Evaluation of Doctoral Thesis

Special functions and polynomials of affine Weyl groups and corresponding Fourier methods

by

Adam Brus

This thesis summarizes a significant contribution to the study of Weyl-orbit functions, related polynomials and Fourier transforms. It focuses on the affine (Kac-Moody) Weyl groups. The work has produced 4 publications 2018-21 all co-authored with Drs Hrivnak and Motlochova. 1 appears in the prestigious mathematical physics journal Journal of Physics A: Mathematical and Theoretical, and the 3 others in open-access journals Symmetry and Entropy.

It was Patera and Klimyk who launched the study of Weyl-orbit functions as special functions around 2006. Hrivnak and Motlochova were a postdoc and a PhD student with Patera, respectively, and have been leaders in the subject for some years now. Brus's thesis topic is current, and co-authored with recognized experts.

For a doctoral candidate, a sole-author publication strengthens their case significantly, as clear evidence of independent research. Brus has only published in collaboration with both Hrivnak and Motlochova. On the other hand, it is no mean feat to work with them in this subject at their level.

A variety of methods were used in the research summarized in this thesis. Analytic calculations were carried out using Fourier transforms, orthogonal polynomials, lattice and other techniques, and a mastery of Lie algebra theory is crucial and apparent. Extensive use of the symbolic computation software Mathematica was also noted.

As stated in the Introduction, a goal of the thesis is to summarize and consolidate the contributions of the author contained in the articles [A1-A4]. In my opinion, the summary would be improved by making the treatment more self-contained. For such a thesis, the reader should certainly expect to have to consult the articles [A1-A4] directly sometimes. However, I found that I had to do so too often.

For example, the term "dot" was first used on page 42, but not defined or explained before. For another example, the hopping function and hopping operator and their different roles were not motivated well. Not everything can be repeated in such a "consolidating summary." It would be better, I think, to motivate the work in [A1-A4] in a general, and perhaps less precise way, leaving some of the concrete details to the including articles. The tradeoff would produce a summary that is more consolidating, I believe.

The results reported are sufficiently fundamental and general that I expect that they will be used in several applications. This is the main scientific value of the research. Many possibilities have been mentioned in the included articles. Already a number of applications have been fleshed out: quantum particles in alcoves and on quantum dots, conformal field theory, and discrete quantum billiard systems, for examples.

The main novelties in the results reported relate to the multivariate Chebyshev-like polynomials. Other work seems to be variations on previous research, although it is nontrivial, demanding and potentially, very useful.

My overall evaluation of the thesis: very good. I recommend it without hesitation for presentation and defense.

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