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OPTIMIZATION AND SIMULATION OF SPECIAL OPTICAL IMAGING SYSTEMS

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## Abstract

## Résumé
Photographic digital technology plays an important role in different areas of both research and industry respectively. The emerging potential digital sensors enables to replace the classical photographic techniques with the modern digital technologies in many sectors of medicine, microscopy, military, security or astronomy. Such a digital technologies help to easily archive, access and search the acquired data. Particularly, the high resolution digital technology and its rapidly growing development has the significant influence in field of astronomy. Progress and testing of the new and more modern technologies in astronomy, brings the possibilities to realize precise measurements and obtain better and more exact results.

The increasing quantity of obtained data has to be taken into account at the same time, which increases the need for automatic data acquisition and evaluation. Therefore, the automatic astronomical optical systems find the growing importance in last several years. The main idea of automatic astronomical optical systems is based on the long term image data collection. These systems are often used as a ground based supporting experiments, such as Burst Observer and Optical Transient Exploring System (BOOTES)\(^*\) and Burst Alert Robotic Telescope (BART)\(^1\), for the special satellites, for example INTEGRAL\(^2\) or BeppoSAX\(^3\). Since this thesis is aimed at scientific (astronomical) image data processing, the image data acquired from the experiment BOOTES, double station video observation of the meteors, and its digital version Meteor Automatic Imager and Analyser (MAIA) are used in this work. While, the BOOTES is equipped with a set Charged Couple Device (CCD) cameras and telescopes with various focal length, each for the different monitoring task, MAIA and the double station video observation system is used to provide the trajectory of observed meteors in the Earth’s atmosphere.

Systems, mentioned above are dedicated to the specific tasks and their use as well as the use of other professional astronomical instrumentation for a general astronomical monitoring is very limited. Practically, a waiting list for astronomical observation is created for all professional telescopes whose network is also very sparse, not to mention the price of such equipment. Moreover, there is a large number of astronomical events which needs to be monitored, such as Optical Transient (OT) of Gamma Ray Bursts (GRBs), Novae, Novae, Novae, Novae, Novae, Novae.

\(^*\)http://www.laeff.esa.es/BOOTES/ing/introd.htm
\(^1\)http://lascaux.asu.cas.cz/en
\(^2\)http://www.esa.int/esaMI/Integral/
\(^3\)http://www.asdc.asi.it/bepposax/
Variable Stars, Active Galactic Nuclei (AGN) and others, and even the fastest optical follow-up telescopes cannot access the times close or identical to times of GRBs.

Since the position of these astronomical events is unpredictable, they can be accessed only by optical monitors from which the all sky monitors offer the best sky coverage. In order to monitor the Field of View (FOV) of the professional systems, an alternative approach using the optical wide field camera can be used. The idea of using the network of the low cost wide field imaging systems in order to monitor the sky is more and more popular due to 24/7 data acquisition opportunity.

However, the processing of image data acquired form wide field systems bring a lot of challenges. Due to low sensitivity, noise properties, optical aberrations and their spatial variance, and the size of the stelar objects, the precise astronomical measurement cannot be performed over entire FOV. Practical measurement evaluates wide field image data only around the midle of FOV, the marginal parts of the images cannot be handled by common measurement techniques, such as Image Reduction and Analysis Facility (IRAF), in term of accuracy. This work deals with the issues caused by optical aberrations of such systems and addresses the accuracy of astronomical measurement performed on wide field image data. More specifically, the two approaches how to improved the astronomical measurement are discussed further in this work. Considering the low cost imaging system, the possibility of using the Adaptive Optics (AO) element in order the determine its wavefront aberration and Point Spread Function (PSF) for any source position, is not real due to its price of such a element.

### 1.1 Aims of the Thesis

The main aims of this thesis is to contribute to the knowledge in the area of wide field astronomical imaging, while the low cost systems are particularly be addressed. More specifically, the individual aims of this thesis are:

- Investigation of the possibilities of an Wide Field Camera (WFC) system and its use for monitoring in astronomy. This includes the description of the low cost all sky monitoring system.
- Design of the Spatially Variant (SV) model of a PSF corresponding to WFC optical system and its simulation. As mentioned above, this model should reflect the optical aberrations of wide field system.
- Improvement of the astronomical measurements performed on the image data obtained with WFC optical systems.
- Proposition and implementation of a suitable novel method for wide field astronomical image data restoration.

### 1.2 State of the Art

This section targets to briefly summarize general findings in the field of optical system modelling and spatially variant image restoration. Optical systems design, modelling and analysis have been solved since the first half on the nineteenth century. At the present
1.2. State of the Art

time the Computer Aided Design (CAD) software, such as Optics Software for Layout and Optimization of optical systems (OSLO) is used to design the optical systems. In order to model the transfer properties of the optical system and thus their aberrations, the two methods based on the Seidel and Zernike polynomials [Bass et al. 2009a, Born et al. 1999, Carvalho 2005, Maeda 2003, Mahajan 1994a] are used. The Seidel or Zernike polynomials are used for precise calculation of the wavefront aberrations of the optical system [Mahajan 1995, Mahajan 1994b, Hopkins 1950]. Seidel polynomials are convenient for serving as a primary aberrations interpreter, while even high order wavefront aberrations can be described using the Zernike polynomials with higher precision [Braat et al. 2011].

Optical observation instruments using wide field lenses are usually designated as Ultra Wide Field Camera (UWFC) and/or WFC [Reeves 2000, Symposium & MacGillivray 1994]. All sky imaging (monitoring) based on fisheye lenses is used in some applications as well. The images from these systems contain a survey data with huge amounts of object with small size. Detection of new objects e.g. novae, supernovae and AGN is well known application of these images analysis.

