CZECH TECHNICAL UNIVERSITY IN PRAGUE

DOCTORAL THESIS STATEMENT

Czech Technical University in Prague
Faculty of Electrical Engineering
Department of Circuit Theory

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Causal Relations in Autoregressive Models

Ph.D. Programme: Electrical Engineering and Information Technology
Branch of Study: Electrical Engineering Theory

Doctoral thesis statement for obtaining the academic title of "Doctor", abbreviated to Ph.D.

Prague, August 2012

The doctoral thesis was produced in full-time manner Ph.D. study at the department of Circuit Theory of the Faculty of Electrical Engineering of the CTU in Prague.

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The doctoral thesis statement was distributed on: ....

The defence of the doctoral thesis will be held on .... at .... before the Board for the Defence of the Doctoral Thesis in the branch of study Electrical Engineering Theory in the meeting room No. .... of the Faculty of Electrical Engineering of the CTU in Prague.
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Chapter 1

Introduction and State of the Art

Identification of statistic causal relations among simultaneously recorded signals is an important problem in the area of multidimensional time series analysis with applications in many domains spreading from biology to economics. It reveals connections among simultaneous time series and identifies not only the strength of the relations but also the direction of the information flow.

A whole new area opened for Granger causality in neurophysiology at the end of 20th century. Human brain is the least explored body organ. Mapping of activity in particular centres contributes not only to understanding its functionality but also can lead to new procedures in the treatment of various diseases. Functional magnetic resonance imaging (fMRI), electroencephalography (EEG) and magnetoencephalography (MEG) (Toga and Mazziotta, 2002) provide data which meets the conditions of causality analysis very well and together with progress of computational technology, it is possible to find causal relations in the large data of brain activity records, e.g., (Liang et al., 2000, Brovelli et al., 2004, Uddin et al., 2009, Zhou et al., 2009).

1.1 Multivariate Autoregressive Model Methods

One very important group of methods of causal relations analysis is based on MVAR models. Once suitable model parameters are fitted to the data, the methods try to estimate the causal connections from these model parameters.

The original Granger causality concept (Granger, 1969) is based only on bivariate pairwise test classifying the strength of the causal connection by one real non-negative number in one direction. The MVAR model is fitted

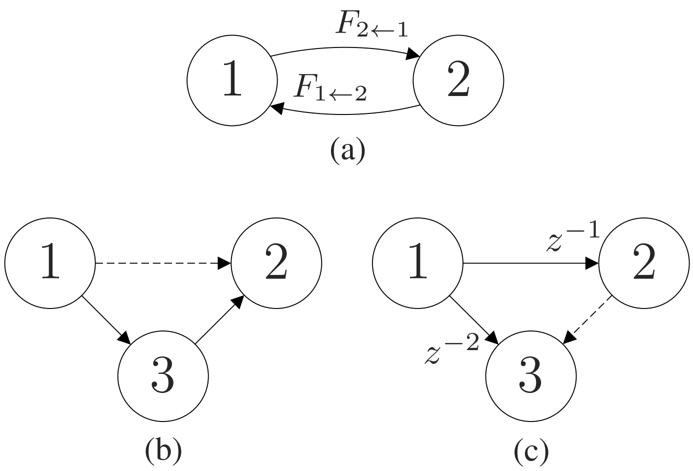

Figure 1.1: Typical properties of causality analysis. (a) Bidirectional connection evaluated via Granger causality, with source variable 1 and target variable 2, giving causal strength $F_{2\leftarrow 1}$. The analysis with source variable 2 and target variable 1 gives another result $F_{1\leftarrow 2}$ as the causal relations are asymmetric, i.e., directional. (b) Sequential driving problem should be detected as direct causal relations $3 \leftarrow 1$ and $2 \leftarrow 3$ however some methods also detect false indirect $2 \leftarrow 1$. (c) Different delay driving should be detected as direct causal relations $2 \leftarrow 1$ and $3 \leftarrow 1$ however some methods also detect false indirect $3 \leftarrow 2$ as 2 contains an information which can help to predict future values of 3 (the z-domain delay represents the causal connection delay).

to the time series and if the variance of residual prediction error of the target variable is lower in the case that the previous values of the source variable are included into the model then we say the source variable causes the target variable. For two variables, the pairwise Granger causality (GC) gives asymmetric values for the respective directions. This is the essential property of Granger causality, pointing the direction of information flow between sources (see Fig. 1.1(a)).

If a data contains more time series, two problems may appear. The first problem, a so called *sequential driving*, occurs when the causal connection from the first to the second variable is completely mediated via the third variable (see Fig. 1.1(b)). In this case, a pairwise test will detect also indirect causal relation from the first to the second variable because it cannot distinguish between direct and indirect connections. The second problem, a *different delay driving*, occurs when the first variable drives both the second and the third variable but the driving of the second variable has a smaller delay than the driving of the third one (see Fig. 1.1(c)). Then, samples of the second variable contain information which helps predict future samples of the third variable. A pairwise method detects a causal relation from the second to the third variable and cannot distinguish that this is an indirect one. Both problems with indirect connections were solved via multivariate extension, referred to as Conditional Granger causality (CGC) (Ding et al., 2006). It works with MVAR model that includes all time series in order to find the direct connections and recognize and eliminate the indirect ones.

In recent years, MVAR models have been used on neurophysiological data such as fMRI, MEG or EEG for analysing causal connections among brain structures. The common task of these applications is to find which frequency rhythms participate in the causal relation. A number of methods for the frequency decomposition have been published.

The Directed Coherence (DC) (Saito and Harashima, 1981, Baccalá et al., 1998) decomposes power spectral density of each MVAR variable to causal components in frequency domain but is defined for a total transfer between each pair of variables and hence, it cannot recognize the direct path of connection. The Directed Transfer Function (DTF) (Kamiński and Blinowska, 1991, Eichler, 2006) also does not distinguish the direct connections from the indirect ones. The Partial Directed Coherence (PDC) (Baccalá and Sameshima, 2001, Cadotte et al., 2009) solves this problem by recognizing only the direct connections, but uses a normalization that causes inability of comparison of the strength of the coupling among variables. The lower value can mean a stronger relation exists (Baccalá et al., 2007). The Direct Directed Transfer Function (dDTF) (Korzeniewska et al., 2003, Liang et al., 2000, Cadotte et al., 2009, Astolfi et al., 2009, Benz et al., 2012) claims to improve the DTF in order to distinguish the indirect connections. A straightforward frequency transform of Granger causality (originally by Geweke (Geweke, 1984)) leads to problems when utilizing three variables, such as occasional occurance of negative values which have no meaning in terms of causality. (Chen et al., 2006) applied a partition matrix technique to overcome the problem. However, this method is limited to three variables only. The Generalized Partial Directed Coherence (GPDC) (Baccalá et al., 2007, Sato et al., 2009, Hoerzer et al., 2010) modifies the PDC using additional normalization to make the values more comparable. Another method, Renormalized PDC (Schelter et al., 2009) provides similar outcomes as GPDC.

