

## EVOLUTION OF NEW REAL AND COMPLEX IMPROPER SOLUTIONS OF THE SLOTLINE DISPERSION EQUATION

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*Abstract* - An attempt is made in this paper to reach a better understanding of the behaviour of open uniplanar circuits, particularly the slotline. A new second leaky wave has been revealed in addition to the dominant bound and leaky waves known up to now. The evolution of its dispersion characteristics in dependence on the line dimensions is presented. Possible excitation of this new leaky wave brings down the upper cut-off frequency of pure bound wave propagation. Consequently the new leaky wave associated with simultaneous leakage into two surface waves can significantly deteriorate circuit performance, especially in the mm-wave band. Conclusions achieved for the slotline can be extended qualitatively to the other printed-circuit lines.

### INTRODUCTION

Applications of microwave and millimetre wave integrated circuits have stimulated ongoing interest in a more profound understanding of their behaviour. Leakage effects on planar transmission lines significantly influence their performance. Shigesawa et al (1) have summarized all earlier findings about effects which may occur in open printed-circuit lines. This particularly concerns simultaneous propagation of bound and leaky dominant waves on printed-circuit lines and evolution of their dispersion characteristics depending on the relative line dimension. Recently in Zehentner et al (2) we have identified a new leaky wave on the slotline and have reported its influence on the upper frequency limit of dominant bound wave propagation.

In this paper we will briefly recapitulate the occurrence of the well known first and also this new second leaky wave on the slotline. The evolution of the second leaky wave dispersion characteristics in dependence on the relative line dimension will be presented here for the first time. Conclusions regarding the slotline have general validity. It will be shown that the new surface leaky wave brings down the upper cut-off frequency of bound wave propagation. A full record of its closed-form formula will appear in Zehentner et al (3). The presented findings are new, elucidate the behaviour of the slotline in the millimetre wave band and determine the frequency applicability of the slotline. They also contribute to the general discussion on planar/uniplanar transmission line characteristics.

### THE FIRST AND THE SECOND LEAKY WAVE ON THE SLOTLINE

The cross-section of the investigated slotline is shown in Fig. 1. The method of moments modified by the Galerkin testing procedure with successive complex root searching was used in calculating the propagation constant. The integration path in the spectral domain was deformed in order to include the residual contributions associated with the propagation of the corresponding surface waves. Our calculations confirmed all results concerning bound and leaky dominant waves and their simultaneous propagation presented in (1). We refer to the wave associated with the leakage into the  $TM_0$  surface wave as the first leaky wave. We denote as the second leaky wave the field which in addition respects leakage into the  $TE_1$  surface wave.

The normalized propagation constant of the bound wave ( $\beta/k_0$ ), the normalized improper real ( $\beta/k_0$ ) solution and also the normalized improper complex (propagation constant  $\beta/k_0$  and leakage constant  $\alpha/k_0$ ) solution of the dispersion equation of the slotline taken over from (1) related to the first and second leaky waves in dependence on the normalized frequency ( $h/\lambda_0$ ) are plotted in Fig. 2. Each of the improper real solutions associated to the second leaky mode breaks off from two respective improper real solutions related to the first leaky mode at the cut-off frequency of the  $TE_1$  surface wave  $f_1$ . The second improper complex solution breaks off from the lower second improper real solution. This complex solution is physical for frequencies greater than  $f_4$  when the phase constant of the second leaky wave becomes lower than the phase constant of the  $TE_1$  surface wave and of course also when it is lower than the phase constant of the  $TM_0$  surface wave. The second leaky wave leaks power into the  $TM_0$  and  $TE_1$  surface waves simultaneously. Its leakage constant  $\alpha_2$  is greater than the leakage constant of the first leaky wave  $\alpha_1$ . Leakage into the  $TE_1$  surface wave occurs at a lower angle than leakage into the  $TM_0$  surface wave.

The spectral gap marked out by the frequency  $f_2$  when the bound wave sets down and by the frequency  $f_3$  when the first leaky wave sets in now evidently disappears. The second leaky wave propagates simultaneously with the bound wave from  $f_4$  to  $f_2$ . Above  $f_2$  the second leaky wave propagates alone up to the frequency  $f_3$  and for even higher frequencies the first and the second leaky waves can propagate simultaneously. Consequently the second leaky wave overlaps the frequency gap and takes the role of the dominant leaky wave primarily attributed to the first leaky wave in (1).

