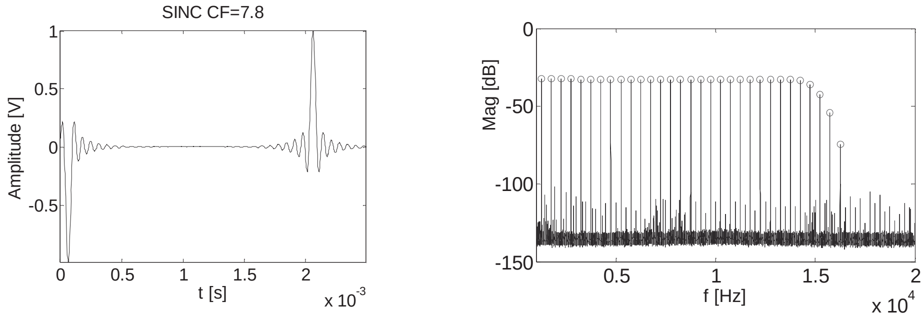

Fig. 13. The time plot of the SINC signal - 31 Tones, *CF* = 7.8, *BW* = 15 kHz.

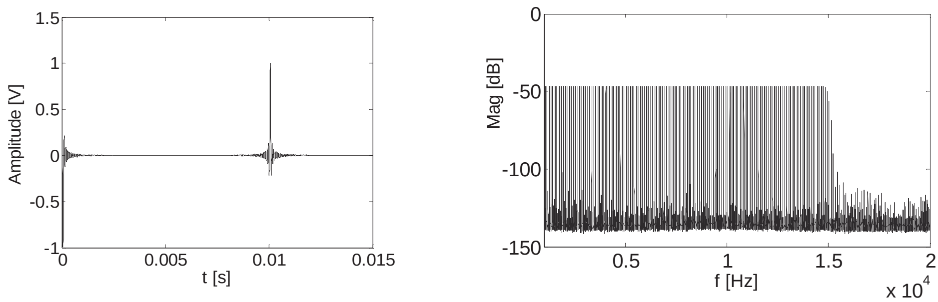

Fig. 14. The time plot of the SINC signal - 143 Tones, *CF* = 17, *BW* =15 kHz.

The internal DAC in the multifunction PXI card from National Instrumests (DAQ NI PXI 6251) was chosen as the DUT. The sampling frequency was set to 400 kSa/s. The bandwidth was limited to the range from 20 Hz to 20 kHz. The results of the tests are summarized in Table 3.

Table 3. Summarized results of DAC testing.

Signal	Frequency	Number of tones	*CF*	*ENOB* (bit)	*SINAD* (dB)
SW	3358.87 Hz	1	1.4	14.3	87.8
SW	7359.87 Hz	1	1.4	14.1	86.5
SW	9784.52 Hz	1	1.4	13.9	85.6
SW	9784.52 Hz	1	1.4	14.3	87.9
Dual Sine	3357.87 Hz, 7359.87 Hz	2	2.0	14.2	87.4
Triple Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz	3	2.5	14.2	87.5
Quad Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz, 15987.41 Hz	4	2.8	14.3	87.6
DSW	170 Hz – 17 kHz	17	1.7	14.6	89.7
DSW	170 Hz – 17 kHz	17	2.6	14.5	89.1
DSW	170 Hz – 17 kHz	17	4.0	14.4	88.5
SINC	170 Hz – 17 kHz	17	5.6	14.2	86.9
SINC	170 Hz – 17 kHz	31	7.8	14.3	87.7
SINC	170 Hz – 17 kHz	141	17.3	14.3	87.6

Metrol. Meas. Syst., Vol. XIX (2012), No. 1, pp. 105-114.