UWFC image data analysis is very difficult in general not only because of presence of the optical aberrations and distortions different kinds. Even with modern optical design method, the UWFC systems without optical aberrations cannot be designed [Smith 2005, Smith 2000, Lam 2001]. The largest amount of optical aberrations and distortions occurs at the edges of FOV. Moreover, the resolution of wide field images decreases toward margins due to the geometric distortion and due to a change of scaling between the projection of spherical surface on the flat surface.

Optical aberrations negatively affect the image quality and imaging system transfer characteristics and cut precision of astronomical measurements. The most important methods for astronomical image data processing consist of photometry and astrometry measurement, false objects detection and their classification, Mean Square Error (MSE), Signal to Noise Ratio (SNR) and subjective evaluation [Berry & Burnell 2005]. As mentioned above, the wide field optical systems contain optical aberrations and distortions, which increase toward margins of FOV. Thus, the edges of wide field images are not evaluated in general.

Due to the aberrations field dependence, the WFC systems has spatially variant PSF. Therefore, in the low cost wide field systems, for the image processing applications (such as deconvolution) the field dependence of PSF must be taken into account. Several approaches to deal with the spatial deconvolution ill-posed problem have been proposed. Most of the proposed algorithms involve narrow application [Robbins & Huang 1972, Sawchuk 1974, Sawchuk & Peyrovian 1975], or the extensive iteration process [Tekalp & Pavlovic 1989, Sezan & Stark 1983, Sezan & Tekalp 1990, McNown & Hunt 1994, Ozkan et al. 1994] or significant increase it terms of complexity [Costello & Mikhail 2003, Trussell & Hunt 1978, Richardson 1972, Schafer et al. 1981]. However, the amount of the publications dealing with spatially variant deconvolution in order to remove the influence of the optical aberrations is very low [Kieweg et al. 2010]. In [Kieweg et al. 2010], low complexity restoration of images degraded by physical optics is presented, authors use sectioning method proposed in [Trussell & Hunt 1978] and optimized Wiener filter taking into account the processing of multimedia images.
Professional astronomical instrumentation, even the fastest one, cannot observe and record the image data from all sky at once. In order to avoid the situation that the important astronomical event is missed, it is very important to monitor all sky using the wide field imaging systems. Even though, the precision and quality of the wide field image data cannot compete with those acquired from professional deep sky telescope, wide field systems are successfully used for initial detection of astronomical objects and events.

Wide field imaging systems can monitor large FOV and thus gives a bulk of information for statistics, such as positions, magnitudes, etc. Therefore, the wide field monitoring of the sky is used in several branches of astronomy. More specifically, the wide field systems are usually used for searching for short OT of astronomical origin (i.e. searching for optical flashes accompanying GRB), studying the fast variable stars, and monitoring activity of blasars and other AGNs.

Astronomical images obtained with high end WFC or UWFC systems have high spatial resolution (up to 4096 x 4096 pixels) and they are usually represented with 16 bits precision in the grey scale. These images contain a survey data with huge amounts of objects with small size, which can be statistically processed and evaluated. The detection of new objects e.g. novae, supernovae, and AGN is often and well known application of analysis of these images. However, analysis and evaluation of an image data acquired from WFC systems is very difficult due to a number of optical aberrations and distortions of different kind contained mainly in their optical part. Moreover, the data reduction from UWFC systems is mostly complicated by low spatial sampling of the signal, which together with a significant optical aberrations influence the PSF and its spatial variance. In order to show the main characteristics of an astronomical image data, the real images from several WFC systems were analysed. Description of several astronomical systems, whose image data were used for our experiments in this work, is presented below.

**Processing of Acquired Image Data**

To show the characteristics of the image data, the real digitalized recording in uncompressed Audio Video Interleaved (AVI) format was processed and analyzed. The suitable video sequence, where at least several stars move in time from the edge to the middle of FOV or vice versa, were chosen for our purposes (see Figure 2.1a and 2.1b). Five different times for five different positions of each object $A, B, C, D$ were selected. At each time $T_1, ..., T_5$, the
several video frames were cut out to form uncorrected testing image, which was determined as frames median to avoid the background noise. Then, the usual astronomical image preprocessing techniques were conducted. Subsequently, the centre of gravity for each object $A, B, C, D$ at each time $T_1, ..., T_2$ was determined and images of size $15\times15$ pixels for each object and time were isolated and used for our measurement.

Detailed analysis of each of the suitable star was performed and some main characteristics showing variability of the stars through FOV are shown in Figure 2.1c, 2.1d, 2.1e and 2.1f. The stars vary not only in their position but also in their brightness and shape, which is the main characteristic of space variant system.

2.1 Aspects of Using the Low Cost Imaging Systems in Astronomy

In this section, the study of potentials of a low cost imaging system is presented, the enhanced version of this study was published in [Páta et al. 2010] by the author and his colleagues. Recent emerging potentials of the utilization of low cost imaging systems enables their use for astronomical measurements, as is has been proved among others for example at the observatory Karlovy Vary. Such systems usually exploit conventional and commercially easily available digital cameras. However, using a consumer level digital camera for astronomical observations brings the following implications.

- **Advantages**
  - Optical light curves of the bright stellar objects. Long term monitoring of bright stars with the wide field digital camera can enhance the information of the object variability. Such a measurement, commonly referred to as a differential photometry, can be effectively done only for the bright stars where larger amount of statistical data is more important than the low precision of the measurements.
  - Fast observation and detection of new bright effects like supernovas, optical counterparts of GRB etc.
  - Net observation of the meteors including astrometry and fast meteor trajectory determination.
  - Light pollution monitoring. In cooperation with precise astronomical instrument can low cost camera help to determine local observational conditions.