Since the time the Granger causality was formulated, several other methods measuring causal relations have been proposed and are either based on the similar idea or a completely different approach. The methods can be split into two groups. The first group returns only a scalar value that represents the total strength of the connection in the given direction. The second group allows for an analysis in individual frequency bands. Some methods work only with a pair of variables, other handle all variables simultaneously in order to distinguish direct and indirect connections when the information flow from the first variable to the second variable is mediated by the third variable. Majority of the used measures are based on autoregressive models but several others exploit different concepts. Some studies utilize non-linear models in order to describe the nature more accurately while others do not recommend this approach.

Majority of causality connection measures are based on MVAR models. In recent years, they have been applied on neurophysiological data. The methods use different approaches and their authors still work on their improvements and modifications. It is hard to answer the question which measure works best on a general data.

If one does not request the exact path analysis and only wants to find related areas, the pairwise GC in time domain or DC in frequency domain may be used. This can lead to simpler visualisation of general information flow in the case of analysis of many variables (like the surface high resolution EEG). A great advantage of DC is that it is defined in meaningful physical terms as power spectra.

PSI, an estimator based on imaginary part of cross-spectra, is an interesting novel approach in the causal connectivity analysis. It provides more reliable results on mathematical models than Granger causality in the case of unidirected connections. A small drawback is its bivariate definition making it impossible to distinguish direct and indirect connections. As mentioned above, this is not necessarily a big issue. However, much more serious drawbak is its inability to recognize bidirectional connections frequently present in the brain. One possible solution is the combination of PSI with other estimators (proposed in the thesis) where PSI will serve as a critical evaluator of the unidirectional results.

Further studies have to be done to demonstrate whether the non-linear estimators deliver any extra information not contained in linear estimators, in conjunction with preserving the estimation reliability.

Although dDTF, PDC and GPDC methods claim to distinguish between direct and indirect causal connections in the frequency domain, the definitions and consequently interpretation of results vary.

(Blinowska, 2008) states: *"Unfavorable feature of PDC is its weak dependence on frequency (practically 'flat' spectrum), which does not permit to distinguish well the role of different rhythms."* and *"DTF detects not only direct but also indirect flows. This feature may be important when estimating transmissions from implanted or subdural electrodes. However in these cases Direct Directed Transfer Function which combines DTF with partial coherence, may be used."*

The dDTF is described by (Korzeniewska et al., 2003): *"The maxima of dDTF may better (in comparison to DTF) reflect a frequency of rhythm being a carrier for information flow."*

The GPDC is explained by (Sato et al., 2009): *"The square modulus of GPDC value from jth time series by ith series can be understood intuitively as the proportion of the power spectra of the jth time series, which is sent to the ith series considering the effects of the other series."* In Sec. 3.0.1, it is shown that this statement is not accurate and the newly proposed Autoregressive Causal Relation (ACR) that uses the power spectral density is the same as the square modulus of GPDC only in the case of two variables with one unidirectional causal relation.

(Faes and Nollo, 2011) specifies: *"Directed coherence (DC) measures causality in meaningful physical terms as power contributions, but cannot separate direct effects from indirect ones; GPDC determines the correct interaction structure in terms of direct causal effects, but its absolute values lack of straightforward interpretability."*

(Hu et al., 2011) mentions a possibility of misleading results of PDC, GPDC, and dDTF methods.

(Wu et al., 2011) summarizes: *"Comparing the DTF, PDC and their derivatives, which of these measures is the most advantageous and accurate is still an open question."*

Chapter 2

Aims of the Doctoral Thesis

2.1 Thesis Outline

Although state-of-the-art methods are used for the real data analysis, many studies point out difficulties with interpretation of their outputs. Therefore, the primary focus of this thesis is in the model data analysis with known causal relations and formulating a methodology that will allow for a clear interpretation of causal relations in meaningful physical terms.

The thesis is organized as follows.

- The *first chapter* introduces the background and motivation for the causal relation analysis using linear multivariate autoregressive models, i.e., the scope of this thesis.

- The *second chapter* brings definitions of state-of-the-art methods of causal relation analysis based mainly on linear multivariate autoregressive model. It also briefly discusses the advantages and problems of these approaches, laying the ground for the final section of objectives and intended goals of this thesis.

- The *third chapter* deals with the original contribution of the thesis, a novel method of spectral matrix decomposition, and proposes criteria allowing for a better interpretation of causal relation analysis in multivariate autoregressive time series.

- The *fourth chapter* demonstrates the ideas of the new method and compares the results with the state-of-the-art methods on illustrative synoptical model data examples.

- The *fifth chapter* summarizes the results and discusses the advantages and disadvantages of the methods with the focus on the future work.

2.2 Objectives of the Work

In conclusion, the DC method definition provides a causal relation analysis in frequency domain and in meaningful physical terms but cannot distinguish the direct connections from the indirect ones. Newer methods have tried to overcome this problem but with the price of problematic interpretation of the strength of the relations.

The primary focus of this work is in the analysis of linear multivariate autoregressive (MVAR) model in frequency domain. The major contributions of this thesis are

- Interpretation of MVAR model in the digital filtration aspect.
- Decomposition of power spectral matrix into separate causal components in meaningful physical terms.
- Definition of novel causal measure respecting direct relations allowing a comparison of strengths of relations in the signal power sense.

Chapter 3

Working Methods

This thesis is focused on methods based on multivariate autoregressive (MVAR) models. It reviews state-of-the-art methods and suggests a new measure Autoregressive Causal Relation (ACR) for evaluation of an absolute and relative causal relation in frequency domain in the context of MVAR models. ACR is based on the interpretation of MVAR model in the digital filtration sense. ACR decomposes diagonal elements of a spectral matrix into separate causal components that show the directions of influence among multivariate time series of an autoregressive character. ACR measures causality in meaningful physical terms as power contributions and can separate direct effects from indirect ones.

Unlike the original Granger causality concept that evaluates the strength of each causal relation with a single non-negative scalar value, the values of ACR can either be positive or negative to model both an increase or a decrease of the power spectral density caused by the causal relation in the analyzed direction. Therefore, the ACR allows one not only to compare the *strength* of the relations, but also to analyze their *effects*.

The experimental section of the thesis studies performance of state-of-the-art methods and compares them with the novel ACR method in synoptic artificial data experiments and discusses advantages of the new approach.

The MVAR model can be written as

$$\sum_{n=0}^{p} A(n)\, X(t-n) = E(t) \tag{3.1}$$

where $A(n)$ are model coefficients, $X(t-n)$ are k real number stationary variables with zero means (e.g., corresponding to particular simultaneous channels), p is the model order and $E(t)$ are vectors of residual errors.

