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Czech Technical University in Prague

F3

Faculty of Electrical Engineering
Department of Economics, Management and Humanities

Software application for energy savings

Petr Jílek

Supervisor: Ing. Jiří Beranovský, Ph.D., MBA

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ZADÁNÍ DIPLOMOVÉ PRÁCE

I. OSOBNÍ A STUDIJNÍ ÚDAJE

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Softwarová aplikace pro úsporu energie

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- Analýza stávajících aplikací pro energetické vyhodnocení objektů.
- Analýza příležitostí pro snížení energetické náročnosti v kontextu mobilních aplikací.
- Návrh a implementace mobilní aplikace pro technicko ekonomické výpočty úsporných opatření.
- Výpočet modelového příkladu.
- Shrnutí, závěry a diskuse.

Seznam doporučené literatury:

[1] Beranovský, J., Pokorný, J. (2014) Je úsporný dům opravdu úsporný? Z čeho postavit úsporný dům? [online] Praha, ? EkoWATT, Centrum pro obnovitelné zdroje a úspory energie. ISBN: 978-80-87333-10-5. Dostupné z http://www.ekowatt.cz/cz/datum-publikace.
[2] Beranovský, J., Jindrák, M., Bejvlová, V. (2017) Efektivní vytápění energeticky úsporných domů. [online] EkoWATT z. s., Praha. ISBN: ISBN 978-80-87333-14-3. Dílo bylo zpracováno za finanční podpory Státního programu na podporu úspor energie na období 2017-2021 - Program EFEKT 2 pro rok 2017. Dostupné z http://ekowatt.cz/cz/publikace/.
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Jméno a pracoviště vedoucí(ho) diplomové práce:

Ing. Jiří Beranovský, Ph.D., MBA   katedra ekonomiky, manažerství a humanitních věd   FEL

Jméno a pracoviště druhé(ho) vedoucí(ho) nebo konzultanta(ky) diplomové práce:

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Ing. Jiří Beranovský, Ph.D., MBA podpis vedoucí(ho) práce	podpis vedoucí(ho) ústavu/katedry	prof. Mgr. Petr Páta, Ph.D. podpis děkana(ky)

III. PŘEVZETÍ ZADÁNÍ

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Seznam použité literatury, jiných pramenů a jmen konzultantů je třeba uvést v diplomové práci.

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Declaration

I declare that the presented work was developed independently and that I have listed all sources of information used within it in accordance with the methodical instructions for observing the ethical principles in the preparation of university theses.

Prague, May 24, 2024 Petr Jílek

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Abstract

The work deals with developing a mobile application for energy savings called Save Energy. The work commences with a comprehensive exploration of energy audit and potential savings strategies, laying a solid foundation of knowledge. Following this, the existing selected applications found on the Google Play store will be examined in terms of their functionality and user interface. The subsequent chapters deal with the mathematical and economic essentials. These concepts are then applied to develop the mathematical application's core, for which a separate chapter is designated. A brief chapter about obtaining irradiance data for evaluating photovoltaic systems is presented in the middle of the work. The work then shifts toward the actual implementation of the application, with a focus on software development aspects, culminating in a detailed exploration of the application's user interface featuring actual screenshots. Finally, the project concludes with a chapter demonstrating the application's utility through a practical case study involving energy consumption analysis of a sample household. Various analyses are presented to illustrate the application's effectiveness in real-world scenarios, along with the outcomes of these analyses.

Keywords: mobile application, flutter framework, energy, energy consumption, economic evaluation, photovoltaic system, energy distribution

Supervisor: Ing. Jiří Beranovský, Ph.D., MBA

Abstrakt

Práce se zabývá vývojem mobilní aplikace pro úspory energie s názvem Save Energy. Práce začíná komplexním průzkumem energetického auditu a potenciálních strategií úspor, čímž se položí pevný základ znalostí. Následně budou stávající vybrané aplikace, nalezené v obchodě Google Play, prověřeny z hlediska jejich funkčnosti a uživatelského rozhraní. Následující kapitoly se zabývají matematickými a ekonomickými základy. Tyto pojmy jsou aplikovány na vývoj matematického jádra aplikace, kterému je určena samostatná kapitola. Uprostřed práce je uvedena stručná kapitola o získávání dat záření pro hodnocení fotovoltaických systémů. Práce se poté přesune k samotné implementaci aplikace se zaměřením na aspekty vývoje softwaru, což vyvrcholí detailním prozkoumáním uživatelského rozhraní aplikace, které je demonstrováno na skutečných snímcích obrazovky. Nakonec projekt uzavírá kapitola demonstrující užitečnost aplikace prostřednictvím praktické případové studie, zahrnující analýzu spotřeby energie na vzorku domácnosti. Jednotlivé analýzy jsou prezentovány spolu s jejich výsledky, které ilustrují efektivitu aplikace v reálných scénářích.

Klíčová slova: mobilní aplikace, flutter framework, energie, spotřeba energie, ekonomické hodnocení, fotovoltaický systém, distribuce energie

Překlad názvu: Softwarová aplikace pro úsporu energe

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Acronyms

CCF Cumulative Cash Flow

CF Cash Flow

DCF Discounted Cash Flow

DPP Discounted Payback Period

EC Escalation Coefficient

EU European Union

IRR Internal Rate of Return

NPV Net Present Value

OTE Electricity Market Operator

PENB Certificate of energy efficiency of the building

PHI Passive House Institute

PHPP Passive House Planning Package

PP Payback Period

US United States

VAT Value Added Tax

Chapter 1

Introduction

Many companies and individuals do energy audits and evaluate energy-saving opportunities. This work aims to develop a mobile application that will analyze the energy consumption of the specified object and evaluate the economic aspects of energy-saving opportunities.

For example, when considering the investment into photovoltaic panels for the house, there are many aspects to consider. The standard approach would be to prepare the model, including the house's electricity consumption with its consumption diagram, to know when it is consumed. This could then be compared with electricity produced from the photovoltaic panels. Also, many other relevant aspects must be assessed, including technical aspects like the efficiency of photovoltaic panels and the efficiency of inverters and economic aspects like electricity price, inflation, etc. Most of these models are done using Excel or other software. Excel offers a wide range of functions and possibilities with good flexibility. Some individuals could also use software that requires programming skills, like Python or R. However, creating such an evaluation without the necessary skills and knowledge can sometimes be challenging. Therefore, the first motivation to develop the mobile application is to provide a tool that will analyze and evaluate energy-saving opportunities with less requirements of having a background in electricity, economics, and programming. Thus, users can create evaluations on their own.

Development of the application is not straightforward because a balance between the complexity and user-friendly interface is needed. When considering the example of a photovoltaic system, the Excel models could be very complex, considering many technical and economic aspects, where the sheets could become significantly large. For example, input parameters could be located on the first sheet, including most technical and economical input parameters. However, when developing the app, it is necessary to obtain the input parameters from the user through some interactive forms. If a significantly complex model is to be implemented in the app, the forms for entering values will get so complex that the user will be overwhelmed. Due to this bewilderment, the user could be discouraged from using the app or misusing it.

The second thing to consider is the app's generics. In the field of energy savings opportunities, there are many options to consider, from replacing basic equipment with a new energy-efficient one to installing a photovoltaic system. The user can have a requirement to analyze all options, including those mentioned above. Both analyses must have specific input parameters, which only sometimes intersect. Here, the app faces that the experienced evaluator could create an Excel model for almost every case and specify any required parameters, but the app has certain limits. When creating the Excel model for a photovoltaic system, the evaluators could, for example, set the input parameters as they want. For example, how the photovoltaic panels are installed, including their azimuth and tilt. They could also make more sets of photovoltaic modules, each with different properties. These input parameters are not generic as they are only used when analyzing the photovoltaic system, but when analyzing the purchase of a new appliance, these parameters are not used. On the other hand, parameters like inflation, discount rate, and electricity price are more generic as they are usually the same in multiple analyses. The appropriate forms must be created to get these inputs so the user can set the inputs for all analyses. The user should be required to enter the generic parameters only once, which will be reused for other analyses. Otherwise, the app would require assigning the same parameters repeatedly, which could frustrate the user. Also, some variants are difficult to include. For the photovoltaic panel's analysis, the evaluator could choose whether the electricity will be used mainly to cover the house appliances' electricity consumption, sold directly to the grid, or stored in the battery. The energy could also be used to heat the water or the house. The complex form would have to be implemented to obtain all necessary input variables from the user and address these options and potential situations. Thus, it is sometimes complicated to include many variables in the app. Hence, considering how the app will be generic could result in higher development complexity and costs. Overall, the app should provide features that both non-experienced users and experienced users can use. The integrated calculations should also be consistent and balanced in complexity and simplicity.

Various software can perform calculations to evaluate photovoltaic systems and house consumption. Most of the software is desktop or web applications. Some mobile apps can help the user create essential house evaluations and evaluations of photovoltaic systems.

In the case of the mobile application, there is another caveat when considering how the information will be presented to the user. The mobile device's screen is much smaller than the screen of the desktop or laptop. Therefore, the user interface should be designed so that the user can see all the relevant information and interact with the app in a user-friendly way. For example, many graphs that are easily readable on the desktop could be complex to read on a mobile device, where sometimes the developers need to consider which information is necessary to be included and which could be somewhere else or not included. In the case of graphs on the desktop, the axis labels and titles are easily presented. However, placing the graph in the mobile app will cover the entire screen and still be small. Therefore, the axis title for the vertical axis sometimes has to be written somewhere else as they do not fit next to the graph.

From this context, the purpose of this work is slightly different from creating a directed model and evaluation. Commonly, the evaluator focuses mainly on correctly estimating specific technical and economic parameters. For example, they focus on estimating the correct value or values for inflation and escalating the prices, affecting the economic results. In this work, rather than estimating correct values, the focus will be on the mathematical representation of those inputs and how these inputs are gathered throughout the app and represented in the mathematical model. As for the example of inflation, the app could have a simple form to let the user input just one value, representing the inflation through the whole evaluation period. However, this approach will be so limited that it may not be enough to estimate the reality, and the user will not use the app but rather make his model. Thus, the app should provide a more complex form, where, for example, the user will enter the first n for the first n years. If the evaluation period is longer, the last entered value will be taken as the inflation for the following years. This approach supports more complex input but could also be enhanced further. For example, with the first n values, the user could choose how the values should continue for the following years. For example, the option could include the last value or the geometric average of previous values, or linear regression could be calculated. This could be obtained if the app provides one field for entering the values and then a dropdown button with the options for how the values should continue. This concept is actually used in the app and is called the values continue.

In conclusion, this work aims not to create a focused model to resolve specific issues but to provide a framework for the users to write the input parameters into the app and create the evaluation themselves. The central part of the work is to think about how the inputs will be mathematically represented and how the final results are calculated.