- **Drawbacks**
  - Low bit depth of the Analog to Digital (A/D) converter. Most of the low cost digital cameras uses the 8 bits representation.
  - The absence of the standardized raw image file data container independent on the camera manufacturer, similar to Flexible Image Transportation System (FITS).
  - Color representation. Conventional digital camera sensors use Color Filter Array (CFA) masque, thus the interpolation techniques has to be exploited or raw data format together with knowledge about sensor spectral response is used.
  - The absence of several optoelectronics parameters documentation. The manufacturer does not provide the important parameters, such as the real effective area of one pixel, quantum efficiency of the spectral response, stabilized read-out noise, and A/D gain for the low cost sensors.
2.1. Aspects of Using the Low Cost Imaging Systems in Astronomy

Figure 2.1: Properties of the double station video observation system. (a) Testing image at the time T1. (b) Testing image at the time T5. (c) Star brightness dependence on radial distance from the middle of FOV. (d) Magnitude error dependence on radial distance from the middle of FOV. (e) FWHM versus radial distance from the middle of FOV. (f) Normalized ellipticity of the object A profile as a function of angle distance from the middle of FOV (zero value means round shape).
– Device inadaptability for strong cooling system.

The astronomical utilization of Digital Single Lens Reflex Cameras (DSLRs) is particularly limited by lens properties (focal length, wavefront aberrations etc.). Astrometry and photometry precision, and shape and distortion of PSF are the important aspects for the astronomical utilization of DSLRs. For precise astronomical measurement, the difference of spectral transparency of RGB and UBVRI filters is restricting. Especially, the noise properties of digital sensor of a low cost cameras and applied CFA mask have the considerable impact on image quality. One of the main handicaps of digital camera astronomical utilization is lower precision of astrometric measurements. On the other hand, one of the main benefits is low price, which allows to construct the setup of several observation stations.

2.2 Optical Systems Modelling and Simulation

The purpose of this section is to describe the modelling of optical aberration and to introduce new approach for PSF estimation. This approach will be used as a novel tool for wide field cameras in astronomy. Two sets of basis function, usually used for modelling of wavefront aberrations are discussed. These models (Seidel and Zernike polynomials) serve us for understanding how the optical aberrations affect the transfer characteristics of optical systems. Furthermore, the issues of spatially variant imaging are discussed its model based on Zernike polynomial is proposed, implemented and simulated. Special attention is paid to the UWFC system such as fisheye lens and its geometric distortion model is discussed and simulated bellow in this chapter. Moreover, the design of fisheye lens has been created and it is also briefly described. Issues related to an astronomical measurements performed on the wide field images are discussed and the method of measurement quality improvement is proposed and tested below. Last part of this chapter deals with the aberrations estimation and proposes a method to estimate the amount of aberrations in the optical system without using adaptive optics. Several measurements of a real optical system PSF were conducted and their results are presented.

An ideal optical system, having all aberration compensated is commonly referred to as a diffraction limited system. PSF of a diffraction limited imaging system is described as the Fraunhofer diffraction pattern of the exit pupil [Goodman 2005]. Realization of such a system practically is not possible, due to the optical aberrations, which negatively affect the image quality and imaging system transfer properties. In the following text, the description of the optical aberration is presented. Assuming the ray optics, the optical path of the light rays through an optical system is determined by an optical design of this system. More specifically, the trajectory of a ray of light can be expressed by the refractive and incident angles on the individual optical surfaces. These angles are formed by an investigated ray and the optical axis. The ray approach describes the aberrations using the deviations of intersection of the geometrical rays from geometrical point projected by an ideal optical system. For each ray passing through the optical system, the ray aberration is computed. The transversal perturbations of each ray with origin field coordinates \((x_1, y_1)\) and intersecting the exit pupil at the coordinates \((\xi_1, \eta_1)\) can be computed. The Seidel (primary) field dependent (i.e. dependent on object point coordinates) third order ray transversal aberrations are expressed in cartesian coordinates as
2.3. Wave Aberrations

Figure 2.2: Simulation of ray aberrations. (a) Ray spot diagram computed for coma aberrated system for different object positions. 1800 rays for each point source are assumed for adapted Monte Carlo simulation. (b) Testing image. (c) Visualization of ray coma aberration. (d) Visualization of ray astigmatism aberration.

Visualization of the image formed by an optical system with a certain number of rays gives so-called spot diagram. The spot diagram for several ideal object points has been simulated. A SV optical system containing only the coma aberration and the regular aiming or 1800 rays (for each point) over the exit pupil have been considered. The resulting spot diagram in shown in Figure 2.2a. The third order aberrations, namely coma and astigmatism, have been simulated and the examples of images, distorted by these aberrations, are shown in Figure 2.2.

2.3 Wave Aberrations

The second approach derives benefits from diffraction theory and applies the approximating polynomials to estimate the wavefront aberration. The aberration influence to the resulting image is expressed by the wave aberration function. The wave aberration function, $W(\xi_1, \eta_1)$, is defined as the distance, expressed in Optical Path Length (OPD) from the reference sphere to the wavefront in the exit pupil, measured along the ray. The wave aberration function is therefore a function of the transverse coordinates $(\xi_1, \eta_1)$. According [Born et al. 1999, Maeda 2003, Smith 2005], the wave aberration function is expressible as a power series expansion or polynomials, such as, for example, a weighted sum of basis functions, monomials or modes, which are functions of the exit pupil coordinates. In order to describe the wavefront aberrations of the optical system precisely, the Seidel and Zernike polynomials are usually used as a basis functions. The Zernike and Seidel polynomials are normally expressed in normalized polar coordinates $(\rho, \theta)$ in exit pupil, where $0 \leq \rho \leq 1$ and $0 \leq \theta \leq 2\pi$.