6. Conclusions

This work comes from the methods using a multi-tone signal with uniform amplitude distribution in the spectral domain for ADC testing. In addition, this paper shows other ways to test or generate the testing signal with significant numbers of the spectral components. An elegant solution is to use a pulsed SINC signal, which has also uniform amplitude displacement of the spectral components. A benefit of those SINC or DSW methods is that you do not need to choose all parameters for every tone in comparison with the sum of sine-wave signals. For a FFT test it is better to have uniformly distributed amplitudes of the spectral components. Then every tone has the same impact on the results. In this case, the *ENOB* obtained from the test expresses the average performance of the DUT in the chosen frequency range. The results show that it is possible to use MT, DSW and SINC signals for DAC testing. After correction, the results are comparable with the standard *SW* FFT test. When the signal is generated by the DDS system, it is necessary to limit the frequency range by a digital filter. This method should be primarily used in a fast audio-codec test in a wide frequency bandwidth. However, the 2-MSamples were acquired during one test. Therefore, future research will be focused on reducing the number of samples with linear or non-linear fit test methods with economic aspects of short testing in the industry area.

To measure such a performance using a Single Sine Wave signal it is necessary to do as many tests as the number of tones of the band signal. For example, a 17-tone signal is used for testing. The standard procedure, which covers this frequency range, will consume 17 times much time in comparison with a band signal test. Another important thing how to save time is to sample a smaller amount of samples. It was proved that it is sufficient to use only 30 kSamples to reach 0.2 bit deviation of the *ENOB* in comparison with results obtained from 4 MSamples [4]. Typical costs for testing a 16-bit ADC are approximately 100 USD per hour. A typical test takes 80 seconds, a 17-tone test takes 22 minutes. The test using a band signal still takes 80 seconds; it means we save 35USD (during one 17-tone test). An interesting observation during the test is that the characteristic of the *ENOB* is slightly rippled. The ripple maximum is 0.4 bits. The question is how it affects the pulse signal test. From its nature, the pulse signal tests the average performance in a chosen frequency range. The average *ENOB* obtained from a set of single tests is 14.2 bits and the ENOB from a *SINC* test is between 14.2 bits and 14.3 bits. Therefore, it possible to say the results follow the idea. In future, it will be interesting to test a device that has a significant drop in the *ENOB* characteristic, for example a low cost audio codec.

Acknowledgment

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Chapter 7 Conclusion

7.1 Achieved Objectives

The complex information is obtained from the test which uses band signals. The average *ENOB* or *SINAD* are obtained from a single measurement in chosen band which save a time. The comparison of measuring DUT's *ENOB* at chosen band can serve as an example. A standard procedure will do i.e. 20 times the same test at 20 different frequencies. The faster measurement will be reached by using i.e. multi-tone signal as a sum of 20 sine waves. The average ENOB will be obtained in one measurement. The same situation will be with sinc signal or with a chirp signal used in Wobbler Test. Those signals have uniformly distributed spectral lines. In other methods using exponential, modulated, damped sine wave signals, spectral lines are not uniformly distributed. Then the impact of each spectral line has different weight. The benefit of this method is that natural signal for a specific system or application could be used.

The process of developing an automated system is described in Chapter 3. The first manual measurements with oscilloscope were time consuming so one of the tasks was to replace the repetitive measuring and generating task by automated process. To achieve this goal several subtasks had to be fulfiled. The first problem was to find suitable hardware. The second one was to program chosen harware.

The PXI platform from National Instrumets has been used for the testing purposes. PXI chassis was equipped with a 24 bits reference digitizer (NI PXI 5922) and a tested devices were 16 bits generator (NI PXI 6251). The system was driven by PC with LabVIEW. The system was designed to produce consistent data when changing testing signals. It allows the comparison of results from different test methods for ADC and DAC testing. The paper *Developing Automated Data Acquisition System* presents several solutions that have been used in this thesis including the PXI measuring systems in Chapter 3. The relevant instruments produced by National Instruments offered a very convenient way to create a custom made application in a short period of time. The main disadvantage of this approach is its enormous prize. This fulfils the goal 3, but the prize is contradictory to the requirement for economical testing as defined in the thesis.

3. *Designing a test system to get consistent results from proposed methods (Chapter 3).*

The research was divided into three main subtopics using 3 different group of signal (Chapter 4 - Chapter 6). The first part is devoted to exponential signal and chirp signal. The second part analyzed modulated signals: AM, FM, QAM signals. The last part devoted to the usage of impulse testing signals: *DSW*, *Sinc* and *Multi-Tone* signals in dynamic testing. This fulfills requirements of the goal 2 of the thesis.