Chapter 2

Energy audit and options for increasing energy efficiency

This chapter is focused on the energy audit and options for increasing energy efficiency. In the first part, we will focus on the energy audit, certificate of energy performance of the building, and appliance energy label. After that, a brief description of energy pricing and how it works will be provided. Finally, we will describe the options for increasing the energy efficiency of the building, such as basic equipment replacement, heat insulation, etc.

2.1 Energy audit

An energy audit is a systematic review and analysis of the energy consumption of the object to obtain information about actual energy management, which provides information about how much the object consumes energy and how much the energy costs [22].

The energy audit also sets a baseline for identifying energy-saving opportunities and provides a basis for their evaluation [23]. It is also recommended to undertake an energy audit before implementing some renewable energy system [23].

The energy audit is performed by an energy auditor who is a professional with the knowledge and skills to evaluate the energy performance of the object [23].

2.1.1 Households

According to [24], the auditor will first explore how the house is built and how well the thermal insulation works. The auditor will check how much energy is lost through drafts or leaks in the building. The measurements will be performed on the heating, hot water, lighting, and appliances. Finally, the auditor will provide a report with recommendations for energy-saving measures with the estimation of the money savings.

2.1.2 Legislation in Czechia

In Czechia, the energy audit is required by the legislation to be performed by enterprises with more than 250 employees, which have a gross annual turnover of more than 1 300 000 000 CZK or annual balance sheet total of more than 1 100 000 000 CZK [25].

The energy audit is required once every four years [25]. However, there are some exceptions. For example, if the enterprise has an annual energy consumption of fewer than 200 MWh, do not have to perform the energy audit [25].

2.1.3 Methodology in Czechia

The methodology for evaluating the energy consumption of buildings is not uniform. According to [26] in the Czechia, the energy assessment of buildings is mainly carried out according to the following methodologies:

- ČSN 73 054010 - Defines low-energy, passive, and zero-energy houses in the Czech Republic.
- TNI 73 0329 for family houses and TNI 73 0330 for apartment buildings.
- PENB = Certificate of energy efficiency of the building.
- PHI = Passive House Institute.

Passive House Institute (PHI) uses Passive House Planning Package (PHPP) tool for energy planings. A brief list of some of the criteria with TNI and PHI requirements is shown in the following table.

Criterion	Unit	TNI	PHI
Heat demand for heating	$\mathrm{kWh}/(\mathrm{m}^2 \cdot \mathrm{year})$	$\leq 20\ (15)$ 1)	≤ 15
Primary energy demand from non-renewable sources	$\mathrm{kWh}/(\mathrm{m}^2 \cdot \mathrm{year})$	≤ 60 2)	≤ 120 3)
Building envelope airtightness	h^{-1}	≤ 0.6	≤ 0.6

Table 2.1: Passive standard according to TNI and PHI [13].

1) The value of 20 $\mathrm{kWh}/(\mathrm{m}^2 \cdot \mathrm{year})$ applies to family houses, 15 $\mathrm{kWh}/(\mathrm{m}^2 \cdot \mathrm{year})$ applies to apartment buildings.
2) Energy for heating, hot water preparation, and technical building systems (excluding lighting and other household appliances).
3) Energy for heating, hot water preparation, technical building systems, lighting, and other household appliances.

To achieve passive house standards, prioritizing effective thermal insulation within the building envelope is essential. The heat transfer coefficient measured in $\mathrm{kWh}/\left(\mathrm{m}^2 \cdot \mathrm{year}\right)$quantifies this insulation's quality. Below is a table presenting the prescribed values per norm ČSN 73 054010, alongside the recommended thresholds for passive houses.

Heat transfer coefficient ($\mathbf{kWh}/\left(\mathbf{m}^2 \cdot \mathbf{year}\right)$)	**Requirement**	**Recommended**
External wall	0.30	0.18 to 0.12
Sloped roof	0.30	0.18 to 0.12
Flat roof	0.24	0.15 to 0.10
Ground floor on terrain	0.45	0.22 to 0.15
Windows	1.5	0.8 to 0.6

Table 2.2: Thermal insulation parameters of selected constructions [14].

2.2 Certificate of energy performance of the building

The Certificate of energy efficiency of the building (PENB) is applied to assess buildings in Czechia. In Czech language the PENB is written as "Průkaz energetické náročnosti budovy". The certificate is a document that provides information about the energy performance of the building. It assesses the energy consumption for heating, cooling, hot water, ventilation, lighting, and other object operations [27]. However, the calculation does not include the consumption of individual appliances inside the house, as this will always change from family to family [28].

Moreover, the energy consumption is multiplied by a primary energy conversion factor, which results in the primary energy estimation [29]. The factor is different for each energy source. The following table shows the conversion factors for each energy source.

Energy source	**Conversion factor**
Natural gas, black coal, brown coal	1.1
Propane-butane, LPG, heating oil	1.2
Electricity	3.0
Wooden pallets	0.2
Piece wood, wood chips	0.1

Table 2.3: Conversion factors for primary energy [15].

In the table above, we can see that electricity is considered the most unecological energy source. It is because electricity is produced by burning coal, gas, or oil with an efficiency of about 30 to 40 % [30]. On the other hand, the coal could be burned directly in the house with much higher efficiency up to 90 % (burning coal for heat) [31]. Roughly, if we divide the 90 % by 30 %, we get 3.0, which is the conversion factor for electricity. In this calculation, we did not include the conversion factor for coal of 1.1, but for basic understanding, it is unnecessary.

Suppose the household has a heating system based on electricity (for example, 2000 kWh per year). In that case, the amount will be multiplied by the conversion factor 3.0, and the final value will be 6000 kWh. This value will then be used to calculate the energy performance of the building. On the other hand, if the household has a heating system based on coal with the same consumption value (2000 kWh per year). The amount will be multiplied by the conversion factor 1.1, and the final value will be 2200 kWh. This value will then be used to calculate the energy performance of the building. As we can see, the difference is significant, and the energy performance of the building will be much better if the heating system is based on coal.

Finally, the primary energies are summed up and then divided by the property's total floor area, and the result is the primary energy requirement per square meter squared (kWh/m^2 per 1 year) [28].

The calculated value then determines to which category the building belongs. The categories are from A to G, where A is the most economical building, and G is the least economical building [28]. The table below shows the energy classes and the required energy consumption per square meter (per year).

Class	Description	Primary energy (kWh/m^2)
A	Extremely economical	≤ 50
B	Very economical	≤ 97
C	Economical	≤ 142
D	Less economical	≤ 191
E	Uneconomical	≤ 205
F	Very uneconomical	≤ 245
G	Extremely uneconomical	> 286

Table 2.4: Energy classes and required energy consumption per square meter [16].

According to [32], the legislation for all buildings requires the certificate of energy performance of the building if the building is:

- Newly built.
- Being sold.
- Being rented.
- Being under reconstruction greater than 25 % of the building cover area.

Today, all new family houses must reach the minimum class B (very economical building) [28].

There are also some exceptions where the PENB is not required. The brief list of exceptions according to [28] is:

- Recreational facilities whose energy consumption does not exceed 25 % of the expected consumption during year-round use.
- Small buildings with an energy reference area of up to 50 m^2.

- Buildings that were built before 1947 and at the same time were not significantly reconstructed after the same year (25 % rule).
- Cultural monuments and buildings in the heritage reserve, or even buildings used for religious purposes, have an annual energy consumption of up to 700 GJ.

The example of the PENB label is shown in the following figure (the language of the label is Czech).

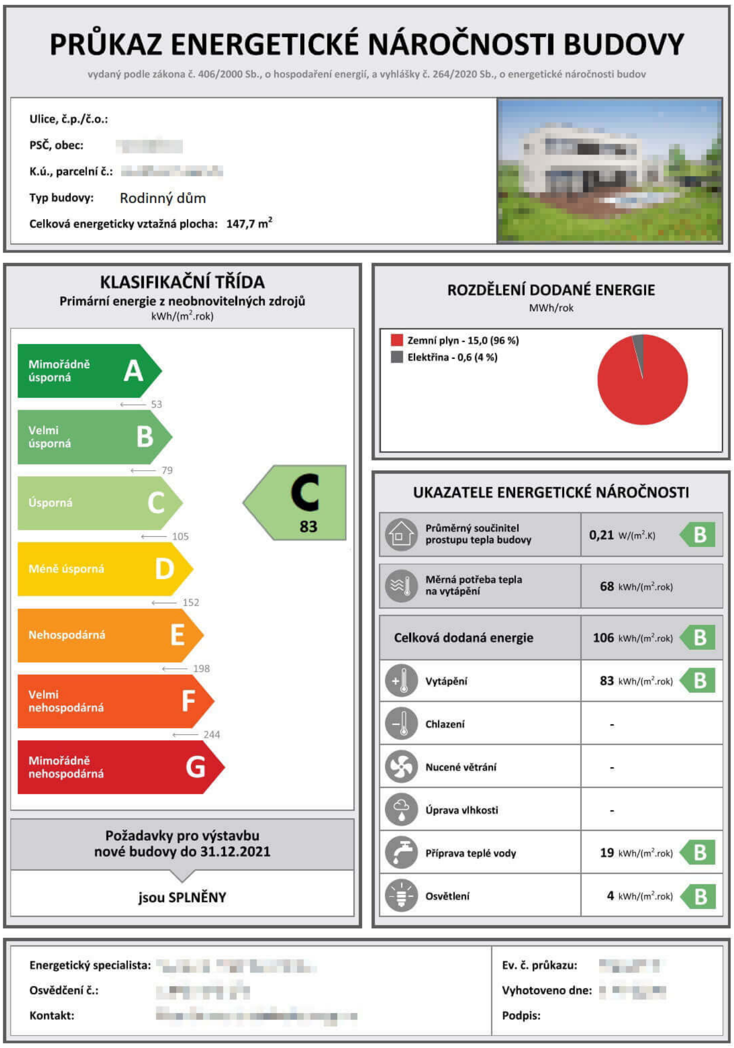

Figure 2.1: PENB label [1].

In the figure above, we can see an example of a PENB label. The PENB is a report of roughly 15 pages, but the energy label covers the basics. On top of this, primary data about the object are provided, such as the address, the object type, and the area. The energy classes are shown in the middle left part of the label, and information to which category the building belongs. The energy consumption in kWh/m^2 per year is written.

Furthermore, the middle right side label contains the distribution chart of energy sources used in the building. Then, information such as the average heat transfer coefficient is provided.

Finally, the label on the bottom contains information about the author, the report's date, and the signature. The PENB is then valid for ten years [1].

2.3 Appliance energy label

According to [2], the appliance energy label was first introduced in 1994 in the European Union. The label is a sticker placed on the appliance and provides information about the appliance's energy consumption. The label aims to help consumers choose the most energy-efficient appliance and save money on energy bills. It should also motivate the manufacturers to produce more energy-efficient appliances [2]. The energy label is part of the Ecodesign directive, a set of mandatory rules for manufacturers to produce more energy-efficient appliances [33].