Seidel Polynomials

Seidel aberration polynomials are convenient for serving as a primary aberrations interpreter. Considering the rotational symmetry of an optical system, the field dependent third order wavefront aberration function $W(\rho, \theta)$ in polar coordinates is expressed as

$$W(\rho, \theta) = W_{040}\rho^4 + W_{131}\rho^3\cos \theta + W_{222}\rho^2\cos^2 \theta + W_{220}\rho^2 + W_{311}\rho^3 \cos \theta \quad (2.1)$$
where $\rho$ and $\theta$ are polar coordinates of the exit pupil, $h$ is the normalized image height and $W_{i,j,k}$ presents Seidel coefficients for primary aberrations. The aberration coefficient are often expressed using Seidel sums $S_I - S_V$ [Malacara 2007] as

$$W(\rho, \theta) = \frac{1}{8} S_I \rho^4 + \frac{1}{2} S_{II} h \rho^3 \cos \theta + \frac{1}{2} S_{III} h^2 \rho^2 \cos^2 \theta + \frac{1}{4} (S_{III} + S_{IV}) h^2 \rho^2 + \frac{1}{2} S_V h^3 \rho \cos \theta. \quad (2.2)$$

Additionally, the Equation (2.1) can be simplified to

$$W(\rho, \theta) = \sum_{i,j,k} W_{i,j,k} \rho^i \cos^k \theta. \quad (2.3)$$

Zernike Polynomials

Zernike polynomials form a complete set of function that are orthogonal over a circle of unit radius and thus convenient for serving as a set of basis functions [Maeda 2003, Mahajan 2003, Mahajan 1995, Wyant & Creath 1992]. This makes them suitable for accurately describing wave aberrations as well as for data fitting. Zernike polynomials are used for description of the high order wavefront aberrations with higher precision. Each Zernike polynomial consists of two factors, the normalization factor $N_m^n$ and the radial polynomial $R_{n}^{m|}(\rho)$. Zernike polynomials are defined as

$$Z_n^m(\rho, \theta) = \begin{cases} N_m^n R_{n}^{m|}(\rho) \cos m \theta & \text{for } m \geq 0, 0 \leq \rho \leq 1, 0 \leq \theta \leq 2\pi \\ -N_m^n R_{n}^{m|}(\rho) \sin m \theta & \text{for } m < 0, 0 \leq \rho \leq 1, 0 \leq \theta \leq 2\pi \end{cases} \quad (2.4)$$

for a given $n$ the number $m$ takes values of $(-n, -n+2, -n+4, \ldots, n)$. The normalization and the radial polynomial factor are defined as

$$N_m^n = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}} \quad (2.5)$$

$$R_{n}^{m|}(\rho) = \sum_{s=0}^{n-|m|} \frac{(-1)^s (n-s)!}{s! [0, 5(n+|m|)-s]! [0, 5(n-|m|)-s]!} \rho^{n-2s} \quad (2.6)$$

where $\delta_{m0} = 1$ for $m = 0$ and $\delta_{m0} = 0$ for $m \neq 0$. Zernike polynomials can also be specified by the single indexing scheme

$$Z_j(\rho, \theta) = Z_n^m(\rho, \theta) \quad (2.7)$$

where

$$j = \frac{n(n+2)+m}{2} \quad (2.8)$$

$$n = \text{round} \left[ -3 + \sqrt{9 + 8j} \right] \quad (2.9)$$

$$m = 2j - n(n+2) \quad (2.10)$$
The Description of Wavefront Aberration Function Using Zernike Polynomials

The wavefront aberration function may be expressed as sum of Zernike polynomial weighted by a corresponding aberration coefficient as

\[
W(\rho, \theta) = \sum_{n}^{k} \sum_{m=-n}^{n} W_n^m Z_n^m(\rho, \theta)
\]

where \(k\) represents the polynomial expansion order and \(W_n^m\) are the coefficients equal to the wavefront Root Mean Square Error (RMSE) for a given mode of \(Z_n^m\).

2.4 Model of Spatially Variant Optical System

The relation between object and image of Linear Shift Invariant (LSI) optical system can be expressed by the convolution in the spatial domain, where the object irradiance distribution is convolved with the impulse response. It has been also shown, that the PSF of an optical system can be computed as a Fourier Transform (FT) of the aperture light distribution.

Considering the real LSI optical imaging with certain amount of aberrations, which are expressed by aberration wavefront function, the PSF will be \[\text{Goodman 2005}\]

\[
PSF(x_2, y_2) = \left| \mathcal{F} \left\{ p(\xi_1, \eta_1) \exp \left( -i \frac{2\pi}{\lambda} W(\xi_1, \eta_1) \right) \right\} \right|^2,
\]

where \(\exp[-i2\pi/\lambda W(\xi_1, \eta_1)]\) accounts for the phase deviation of the wavefront from a reference sphere and \(p(\xi_1, \eta_1)\) defines the shape, size and transmission of exit pupil.

In practice, imaging systems are seldom spatially invariant over the entire FOV and the input output relationship is expressed in general by the Fraunhofer diffraction formula. However, for a rotational symmetric systems, the field dependence of optical aberrations on image height \(h = \sqrt{x_1^2 + y_1^2}\) can by exploited to express the SV PSF. Therefore, the PSF of a SV optical system will be

\[
PSF(x_2, y_2, x_1, y_1) = \left| \mathcal{F} \left\{ p(\xi_1, \eta_1) \exp \left( -i \frac{2\pi}{\lambda} W(\xi_1, \eta_1, h) \right) \right\} \right|^2,
\]

According to \[\text{Lam 2001}\], the transverse aberration types are characterized by their field dependence upon image height \(h\) and by dependence upon exit pupil coordination \(\rho\). Aberrations depending on an odd power of \(\rho\) are astigmatic, while even power of \(\rho\) determines the comatic aberrations. The order of the transverse aberration can be determined by the sum of the powers of \(h\) and \(\rho\) \[\text{Lam 2001}\]. According to the statement from \[\text{Malacara & Malacara 2003}\], which basically says that the power of \(\rho\) for wavefront aberrations is greater by 1 than power of \(\rho\) for the transverse aberrations, the Zernike polynomials can be separated to correspond to the order of aberrations, see for example \[\text{Wyant & Creath 1992}\]. Thus, the field dependency of Zernike polynomials, corresponding to basic aberrations, upon the image polar coordinates \((h, \vartheta)\) is proposed.
**Figure 2.3:** Simulation of wavefront aberrations. (a) Original image. (b) Image distorted by coma aberration. (c) Image distorted by astigmatism aberration.

