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REPORT OF A THESIS ADVISOR

Dissertation title: "Higher-order derivative gravity"
Candidate: Aneta Pjatkanová
Institution: Department of Physics, Czech Technical University in Prague, Faculty of Nuclear Science and Physical Engineering, Břehová 7, 115 19 Czech Republic
Assessment: A (výborně)

It is a pleasure to report on Bachelor's Thesis of Aneta Pjatkanová. Ms. Pjatkanová worked under my supervision throughout the period: 2023-2024. During this time, she focused her attention on technical aspects of classical higher-order derivative gravity with an emphasis on prospective applications in cosmology and astrophysics.

The concept of higher-order derivative gravity (or simply higher-derivative gravity) was originally introduced in a series of papers by H. Weyl in 1918-20, even though the incentives can be traced back to A. Einsteins' 1915 general relativity. Broadly speaking, higher-derivative gravity theories represent a class of gravity theories which, in addition to the conventional general coordinate invariance, include terms higher than the second derivative of the metric tensor. Apart from interesting new implications in cosmology and astrophysics, such theories are often considered serious candidates for consistent quantum gravity, due to their power-counting renormalizability.

In her work, Ms. Pjatkanová was inspired by a number of papers on extended theories of gravity [1,2,3] as well as pioneering works by P.D. Mannheim *at al.* [3,4,5] on conformal gravity. The Thesis itself is divided into 3 chapters and 3 technical appendices. After the introductory chapter (Introduction), in Chapter 1 Ms. Pjatkanová first discusses the need for extending Einstein's general theory of relativity. She then goes on to discuss the classes of higher-derivative theories of gravity with their respective pros and cons. This material, apart from being quite informative (for example, I learned about $f(R, G)$ gravity), is needed in the main body of the text. This is followed by Chapter 2, in which external Schwarzschild types of solutions are derived for two important classes of higher-derivative theories of gravity, namely $f(R)$ gravity and conformal (or

Weyl) gravity. In this chapter Ms. Pjatkanová demonstrated both mathematical and technical skills in deriving and solving often technically involved equations. In Chapters 3, Ms. Pjatkanová focused on axially symmetric solutions. She derives and discuses Kerr-types of solutions both in the $f(R)$ gravity and conformal gravity. It is expected, that issues from Chapters 2 and 3 will be further explored in more detail during Ms. Pjatkanová's MSc research project. The Thesis ends with a brief concluding chapter.

For technical completeness, there are 3 appendices discussing more detailed technical aspects and derivations related to $f(R)$, $f(R,G)$, and conformal gravity. These clearly illustrate the technical grasp of the student. Thesis also provides an extensive bibliography, which pretty well reflects the current state of the art in the field. Unfortunately, unlike the text itself, there are a number of typos in Bibliography. For example, Ref. [3] says "september" instead of "September", in Ref. [5] a page is missing, in Refs. [12] and [19] a journal reference is missing, in Refs. [17] and [21] is "kerr" instead of "Kerr", etc. I suspect that these typos are most likely a consequence of the last-minute rush before submission.

All in all, in my opinion, the Bachelor's thesis of Ms. Pjatkanová has a very good quality. It offers a representative selection and subsequent discussion of topics that are indispensable for a coherent research in the field of extended classical and quantum gravity. Key aspects of classical higher-derivative gravity are logically and clearly worked out. Last but not least, higher-derivative gravity is a conceptually and numerically challenging endeavour within the ever-expanding field of quantum gravity, and I am sure that expertise gained by Ms. Pjatkanová will serve her well in the years to come.

[**1**] V. Faraoni and S. Capozziello, *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*, (Springer, Dordrecht, 2011).

[**2**] S. Capozziello and F. Bajardi, *Nonlocal gravity cosmology: An overview*, International Journal of Modern Physics D, **31** (2022) 2230009.

[**3**] P. Li, Y.-q. Liu, J.-h. Yang, S. Xu, and X.-h. Zhai., *Analytical calculation of Kerr and Kerr-AdS black holes in $f(r)$ theory.*, EPJC **84** (2024) 704.

[**4**] P.D. Mannheim and D Kazanas, *Exact vacuum solution to conformal Weyl gravity and galactic rotation curves*, Astrophysical Journal **342** (1989) 635.

[**5**] P. D. Mannheim, *Some exact solutions to conformal Weyl gravity*, Annals of Phys. **631** (1991) 194.

[**6**] P.D. Mannheim and D Kazanas, *Solutions to the Reissner-Nordström, Kerr, and Kerr-Newman problems in fourth-order conformal Weyl gravity*, Phys. Rev. D **44** (1991) 417.

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