The old energy labels were from A+++ to G, where A+++ was the most energy-efficient appliance, and G was the least energy-efficient appliance. However, the old labels confused the consumers because most appliances were in category A. Therefore, the new energy labels were introduced in 2021. The new labels are from A to G, where A is the most energy-efficient appliance, and G is the least energy-efficient appliance [34]. In the following parts, we will focus on the new energy labels.

Several types of appliances have the energy label. The most common appliances are washing machines, dishwashers, refrigerators, freezers, televisions, light bulbs, etc. The example label for the fridge with freezer is shown in the following figure.

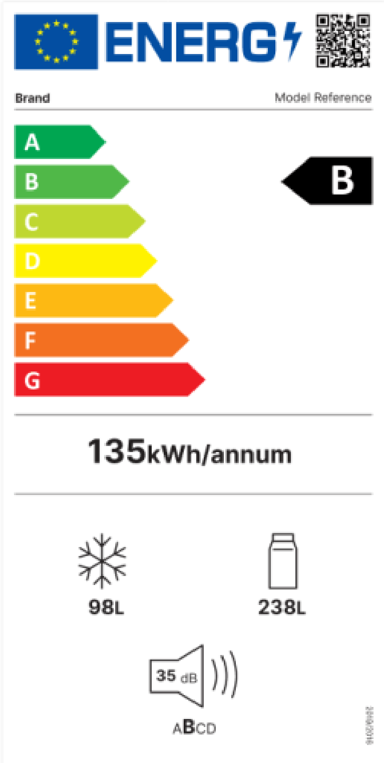

Figure 2.2: Example energy label for fridge with freezer [2].

The energy labels are divided into several categories. On the to the title, the QR code is placed. The QR code is used to get more information about the appliance. Then, the energy class is shown. The energy class is from A to G, where A is the most energy-efficient appliance, and G is the least energy-efficient appliance. The following parts differ for each appliance. The label above is for the fridge, where primarily the electricity consumption in kWh per year for the fridge with freezer is shown. For the fridge, the electricity consumption is calculated for 24 hours per day and 365 days per year usage of the appliance [34]. However, the consumption calculation differs for each appliance. For example, for the washing machine, the electricity consumption is represented in kWh per 100 washing cycles [35]. At the bottom, additional information about the appliance is provided. In this example, the fridge, freezer volume, and noise level are shown [35]. The energy label for the light bulb is shown in the following figure.

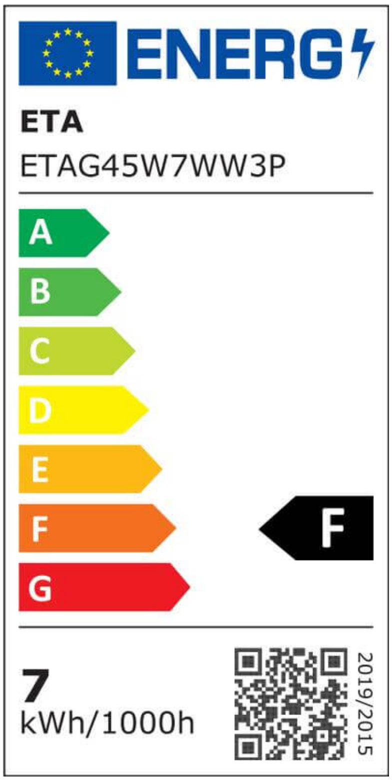

Figure 2.3: Example energy label for light bulb [3].

The label above is for an LED light bulb with 600 lumens and 7 W [3]. Therefore, the consumption is 7 kWh per 1000 hours. Now, we can calculate and assign the energy class. Firstly, we need to calculate the total main efficiency as follows:

$$\eta_{TM} = \frac{\Phi_{use}}{P_{on}} \cdot F_{TM}, \tag{2.1}$$

where:
η_{TM} is total mains efficiency (lm/W),
Φ_{use} is the useful luminous flux of the light in lumens (lm),
P_{on} is a factor that depends on the lamp type (directional/non-directional, operating on mains/ not operating on mains) [17].

As we are calculating the classic light, which is non-directional and operating on mains, the factor F_{TM} is 1.0. The Φ_{use} is 600 lm and P_{on} is 7 W. Therefore, the total main efficiency is:

$$\eta_{TM} = \frac{600}{7} \cdot 1.0 \approx 85.71 \text{ lm/W}.$$

The next step is to assign the energy class. The energy class is assigned according to the total mains efficiency. The following table shows the energy classes and the required total mains efficiency:

Energy class	Total mains efficiency (lm/W)
A	$210 \leq \eta_{\mathrm{TM}}$
B	$185 \leq \eta_{\mathrm{TM}} < 210$
C	$160 \leq \eta_{\mathrm{TM}} < 185$
D	$135 \leq \eta_{\mathrm{TM}} < 160$
E	$110 \leq \eta_{\mathrm{TM}} < 135$
F	$85 \leq \eta_{\mathrm{TM}} < 110$
G	$\eta_{\mathrm{TM}} < 85$

Table 2.5: Energy classes and required total mains efficiency [17].

For the calculated total mains efficiency of 85.71 lm/W , the energy class is F.

2.4 Energy pricings

This section will briefly describe the energy pricings in Czechia for electricity and gas.

2.4.1 Electricity pricing

There are two types of divisions in electricity pricing. The first division is the division into unregulated and regulated parts. The unregulated is the price for the electricity as an energy (could be fixed and variable) [36]. Energy Regulatory Office regulates the regulated parts [37]. According to [36], the regulated parts are the following:

- Distribution fee.
- Charge for reserved power.
- Subsidy for the production of renewable energy sources.
- Fee for system services.
- Fee to the market operator.

The subsequent division is into the fixed and variable parts. The fixed part is the amount that is paid regardless of the amount of consumed electricity (commonly per month). In contrast, the variable part is the amount paid according to the consumed electricity consumed (commonly per MWh or kWh) [38]. Finally, the taxes are applied as the tax on electricity and the VAT [39].

2.4.2 Gas pricing

In gas pricing, the fixed and variable parts are also applied. The price mainly varies according to the amount of gas consumed. Commonly, the fixed price increases when the amount of consumed gas is higher. On the other hand, the variable price goes down when the amount of consumed gas is higher [40].

2.5 Options for increasing energy efficiency

There are several ways how to increase the energy efficiency of the building. The most common ways are switching off the equipment, basic equipment replacement, and heat insulation. However, other ways exist, such as heat pumps, photovoltaic systems, or other renewable energy sources.

2.5.1 Switching off appliances

The most straightforward way to save energy is to switch off the appliances when they are not used. The appliances in standby mode still consume energy. The most common appliances that are in standby mode are TVs, computers, printers, etc. [41]. However, switching off the appliances will not increase the energy performance of the building, as these appliances are not calculated in the PENB [28].

2.5.2 Basic equipment appliances

Essential appliance replacement is a common way to increase the energy efficiency of a building. The most common appliances that are replaced are refrigerators, washing machines, dishwashers, etc. The new appliances are more energy efficient and consume less energy. Also, replacing old light bulbs with LED bulbs is an excellent way to save energy [42].

2.5.3 Heat insulation

Heat insulation is an excellent way to save energy. Heat insulation is used to prevent heat loss in the building. It could be done on walls that are covered with heat insulation material such as polystyrene or mineral wool [43]. The heat insulation could also be done on the roof, which is covered with heat insulation material such as mineral wool or foam. The critical step is also thinking about replacing the windows. The new windows could be more energy efficient and lower the heat loss. The heat insulation should reduce the average heat transfer coefficient of the building envelope [43].

Chapter 3

Existing applications

This chapter will focus on assessing the applications for energy evaluation. The main focus will be on applications that provide calculation techniques for energy consumption, energy costs, and energy savings. A total of four applications will be reviewed in this chapter. The applications are Electricity Cost Calculator, Energy Cost Calculator, Home Electricity Calculator, and PV Calculator. The features of the applications and the user interface will be assessed for each application. All applications are available on the Google Play Store for Android devices.

3.1 Electricity Cost Calculator

The first assessed application is Electricity Cost Calculator [44]. The application has over 10 000 downloads on the Google Play Store with a rating of about 4.5 stars.

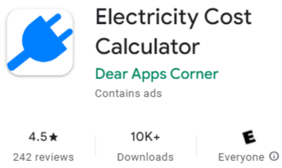

Figure 3.1: Electricity Cost Calculator play store [4].

Overall, the application is straightforward, with only one feature integrated. The feature is presented in the following figures, where the application has three text fields for input values for hours per day, power use in watts, and price per kWh. The application then calculates the cost per hour, cost per day, cost per month, cost per year, and kWh per day. These fields use just basic math formulas to calculate the values. The following figures fill the fields with example values, and the outputs for those values are also shown. Overall, the application could be helpful for quick calculations of energy costs for a specific appliance to get a rough idea of the costs.

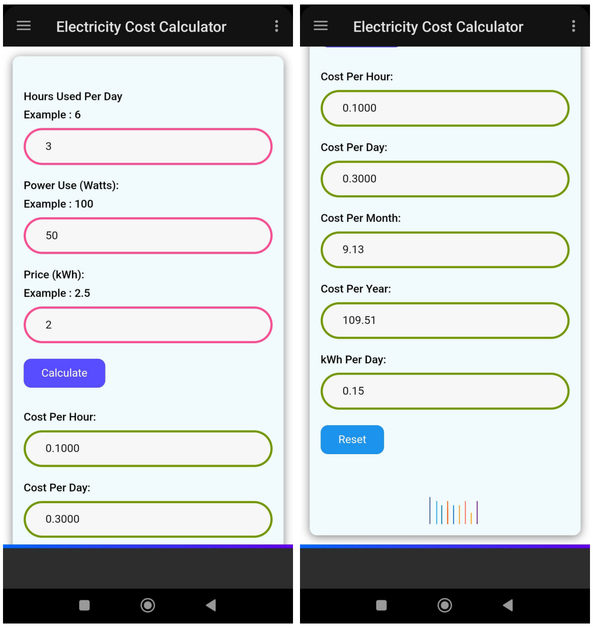

Figure 3.2: Electricity Cost Calculator screenshots [4].

3.2 Energy Cost Calculator

The second assessed application is Energy Cost Calculator [45]. The application has over 10 000 downloads on the Google Play Store with about 3.6 stars rating.

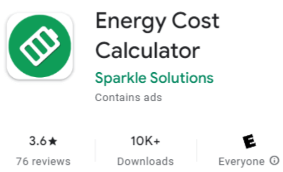

Figure 3.3: Energy Cost Calculator play store [4].

Overall, the application is very similar to the previous one. The application consists of one calculator that calculates energy consumption and costs. In the figure below, the screenshot of the application is shown. It has three input fields: consumption per hour, hours per day, and cost per kWh. The application then calculates the energy consumption and cost per day, week, month, year, and cost for those periods. The application is straightforward and could be helpful for quick calculations, similar to the previous one.