The frequency transform of the MVAR model can be expressed as

$$A(f)\,X(f) = E(f)\,. \tag{3.2}$$

This can be rewritten as (Korzeniewska et al., 2003)

$$X(f) = A^{-1}(f)\,E(f) = H(f)\,E(f) \tag{3.3}$$

where

$$H(f) = \begin{bmatrix} H_{11}(f) & \cdots & H_{1k}(f) \\ \vdots & \ddots & \vdots \\ H_{k1}(f) & \cdots & H_{kk}(f) \end{bmatrix} = A(f)^{-1} \tag{3.4}$$

is the transfer function of the MVAR system. A power spectral density representation, commonly reffered to as the spectral matrix, is

$$\begin{aligned} S(f) &= \lim_{N\to\infty} E\left[\frac{1}{N} X(f)\,X^{*}(f)\right] = \\ &= \lim_{N\to\infty} E\left[\frac{1}{N} H(f)\,E(f)\,H^{*}(f)\,E^{*}(f)\right] = \\ &= H(f)\,V H^{*}(f) \end{aligned} \tag{3.5}$$

where V is a diagonal matrix with variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2$. V is diagonal because the covariances between residual noises are zero.

By reason of clear separation of the causal relation, and thereby its explicit detection, a model without causal connection is defined and then, a causal relation is added. The influence of this modification is analyzed and interpreted in the scope of linear time invariant (LTI) digital filters. The concept is explained on three synoptical artificial data experiments. The first idea of this approach was published in (Bořil and Sovka, 2010a) and it was fully developed developed in (Bořil and Sovka, 2011).

The criteria of causal influence analysis based on preceeding discussion, are as follows.

Absolute value of causal influence

$$\hat{S}_{2\leftarrow 1}(f) = \hat{S}_{22}(f) - \hat{S}'_{22}(f) \tag{3.6}$$

where $\hat{S}'_{22}(f)$ corresponds to the signals without the causal relation, $\hat{S}_{22}(f)$ contains the causal relation.

Relative value of causal influence

$$\hat{S}_{2\leftarrow 1REL}(f) = \left(\hat{S}_{22}(f) - \hat{S}'_{22}(f)\right) / \hat{S}_{22}(f)\,. \tag{3.7}$$

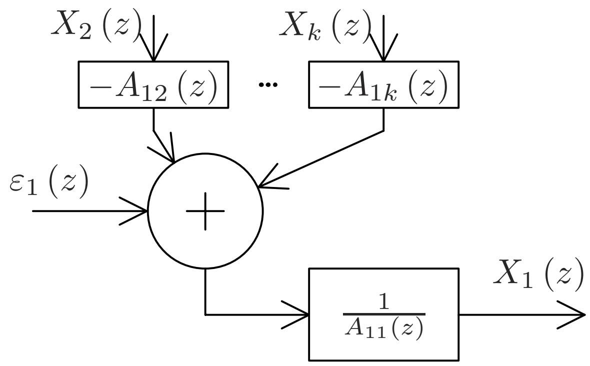

Figure 3.1: MVAR variable $X_1(z)$ filtering diagram.

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The problem is that in general case, such an easy and intuitive separation of the causal influence is not possible.

In this section, we define a new method of a causal relation analysis using the MVAR model. In comparison with the DC method, it decomposes the diagonal values of the spectral matrix as well. In addition, and instead of the DC, it decomposes the PSD components in the direct causal path, allowing one to track the true causal route of the information flow via mediating variables.

This MVAR model can be transformed to z-domain

$$X_1(z) = \frac{1}{A_{11}(z)} \left(\varepsilon_1(z) - \sum_{j=2}^{k} A_{1j}(z) X_j(z) \right) \tag{3.8}$$

where $\frac{1}{A_{11}(z)}$ is an all-pole infinite impulse response (IIR) filter and $-A_{1j}(z)$ are finite impulse response (FIR) filters. Corresponding diagram is depicted in Fig. 3.1. The advantage of this approach is that it does not detect false indirect connections.

With substitution

$$T_{imn}(f) = A_{im}(f) A_{in}^{*}(f) S_{mn}(f), \tag{3.9}$$

and

$$U_{im}(f) = A_{im}(f) A_{ii}^{*}(f) S_{mi}(f). \tag{3.10}$$

ine

$$F_{i \leftarrow i}(f) = \frac{\sigma_i^2}{|A_{ii}(f)|^2}, \tag{3.11}$$

$$F_{i\leftarrow m,n}(f) = \frac{-2\mathrm{Re}\left(T_{imn}(f)\right)}{\left|A_{ii}(f)\right|^2}, \tag{3.12}$$

and

$$F_{i\leftarrow m}(f) = \frac{-\left|A_{im}(f)\right|^2 S_{mm}(f) - 2\mathrm{Re}\left(U_{im}(f)\right)}{\left|A_{ii}(f)\right|^2}. \tag{3.13}$$

The final form of PSD decomposition of $S_{ii}(f)$ to individual causal components can be written in the form of

$$S_{ii}(f) = F_{i\leftarrow i}(f) + \sum_{\substack{m=1\\ m\neq i}}^{k-1} \sum_{\substack{n=m+1\\ n\neq i}}^{k} F_{i\leftarrow m,n}(f) + \sum_{\substack{m=1\\ m\neq i}}^{k} F_{i\leftarrow m}(f). \tag{3.14}$$

The *absolute autoregressive causal relation* (absolute ACR) corresponds directly to the portion of the PSD of the target variable $X_i(t)$ not caused by other variables or the portion of the PSD caused by individual source variables $X_m(t)$

$$ACR_{i\leftarrow mABS}(f) = \begin{cases} F_{i\leftarrow i}(f), & \text{for } m = i \\ F_{i\leftarrow m}(f), & \text{for } m \neq i \end{cases} \tag{3.15}$$

and the portion of the PSD caused by couples of source variables $X_m(t)$ and $X_n(t)$, present only in the case that both variables have direct causal relation with the $X_i(t)$ (nonzero $A_{im}(f)$ and $A_{in}(f)$) and the couple of source variables has nonzero CPSD $S_{mn}(f)$

$$ACR_{i\leftarrow m,nABS}(f) = F_{i\leftarrow m,n}(f), \ m \neq i, n \neq i, n > m. \tag{3.16}$$

The key property of this approach is, as it can be seen from (3.14), for every target variable $X_i(t)$, the summation of its absolute components equals to the PSD of the variable

$$\sum_{m=1}^{k} ACR_{i\leftarrow mABS}(f) + \sum_{\substack{m=1\\ m\neq i}}^{k-1} \sum_{\substack{n=m+1\\ n\neq i}}^{k} ACR_{i\leftarrow m,nABS}(f) = S_{ii}(f), \quad \forall i = 1\ldots k. \tag{3.17}$$

This feature, along with the ability of modeling both increase and decrease of the PSD caused by direct causal relations, are the main advantages of the proposed measure as compared to state-of-the-art methods.