2. *Finding out the set of suitable signals (Chapter 4 - Chapter 6).*

A paragraph with analysis of a chosen group of signals is included in Chapter 4 - Chapter 6. This is in accordance with requirements of the goal 1 of the thesis.

In summary of Chapter 4 - Chapter 6 the measured results were compared with a corresponding standard method. This corresponds to the requirements of the goal 8 of the thesis.

As required by goal 7 of the thesis, the main conlusions are stated in following paragraphs.

1. *Theoretical analysis (see Analysis in Chapter 4 - Chapter 6).*
7. *Each chaper is also focused on evaluation methods, finding out their benefits and disadvantages (see Summary in Chapter 4 - Chapter 6).*
8. *Each chaper also includes comparison of new methods with standardized method (see Summary in Chapter 4 - Chapter 6).*

There were two signals (an exponential and a chirp signal) mentioned in Chapter 4. Parameters of an Exponential Fit Test and Wobbler Test were analysed. ADCs were evaluated in time domain using fitting algorithms. Both of the signal decribed in the Chapter 4.are suitable for fast ADC testing. Usage of such a test can be in end of line testing in manufacturing process. This fulfils the goal 4 of the thesis.

4. Analyzing and verifying non-traditional tests using fitting algorithm using exponential and chirp signals (Chapter 4).

FFT methods using modulated signals were presented in Chapter 5. The results of the test (computed *ENOB*) were significantly infuenced by chosen signal. AM signal differed from single sine wave signal. On the other hand FM signal and single sine wave signal gave comparable results. The standard equation for computing *ENOB* and *SINAD* are derived for sine wave signal. In case of windowing or using signal with different crest factor than sine wave results differed in comparison with standard sine wave FFT test. An elegant solution was proposed using a *Crest Factor* (CF) correction. After corrections a compact results were collected and it was possible to compare results from AM.

FM and single sine wave test. AM test is easier for signal processing, its spectrum contains only three tones. On the other hand with FM test the DUT can be tested in much wider frequency range. Because the tones in AM and FM signals are uniformly distributed, the problem with some of the intermodulation products masking the frequencies of AM or FM signal occurs.

A QAM signal for testing in the paper FM and QAM signals for ADC testing is mentioned in Chapter 5.

It showed up that QAM signals was not a good candidate for a dynamic DUT's testing. It contain discontinuities such as phase steps amplitude steps which produces higher harmonics distortions and they might violate the condition that *Slew Rate* of a DUT has to higher than maximum *SR* of the signal. This fulfilled the goal 5 of the thesis.

5. *Analyzing and verifying non-traditional tests using modulated signals in frequency domain (Chapter 5).*

There are three signals of impulse type (DSW, sinc, Multi-Tone signals) included in In Chapter 6. The paper *ADC and DAC testing using impulse signals in frequency domain* included in this chapter follows the previous research using different set of signal such as sinc signal and damped sine wave signal. The *CF* correction was already solved in Chapter 5. There were other challenges solved in Chapter 6. The first one was a discretization of amplitude spectrum using repetition of the signal segments. The second one was filtering higher frequencies using digital filter. The third one was to remove discontinuities occuring when generating a signal by repeating of its segments by symmetrization of its shape in time domain.

Benefits of DSW and sinc signals consists in the fact that they have wide frequency spectra inspite of requirement of only few parameters needed for their definition.. It it necessary to limit their frequency range and symmetrize them. It is possible to create desired waveform only once and used it many times. A sinc signal which has uniformly distributed amplitude spectral lines is one of the most promissing testing signal.

The main disadvantage of Uniformly Distributed Multitone signal lies in the fact that at least three parameters for each tone has to be defined. The advantage of such signal is its simplicity with no necessity of digital preprocessing (filtering or signal‘s symmetrization). By this fact the goal 6 of thesis is fulfilled.