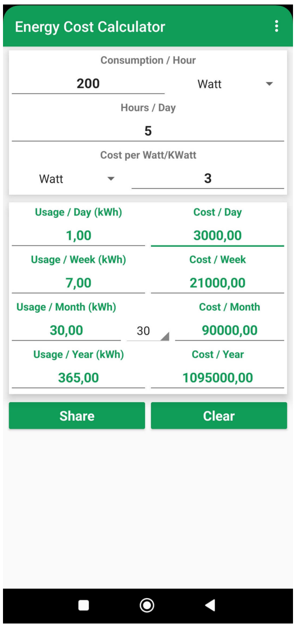

Figure 3.4: Energy Cost Calculator screenshot [4].

3.3 Home Electricity Calculator

Another application is Home Electricity Calculator [5]. The application has over 100 000 downloads on the Google Play Store and has a rating of about 4.0 stars.

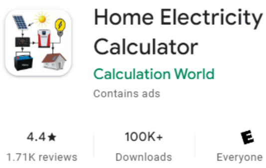

Figure 3.5: Home Electricity Calculator play store [4].

There are two versions of the application. The first is free with ads, and the second is a premium version. The premium version costs 3.99 USD. The further screenshots will be from the free version of the application.

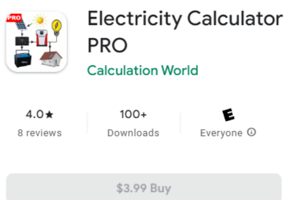

Figure 3.6: Electricity Calculator PRO play store [4].

The home page is shown in the figure below. The application has multiple integrated features. Most are just basic calculators presented from "solar plants" for the "water pump total head." Users can set the appliances in the first tab (home generator design) and other facilities in the second tab. Let us take a look at the appliances tab.

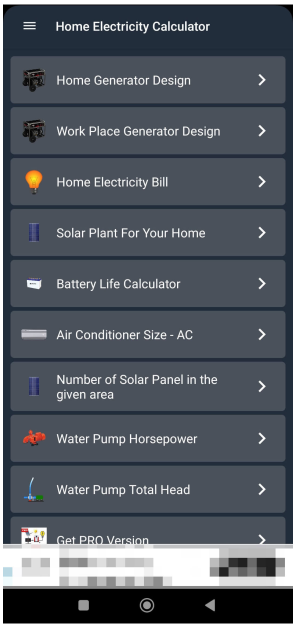

Figure 3.7: Home Electricity Calculator home page [5].

A screenshot of the appliances page is displayed in the figure below. The user can set the appliances from the provided list, such as a refrigerator, washing machine, television, etc. For the next part, we can add television.

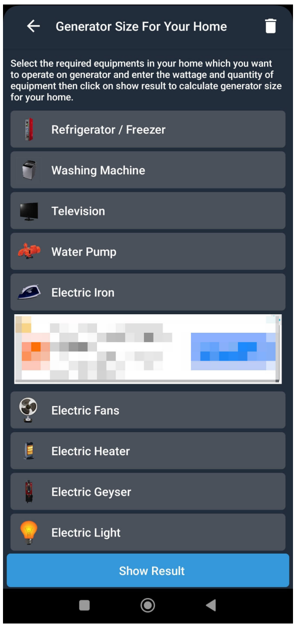

Figure 3.8: Home Electricity Calculator appliances page [5].

When we tap on the television, the application will show the page where we can set the power and quantity. The power could be either set by the dropdown (figure on the right), where there are predefined values, or manually by the user using the text field. After finishing the form, just hit the save button, and the appliance will be added.

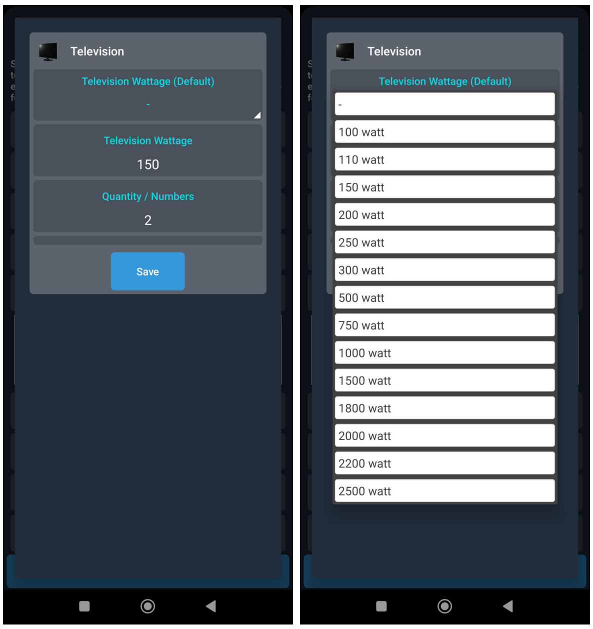

Figure 3.9: Home Electricity Calculator add appliance page [5].

Finally, we can show the result page with the added television as an appliance by clicking the "show result" button on the screenshot at the top. There are different result pages. On the left side is the energy cost analysis, which we can get by tapping the home electricity bill tab on the home page. Then, the appliances will be shown, and tapping on the show result will display the energy costs. Essential information like electricity costs for the week, month and year.

If we go through the first tab on the home page and then tap on the show result, the page will show how big we would need an electricity generator to power all the appliances. This is shown on the right side of the figure below.

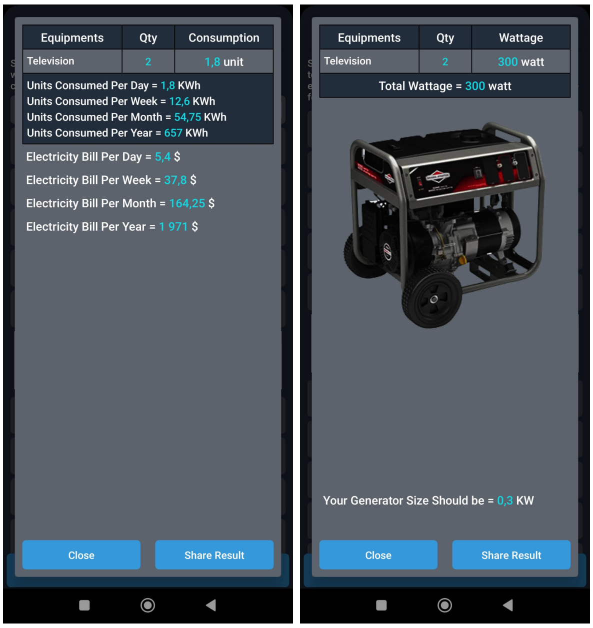

Figure 3.10: Home Electricity Calculator results pages [5].

3.4 PV Calculator

The application PV Calculator [6] has over 50 000 downloads on the Google Play Store with about 4.3 stars rating. The application is for the calculation of photovoltaic systems.

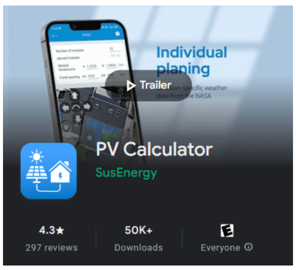

Figure 3.11: PV Calculator play store [4].

The screenshot of the home page is shown in the figures below. On the home page, there is an option to pick the location on the map and load the irradiance data by clicking the "load data" button. On the left figure, there is just a picked point on the map without data loaded, whereas, on the figure on the right, the data are loaded with the graph of irradiation values (kW/m^2) for the whole year. On the graph, the peak values are in months like May, June, July, and August, and the lowest values are in December, January, and February, which is typical for Europe.

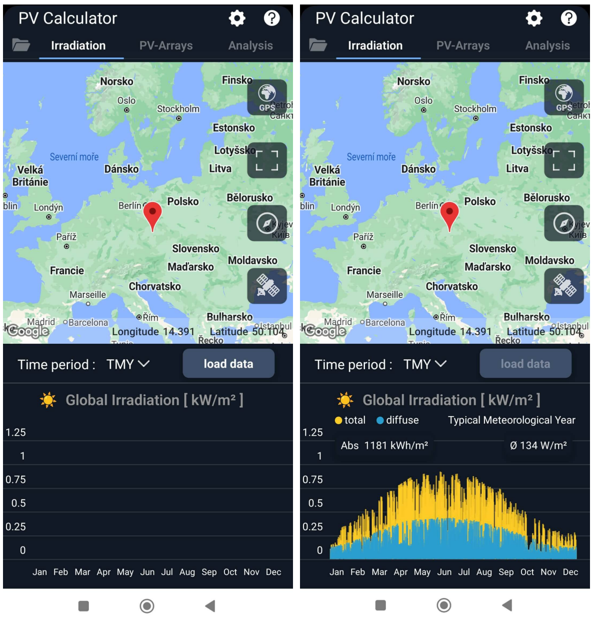

Figure 3.12: PV Calculator map [6].

To move forward, tap on the PV-Arrays tab. The content of that tap is shown in the figures below. Here, the user defines the photovoltaic system properties such as power inverter output power, efficiency, inclination, azimuth, etc. On the left figure is a list of tabs with defined PV modules, whereas on the right figure, the page for editing the PV module is shown.

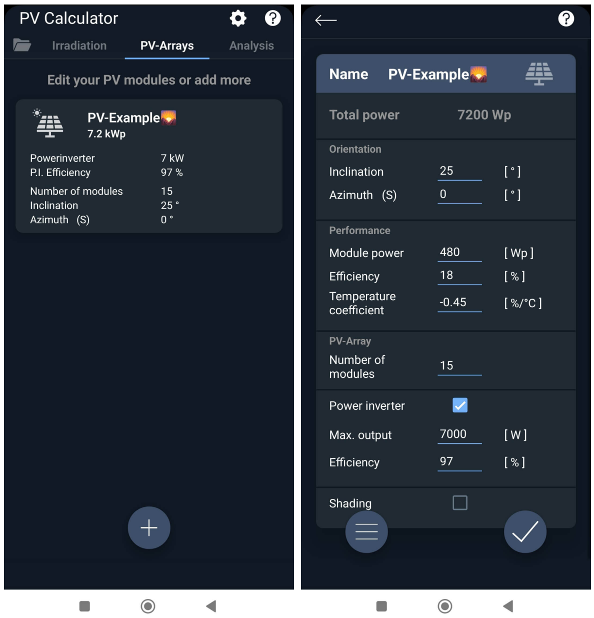

Figure 3.13: PV Calculator PV module [6].

Only one PV module definition is allowed in the free version of the application. The premium version allows multiple PV module definitions. The popup for the premium version is shown in the figure below.