The *relative autoregressive causal relation* normalizes the absolute ACR by the PSD of the target[1] $S_{ii}(f)$ and hence, gives a quantity of the fraction of the contribution to the target variable

$$ACR_{i\leftarrow mREL}(f) = \begin{cases} \frac{F_{i\leftarrow i}(f)}{S_{ii}(f)}, & \text{for } m = i \\ \frac{F_{i\leftarrow m}(f)}{S_{ii}(f)}, & \text{for } m \neq i \end{cases} \tag{3.18}$$

and

$$ACR_{i\leftarrow m,nREL}(f) = \frac{F_{i\leftarrow m,n}(f)}{S_{ii}(f)}, \;\; m \neq i, n \neq i, n > m. \tag{3.19}$$

For every target variable $X_i(t)$, the summation of its relative components equals to 1

$$\sum_{m=1}^{k} ACR_{i\leftarrow mREL}(f) + \sum_{\substack{m=1 \\ m\neq i}}^{k-1} \sum_{\substack{n=m+1 \\ n\neq i}}^{k} ACR_{i\leftarrow m,nREL}(f) = 1, \;\; \forall i = 1 \ldots k. \tag{3.20}$$

This allows one to compare the strength of individual causal components in percentage. Compared with DC squared, ACR also measures causality in meaningful physical terms as power contributions but moreover, the advantage of ACR is that it can also separate direct effects from indirect ones.

3.0.1 Comparison of ACR and GPDC: Two Variables Case

As mentioned above, the square modulus of GPDC is often interpreted as the proportion of the power spectra of the target variable caused by the causal relation (Sato et al., 2009). To illustrate the fact that this statement may not be accurate, a comparison with the relative ACR on a two-variables case is conducted in the following paragraph.

A general equation for causal relation in the direction $2 \leftarrow 1$ can be obtained

$$\mathrm{GPDC}_{21}^{2}(f) = \frac{\frac{1}{\sigma_2^2}|A_{21}(f)|^2}{\frac{1}{\sigma_1^2}|A_{11}(f)|^2 + \frac{1}{\sigma_2^2}|A_{21}(f)|^2} = \frac{\sigma_1^2|A_{21}(f)|^2}{\sigma_2^2|A_{11}(f)|^2 + \sigma_1^2|A_{21}(f)|^2}. \tag{3.21}$$

[1] A normalization by the PSD of the source variable does not make sense in terms of dividing a total power to its parts because sources in the MVAR model definition are copied to targets, not divided.

Using the (3.18) (the component (3.19) is not present for $k = 2$), one gets

$$ACR_{2\leftarrow 1REL}(f) = \frac{-|A_{21}(f)|^2 S_{11}(f) - U_{21}(f) - U_{21}^*(f)}{|A_{22}(f)|^2 S_{22}(f)} \tag{3.22}$$

where

$$U_{21}(f) = A_{21}(f)\, A_{22}^*(f)\, S_{12}(f)\,, \tag{3.23}$$

this together with (3.5) results in

$$\begin{aligned} &ACR_{2\leftarrow 1REL}(f) = \\ &= \frac{1}{|A_{22}(f)|^2 \left(\sigma_2^2 |A_{11}(f)|^2 + \sigma_1^2 |A_{21}(f)|^2\right)} \Big(\sigma_1^2 |A_{21}(f)|^2 |A_{22}(f)|^2 - \\ &- \sigma_2^2 |A_{12}(f)|^2 |A_{21}(f)|^2 + \sigma_2^2 A_{11}(f)\, A_{22}(f)\, A_{12}^*(f)\, A_{21}^*(f) + \\ &+ \sigma_2^2 A_{11}^*(f)\, A_{22}^*(f)\, A_{12}(f)\, A_{21}(f)\Big)\,. \end{aligned} \tag{3.24}$$

Let us consider two scenarios. (a) The model does not contain a causal relation $1 \leftarrow 2$, i.e., $A_{12}(f) = 0$. Then (3.24) can be reduced to

$$ACR''_{2\leftarrow 1REL}(f) = \frac{\sigma_1^2 |A_{21}(f)|^2}{\sigma_2^2 |A_{11}(f)|^2 + \sigma_1^2 |A_{21}(f)|^2} \tag{3.25}$$

and $ACR''_{2\leftarrow 1REL}(f) = \mathrm{GPDC}_{21}^2(f)$.

(b) The model contains the causal relation $1 \leftarrow 2$. In contrast to the ACR, the GPDC does not respect this fact; $ACR_{2\leftarrow 1REL}(f) \neq \mathrm{GPDC}_{21}^2(f)$.

3.1 Statistical Evaluation

Since these causal measures have a highly nonlinear relation to the time series data from which they are derived and distributions of their estimators are not well established (Gourévitch et al., 2006), the use of a surrogate data method is recommended.

The *epoch surrogates* (Kamiński et al., 2001) and *FFT surrogates* (Theiler et al., 1992) methods are used. Data are shuffled in order to obtain surrogate data with a very similar amplitude spectra but destroyed causal relations. The causal measures are then calculated from the surrogate data. By repeating this process a number of times, an empirical estimate of a probability density function (histogram) is constructed, which corresponds to the null hypothesis that there is no causal connection.

Chapter 4

Results

The thesis demonstrates the state-of-the-art methods on model data examples and illustrates the proposed ACR method. The goal is to compare their behavior in various situations, understand their properties, and be able to interpret their outputs.

In this statement, we will focus on partion of such a one example.

4.1 Comprehensive MVAR Model

In order to compare results of ACR, DC, dDTF and GPDC, an advanced five-variables MVAR model of the third order used in (Baccalá and Sameshima, 2001, Ding et al., 2006) was chosen (see Fig. 4.1)

$$\begin{aligned}
X_1(t) &= \varepsilon_1(t) + 0.95\sqrt{2}X_1(t-1) - 0.9025X_1(t-2)\,, \\
X_2(t) &= \varepsilon_2(t) + 0.5X_1(t-2)\,, \\
X_3(t) &= \varepsilon_3(t) - 0.4X_1(t-3)\,, \\
X_4(t) &= \varepsilon_4(t) - 0.5X_1(t-2) + 0.25\sqrt{2}X_4(t-1) + 0.25\sqrt{2}X_5(t-1)\,, \\
X_5(t) &= \varepsilon_5(t) - 0.25\sqrt{2}X_4(t-1) + 0.25\sqrt{2}X_5(t-1)
\end{aligned} \tag{4.1}$$

where ε_1, ε_2, ε_3, ε_4, and ε_5 are Gaussian white noises with zero means and variances of 0.6, 0.5, 0.3, 0.3, and 0.6 respectively.

In addition to the bidirectional connection $4 \leftrightarrow 5$, the model also contains indirect causal connections. The sequential driving $5 \leftarrow 4 \leftarrow 1$ can cause a false detection of the indirect relation $5 \leftarrow 1$. And the differently delayed drivings $2 \leftarrow 1$ and $3 \leftarrow 1$ can cause a false detection of indirect $3 \leftarrow 2$ because

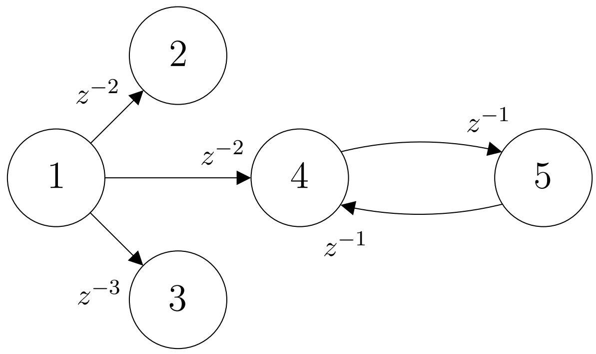

Figure 4.1: Comprehensive MVAR model; diagram of model data connection. The z-domain delay represents the connection delay (the minimal value of the time shift in the equation).

the samples of 2 contain an information which helps with the prediction of 3, although in fact this is the information transferred from 1.