5. *Analyzing and verifying non-traditional tests using Impulse signals in frequency domain (Chapter 6).*

There were several subtasks included in the solution of this problem. One task was to do theoretical analyses. This point was accomplished with help of the program Wolfram Mathematica[K4]. Another one was the analysis of the huge amount of data that after each measurement.. Matlab was the answer how to evaluate really easily even excessive amounts of data.

Three articles describing the thesis results were published in journal with impact factor The partial results were presented at 7 prestigious conferences and 5 other important conferences. An authorised software was designed and one lecture at Beihang University waspresented. The author papers were cited 5 times by foreign authors.

During measurement using soundcard, oscilloscopes, generators, digitizers many experiences were collected. Programming in LabVIEW led to achieving certificate of LabVIEW developer in 2012. Knowledge of signal processing in Matlab helped me in my primary reseach and also in reseach on other project which was rewarded with publishing other papers not related to the thesis. [C.2]

7.2 Future Research

Band signals in general save a time of measuring overall score of DUT in chosen band. There is possible to optimize the time reduction even more reducing number of samples acquired during a test. This optimization how many samples are needed for chosen signal could be done in some future research.

Generally *ENOB*(*f*) as a function of frequency is not flat function. For enhancement of DUT testing to find a method showing how to simply evaluate the average *ENOB* computed from test using band signal. I assume that there might be differences using signal with uniformly and non-uniformly distributed spectral lines. The weights of non-uniformly distributed spectral lines could change their impact in average *ENOB.*

When evaluating an *ENOB* in i.e. an end of line test, this parameter does not discover particular anomalies defects like missing code word of ADC. Thus the detectability of such an error is low.

I have proposed fitting algorithms showing another approach leading to improvement of detectability of irregularities of DUT transfer characteristic using precise fitting in time domain allows to detect differences between measured and fitted signal. Assuming the imperfection of the measured signal is caused by DUT not by a testing device detection of anomalies is at least in principle feasible. So my vision in future research is to search for suitable fitting algorithms for non-traditional testing signals (SINC, DSW) based on the following ideas:

The fitting methods are commonly used to measure *ENOB* using two approaches. First option is implementation of linear fitting method using least minimum square error. This method is suitable for AM and multi-tone signal composed from sum of sine waves. Analogically FM signal is also composed from sum of certain tones and allows to implement linear fitting methods..

Second approach is application of non-linear fitting method. This method in general suffers by lack of convergence in case of badly chosen initial conditions and in case of noise and non-linearity of the signal etc.

For the this type of signal the main task is finding a suitable sampling frequency, carefully choosing initial conditions, predict maximum iterations of the fitting algorithm, avoiding minimum and maximum steps during iterations. First measurements and evaluations in an ideal case corresponded to Single Sine Wave FFT Test [K12]. However this relatively simply implemented method is not robust enough for application in automated testing. For evaluations of the results a MATLAB toolbox CFTOOL (Curve Fitting Toolbox) was used [K11].

Also there is challenge to implement those fitting algorithms in signal processor for fast evaluation of the DUTs.

I have made a several measurements on a multifunctional card NI PXI 6251 [B6] with reference device NI PXI 5922 [B5]. The results are shown in table 9. As the reference method best sine wave fit test was used. A huge difference between sinc signal and sine-wave signal is probably caused by uncertainty of estimating *CF*. In case of DSW the results are much worse because chosen fitting algorithm failed.

Table 9 Results from Sinc Fitting Tests

Signals	*CF*	*ENOB* (bit)	$SINAD_{corr}$ (dB)
Sinewave (f_n = 10 kHz)	1.41	10.6	65.9
Sinc(x) (f_n = 1 kHz, f_r = 500 Hz	24.5	11.7	47.5
Sinc(x) (f_n = 15kHz, f_r = 1 kHz)	10.9	12.9	61.4
Sinc(x) (f_n = 15k Hz, f_r = 500 Hz)	7.7	12.6	62.9
Sinc(x) (f_n = 15 kHz, f_r = 100 Hz)	5.4	10.6	53.7
DSW (*d* = 0.016, f_n = 5kHz, f_r = 130 Hz)	3.7	10.9	58.9
DSW(*d* = 0.032, f_n = 5k, f_o = 260 Hz	3.7	8.6	44.9
DSW(*d* = 0.064, f_n = 5k Hz, f_r = 260 Hz	5.0	9.1	45.5
DSW(*d* = 0.127, f_n = 5 kHz, f_r = 1kHz)	3.2	8.6	46.3
DSW(*d* = 0.255, f_n = 5 kHz, f_r = 900 Hz	4.1	6.6	32.3

where f_n is a frequency of the signal and f_r is a frequency of the repetition of signals segment

Another approach in decreasing computational load and duration of DUT test could be an implementation of Field Programmable Gates Arrays (FPGA).