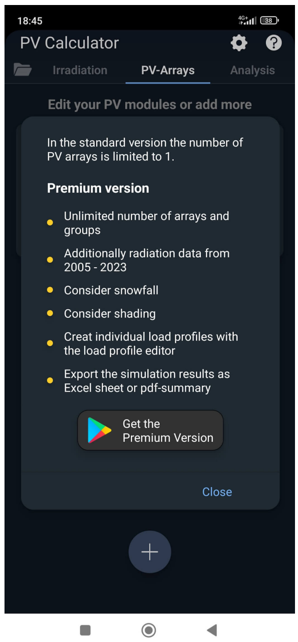

Figure 3.14: Home Electricity Calculator premium [6].

As we progress through the application to the analysis tab, the user can set the PV module to be analyzed along with system efficiency, whether grid-tied or off-grid. Also, the impact of temperature and snowfall could be considered, which is an advanced feature. Below are three screenshots of the analysis tab. The screenshots were taken on the single tab while scrolling down. Therefore, they are from left to right on the same tab as when scrolling down.

The user can set the daily production distribution on the first screenshot, which could be compared with energy demand. The user could also set the distribution of the demand. Here, we use just simple constant demand distribution. The daily production distribution is less for winter and more for summer, which is common in Europe.

The second screenshot shows the energy yield for the months, with two more charts representing photovoltaic power across the year (output power depending on the irradiation) and the energy balance, where we can compare the electricity produced with the demand. If the produced electricity does not wholly cover the demand, the user has to purchase the electricity from the grid.

The third screenshot demonstrates the card of final yearly values for produced power from the PV system, the power that is fed to the grid, and the power purchased from the grid. On the final chart, we can see these values across the months.

Figure 3.15: PV Calculator analysis first part [6].

The following screenshots are the continuation of the analysis tab. The first screenshot shows the option to set the battery storage. The battery has several parameters, such as capacity, efficiency, self-discharge, etc. Also, the number of complete cycles per year is presented on the same card. The following chart shows the charging and discharge cycles of the battery.

In the middle screenshot, the user could set the energy prices for the electricity purchased from the grid and the electricity fed to the grid, along with its percentage changes per year. Then, the user can see the savings for the first year in USD.

In the final screenshot, there is an amortization card where the user can set the price of the PV system and parameters like the rate of change, period of credit, and whether the PV panels will be funded with a loan. The app will then calculate the annual installment, total payment, and amortization period. On the final chart, there are two lines. The red one represents the investments. The line looks straight, but it has a slight positive slope because the set rate of change is only 0.5 %, which is very low for the load. The green line represents the cumulative savings, increasing with time. The intersection of the two lines is the amortization period.

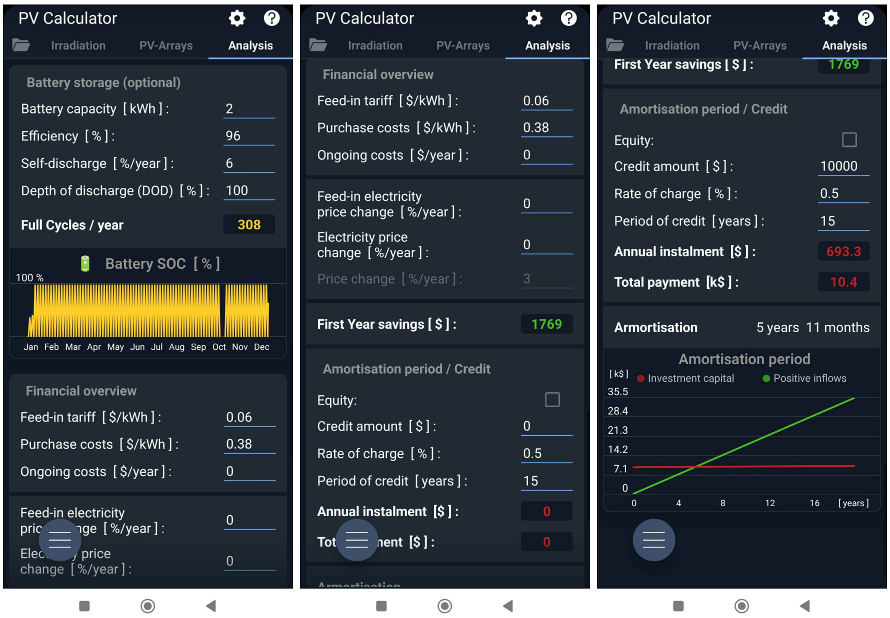

Figure 3.16: PV Calculator analysis second part [6].

All aforementioned items could be saved in the project within the app. The app also supports creating multiple projects. The projects are shown in the figure below:

Figure 3.17: PV Calculator projects.

Finally, the app supports German and English language. There are also two themes: light and dark. The settings are shown in the figure below:

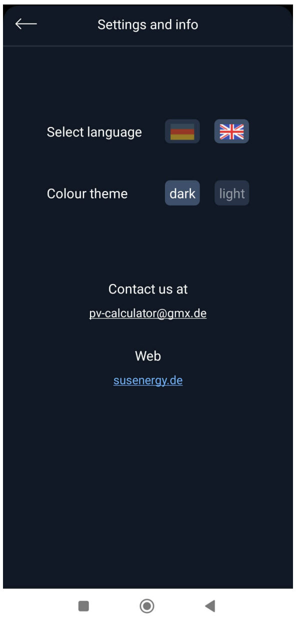

Figure 3.18: PV Calculator settings.

Overall, this app supports many features for analyzing photovoltaic systems, starting from the location and irradiation data through the PV module definition, system analysis, battery storage, energy prices, savings, amortization, etc. The app is handy for quick and detailed analysis of the photovoltaic system. The app is also very user-friendly and easy to use.

Chapter 4

Mathematical background

This chapter will deal with the mathematical background of the work. The standard mathematical background will be described in more depth in the appendix.

4.1 Vectors

The vectors will be written with an arrow above the letter, for example, $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{CF}$, $D\vec{C}F$, etc. The column vector notation will be mainly used. For example $\vec{a}$ will be written as:

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.1}$$

The domain of the vector elements will be mostly the set of real numbers $\mathbb{R}$. Therefore, the note about real numbers will mostly be omitted. Usually, the second element will also be omitted. Therefore, the vector $\vec{a}$ will be written as:

$$\vec{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.2}$$

4.1.1 Dot product

If we have two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, we can calculate the dot product of the two vectors by multiplying the corresponding elements of each vector and summing the results. According to [46], the following formula does this:

$$\langle \vec{a} \mid \vec{b} \rangle = \langle \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \mid \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \rangle = a_1 \cdot b_1 + \cdots + a_n \cdot b_n \in \mathbb{R}. \tag{4.3}$$

4.1.2 Pice-wise multiplication

If we have two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, we can multiply the vectors by multiplying the corresponding elements of each vector. This operation is known as the Hadamard product [47]. According to [47], the following formula does this:

$$\vec{a} \circ \vec{b} = \begin{pmatrix} a_1 \cdot b_1 \\ \vdots \\ a_n \cdot b_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.4}$$

4.1.3 Pice-wise division

If we have two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, we can divide the vectors by dividing the corresponding elements of each vector. According to [48], the following formula does this:

$$\vec{a} \oslash \vec{b} = \begin{pmatrix} a_1/b_1 \\ \vdots \\ a_n/b_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.5}$$

4.2 Matrices

The matrixes will be written in bold font, for example, $\mathbb{A}$, $\mathbb{B}$, $\mathbb{CF}$, $\mathbb{DCF}$, etc. For example the matrix $\mathbb{A}$ will be written as:

$$\mathbb{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}, \tag{4.6}$$

where:
$\mathbb{A}$ is a matrix of dimension $n \times m$,
a_{ij} is the element of the matrix $\mathbb{A}$ at the i-th row and j-th column.

The domain of the matrix elements will be mostly the set of real numbers $\mathbb{R}$. Therefore, the note about real numbers will mostly be omitted. Usually, the second element will also be omitted. Therefore, the matrix $\mathbb{A}$ will be written as:

$$\mathbb{A} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.7}$$

4.2.1 Pice-wise multiplication

The pice-wise multiplication between two matrixes $\mathbb{A}$ and $\mathbb{B}$ of the same dimensions $n \times m$ is done by multiplying the corresponding elements of each matrix. This operation is known as the Hadamard product [47]. The following formula does this:

$$\begin{aligned}\mathbb{A} \circ \mathbb{B} &= \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \circ \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{pmatrix} = \\ &= \begin{pmatrix} a_{11} \cdot b_{11} & \cdots & a_{1m} \cdot b_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} \cdot b_{n1} & \cdots & a_{nm} \cdot b_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}.\end{aligned} \tag{4.8}$$

4.2.2 Pice-wise division

The pice-wise division between two matrixes $\mathbb{A}$ and $\mathbb{B}$ of the same dimensions $n \times m$ is done by dividing the corresponding elements of each matrix. The following formula does this [48]:

$$\begin{aligned}\mathbb{A} \oslash \mathbb{B} &= \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \oslash \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{pmatrix} = \\ &= \begin{pmatrix} a_{11}/b_{11} & \cdots & a_{1m}/b_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1}/b_{n1} & \cdots & a_{nm}/b_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}.\end{aligned} \tag{4.9}$$

4.2.3 Creation by vectors

If we have a list of vectors $\vec{a}_1, \ldots, \vec{a}_m$ of the same dimension n, we can create a matrix $\mathbb{A}$ by putting the vectors as columns of the matrix. The following formula does this:

$$\mathbb{A} = \begin{pmatrix} \vec{a}_1, & \cdots, & \vec{a}_m \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.10}$$

In the same way, we can create a matrix $\mathbb{B}$ by putting the vectors as rows of the matrix. The following formula does this:

$$\mathbb{B} = \begin{pmatrix} \vec{a}_1^T \\ \vdots \\ \vec{a}_m^T \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & \ddots & \vdots \\ a_{1m} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{m \times n}. \tag{4.11}$$

4.3 Cumulative matrix

The cumulative matrix is a matrix $\mathbb{W}$, which is a lower triangular matrix with ones on the diagonal and the lower part of the matrix. The cumulative matrix is used to calculate the cumulative sum of the vector. The cumulative matrix $\mathbb{W}$ is defined by the following formula [49]:

$$\mathbb{W} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}. \tag{4.12}$$

4.4 Values continue

Consider that we have a vector $\vec{a}$ of known values with dimension n (the count of known values):

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.13}$$

However, for example, if we want for some operation higher dimension of the vector, we can continue the values of the vector $\vec{a}$ by certain formulas. Therefore, the previous vector $\vec{a}$ could be extended to a new vector $\vec{b}$ with the dimension m, where $m > n$. The new vector will have the first n elements the same as the vector $\vec{a}$. The vector $\vec{b}$ will be defined by the following formula:

$$\vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ a_{n+1} \\ a_{n+2} \\ \vdots \\ a_m \end{pmatrix} \in \mathbb{R}^m. \tag{4.14}$$

The rest of the elements will be calculated according to the selected formula. The required dimension could also be smaller than n. Here, the vector will just be truncated to the selected dimension. For example, let us denote the natural positive number $p \in \mathbb{N}^+$, where $p < n$. The vector $\vec{b}$ with the dimension p will be defined by the following formula:

$$\vec{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_p \end{pmatrix} \in \mathbb{R}^p. \tag{4.15}$$

Now, we have to create specific rules for extending or truncating the general vector. The values continue is a set of vector $\vec{a} \in \mathbb{R}^n$ and function $\mathrm{f}^{\,\mathrm{c}} : \mathbb{R}^n \times \mathbb{N}^+ \to \mathbb{R}^m$, where $m \in \mathbb{N}^+$. If the dimension is less than n, the function will just truncate the vector. Therefore, these conditions will not be further described as they will apply to all defined values continue functions. However, if the dimension is higher than n, the function will extend the vector by certain formulas described in the following section. The values continue principle will be primarily used in broadcasting. How it will work will be described in the broadcasting section.

