

Supervisor's review of the bachelor thesis

Daria Kuznetsova:

**Modelling Dependence of Random
Variables by Copulas**

Daria Kuznetsova has education in economy and she tried to combine it with statistical methods studied here. The point of intersection is the notion of copula, which describes the dependence of random variables in a form independent of their individual (marginal) distributions. This required to study copulas, which are not taught in our courses, and understand them to the extent allowing own scientific research. Daria Kuznetsova succeeded and contributed to this field by new results.

One direction was the copula of mixtures of random vectors with independent components (not the mixture of copulas, which is simply a convex combination). The question was whether such copulas satisfy some specific properties. The result is a characterization, supported by experimental verification on simulated data. It allows to estimate the number of conditionally independent components.

In economy and other areas, copulas are used to predict simultaneous occurrence of extreme values of random variables. For their description, the asymptotic behaviour of the copula near extremes of its domain is crucial. We found contradictory statements on this theme in the literature. The thesis corrects some wrong sources and presents results based on rather complex limits.

We planned to combine both topics—examine asymptotic behaviour of copulas describing mixtures of Gaussian distributions. This task was not included in the thesis; a thorough study of a single Gaussian distribution resulted in the observation that the limit of density in an extreme is either zero or does not exist. Similar conclusion, not leading to a specific value, was expected in the more general case; this represents much effort without adequate gain.

Daria Kuznetsova proved all expected skills of a graduate of our university. Her thesis is a significant theoretical contribution with applicable consequences. I evaluate the thesis by the grade

A (excellent).

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