As it is well known, the operations required by testing procedures can be performed by hardware in FPGA, i.e. by programmable configuration on the logic gates level. As clock frequency in FPGA can reach several hundreds of MHz, the speed of evaluation could be very high and even most complicated testing algorithms can be included in category of economical ADC testing.

Appendix A List of symbols, abbreviations and variables

A.1. List of symbols / abbreviations

ADC	Analogue to Digital Converter
AM	Amplitude modulation / amplitude modulated
DAC	Digital to Analogue Converter
dB	Decibels
dBc	Decibels relative to the carrier
dBFs	Decibels relative to full scale
DC	Direct Current
DFT	Discrete Fourier Transform
DSW	Damped Sine Wave
DUT	Device Under Test
FFT	Fast Fourier Trasformation
FM	Frequency Modulation
j	Imaginary unit
MSE	Minimum Square Error
MT	Multi-Tone
PXI	PCI eXtensions for Instrumentation
QAM	Quadrature Amplitude Modulation
SINC	Function sin(ωx)/(ωx)
SW	Sine Wave
UDMT	Uniformly Distributed Multi-Tone

A.2. List of variables

A,B,C,X,U	*Amplitutes in time domain*
CF	*Crest Factor*
d	*Damping ratio (damped sinwave)*
D	Matrix notation of a fitted model
DR	*Dynamic range*
e,u,x	Error vector, data series, fitted parameter vector
e	Exponential function
ENOB	*Effective number of bits*
e_{RMS}	*Root mean square of residuals obtained from fitting algorithm*
f	*frequency* [Hz]
FS or FSR	*Full Scale or Full Scale Range*
i,k	*Indexes used in DFT, FFT formulas in general it is used with series and in sigma notation*
IMD	*Intermodulation Distortion*

φ	*Phase of the signal*
LSB	*Least Significant Bit*
M	*Number of samples by DFT*
m	*Number of multi-tone components*
m_{AM}	Modulation depth (AM)
m_{FM}	Modulation index (FM)
n	*Nominal number of bits* - DUT resolution [bits]
ND	*Noise and Distortion*
NFL	Noise Floor
Q	*Ideal code bin width*
RMS	*Root Mean Square*
RSS	*Root Sum Square*
σ_n	*Standard deviation of a noise*
σ_x	*Standard deviation of a signal*
SINAD	*Signal Noise and Distortion*
SFDR	*Spurious Free Dynamic Range*
SNHR	*Signal Non-harmonic Distortion*
SNR	*Signal to Noise Ratio*
SR	*Slew Rate*
t	*Time*
THD	*Total Harmonic Distortion*
$u(i)$,$w(i)$	A window function, a data series representing testing function
ω	*Angular frequency*
$W[k]$	Code bin width
$X[k]$	*Sequence of complex numbers obtained from* DFT
X_{pp}	Peak to peak value of a signal

Appendix B Lists of references

B.1. References Chapter 1 and 2

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Appendix C Author's Publications and Grants

C.1. Author's Publications Related to the Thesis

C.1.1 Publications in Journals with Impact Factor

[1] Fexa, P. (70%), Svatoš, J. (15%), Vedral, J. (15%), (2009). Methods for economical test of dynamic parameters ADCs. *Metrology and Measurement Systems*, *16* (1), 161-170. ISSN 0860-8229. *Number of citations: 1*

[2] Fexa, P. (70%), Svatoš, J. (15%), Vedral, J. (15%), (2011). DAC testing using modulated signals. *Metrology and Measurement Systems*, *18*(2), 283-294. ISSN 0860-8229. *Number of citations: 3*