4.4.1 Last value

The last value is a function $\mathrm{f}^{\,\mathrm{c,lv}}$ is one of the simplest ones. The function will just repeat the last value of the vector. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where $m > n$, the function $\mathrm{f}^{\,\mathrm{c,lv}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,lv}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ a_n \\ a_n \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^m. \tag{4.16}$$

4.4.2 Aritmetic mean

Function $\mathrm{f}^{\,\mathrm{c,am}}$ will extend the vector by the arithmetic mean of the previous values. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where

$m > n$, the function $\mathrm{f}^{\,\mathrm{c,am}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,am}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ \overline{a} \\ \overline{a} \\ \vdots \\ \overline{a} \end{pmatrix} \in \mathbb{R}^m. \tag{4.17}$$

Where:

$$\overline{a} = \frac{1}{n}\sum_{i=1}^{n} a_i. \tag{4.18}$$

4.4.3 Geometric mean

Function $\mathrm{f}^{\,\mathrm{c,gm}}$ will extend the vector by the geometric mean of the previous values. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where $m > n$, the function $\mathrm{f}^{\,\mathrm{c,gm}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,gm}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ \overline{a} \\ \overline{a} \\ \vdots \\ \overline{a} \end{pmatrix} \in \mathbb{R}^m. \tag{4.19}$$

Where:

$$\overline{a} = \sqrt[n]{\prod_{i=1}^{n} a_i}. \tag{4.20}$$

4.4.4 Harmonic mean

Function $\mathrm{f}^{\,\mathrm{c,hm}}$ will extend the vector by the harmonic mean of the previous values. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where

$m > n$, the function $\mathrm{f}^{\,\mathrm{c,hm}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,hm}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ \overline{a} \\ \overline{a} \\ \vdots \\ \overline{a} \end{pmatrix} \in \mathbb{R}^m. \tag{4.21}$$

Where:

$$\overline{a} = \frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}}. \tag{4.22}$$

4.4.5 Linear regression

Function $\mathrm{f}^{\,\mathrm{c,lr}}$ will extend the vector by the linear regression model. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where $m > n$, the function $\mathrm{f}^{\,\mathrm{c,lr}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,lr}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ \beta_0 + \beta_1 \cdot (n+1) \\ \beta_0 + \beta_1 \cdot (n+2) \\ \vdots \\ \beta_0 + \beta_1 \cdot (m) \end{pmatrix} \in \mathbb{R}^m, \tag{4.23}$$

where β_0 and β_1 are the linear regression coefficients calculated by the ordinary least squares method.

4.4.6 Linear regression nonnegative

Function $\mathrm{f}^{\,\mathrm{c,lrnn}}$ will extend the vector by the linear regression model with nonnegative values. For vector $\vec{a} \in \mathbb{R}^n$ and the natural positive number $m \in \mathbb{N}^+$, where $m > n$, the function $\mathrm{f}^{\,\mathrm{c,lrnn}}$ is defined by the following formula:

$$\mathrm{f}^{\,\mathrm{c,lrnn}}(\vec{a}, m) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ \max\left(0, \beta_0 + \beta_1 \cdot (n+1)\right) \\ \max\left(0, \beta_0 + \beta_1 \cdot (n+2)\right) \\ \vdots \\ \max\left(0, \beta_0 + \beta_1 \cdot (m)\right) \end{pmatrix} \in \mathbb{R}^m, \tag{4.24}$$

where β_0 and β_1 are the linear regression coefficients calculated by the ordinary least squares method.

4.5 Productor function

The production function is a function that maps the vector to a vector in which elements are the cumulated products. We will use two productors: the first productor Π_1 and the second productor Π_2. The following formula defines the first productor Π_1:

$$\Pi_1(\vec{a}) = \Pi_1(\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}) = \begin{pmatrix} a_1 \\ a_1 \cdot a_2 \\ a_1 \cdot a_2 \cdot a_3 \\ \vdots \\ a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.25}$$

The second productor Π_2 is defined by the following formula:

$$\Pi_2(\vec{a}) = \Pi_2(\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}) = \begin{pmatrix} 1 \\ a_1 \\ a_1 \cdot a_2 \\ \vdots \\ a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_{n-1} \end{pmatrix} \in \mathbb{R}^n. \tag{4.26}$$

4.6 Broadcasting

Broadcasting is a technique to simplify the operations between two objects with incompatible dimensions [50]. The broadcasting is done by repeating the smaller object's elements to match the larger object's dimensions. Broadcasting is not mainly used in math but instead in programming. In this work, the broadcasting will be used to simplify some operations. In the following example, we will see how the broadcasting is done.

4.6.1 Brodcasting function

The broadcasting function is a function that takes an object as a first argument and a dimension as a second argument. The object could be a scalar, vector, or matrix. The function returns the object broadcasted to the selected dimension. For further explanation, let's define the broadcasting function broad that takes an object and a dimension $m \in \mathbb{N}^+$ (natural positive numbers, where zero is not presented) as arguments. The function returns the object broadcasted to the dimension m. The following formulas define the function for scalar and vector inputs.

Scalar

For the scalar $a \in \mathbb{R}$ and the dimension $m \in \mathbb{N}^+$. The following formula defines the scalar a broadcasted to the dimension m:

$$\text{broad}\,(a, m) = \begin{pmatrix} a \\ a \\ \vdots \\ a \end{pmatrix} \in \mathbb{R}^m. \tag{4.27}$$

Vector

For the vector $\vec{a} \in \mathbb{R}^n$ and the dimension $m \in \mathbb{N}^+$. The vector $\vec{a}$ broadcasted to the dimension m is defined by the following formula:

$$\text{broad}\,(\vec{a}, m) = \begin{pmatrix} \vec{a} & \vec{a} & \cdots & \vec{a} \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.28}$$

Row vector

For the vector $\vec{a} \in \mathbb{R}^n$ and the dimension $m \in \mathbb{N}^+$. The row vector $\vec{a}^T$ broadcasted to the dimension m is defined by the following formula:

$$\text{broad}\left(\vec{a}^T, m\right) = \begin{pmatrix} \vec{a}^T \\ \vec{a}^T \\ \vdots \\ \vec{a}^T \end{pmatrix} \in \mathbb{R}^{m \times n}. \tag{4.29}$$

4.6.2 Implicit broadcasting

Often, the broadcasting is done implicitly, meaning the broad function is not used and the broadcasting is still done. The operation does the implicit broadcasting between two objects with incompatible dimensions. The smaller object is broadcasted to match the larger object's dimensions. In the following example, we will see how the implicit broadcasting is done.

Scalar

Let us denote the scalar $a \in \mathbb{R}$ and the vector $\vec{b} \in \mathbb{R}^n$. When the operation requires the object with a smaller dimension to be broadcasted to the larger object's dimension, the scalar a is broadcasted to the higher dimension. The following formula shows the implicit broadcasting for the addition operation (in the same way it is done for the subtraction and piece-wise multiplication):

$$a + \vec{b} = \begin{pmatrix} a \\ \vdots \\ a \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a + b_1 \\ \vdots \\ a + b_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.30}$$

For the dot product operation, the formula is the following:

$$\langle a \mid \vec{b} \rangle = \langle \begin{pmatrix} a \\ \vdots \\ a \end{pmatrix} \mid \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \rangle = a \cdot b_1 + a \cdot b_2 + \cdots + a \cdot b_n \in \mathbb{R}. \tag{4.31}$$

Sometimes, if a is scalar, we can write the $\vec{a}$ to note that the scalar is broadcasted to the vector. For example:

$$\vec{a} + \vec{b} = \begin{pmatrix} a \\ \vdots \\ a \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a + b_1 \\ \vdots \\ a + b_n \end{pmatrix} \in \mathbb{R}^n. \tag{4.32}$$

For another example we denote the matrix $\mathbb{A}$ as:

$$\mathbb{A} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.33}$$

To create a row sum vector, we can use the following formula:

$$\vec{a}_r = \mathbb{A} \cdot 1 = \mathbb{A} \cdot \vec{1} = \begin{pmatrix} a_{11} + \cdots + a_{1m} \\ \vdots \\ a_{n1} + \cdots + a_{nm} \end{pmatrix} \in \mathbb{R}^n. \tag{4.34}$$

To create a column sum vector, we can use the following formula:

$$\vec{a}_c = \vec{1}^T \cdot \mathbb{A} = \begin{pmatrix} a_{11} + \cdots + a_{n1} \\ \vdots \\ a_{1m} + \cdots + a_{nm} \end{pmatrix} \in \mathbb{R}^m. \tag{4.35}$$

Note that in broadcasting, we can use vector notation with transposition to show that the scalar will be broadcasted to a row vector.

Vector

To show how vector could be implicitly broadcasted, let us denote the matrix $\mathbb{A}$ as:

$$\mathbb{A} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.36}$$

Vector $\vec{b}$ as:

$$\vec{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \in \mathbb{R}^{n}. \tag{4.37}$$

And vector $\vec{c}$ as:

$$\vec{c} = \begin{pmatrix} c_1 \\ \vdots \\ c_m \end{pmatrix} \in \mathbb{R}^{m}. \tag{4.38}$$

The boradcasting in sum operation between the matrix $\mathbb{A}$ and the vector $\vec{b}$ is done by the following formula:

$$\mathbb{A} + \vec{b} = \begin{pmatrix} a_{11} + b_1 & \cdots & a_{1m} + b_1 \\ \vdots & \ddots & \vdots \\ a_{n1} + b_n & \cdots & a_{nm} + b_n \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.39}$$

The broadcasting in sum operation between the matrix $\mathbb{A}$ and the vector $\vec{c}$ transposed is done by the following formula:

$$\mathbb{A} + \vec{c}^T = \begin{pmatrix} a_{11} + c_1 & \cdots & a_{1m} + c_m \\ \vdots & \ddots & \vdots \\ a_{n1} + c_1 & \cdots & a_{nm} + c_m \end{pmatrix} \in \mathbb{R}^{n \times m}. \tag{4.40}$$

4.6.3 Priority broadcasting

When writing the broadcasting with function broad, then there is no ambiguity. However, the situation is different when the broadcasting is done implicitly between two objects, which both could be broadcasted. As a general rule, the variable more on the right side of the operation is broadcasted primarily. However, some objects have priority broadcasting, which overrides the general rule. The objects with priority broadcasting are the cumulative matrix, and the values continue. However, if two objects with priority broadcasting are in the operation, the object on the right side of the operation has priority broadcasting. The following sections will describe the priority broadcasting for the cumulative matrix and the values continue.