**Spatially Variant Imaging Simulation**

The above described model of an optical system base on SV Zernike polynomials is used for our simulations. Let’s assume the testing image (see Figure 2.3a) as an input to our model of an real optical system containing a certain number of aberrations. The PSF of such a system is computed according to the (2.13) with a simulated amount of aberrations for each object point (input image pixel). The influence of CCD and its sampling is taken into account afterwards. The contribution of each object point to the resulting distorted image is given as a convolution of the input and the corresponding PSF. The results of spatially variant convolution (i.e. real optical imaging) for two different aberrations (coma and astigmatism) are shown in Figure 2.3b and 2.3c.

**Precise Astronomical Measurements Using SV PSF Model**

Astronomical measurements performed in previous experiment rely on aperture photometry, however the stelar photometry using fitting functions commonly used in IRAF is rather more precise. These fitting algorithms uses Gaussian, Moffat, Lorentzian and Penny distributions. These algorithms doesn’t take into account the spatially variant influence of optical aberrations which occurs especially when wide field imaging system is used. Thus, for wide field systems, the fitting method based on the physical optics model has been proposed and simulated on a testing data. Testing image has been created with a method described in 2.4. The original image and the image which simulates the distortion of a real system are shown in Figure 2.4a and 2.4b respectively. Additionally to the noiseless testing image, in order to simulate the proces of photon detection, the photon noise is modeled using Gaussian distribution, whose variance depends on the expected photon count. This is a usual procedure [Švihlík & Páta 2008, Liu et al. 2008, Švihlík et al. 2011] in noise modelling theory even though the incident photon count follows the Poisson distribution.

Presence of coma and astigmatism is only assumed for our simulations. Knowing the amount of the optical aberrations, the PSF of an optical system for different stelar profile position can be computed and used for stelar profile fitting directly in IRAF. Exploiting a
stellar photometry package of IRAF, commonly referred to as a DAOPHOT*, enables to load own fitting function and use it for stelar object fitting. Details of this procedure are described in detail in [Massey & Davis 1992].

Basic astronomical measurement (astrometry and photometry) has been conducted on a testing image using several methods. Comparison of measurement results provided by commonly available methods, such as Gauss, Moffat, etc. and the results provided by proposed algorithm are shown in Figure 2.5.

Analysis of the results shows, that the aperture photometry gives, as expected and mentioned above, less precise results than the other algorithms. The rest of the results confirms that the common fitting algorithms fail for wide field image evaluations. The model of PSF based on SV properties of wide field optics provides the most accurate results especially at the edges of FOV for both, astrometry and photometry respectively. It is worth mentioning the drop of astrometry error on the noisy image at the edge of FOV. It is probably caused by noise itself.

Estimation of Optical Aberrations

If we assume that the optical system does not change or rather change its properties slowly, then laboratory measurements of the PSF before the real astronomical use can be performed and the knowledge of PSF used later for modelling of astronomical measurement. This measurements is usually done by the collimated illumination through the whole optical system with one major drawback. Even though such a measurement provides the information about the optical aberrations and dispersion on the optical elements, it doesn’t deal with the atmospheric variations, which play an important role for ground based astronomical measurements. Thus an artificial laser stars is used for PSF and air condition measurement. However, this procedure is used in combination with the AO system only on the largest terrestrial observatories. Since the PSF of an optical system is known afterwards, profile photometry can be applied producing even better results and precision.

The idea of having low cost imaging system or the network of such systems for astronomical observations doesn’t include the expensive AO equipment or artificial star procedure. Thus, the method of aberration estimation has been proposed, implemented and tested. The generalized block diagram of proposed method is shown in Figure 2.6. The estimation is based on the comparison of real stelar profile and its model. Model of a stelar profile at the certain position if FOV is given by the convolution of its optical axis profile and the

*http://iraf.noao.edu/scripts/irafhelp?daophot
Figure 2.5: Results of IRAF astronomical measurement for each star position from image shown in Figure 2.4b. (a) Magnitude error. (b) Magnitude error for noisy image. (c) Astrometry (position) error. (d) Astrometry (position) error for noisy image.
PSF of optical system, which is simulated using Zernike polynomials. The idea is to find the aberration coefficients, and to produce the model which will be closest to a real object. The criterium for model suitability evaluation is the RMSE between real profile and the model aligned one to another according their centre of gravity.