In addition, we have identified a third leaky wave above the cut-off frequency of the  $TM_2$  surface wave. This wave leaks power into the  $TM_0$ ,  $TE_1$  and  $TM_2$  surface waves simultaneously. Since this effect occurs at extremely high frequencies it is insignificant from the practical point of view. On the other hand it demonstrates the multi-valued feature of the solution of the slotline dispersion equation.

#### EVOLUTION OF NEW REAL AND COMPLEX SOLUTIONS OF THE DISPERSION EQUATION

Dramatic variation of the dispersion characteristics of the slotline in dependence on its normalized dimension  $w/h$  is shown objectively in the sequence of Figs. 2-8.

The increase of  $w/h$  to 0.533, as in Fig. 3, draws the lower and upper improper real solutions close together. Moreover, the improper complex solution for the second leaky wave becomes lower than  $k_0$  within the interval from  $f_5$  to  $f_6$  where the solution is nonphysical. This effect becomes more expressive towards greater  $w/h$  and greatly reduces the range of the physical solution. However, this behaviour is not typical when the substrate permittivity is greater, e. g.  $\epsilon_r=10.8$ .

In Fig. 4, when  $w/h$  equals 0.535, the upper and lower improper real solutions have joined and created the new left dashed line from which the second improper real solutions splits off. This line is separated from the analogous new right dashed line by a section of the first nonphysical improper complex solution. This situation is also kept in Fig. 5, where  $w/h=0.552$ . Now the two split-off points at  $f_1$  have nearly merged and the frequency band of the first improper complex solution has widened.

When  $w/h$  grows to 0.6 a portion of the new second improper complex solution splits off from the first improper complex solution at  $f_1$  and sets down when it touches the second improper real solution as is seen in the inset of Fig. 6. The corresponding attenuation constant is  $\alpha'_2$  and the solution is nonphysical. The second portion of this second improper complex solution breaks off again from the second improper real solution at a slightly higher frequency, while  $\alpha_1$  is now greater than  $\alpha_2$ .

A further grow of  $w/h$  to 0.8 as in Fig. 7 means that the second improper complex solution splits off from the first improper complex solution, then crosses the second improper real solution, the first improper complex solution, the bound wave solution and continuously decreases through  $f_4$  and  $f_5$ . On a wide slotwidth the first leaky wave dominates, as is seen in Fig. 8 when  $w/h$  equals 1. However this is not the case when the substrate permittivity is greater.

Transition from the operation mode with the spectral gap to the mode of simultaneous propagation of the bound wave and the first leaky wave, when power leaks into the  $TM_0$  surface wave which has zero cut-off frequency, is explained in (1) in terms of the same number of solutions of the dispersion equation in both the bound and the leaky regions. However, the same is not applicable to the second leaky mode. Presence of the second leaky wave is possible only above the non-zero cut-off frequency of the  $TE_1$  surface wave. Consequently, above  $f_1$  the total number of solutions doubles in comparison with when only the first leaky wave is accounted.

#### FREQUENCY BAND OF PURE BOUND WAVE PROPAGATION

Considering only the first leaky wave in the operation mode with the frequency gap, the frequency band of bound wave propagation is limited from above by the frequency  $f_2$  when the bound wave sets down, Fig. 9a. In the mode of simultaneous propagation of the bound wave and the first leaky wave the frequency  $f_3$  limits pure bound wave propagation, Fig. 9b. When the second leaky wave is taken into consideration the upper cut-off frequency of pure bound wave propagation is identical with the frequency  $f_4$ , Fig. 9c.

For easy assessment of the upper cut-off frequency of pure bound wave propagation on the slotline the closed-form formulae of its normalized values ( $h/\lambda_c$ ) have been proposed as an approximation of the calculated data by the least squares method (2). A full reading will appear in (3).

The second leaky wave asserts itself on the slotline with relatively narrow slotwidth and higher substrate permittivity. The first leaky wave dominates when the slotline has a relatively wider slotwidth and lower substrate permittivity.

#### CONCLUSIONS

The new effects revealed on the slotline may also occur on the other open printed-circuit lines and possess generality encoded in the multi-valued solution of their dispersion equations. This paper presents new findings regarding the behaviour of the slotline in the mm-wave band. The dispersion characteristics of the slotline depend strongly on the dimensions of the line. Their evolution provides a better insight into the mechanism of the wave processes. The new second leaky wave brings down the upper cut-off frequency of pure bound wave propagation, particularly when the slotline has lower characteristic impedance and is made on a higher permittivity substrate.