Cumulative matrix

In every operation, the matrix $\mathbb{W}$ is broadcasted implicitly to match the dimensions of the other object. For example, if we have a vector $\vec{a} \in \mathbb{R}^n$, the cumulative matrix $\mathbb{W}$ and the vector $\vec{a}$ can be composed by the following formula:

$$\begin{aligned}\mathbb{W} \cdot \vec{a} &= \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} = \\ &= \begin{pmatrix} 1 \cdot a_1 + 0 \cdot a_2 + 0 \cdot a_3 + \cdots + 0 \cdot a_n \\ 1 \cdot a_1 + 1 \cdot a_2 + 0 \cdot a_3 + \cdots + 0 \cdot a_n \\ 1 \cdot a_1 + 1 \cdot a_2 + 1 \cdot a_3 + \cdots + 0 \cdot a_n \\ \vdots \\ 1 \cdot a_1 + 1 \cdot a_2 + 1 \cdot a_3 + \cdots + 1 \cdot a_n \end{pmatrix} \in \mathbb{R}^n.\end{aligned} \tag{4.41}$$

Values continue

For values continue, the broad function takes one more argument, and values continue function described in the section above. The definition of broad function is the following:

$$\text{broad}\left(\vec{a}, \mathrm{f}^{\,\mathrm{c}}, m\right) = \mathrm{f}^{\,\mathrm{c}}\left(\vec{a}, m\right). \tag{4.42}$$

Usually, the values continue are broadcasted implicitly to match the dimensions of the other object, and the function will be omitted as it will be known from the context. We will only write a vector to represent the values continue.

4.7 Dot product identity

For two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, the dot product identity is defined by the following formula:

$$\langle \vec{a} \mid \vec{b} \rangle = \langle \vec{a} \circ \vec{b} \mid 1 \rangle, \tag{4.43}$$

where:
$\vec{a} \circ \vec{b}$ is the Hadamard product of the vectors $\vec{a}$ and $\vec{b}$,
1 is the broadcasted 1 to the dimension n.

This identity could be easily proven by comparing the formulas. The left side of the equation is straightforward:

$$\langle \vec{a} \mid \vec{b} \rangle = a_1 \cdot b_1 + \cdots + a_n \cdot b_n. \tag{4.44}$$

The right side of the equation equals to:

$$\langle \vec{a} \circ \vec{b} \mid 1 \rangle = \langle \begin{pmatrix} a_1 \cdot b_1 \\ \vdots \\ a_n \cdot b_n \end{pmatrix} \mid \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \rangle = a_1 \cdot b_1 + \cdots + a_n \cdot b_n. \tag{4.45}$$

From the comparison of the formulas, we can see that the left side of the equation equals the right side.

Chapter 5

Economical background

This chapter will introduce the basic economic concepts used in the following chapters. Before we start, we need to state that the currency units will not be denoted as for some specific currency but as a general currency as CCY.

5.1 Inflation

Inflation is a general increase in prices in an economy. It also means an erosion of a given currency's real values (purchasing power) according to goods and services [51].

There are two types of values: nominal and real. The nominal value is the unadjusted value of a good or service, without taking into account inflation [52]. The real value is the nominal value adjusted for inflation. The real value is expressed in terms of purchasing power in a given year according to base year [53].

For example, if the nominal value in the third year is 1000 and the yearly inflation rate in every year is 10 % (yearly compounded), the real value for base year 0 is calculated as follows:

$$\text{Real value} = \frac{\text{Nominal value}}{(1 + \text{Inflation rate})^{\text{Number of years}}} = \frac{1000}{(1+0.1)^3} \approx 751.$$

The inflation has properties of values continue, and the known values are defined as:

$$\alpha = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{T-1} \end{pmatrix} \in \mathbb{R}^T, \qquad (-) \quad (5.1)$$

where:
α_t is the inflation rate for year t (–).

5.2 Escalation

Escalation is an increase in certain values over time. Here, the escalation is not intertwined with inflation but is used to escalate values, such as energy consumption, the efficiency of devices, etc. The escalation is denoted as $\gamma \in \mathbb{R}$ and is in pair with some value, such as energy consumption, efficiency, etc., defined for year 0. For example, if the value for year zero is $a_0 \in \mathbb{R}$ and the escalation is aforementioned γ, then we can calculate the value for next year as:

$$a_{t+1} = a_t \cdot \gamma, \tag{5.2}$$

where:
a_t is the value for year t,
a_{t+1} is the value for year $t + 1$.

5.3 Escalation coefficient

The Escalation Coefficient (EC) is similar to escalation but is intertwined with inflation. It is mostly used to escalate the prices of commodities. The escalation coefficient for some value $a_0 \in \mathbb{R}$ is denoted as $\delta \in \mathbb{R}$. The value for year $t + 1$ is calculated as:

$$a_{t+1} = a_t \cdot (1 + \delta \cdot \alpha_t), \tag{5.3}$$

where:
a_t is the value for year t,
a_{t+1} is the value for year $t + 1$,
α_t is the inflation rate for year t.

The escalation coefficient for a specific commodity assumes some correlation between inflation and the escalation of the specific commodity. If the escalation coefficient equals 1, the commodity's price rises with the inflation rate. If the escalation coefficient exceeds 1, the commodity's price rises faster than the inflation rate. If the escalation coefficient is less than 1, the commodity's price rises slower than the inflation rate. If the escalation coefficient equals 0, the commodity price does not rise. Moreover, if the escalation coefficient is negative, the commodity's price falls for a positive inflation rate.

5.4 Escalation function

As mentioned before, if we have the escalation γ paired with the value a_0 for year 0, we can calculate all other values for other years. For further usage, we can define the escalation function, which has three inputs: the value a_0 for year 0, the escalation γ, and the number of years T. The output is the vector of values for years 0 to $T - 1$:

$$\mathrm{f}^{\gamma}(a_0, \gamma, T) = a_0 \cdot \Pi_2(\mathrm{broad}(\gamma, T)) = \begin{pmatrix} a_0 \\ a_0 \cdot \gamma \\ a_0 \cdot \gamma^2 \\ \vdots \\ a_0 \cdot \gamma^{T-1} \end{pmatrix} \in \mathbb{R}^T, \tag{5.4}$$

where:
Π_2 is the productor function of the second type,
$\vec{\gamma}$ is the vector created form broadcasting γ to the length of T.

Usually, we will work with a vector of values $\vec{a_0}$ for year 0 with dimension of n, which has corresponding escalation vector $\vec{\gamma}$ also with dimension of n. Therefore, we can define the escalation function for vectors as follows:

$$\mathrm{f}^{\vec{\gamma}}(\vec{a_0}, \vec{\gamma}, T) = \begin{pmatrix} \mathrm{f}^{\gamma}(a_{0,0}, \gamma_0, T)^T \\ \mathrm{f}^{\gamma}(a_{0,1}, \gamma_1, T)^T \\ \vdots \\ \mathrm{f}^{\gamma}(a_{0,n}, \gamma_n, T)^T \end{pmatrix} = \\ = \begin{pmatrix} a_{0,0} & a_{0,0} \cdot \gamma_0 & \cdots & a_{0,0} \cdot \gamma_0^{T-1} \\ a_{0,1} & a_{0,1} \cdot \gamma_1 & \cdots & a_{0,1} \cdot \gamma_1^{T-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{0,n} & a_{0,n} \cdot \gamma_n & \cdots & a_{0,n} \cdot \gamma_n^{T-1} \end{pmatrix} \in \mathbb{R}^{n \times T} \tag{5.5}$$

Usually, the input T will be omitted because it will be clear from the context. Thus the function will be denoted as $\mathrm{f}^{\gamma}(a_0, \gamma)$ and $\mathrm{f}^{\vec{\gamma}}(\vec{a_0}, \vec{\gamma})$.

5.5 Escalation coefficient function

Similar to the escalation function, we can define the escalation coefficient function. The escalation coefficient function has four inputs: the value a_0 for year 0, the escalation coefficient δ, the inflation vector $\vec{\alpha}$, and the number of years T. The output is the vector of values for years 0 to $T-1$:

$$\mathrm{f}^{\delta}(a_0, \delta, \vec{\alpha}, T) = a_0 \cdot \Pi_2(1 + \mathrm{broad}\,(\delta, T) \circ \vec{\alpha}) = \\ = \begin{pmatrix} a_0 \\ a_0 \cdot (1 + \delta \cdot \alpha_0) \\ a_0 \cdot (1 + \delta \cdot \alpha_0) \cdot (1 + \delta \cdot \alpha_1) \\ \vdots \\ a_0 \cdot (1 + \delta \cdot \alpha_0) \cdot \ldots \cdot (1 + \delta \cdot \alpha_{T-1}) \end{pmatrix} \in \mathbb{R}^T, \tag{5.6}$$

Note: The 1 and vector $\vec{\alpha}$ are implicitly broadcasted to the length of T ($\vec{\alpha}$ is boradcasted accordind to properties of values continue).

Similar to the escalation function, we can define the escalation coefficient function for vectors as follows:

$$\mathrm{f}^{\vec{\delta}}(\vec{a_0}, \vec{\delta}, \vec{\alpha}, T) = \begin{pmatrix} \mathrm{f}^{\delta}(a_{0,0}, \delta_0, \vec{\alpha}, T)^T \\ \mathrm{f}^{\delta}(a_{0,1}, \delta_1, \vec{\alpha}, T)^T \\ \vdots \\ \mathrm{f}^{\delta}(a_{0,n}, \delta_n, \vec{\alpha}, T)^T \end{pmatrix} \tag{5.7}$$

Usually, the input of inflation vector $\vec{\alpha}$ and the number of years T will be omitted because it will be clear from the context. Thus the function will be denoted as $\mathrm{f}^{\delta}(a_0, \delta)$ and $\mathrm{f}^{\vec{\delta}}(\vec{a_0}, \vec{\delta})$.

5.6 Inflation coefficient

The economic model offers several strategies for including inflation and escalation, each with specific advantages and limitations. Below is an overview of three approaches to modeling inflation and escalation that we have evaluated in detail.

In the following section, we will focus on the concept of the inflation coefficient. We will explain in more detail why we decided to adopt this approach, how it is integrated into the model, and what advantages and challenges it brings.

First, we introduce the price $c_{i,t}$, which represents the price of the i-th product in the t-th year.

Method 1

- There is only one inflation value $\alpha \in \mathbb{R}$ in the model.
- Partial escalations are not considered.