Considering the fact, that the optical part of the wide field imaging system has the biggest impact on resulting image quality, the influence of other parts of imaging chain, including atmospherical turbulence and sensor characteristics (except the noise properties, as explained above), was not taken into account in our experiments. The functionality of proposed algorithm for aberration estimation was tested on a real image data, acquired from double station video observation system. The estimation algorithm was developed in Matlab and is very time consuming, especially for higher number of aberration coefficients. The bigger the number of aberration coefficients one wants to retrieve, the bigger the number of parameters one will have to optimize in the matching algorithm. Therefore, even with an analysis limited to the $6^\text{th}$ radial order of Zernike polynomial, the number of parameters to optimize only for the aberration coefficients is 24 (the first 3 polynomials are not considered). The results of estimation, while only two main aberrations (coma and astigmatism) in an optical system are assumed, are shown in Figure 2.7.
2.5 Wide Field Image Data Reconstruction

Deconvolution is a computational technique for improving the contrast and resolution of digital images. It includes a suite of methods that seek to remove or reverse the blurring present in images caused by the e.g. distortions, noise, aberration etc. In wide field optical systems, the optical aberrations play an important role and determine the field dependence of the transfer characteristic of whole imaging system. Thus, it is crucial to take into account the optical aberrations and their influence on a PSF when the image processing applications using deconvolution is carried out.

This chapter briefly reviews the principles of common deconvolution algorithms, with special focus of those which are used usually in astronomy. Then, the description of a deconvolution problem for SV systems is discussed and a spatially variant deconvolution method is proposed. The proposed method uses the tailored sectioning which is based on the model of the optical aberrations described in Chapter 2.2. Several deconvolution algorithm using the proposed sectioning method are implemented and their mutual comparison if performed on a testing image simulating the astronomical image data.

Model of Spatially Variant Deconvolution

The principal difficulty in SV systems is that Fourier approach to restore image data cannot be used. Spatial variance of PSF of an optical aberrations makes the deconvolution process even more complicated. Common non iterative approaches fails while trying to restore spatially variant image data and the iterative algorithms need a high number of iterations for each object position, especially for wide FOV.

The proposed method to restore the image data obtained from the spatially variant wide field system is based on sectioning technique [Trussell & Hunt 1978]. This method divides the FOV into the sections, commonly referred to as isoplanatic patches (see Figure 2.8), in which the spatial invariance is assumed (i.e. the PSF doesn’t change).

Our method proposes the optimized division of FOV in a way that it relies on the known model of optical aberrations based on Zernike polynomials and their field dependence. The Equation (2.13) is used to compute the model of the PSF for each isoplanatic patch and this model is used for image deconvolution of whole image. At the end, all the deconvolved images corresponding to each isoplanatic patches are mutually composed. The final composition can be described by a masking process defined as

\[
SV_{deconv}(i, j) = \sum_{m=0}^{n} Mask_m(i, j)O_m(i, j),
\]

where \(SV_{deconv}\) is a spatially deconvolved image, \(O_m\) is a result of a spatially invariant deconvolution for \(m\)th patch and \(Mask_m\) is a masking filter for \(m\)th isoplanatic patch defined as

\[
M_m(i, j) = \begin{cases} 
1 & \text{if } (i, j) \text{ is inside the isoplanatic}, \\
0 & \text{otherwise} 
\end{cases}
\]

In order to improve results of composition process, the values of the masking filter are weighted in the sense that the isoplanatic patches overlaps. Three different weighting have been used in our algorithm, logic, linear and cosine (see Figure 2.9). Logic weighting doesn’t assume the patches overlapping. Additionally, three different methods of FOV division, in
2.5. Wide Field Image Data Reconstruction

In order to optimize the deconvolution algorithm, were proposed and implemented. Examples of a linear and cubic division (for linear and cosine weighting) in radial direction of FOV are illustrated in Figure 2.8a and 2.8b.

Three, above mentioned, deconvolution algorithm and several, not only astronomical, measurements have been exploited in order to test the proposed method. The Matlab implementation for Wiener and blind (maximum likelihood) deconvolution were used. The third, Lucy-Richardson algorithm uses own implementation.

Image, illustrated in Figure 2.10a was used for our simulations. The distortion caused by two different aberrations (coma and astigmatism) was simulated. The resulting image containing the spatially variant coma distortion is shown in Figure 2.10b and results of three different deconvolution algorithms in presented in Figure 2.10c, 2.10d and 2.10e. Visual comparison of these results shows that the Lucy-Richardson algorithm performs the best.

In the ideal case, the deconvolution process should be repeated as many times as pixels in the input image. This would lead to the very time consuming computation. In order to determine, how many isoplanatic patches is needed to achieve reasonable results (in terms of accuracy), the various number of isoplanatic patches has been used for each deconvolution condition (masking, weighting, algorithm). Then the RMSE as a quality measurement has been computed. The example of RMSE dependency on the number of patches is shown in Figure 2.11a. In order to investigate the spatial properties of each deconvolution algorithm, the size of the of RMSE is computed within whole image, and then within the areas (green and red rectangle) illustrated in Figure 2.11b.

Obviously, when the tailored image splitting is used, the number of patches needed to cover whole FOV is lower. Results shows, that the tailored splitting gives the better or comparable results (see Figure 2.12a) as the regular splitting. However, using the tailored splitting saves the computation time by 25% due to decreased number of exploited

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.8.png}
\caption{Image splitting into isoplanatic patches. (a) Linear splitting in radial direction. (b) Nonlinear cubic splitting in radial direction.}
\end{figure}
Figure 2.9: Image masks for cubic image splitting. (a)-(d) Image masks with linear weighting profile in radial and angular direction. (e)-(h) Image masks with cosines nonlinear weighting profile in radial and angular direction.

Figure 2.10: (a) Original image. (b) Image distorted by coma. (c) Result of Wiener SV deconvolution. (d) Result of Lucy-Richardson SV deconvolution. (e) Result of blind SV deconvolution.
Figure 2.11: (a) Examples of the results showing the RMSE value for each combination of FOV splitting in both radial and angular direction. (b) Areas used for evaluation. The whole image, the stripe at the edge (green rectangle) and corner (red rectangle) were used as areas of interest for RMSE computations.