4.1.1 Causal Measures from Exact MVAR Model Coefficients

Let us focus on measures in frequency domain computed from exact coefficients of the MVAR model of the order of 3.

The absolute ACR (see Fig. 4.2 and 4.4(a)) shows PSD of the contribution of the causal relation to the target variable. The main diagonal values correspond to the PSD of the variable which is not caused by other variables. The summation of all contributions to the target variable is equal to the total power spectral density of the variable (3.17), a diagonal value of the spectral matrix (3.5).

The advantage of ACR is that it correctly detects only direct relations and excludes false indirect ones (e.g., $ACR_{5\leftarrow 1ABS}(f)$ or $ACR_{3\leftarrow 2ABS}(f)$ in Fig. 4.2 are zero) and therefore uncovers the correct connection diagram of causal relations among time series.

The relative ACR (see Fig. 4.3 and 4.4(b)) is the absolute ACR normalized by the PSD of the target variable $S_{ii}(f)$, thus depicts the relative amount of the contribution of the causal relation and the summation of all contributions equals to 1. The main diagonal values correspond to the relative part of the power spectral density of the variable which is not caused by other variables.

The suggested use is to evaluate the absolute ACR in order to get an overview of the power of the contributions at each frequency, and then compare the values with the total PSD of the target variable via the relative ACR. The use of the relative ACR alone cannot be generally recommended because such results can return high values at frequencies where signals have a very low power because of the ratio of the two values being close to zero.

The DC squared (see Fig. 4.5) is very similar to the relative ACR (see Fig. 4.3) as it also decomposes diagonal values of the spectral matrix to causal components and the summation of all contributions to the target variable equals to 1. However, it reflects the total causal relation from the source to the target variable without considering the real direct route of the causal influence. While it has no problem with the different delay driving ($3 \leftarrow 2$ is zero), the $5 \leftarrow 1$ is nonzero caused by the sequential driving mediated by 4. This is the main difference between DC squared and relative ACR.

The dDTF (see Fig. 4.6) often returns shapes similar to the absolute relations (see Fig. 4.2), however this is not always the case (e.g., relations $5 \leftarrow 4$ and $4 \leftarrow 5$). The strength of the relations values is difficult to compare and interpret. The dDTF falsely detects the indirect causal relation $5 \leftarrow 1$ with the same order of magnitude as the correct direct relation $5 \leftarrow 4$.

The GPDC (see Fig. 4.7) normalizes the values of the relation strength in the range from 0 to 1. The squared modulus of the GPDC (see Fig. 4.7) partially resembles the relative ACR (see Fig. 4.3) but appears more flattened. It reaches high values even for frequencies where signals have a very low power. On the other hand, the maxima are lowered in comparison with the relative ACR. These facts complicate the interpretation of the GPDC results. In comparison to the dDTF, the advantage of GPDC (as well as of ACR) is that it detects only the direct causal connections, excluding the false indirect ones. This is caused by the numerator of the GPDC definition which uses only the MVAR coefficients of the direct connection.

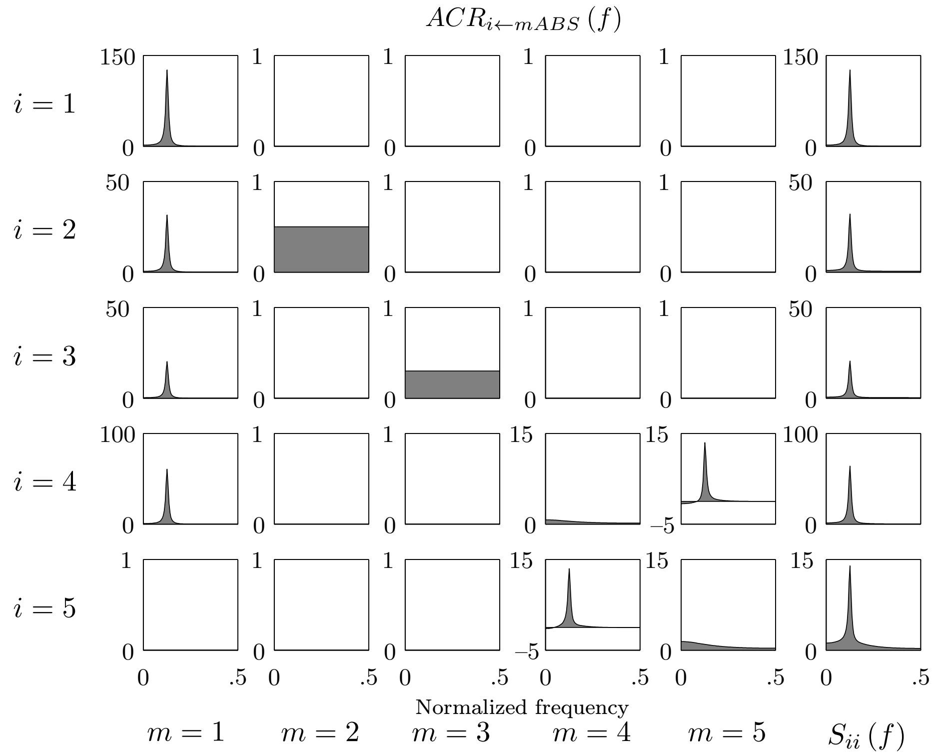

Figure 4.2: Absolute ACR from exact coefficients; causal relation from source $X_m(t)$ to target $X_i(t)$. The summation of all components creating the target variable including the relation from couple of sources in Fig. 4.4(a) is equal to the PSD of the variable $S_{ii}(f)$ depicted in the 6th column.

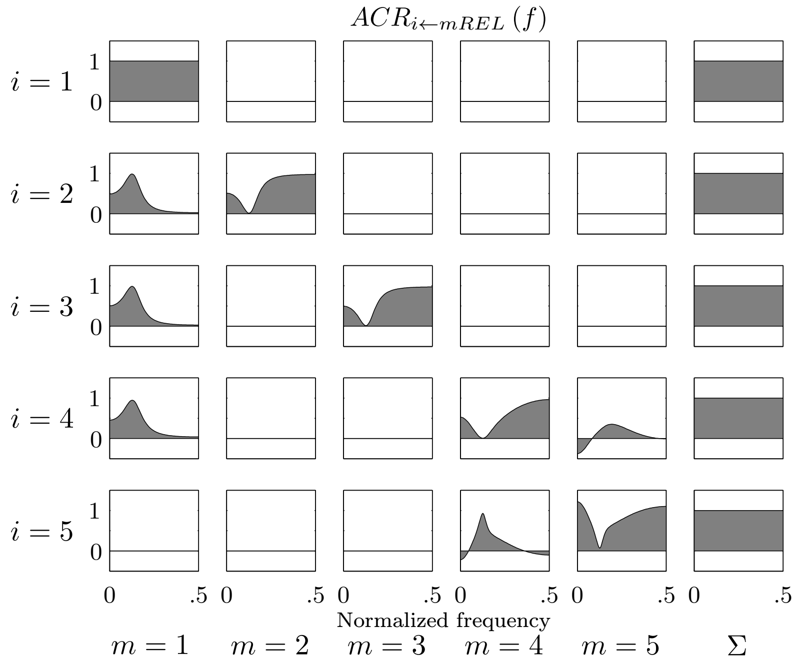

Figure 4.3: Relative ACR from exact coefficients; causal relation from $X_m(t)$ to $X_i(t)$. The summation of all components of the target variable including the relation from couple of sources in Fig. 4.4(b) is always one (see the 6th column).