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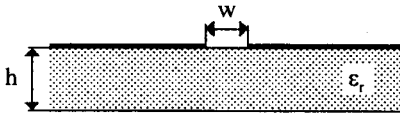


Fig. 1 Cross-section of the slotline

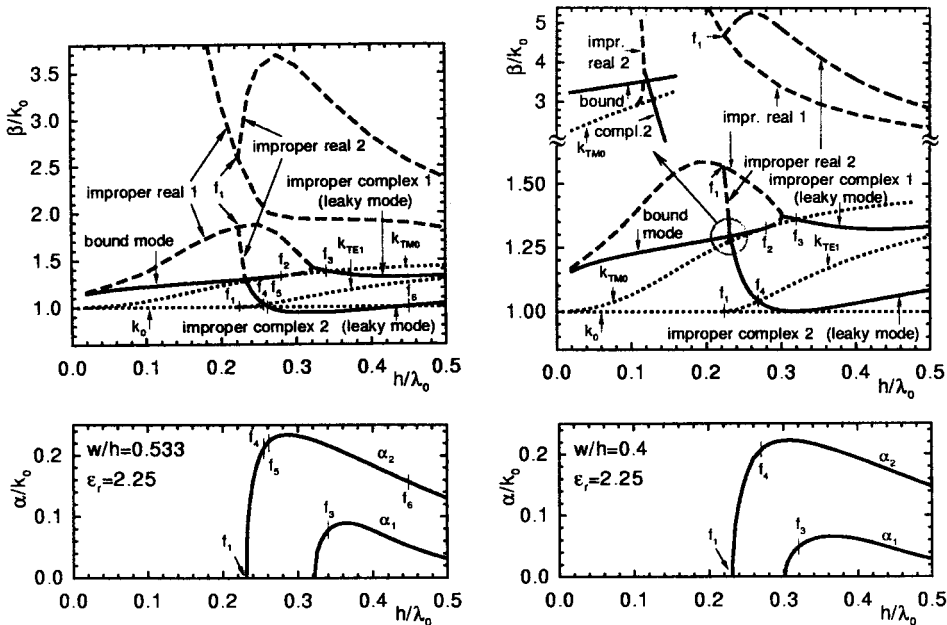


Fig. 3 A plot similar to that in Fig. 2 but for a wider slotwidth  $w/h=0.533$

Fig. 2 The normalized phase and leaky constants for the slotline with  $w/h=0.4$  and  $\epsilon_r=2.25$  as a function of normalized frequency ( $h/\lambda_0$ )

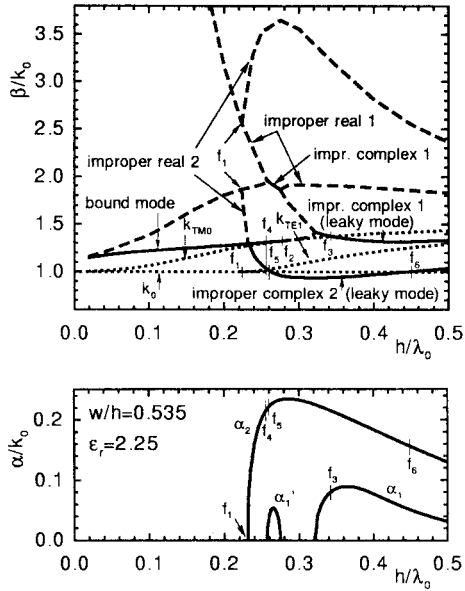


Fig. 4 A plot similar to that in Fig. 3 for a slightly wider slotwidth  $w/h=0.535$

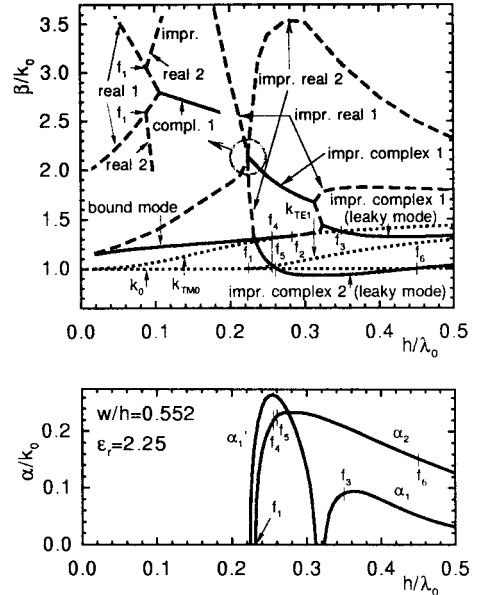


Fig. 5 A plot similar to that in Fig. 4 but for a still wider slotwidth  $w/h=0.552$