The astronomical measurements conducted on the deconvolved data show that Lucy-Richardson algorithm outperforms the others in each directions, as shown in Figure 2.13. The results from all our measurement show that the influence of optical aberrations on the image quality can be partially suppressed by using the spatially variant deconvolution model, which takes them into account. The number and distribution of isoplanatic patches should be chosen regarding to the main distortion, image data and also the position of the objects of interest.

In order to estimate the number of iterations needed to achieve the accurate results, the following experiment has been conducted. The computation for best performing algorithm and different number of iterations has been performed. The dependence of the RMSE, for all weighting and masking conditions, on the number of iteration is shown in Figure 2.14 algorithms. Based on the results, the compromise between time consumption and accuracy has been chosen and 10 iterations were used in iterative deconvolution algorithms.
Chapter 2. Working Methods

Figure 2.12: (a) Dependence of RMSE on actual number of patches used for deconvolution. (b) Number of patches, which are used for deconvolution versus an amount of patches in diagonal radial direction for different number of patches in angular direction and for all splitting distribution.

Figure 2.13: Results of measurements conducted on image data deconvolved by Lucy-Richardson when coma aberration (Z7 and Z8) and maximal image division was used. (a) Ellipticity versus FOV position. (b) FWHM. (c) Astrometry error. (d) Magnitude error.

Figure 2.14: RMSE versus number of iterations for L-R deconvolution. (a) Logic masking. (b) Linear masking. (c) Cosine masking.
This work deals with the methods of optical design and modelling based on Zernike and Seidel polynomials, and with methods of SI astronomical image restoration. Precise modelling of a wide field SI optical system is very important in order to remove the influence of optical aberrations in the astronomical measurement and image data evaluations. Two different approaches are applied to avoid the astronomical measurement error. While first one is using the proposed model of physical PSF as a fitting function in IRAF, the second one exploits the different deconvolution techniques in order to get an undistorted image, on which the standard measurement techniques can be performed.

3.1 Summary of Results

This work specially addressed the wide field optical imaging systems used in astronomy. Such systems have a SI properties in the terms of field dependent PSF. The closer description of the properties of these systems is presented and the detailed astronomical measurement upon the image data obtained from real observation systems is conducted. The measured characteristics of a real data are closely discussed and support the main motivation (the need of accurate model for wide field astronomical measurement) of this work in general. Also, the possibilities of using the low cost commercially available digital camera for an astronomical observations is analyzed in detail and its main properties are compared to those of professional systems.

There are optical aberrations models for Spatially Invariant (SI) and SV systems presented in this thesis. These models, based on Seidel and Zernike polynomial, serve us as suitable tools for understanding how to estimate and fit the wavefront aberration of real optical system. On the basis of Zernike field dependent polynomials, the SI PSF model is proposed and simulated. Influence of optical aberrations on both, profile of the PSF and astrometry measurement respectively, is investigated and discussed. A novel method of astronomical measurement improvements is proposed and tested on simulation data and a extensive comparison of the proposed method and standard measurement techniques is conducted. Proposed model of an SV PSF, as well as the aberration estimator is implemented in Matlab. Moreover, the simulation of geometric distortion of a real optical system and its correction is presented together with an own design of UWFC optical system.

General issues of SI and SV image restoration has been addressed. A deconvolution
method taking into account the SV properties of an optical system caused by its field dependence aberrations is proposed. The three deconvolution algorithms were implemented considering the tailored sectioning approach. Several distributions of isoplanatic patches over entire FOV have been proposed and simulated as well as various weighting functions. The number of patches, needed to get results with satisfactory accuracy, was compared for each sectioning and weighting condition.

3.2 Main Contributions

The main contribution of this work lies in the design of the physical model of SV optical system, which takes into account the field dependence of the optical aberrations, and its use for optimization of astronomical measurement and deconvolution algorithms. The particular contributions of the dissertation are:

- Design and implementation of SV model of real optical system.
- Proposition of a novel method for optimized astronomical measurement on wide field and all sky monitoring systems.
- Creation of an own optical design of the all sky optical system (fisheye).
- Design and implementation of an optimized method for restoration of the SV wide field optical system.
- Study of a low cost digital camera potentials while using it as a astronomical observation and monitoring system.

3.3 Possible Extensions in Science and Practical Applications

This work proposes a method to estimate the aberrations of a wide field optical imaging system. However, the implementation of method is done in Matlab doesn’t have a proper performance in term of computation time. As mentioned in Chapter 2.2, there are many parameters to be estimate (Zernike coefficients up to required order). If a large number of coefficients has to be optimized, a genetic algorithm is often used. Thus, the possible extension of this work would be to use such an algorithm in order to estimate the aberration coefficients. Then the extensive verification of the model on a real data should be performed. Another possibility to transform this work into a real life is the implementation of the fitting algorithm into IRAF in form of additional plugin. Also, an extension of proposed model by adding the sensor noise properties and the model of the atmosphere turbulence would make this model more robust and it is in consideration for the next project phase.
Bibliography


Personal Publications

5.1 Publications Related to the Topic of the Dissertation

Articles in Impacted Journals


Articles in Reviewed Journals


Publications Indexed by WoS


5.1. Publications Related to the Topic of the Dissertation

Other Publications


5.2 Publications Unrelated to the Topic of the Dissertation

Publications Indexed by WoS


Book Chapters


Other Publications


The contributions of all authors to the each individual publication, mentioned above, are equal.

Martin Řeřábek
6.1 Known Citations

The listed citations do not include self-citations and citations by the coauthors or colleagues.