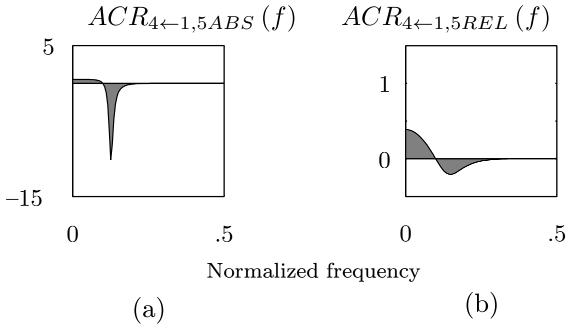

Figure 4.4: ACR – coupled relations from exact coefficients. (a) absolute ACR and (b) relative ACR causal relation from couple of sources $X_1(t)$ and $X_5(t)$ to target $X_4(t)$. This is the only nonzero coupled causal relation because of two direct causal connections to the same target from sources with nonzero CPSD.

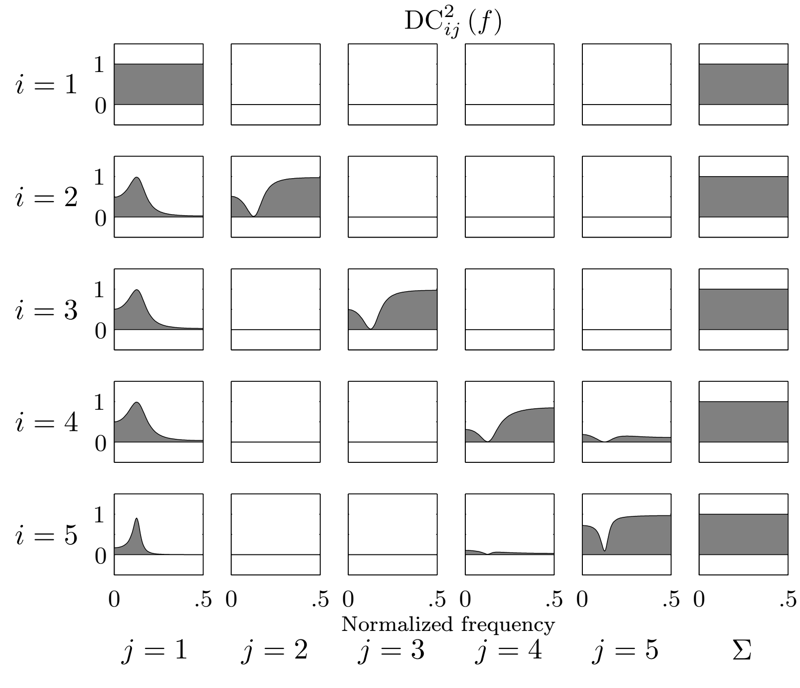

Figure 4.5: $\mathrm{DC}_{ij}^2\,(f)$ *from exact coefficients; causal relation from $X_j(t)$ to $X_i(t)$. The 6th column is summation of all values in the row, always equal to one.*

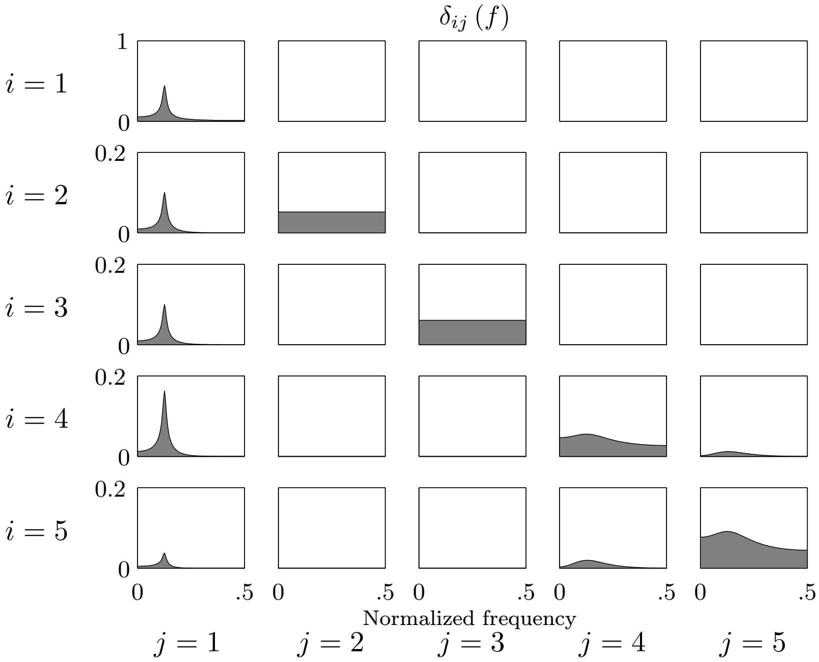

Figure 4.6: dDTF from exact coefficients; causal relation from $X_j(t)$ to $X_i(t)$.

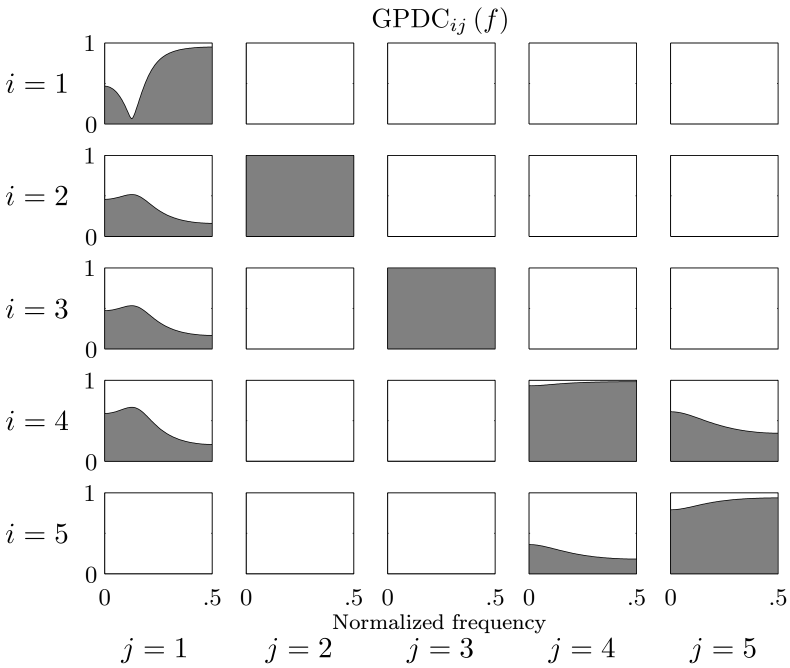

Figure 4.7: GPDC from exact coefficients; causal relation from $X_j(t)$ to $X_i(t)$.