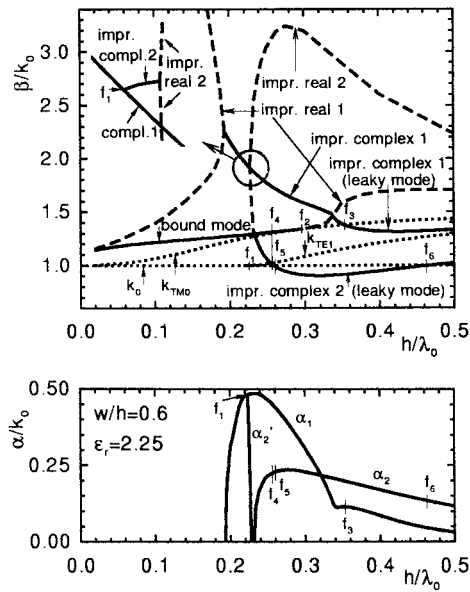


Fig. 6 A plot similar to that in Fig. 5 but for the slotwidth increased to  $w/h=0.6$

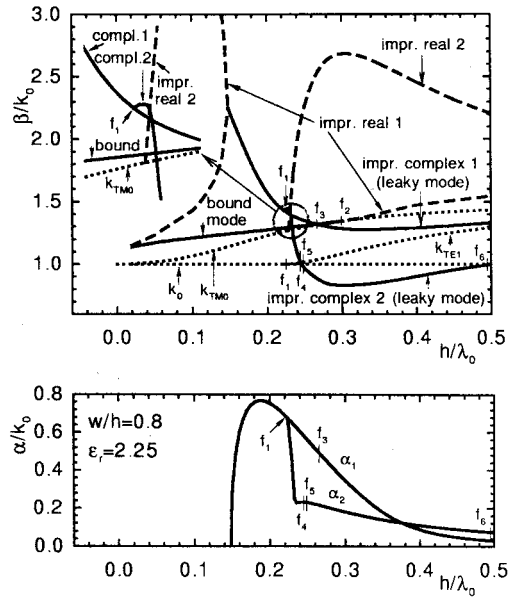


Fig. 7 A plot similar to that in Fig. 6 but for a very wide slot with  $w/h=0.8$

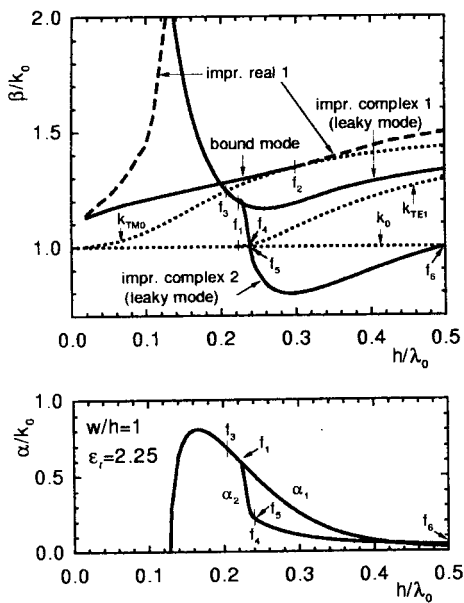


Fig. 8 A plot similar to that in Fig. 7 but for even wider slotwidth  $w/h=1$

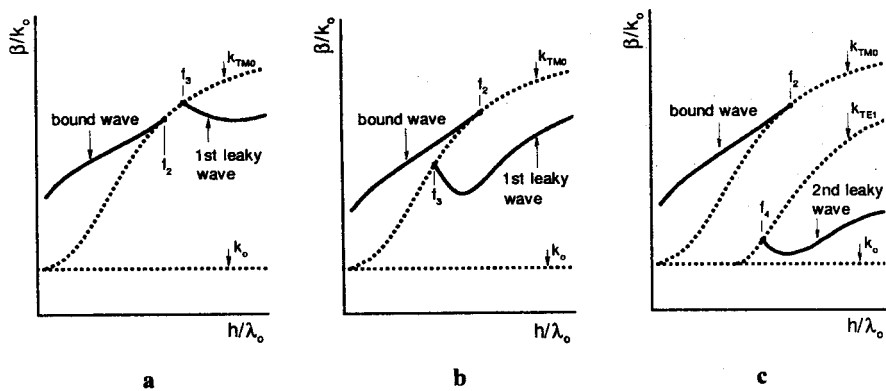


Fig. 9 Upper cut-off frequency of pure bound wave propagation in different operation modes