[3] Fexa, P. (75%), Vedral, J. (25%), (2012). DAC testing using impulse signals. *Metrology and Measurement Systems*, *19*(1), 105-114. ISSN 0860-8229. *Number of citations: 0*

C.1.2 Author's Publications in ISI

[4] Fexa, P. (10%), Svatoš, J. (10%), Vedral, J. (80%), (2009). Economical test of internal ADC in embedded systems. In *Proceedings of XIX IMEKO World Congress, Lisbon, Portugal* (pp. 702-705). ISBN 978-963-88410-0-1. *Number of citations: 1*

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[7] Fexa, P. (50%), Svatoš, J. (25%), Vedral, J. (25%), (2010, September). FM and QAM signals for ADC testing. In *2010 International Conference on Applied Electronics*. Pilsen: University of West Bohemia, p. 359-362. ISBN 978-80-7043-781-0. *Number of citations: 0*

[8] Fexa, P. (25%), Svatoš, J. (25%), Vedral, J (50%), (2010, May). Using of AM and FM signal for ADC testing. In *Instrumentation and Measurement Technology Conference (I2MTC), 2010 IEEE* (pp. 508-511). Austin, Texas, 2010, p. 508-511. *Number of citations: 0*

[9] Fexa, P. (50%),Vedral, J. (50%), (2011, September). Developing automated data acquisition system for ADC and DAC testing. In *Intelligent Data Acquisition and Advanced Computing Systems (IDAACS), 2011 IEEE 6th International Conference on* (Vol. 1, pp. 39-42). ISBN 978-1-4577-1426-9 *Number of citations: 0*

[10] Fexa, P.(50%), Vedral, J. (50%), (2011, September). Using of pulse signals for DAC testing. In *Applied Electronics (AE), 2011* (pp. 1-4). ISBN 978-80-7043-987-6. *Number of citations: 0*

C.1.3 Author's Other Publications

[11] Fexa, P., Svatos, J., Vedral, J. (2009). Dynamic Testing of Audio Codec. In: *Electronic Devices and Systems, IMAPS CS International Conference 2009 Proceedings*. Brno: VUT v Brně, FEI, 2009, vol. 1, p. 63-68. ISBN 978-80-214-3933-7. *Number of citations: 0*

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[14] Vedral, J. (50%), & Fexa, P. (50%), (2011, May). ADC and DAC testing using impulse signals. In *Mixed-Signals, Sensors and Systems Test Workshop (IMS3TW), 2011 IEEE 17th International* (pp. 96-99). IEEE. *Number of citations: 1*

[15] Vedral, J. (50%), & Fexa, P. (50%), Using of Pulse Signal Sinx/x for DAC Testing. In *IMEKO TC4 International Workshop on ADC Modelling, Testing and Data Converter Analysis and Design [CD-ROM]* (pp. 42-45). *Number of citations: 0*

C.2. Author's Publications Non-related with Thesis

C.2.1 Author's Publications in Journals with Impact Factor

[16] Svatoš, J.(70%), Vedral, J.(15%), Fexa, P. (15%) (2011). Metal detector excited by frequency-swept signal. *Metrology and Measurement Systems, 18*(1), 57-68. ISSN 0860-8229. *Number of citations: 3*

C.2.2 Author's Publications in ISI

[17] Svatoš, J. (50%), Fexa, P. (25%), Vedral, J. (25%), (2010, September). Metal detector excited by polyharmonic signals. In *Applied Electronics (AE), 2010 International Conference on* (pp. 1-4). IEEE. ISBN 978-80-7043-865-7. *Number of citations: 0*

C.2.3 Author's Other Publications

[18] Fexa, P. (2012). Automatizovaný systém pro testovaní Č/A převodníků. In: *NI Days 2012*. "ftp://ftp.ni.com/pub/branches/ee/2012/nidays/fup/cz-nidays-2012.zip". *Number of citations: 0*

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Appendix D Inserted Articles

Inserted articles had to be rescaled to fit the thesis layout this reduce readability, so the work contains a DVD with electronic form of papers which were inserted into this thesis.