Chapter 5

Conclusion

The primary focus of this work is in the analysis of linear multivariate autoregressive (MVAR) models in the frequency domain. It reviews well-accepted methods based on MVAR modeling for causal relation analysis of multichannel signals with AR characteristics.

CGC performs well in distinguishing direct and indirect causal relations (Bořil, 2009). A combination of CGC with pairwise PSI is proposed in order to get more reliable results in noisy conditions, (Bořil and Sovka, 2010b).

In many applications, a frequency decomposition of causal relations is desirable. It allows to analyze which frequency rhythms participate in the information flow. DC decomposes spectral matrix to separate causal components and reflects the total causal relation from the source to the target variable. This approach does not consider the real direct route of the causal influence and cannot recognize a connection diagram of sequentially driven systems.

GPDC overcomes this problem but loses the advantage of the interpretation of the strength of causal relations in meaningful physical terms (Bořil and Sovka, 2010a), dDTF also tries to detect only direct relations but it fails in some cases as shown in this work.

We have suggested criteria of evaluation of causal relations in frequency domain (Bořil and Sovka, 2011) and based on these criteria, we have proposed a novel measure ACR in order to recognize the direct route of the causal influence and preserve the advantageous of the PSD interpretation. ACR decomposes diagonal elements of a spectral matrix into separate causal components that show directions of influence among multivariate time series of an autoregressive character. ACR measures causality in meaningful physical terms as power contributions and can separate direct effects from indirect ones. This easily interpretable definition allows one to evaluate the performance of state-of-the-art methods. The concurrent use of both absolute and relative ACR is always recommended for obtaining a complete representation of absolute values of causal relations and also the proportion of their impact on the target. The ACR is focused on the real impact of each connection, i.e., what part of power is transferred via the connection.

As the time series have random character, the PSD approach must be applied. As a result of the PSD utilization, the ACR contains extra coupled causal relations from two sources together. However, these components are present only in the case when both source variables have a nonzero cross power spectral density and both have direct causal influence to the target variable, so the presence of such components does not affect the causal relations connection diagram.

Unlike the original Granger causality concept that evaluates the strength of each causal relation with a single non-negative scalar value, the values of ACR can be positive or negative in order to model both an increase or a decrease of the power spectral density caused by a causal relation in the analyzed direction. Therefore, the ACR allows one not only to compare the strength of the relations, but also to analyze the effect of the relation.

The advantage of the ACR is that the summation of components creates the total PSD of the target variable. This allows one to compare and clearly interpret the strength of the causal components.

The experimental section of the thesis studies performence of state-of-the-art methods and compare them with the novel ACR method in synoptic artificial data experiments and explains advantages of the new approach. When analyzing artificial signals with known generating equations, ACR may serve for evaluating other methods.

In the case of estimation of MVAR model coefficients, two methods of estimation of statistical properties are examined. Based on surrogate data analysis, emperical distribution functions are obtained. ACR is found to be more sensitive to inaccurate estimates of the MVAR model coefficients. DC is less sensitive to inaccuracies and can help to review ACR results. In the case where DC returns zero, ACR should be also zero because ACR detects only the direct causal connections and DC detects both direct and indirect ones.

The contributions of the thesis are in interpretation of MVAR model in the digital filtration aspect. A novel method was proposed, decomposing of power spectral matrix into separate causal components respecting the direct route of the causal relations. Experiments featuring the method were statistically analysed using a surrogate data technique. These contributions can serve to advance both knowledge and algorithm development for causal relations analysis in MVAR models.

While this thesis suggests new methodology of interpretation of MVAR model in term of causal relations in the frequency domain, future research should be focused on behaviour of signals not strictly satisfying the MVAR conditions. Elaborate study of numerical stability in such cases should be also examined.

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Chapter 6

Selected Publications

Relating to the Thesis

Articles in Impacted Journals

Articles in Reviewed Journals

1. Bořil, T. – Sovka, P.: Metody pro analýzu kauzálních vztahů v EEG. Elektrorevue [online in Czech language]. 2010, roč. 2010, č. 126, s. 1–6. ISSN 1213-1539. Reported contributions: 70 % – 30 %.

Patents

Publications Indexed by WOS

Other Publications

1. Bořil, T. – Sovka, P.: System Interpretation of Causality Measures in Frequency Domain Used in EEG Analysis. In 19th European Signal Processing Conference (EUSIPCO 2011) [CD-ROM]. Herent: EURASIP, 2011, p. 1539–1543. ISSN 2076-1465. Reported contributions: 70 % – 30 %.

2. Bořil, T. – Sovka, P.: Performance study of causality measures. In Technical Computing Bratislava 2010 [CD-ROM]. Bratislava: RT systems, s.r.o, 2010, p. 1–5. ISBN 978-80-970519-0-7. Reported contributions: 70 % – 30 %.

3. Bořil, T.: Multivariate Autoregressive Modelling of Causal Connections in EEG. In Analýza a zpracování řečových a biologických signálů - sborník prací 2010. Praha: České vysoké učení technické v Praze, 2010, p. 9-13. ISBN 978-80-01-04680-7. Reported contributions: 100 %.

4. Bořil, T.: Toolkit for EASYS2 EEG data format processing in Matlab, EEGLAB and sLoreta environment. In Proceedings of the 8th Czech-Slovak Conference Trends in Biomedical Engineering [CD-ROM]. Bratislava: Slovak University of Technology in Bratislava, 2009, ISBN 978-80-227-3105-8. Reported contributions: 100 %.

5. Bořil, T.: Revealing of Relations in EEG via Granger Causality. In 13th International Student Conference on Electrical Engineering [CD-ROM]. Prague: CTU, Faculty of Electrical Engineering, 2009. Reported contributions: 100 %.

6. Bořil, T. – Sovka, P.: Active Brain Centres Selection for Function Connection Analysis. In Technical Computing Prague 2009 [CD-ROM]. Prague: HUMUSOFT, 2009, p. 18–21. ISBN 978-80-7080-733-0. Reported contributions: 70 % – 30 %.
7. Bořil, T.: Grangerova kauzalita a EEG. In Analýza a zpracování řečových a biologických signálů - sborník prací 2009. Praha: České vysoké učení technické v Praze, 2009, s. 30-37. ISBN 978-80-01-04474-2. Reported contributions: 100 %.
8. Bořil, T. – Sovka, P.: Performance Study of Bivariate Granger Causality. In Digital Technologies 2008 [CD-ROM]. Žilina: University of Žilina, Fakulty of electrical engineering, 2008, vol. 1, ISBN 978-80-8070-953-2. Reported contributions: 70 % – 30 %.

Unpublished Articles

Known Responses

The list does not include self-citations and citations by the coathors or colleagues.

