Thesis reviewer's report

Thesis title:	**Modelling Dependence of Random Variables by Copulas**
Author's name:	**Daria Kuznetsova**
Type of thesis :	Bachelor's thesis
Faculty/Institute:	Faculty of Electrical Engineering
Department:	Department of Cybernetics
Thesis reviewer:	doc. Ing. Tomáš Bacigál, PhD.
Reviewer's department:	Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava

The thesis discusses joint distribution of a vector of continuous random variables modeled by decomposition to copula and marginal distributions. It specifically focuses on copula density analysis in concurrent extremes and detecting number of normally distributed variables hidden in a mixture of independent pairs, providing Matlab code for all calculations.

Stylistically it is mostly well written, also structured with proper citations and cross-references. On the other hand its content is a bit inconsistent and purpose of some parts unclear:

- In the introduction a conditional independence is stressed as crucial for modeling dependence. However, it is not clear what it means since independence of some random variables conditional on their relationship to other variables is not treated directly and the only reference is given implicitly by the chapter 3 devoted to finding a number of levels for common factor present in the mixture. I would expect to see more explicit theory of conditional probability in the thesis and fulfilling the promise given in the third paragraph on page 1.
- Computation of limits in copula tails is analytically interesting, but no practical example is given of where it might be relevant for applications, there is only a very brief notion of VaR on page 23. (I suggest the student to illustrate at the defense how density evaluated in extreme points could be utilized, e.g., for calculation of VaR.) When probability or level curves are calculated near the corners [0,0] or [1,1] one must be aware of extrapolation uncertainty/hazard. I also wonder, why tail dependence coefficient was not mentioned in this context.

Moreover I have several minor comments to the text, ordered here by appearance:

- page 1/paragraph 2: Variables can be considered independent thanks to – not despite – the presence of common factors. But this is kind of philosophical remark.
- p.2/line -5: Formulation "data located in the copula are simulated and predefined" is not clear.
- p.10/l.7: "Strictness" not defined above.
- p.18/Remark 2.3: What does "only up to a set of measure zero" mean?
- p.24/l.7: What "the test statistics"? It should be put into context with the cited paper, such as "statistics used in the Gaussianity test".
- p.24/l.13: While up to this point tail dependence may have been mentioned in general, here the sentence "tail dependence neglected by the Gaussian copula can be as large as 0.6" refers to a particular measure. Which one?
- p.25/l.-6: The focus in this chapter was not "on a conditionally independent distribution" specifically, just generally on distribution densities. The topic of conditionally independent distribution is a matter in the next chapter.
- p.27/l.-7: It is desirable to formally define "mixture of Z1 and Z2 with coefficients c, 1-c".

- p.35/above: It would be illustrative to show the result of the code (discretization) before passing to decomposition.
- p.37/l.5: The value is even smaller, 0.02 not 0.2. How do you judge significance of diagonal values in matrix S of SVD? Is there a standard routine?
- p.39/par.-2: So you are saying that the economists' mistake when predicting market behaviour was not in choosing copula with zero tail dependence but in choosing copula with no limit in tails?! It needs either proper citation or an experiment showing differences in predictions.

Overall, the topic is quite complex for bachelor thesis so the above comments does not lower the value of student's work. She may decide which of them to address at the defense.

In summary, I acknowledge that this **work by Daria Kuznetsova satisfies the requirements for bachelor thesis** and I evaluate it by the grade **B - very good**.

In Bratislava, June 3, 2024

doc. Ing. Tomáš Bacigál, PhD.