- Paper [10] has been cited in:

- Paper [15] has been cited in:

- Paper [35] has been cited in:

6.2 Awards

- Paper [22] has been awarded best paper in section Electronics and Instrumentation of the International Student Conference on Electrical Engineering - Poster 2009

- Paper [25] has been awarded third place in section Electronics and Instrumentation of the International Student Conference on Electrical Engineering - Poster 2008

6.3 Reviewer of Conference Papers

- International Conference on Pattern Recognition (ICPR), http://www.icpr2012.org/
  - since 2012, 3 reviews
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  - Applicant of the grant project Prof. Vladimír Šebesta.

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  - Applicant of the grant project Prof. Miloš Klíma.

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  - Applicant of the grant project Prof. Dr. Touradj Ebrahimi.

  - Applicant of the grant project Prof. Dr. Touradj Ebrahimi.

  - Applicant of the grant project Prof. Dr. Touradj Ebrahimi.
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  - Applicant of the grant project Prof. Dr. Touradj Ebrahimi.
Abstract

This thesis focusses on a narrow field of research related to the modelling of wide field optical systems used for astronomical image data acquisition, and presents a contribution in the area of astronomical measurement and astronomical image data reconstruction.

Wide field optical imaging brings, not only in astronomy, a several challenges for general image processing, mainly due to the properties of an optical system, which decreases the resulting image quality. However, it is especially astronomy, where the image quality is essential for a precise image data measurements and evaluation. The image quality criteria can be usually followed when the professional imaging system (deep sky) is used. On the other hand, wide field and all sky monitoring systems, commonly referred to as Wide Field Camera (WFC) and Ultra Wide Field Camera (UWFC) systems, provides worse performance in terms of transfer characteristics, which are distorted by the wavefront aberrations.

This thesis describes the wavefront aberration using Zernike analysis in order to create the physical model of the Point Spread Function (PSF) of real optical system. The goal of developing a physical model of the PSF is to obtain a forward model allowing to match a measured PSF with a theoretical one and estimate the aberration coefficients. With the knowledge of the coefficients for individual Zernike polynomials, the output of Spatially Invariant (SI) optical system can be simulated.

Practically, all real systems (especially wide field) are Spatially Variant (SV), which means that the Zernike polynomials describing the wavefront aberrations are field dependent. The field dependence of Zernike polynomials must be know to model the PSF of SV system. Such a field dependency together with the SV model of a real imaging system is presented and simulated in this work. Exploiting of this model, in order to improve the astronomical measurements made upon a wide field image data, is proposed. This optimized approach is tested within the professional software package Image Reduction and Analysis Facility (IRAF) and its results are compared to the results of standard measurement procedures.

Another approach to get a better image quality in terms of spatial invariance is to use image restoration reflecting the field dependence of optical aberrations. A novel method of an optimized SV myopic deconvolution to deal with field dependent distortions is proposed and described. The standard measurements have been conducted upon the deconvolved images and a series of comparison of three different deconvolution algorithms for plenty of spatially variant conditions are performed and described.

Keywords: Optical Aberrations, Zernike Analysis, Deconvolution, Spatially Variant System, Optical System Design.
Résumé

Tato práce se soustředí na úzkou oblast týkající se modelování širokoúhlých astronomických zobrazovacích systémů a dále prezentuje přínos v oblasti měření, hodnocení a rekonstrukce vědeckých (astronomických) obrazových dat.

Širokoúhlé zobrazování přináší mnoho úzkalí nejen v astronomii. Zejména při zpracování širokoúhlého snímku se projeví negativní vlastnosti samotné optiky a kvalita obrazu rychně klesá. Astronomie je jednou z oblastí, kde kvalita zpracovávaného obrazu hraje velmi důležitou roli, zejména pro měření a zpracování astronomických vědeckých snímků. Zatímco, profesionální pozorovací stanice (deep sky) splňují kritérium požadované kvality, širokoúhlé a celooblohové systémy vykazují zhoršené přenosové vlastnosti a tudíž kvalitu výsledného obrazu. To je způsobeno především optickými vadami, které jsou v takových systémech přítomné a tudíž zkracují vlnoplochu ve výstupní pupile a tím i tvar a velikost impulsní odevy.

V této práci je uveden popis aberované vlnoplochy pomocí Zernikovy analýzy. Na základě Zernikových polynomů je vytvořen fyzikální model impulsní odevy reálné optické soustavy. Vytvořením fyzikálního modelu optického systému je možno získat dopředný model impulsní odevy a tím i informace o aberačních koeficientech, které jsou v daném systému obsažené. Znalosti aberačních koeficientů je možné použít k simulaci k definování výstupu lineárního prostorově invariantního systému.

Praktické širokoúhlé systémy vykazují určitý stupeň prostorové variantnosti, což znamená, že jsou Zernikovy polynomy, které aberační vlnoplochu popisují, závislé rovněž na souřadnicích předmětu. Abychom definovali impulsní odevzu prostorově variantního systému, závislost jednotlivých Zernikových polynomů na souřadnicích objektu musí být určena. Právě taková prostorová závislost Zernikových polynomů na výšce objektu je definována a prostorově variantní model reálného širokoúhlého optického systému je navržen a popsán. Takový model je pak možné použít pro přesné astronomické měření. Porovnání použití navrženého prostorově variantního modelu se standardními měřicími metodami je rovněž součástí této práce.

Dalším přístupem k vylepšení kvality obrazu na výstupu reálné soustavy je použití dekonvolučního algoritmu, který bere v úvahu prostorové závislosti jednotlivých Zernikových polynomů. Prostorově variantní algoritmus myopické dekonvoluce, je navržen a popsán v rámci této práce. V závěrečné části je uvedeno srovnání výsledků použitých dekonvolučních algoritmů pro prostorově variantní systém, jehož vlastnosti jsou popsány vytvořeným modeltem impulsové soustavy.

Klíčová slova: Optické Aberace, Zernikeho analýza, Dekonvoluce, Prostorově Variantní Systémy, Optický návrh.