Other Known Responses

The list includes citations by colleagues.

1. Bořil, T.: Toolkit for EASYS2 EEG data format processing in Matlab, EEGLAB and sLoreta environment. In Proceedings of the 8th Czech-Slovak Conference Trends in Biomedical Engineering [CD-ROM]. Bratislava: Slovak University of Technology in Bratislava, 2009, ISBN 978-80-227-3105-8. Reported contributions: 100 %.

   WoS

   (a) Doležal, J., Šťastný, J., Sovka, P.: Exploiting temporal context in high-resolution movement-related EEG classification, Radioengineering, Volume 20, Issue 3, 2011, Pages 666-676, ISSN: 12102512.

   Other

Member of Grant Teams

1. SGS10/176/OHK3/2T/13. “Brain activity mapping and analysis” of the Czech Technical University in Prague. Applicant of the grant project Tomáš Bořil
2. GD102/08/H008 “Analysis and modelling of biomedical and speech signals” of the Czech Grant Agency. Applicant of the grant project Roman Čmejla.

Not Relating to the thesis

Articles in Impacted Journals

Articles in Reviewed Journals

Patents

Publications Indexed by WOS

Other Publications

1. Bořil, T.: The use of Matlab and Simulink in signal and system theory course. In Technical Computing Bratislava 2010 [CD-ROM]. Bratislava: RT systems, s.r.o, 2010, p. 1–4. ISBN 978-80-970519-0-7. Reported contributions: 100 %.
2. Bořil, H. – Bořil, T. – Pollák, P.: Methodology of Lombard Speech Dabase Acquisition: Experiences with CLSD. In Proceedings of 5th International Conference on Language Resources and Evaluation. Paris: ELRA – European Language Resources Association, 2006, vol. 1, p. 1644–1647. ISBN 2-9517408-2-4. Reported contributions: 50 % – 20 % – 30 %.
3. Bořil, H. – Bořil, T. – Pollák, P.: Design of Lombard Effect Speech Database. In Radioelektronika 2005 - Conference Proceedings. Brno: VUT v Brně, FEI, Ústav radioelektroniky, 2005, p. 144–147. ISBN 80-214-2904-6. Reported contributions: 40 % – 20 % – 40 %.

Unpublished Articles

Known Responses
The list does not include self-citations and citations by the coathors or colleagues.

1. Bořil, H. – Bořil, T. – Pollák, P.: Methodology of Lombard Speech Dabase Acquisition: Experiences with CLSD. In Proceedings of 5th International Conference on Language Resources and Evaluation. Paris: ELRA – European Language Resources Association, 2006, vol. 1, p. 1644–1647. ISBN 2-9517408-2-4. Reported contributions: 50 % – 20 % – 30 %.

   WoS

   (a) Youyi Lu and Martin Cooke (2008). Speech production modifications produced by competing talkers, babble and stationary noise, The Journal of the Acoustical Society of America, 124(5), November, pp. 3261–3275.

   Other

   (a) Damjan Vlaj, Aleksandra Zögling Markus, Marko Kos, Zdravko Kacic (2010). Slovenian Speech Database with Lombard Effect – SiLSD, in Proc. of 13th International Multiconference Information Society/7th Language Technologies Conference, pp. 20–23, Ljubljana, Slovenia.
   (b) Youyi Lu (2010). Production and Perceptual Analysis of Speech Produced in Noise, PhD thesis, University of Sheffield, UK.
   (c) Damjan Vlaj, Aleksandra Zögling Markus, Marko Kos, Zdravko Kacic (2010). Acquisition and Annotation of Slovenian Lombard Speech Database, Proceedings of the International Conference on Language Resources and Evaluation (LREC 2010), May, Valletta, Malta.

(d) Marteen Brouwers (2008). The Influence of the Auditory Environment on the Emotional Perception of Speech. Master Thesis, Philips Research Europe/Technische Universiteit Eindhoven, May, 55 pages.

Member of Grant Teams

Chapter 7

Summary

Identification of statistic causal relations among simultaneously recorded signals is an important problem in the area of multidimensional time series analysis with applications in many domains spreading from biology to economics. It reveals connections among simultaneous time series and identifies not only the strength of relations but also the direction of the information flow.

This thesis is focused on methods based on multivariate autoregressive (MVAR) models. It reviews state-of-the-art methods and suggests a new measure Autoregressive Causal Relation (ACR) for evaluation of an absolute and relative causal relation in frequency domain in the context of MVAR models. ACR is based on the interpretation of MVAR model in the digital filtration sense. ACR decomposes diagonal elements of a spectral matrix into separate causal components that show the directions of influence among multivariate time series of an autoregressive character. ACR measures causality in meaningful physical terms as power contributions and can separate direct effects from indirect ones.

Unlike the original Granger causality concept that evaluates the strength of each causal relation with a single non-negative scalar value, the values of ACR can either be positive or negative to model both an increase or a decrease of the power spectral density caused by the causal relation in the analyzed direction. Therefore, the ACR allows one not only to compare the *strength* of the relations, but also to analyze their *effects*.

The experimental section of the thesis studies performance of state-of-the-art methods and compares them with the novel ACR method in synoptic artificial data experiments and discusses advantages of the new approach.

Chapter 8

Résumé

Důležitým problémem v oblasti analýzy vícerozměrných časových řad je identifikace statistických kauzálních vztahů mezi simultánně zaznamenanými signály, aplikaci nachází v mnoha oblastech od biologie po ekonomii. Taková analýza totiž odkrývá vazby mezi souběžnými časovými řadami a vyhodnocuje nejen jejich sílu, ale také směr šíření informace.

Tato disertace je zaměřena na metody vycházející z vícerozměrných autoregresních (MVAR) modelů. Seznamuje se současným stavem v této problematice a navrhuje novou metodu Autoregresní kauzální vazba (Autoregressive causal relation – ACR), která také vychází z MVAR modelu a umožňuje vyhodnocení kauzálních vztahů v absolutní i relativní míře, a to ve frekvenční oblasti. Myšlenka metody ACR vychází z interpretace MVAR modelu z pohledu číslicové filtrace. ACR rozkládá diagonální prvky spektrální matice na jednotlivé kauzální komponenty ukazující směr vlivu mezi souběžnými signály s autoregresním charakterem. ACR měří míru kauzality v rozumném fyzikálním smyslu jako výkonový příspěvek. Navíc dokáže odlišit přímou cestu šíření od nepřímé.

Narozdíl od Grangerovy kauzality, která vyhodnocuje sílu každé vazby nezáporným skalárním číslem, hodnoty ACR mohou být jak kladné, tak záporné. Z toho důvodu umožňují modelovat jak přírůstky, tak poklesy výkonové spektrální hustoty způsobené kauzální vazbou v daném směru. ACR tak umožňuje porovnávat nejen *sílu* vazeb, ale rovněž jejich skutečný *dopad.*

Experimentální část této práce studuje chování moderních uznávaných metod na přehledných modelových příkladech a porovnává je s nově navrženou metodou ACR, diskutuje rozdíly a výhody nového přístupu.