It follows:

$$c_{i,t+1} = c_{i,t} \cdot (1 + \alpha). \tag{5.8}$$

Advantages:

- Ability to create a what-if analysis on α.

Disadvantages:

- All prices are adjusted for inflation, which is not always realistic.
- Does not take into account that inflation can change over time.

Method 2

- There is only one inflation value $\alpha \in \mathbb{R}$ in the model.
- Partial escalations are considered, $e_i \in \mathbb{R}$ is the partial escalation of the ith product.

It follows:

$$c_{i,t+1} = c_{i,t} \cdot (1 + \alpha \cdot e_i). \tag{5.9}$$

- If $e_i = 1$, then the price is exactly correlated with inflation.
- If $e_i < 1$ means the price grows more slowly than inflation.

- If $e_i = 0$, the price does not change.
- If $e_i < 0$, the price decreases with positive inflation.
- If $e_i = -1$, the price negatively correlates with inflation.

Advantages:

- Ability to create a sensitivity analysis on α.
- Consideration of partial escalation.

Disadvantages:

- Does not take into account that inflation can change over time.

Method 3

- In the model, the inflation value is $\alpha_t \in \mathbb{R}$ for year t.
- Partial escalations are considered.

It follows:

$$c_{i,t+1} = c_{i,t} \cdot (1 + \alpha_t \cdot e_i). \tag{5.10}$$

Advantages:

- Consideration of partial escalation.
- Consideration of changes in inflation over time.

Disadvantages:

- Unable to create a sensitivity analysis on α.

Method with inflation coefficient

- In the model, the inflation value is $\alpha_t \in \mathbb{R}$ for year t.
- Partial escalations are considered.
- Use the inflation coefficient $k_\alpha \in \mathbb{R}$.

It follows:

$$c_{i,t+1} = c_{i,t} \cdot (1 + \alpha_t \cdot e_i \cdot k_\alpha). \tag{5.11}$$

It is a fact that:

- If $k_\alpha = 1$, then inflation is not changed.
- If $k_\alpha \neq 1$, so inflation is changed over a time horizon.

Advantages:

- Consideration of partial escalation.

- Taking into account changes in inflation over time.
- Option to create a what-if analysis on k_α.

In this way, every price escalation, which is intertwined with inflation, is also intertwined with the inflation coefficient, which has the advantage that by changing only one value, we can explore the impact of a whole model.

5.7 Cash flow

The Cash Flow (CF) is a concept used in accounting and finance. It represents the amount of money that flows in or out. The cash flow can be either standard, where the income has a positive sign and the costs have a negative sign, or inverted, where the income has a negative sign and the costs have a positive sign. For calculation purposes, we denote the cash flow as:

$$\vec{CF} = \begin{pmatrix} CF_1 \\ CF_2 \\ \vdots \\ CF_T \end{pmatrix} \in \mathbb{R}^T, \qquad \text{(CCY)} \quad (5.12)$$

where:
CF is the cash flow vector (CCY),
CF_t is the cash flow for year t (CCY),
T is the number of years.

We can also calculate the Cumulative Cash Flow (CCF) vector as:

$$C\vec{CF} = \mathbb{W} \cdot \vec{CF}, \qquad \text{(CCY)} \quad (5.13)$$

where:
$\mathbb{W}$ is the cumulative sum matrix.

5.8 Discounting

Discounting is a method used to evaluate the value of future cash flows in today's terms. The discount rate is the rate at which future cash flows are discounted. The discount rate is usually expressed as a percentage. The discount rate is denoted as r [54].

5.9 Real and nominal discount rate

The discount rate can be either real or nominal. The nominal discount rate is the rate where inflation is included in the discount rate. The real discount rate is the rate where inflation is not included in the discount rate. The conversion equation is as follows:

$$1 + r_n = (1 + r_r) \cdot (1 + \alpha), \qquad (5.14)$$

where:
r_n is the nominal discount rate (–),
r_r is the real discount rate (–),
α is the inflation rate (–) [55].

5.10 Economic nabla

We can define the economic nabla to further simplify calculation values for Discounted Cash Flow (DCF) and Net Present Value (NPV). The economic nabla is a vector of values for years 1 to T and is defined as:

$$\vec{\nabla}_E(r,T) = 1 \oslash \Pi_1\left(\mathrm{broad}\left(1+r,T\right)\right) = \begin{pmatrix} \frac{1}{1+r} \\ \frac{1}{(1+r)^2} \\ \vdots \\ \frac{1}{(1+r)^T} \end{pmatrix} \in \mathbb{R}^T, \quad (-) \quad (5.15)$$

where:
r is the discount rate (–),
T is the number of years.

The parameters r and T will usually be omitted because they will be apparent from the context. Thus, the economic nabla will be denoted as $\vec{\nabla}_E$.

Here, the nabla has a real discount rate inside. However, for real calculations, we will use the nominal discount rate. Therefore, we can define the economic nabla with a nominal discount rate as:

$$\vec{\nabla}_{E\alpha}(r,\vec{\alpha},T) = \vec{\nabla}_E\left(r,T\right) \oslash \Pi_1\left(\mathrm{broad}\left(1+\vec{\alpha},T\right)\right) \in \mathbb{R}^T, \quad (-) \quad (5.16)$$

Similarly, the parameters r, $\vec{\alpha}$, and T will be omitted because they will be apparent from the context. Thus, the economic nabla with nominal discount rate will be denoted as $\vec{\nabla}_{E\alpha}$.

Now, to discount all parts of the cash flow vector, we use the Hadamard product:

$$D\vec{C}F = \vec{CF} \circ \vec{\nabla}_{E\alpha}, \quad (\mathrm{CCY}) \quad (5.17)$$

We can also easily calculate the cumulated discounted cash flow vector as:

$$CD\vec{C}F = \mathbb{W} \cdot D\vec{C}F, \quad (\mathrm{CCY}) \quad (5.18)$$

where:
$\mathbb{W}$ is the cumulative sum matrix.

5.11 Economic criteria

The economic criteria are methods used to evaluate the profitability of an investment. The most common economic criteria are the NPV, Internal Rate of Return (IRR), Payback Period (PP), and Discounted Payback Period (DPP). We will describe each of them in more detail in the following subsections.

5.11.1 Net present value

The NPV is a method used to evaluate the profitability of an investment. The common formula for NPV is:

$$NPV = -I + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}, \qquad \text{(CCY)} \quad (5.19)$$

where:
I is the initial investment (CCY),
CF_t is the cash flow for year t (CCY),
r is the discount rate (–),
T is the number of years.

We will use the NPV with a nominal discount. Therefore, the formula will be:

$$NPV = -I + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t \cdot (1+\alpha)^t}, \qquad \text{(CCY)} \quad (5.20)$$

where:
I is the initial investment (CCY),
CF_t is the nominal cash flow for year t (CCY),
r is the real discount rate (–),
α is the inflation rate (–),
T is the number of years.

The limitation in this equation is that the inflation rate α must be constant throughout the evaluation time. However, if we want to address that the inflation could change over time, we need to rewrite the formula as follows:

$$NPV = -I + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t \cdot \prod_{i=1}^{t}(1+\alpha_{i-1})}, \qquad \text{(CCY)} \quad (5.21)$$

where:
I is the initial investment (CCY),
CF_t is the nominal cash flow for year t (CCY),
r is the real discount rate (–),
α_{i-1} is the inflation rate for year $i-1$ (–),
T is the number of years.

Finally, we will make a final simplification by using the economic nabla with a nominal discount rate:

$$NPV = \langle \vec{CF} \mid \vec{\nabla}_{E\alpha} \rangle - I = \\ = \langle \vec{CF} \circ \vec{\nabla}_{E\alpha} \mid 1 \rangle - I = \langle D\vec{CF} \mid 1 \rangle - I, \quad \text{(CCY)} \quad (5.22)$$

where:
$\vec{CF}$ is the vector of nominal cash flows (CCY),
$D\vec{CF}$ is the vector of discounted nominal cash flows (CCY),
$\vec{\nabla}_{E\alpha}$ is the economic nabla with nominal discount rate (–),
I is the initial investment (CCY).

5.11.2 Internal rate of return

The IRR is a method used to evaluate the profitability of an investment. If we denote the NPV as a function of the discount rate r as $\text{NPV}(r)$, the IRR is the value of r for which $\text{NPV}(r) = 0$:

$$\text{NPV}(IRR) = 0. \quad (-) \quad (5.23)$$

Commonly, finding the IRR analytically is difficult. Therefore, we use numerical methods to find the IRR, such as the Newton method described in the previous chapter.

5.11.3 Payback period

The PP is the time it takes for the investment to return the initial investment. The common formula for the payback period is:

$$PP = \min\{t \in \mathbb{N} \mid CCF_t - I \geq 0\}, \quad \text{(year)} \quad (5.24)$$

5.11.4 Discounted payback period

The DPP is the time it takes for the discounted cash flow to return the initial investment. The common formula for the discounted payback period is:

$$DPP = \min\{t \in \mathbb{N} \mid CDCF_t - I \geq 0\}, \quad \text{(year)} \quad (5.25)$$

Chapter 6

Core of the application

The current chapter describes the core of the application. The core is the part of the application responsible for the mathematical calculations based on the user-defined appliances and settings.

6.1 Energy source type

Energy source type is a type of energy source that can be used to cover the household's energy demand. Each energy source type has its name and index. The index will be used to identify the energy source type later in the work. The application supports the following energy source types:

1. Electricity
2. Heat
3. Gas
4. Oil
5. Coal
6. Biomass

Note: The number of the energy source type is the index.

The number of the energy source types is denoted as n_{est}:

$$n_{est} = 6. \tag{6.1}$$

Electricity

Electricity is the most common energy source type used in households. Electricity is used for various purposes, such as lighting, heating, cooling, cooking, etc.

Heat

The heat energy source type means the heat is purchased directly from the supplier. This could be, for example, the heat from the district heating system.

Gas

The gas energy source type is mainly used for heating and cooking. The gas is purchased directly from the supplier.

Oil

The oil energy source could be used to heat or to power the generator.

Coal

The coal energy source type is often used for heating in the form of coal briquettes or coal pellets.

Biomass

The biomass energy source type is used for heating. The biomass could be wood, wood pellets, or other types of biomass.

6.2 Energy distibution

The energy distribution is used to model the energy consumption of the household. Generally, the distribution is characterized by 8760 values, from which we can estimate the energy consumption for each hour of the year.

For example, if we have list $k_1, k_2, \ldots, k_{8760}$, where k_i is the energy distribution coefficient for i-th hour of the year, and we have the energy consumption for the whole year E, then the energy consumption for i-th hour of the year is:

$$E_i = E \cdot \frac{k_i}{\sum_{j=1}^{8760} k_j}, \qquad \text{(kWh)} \qquad (6.2)$$

where:
E_i is the energy consumption for i-th hour of the year (kWh),
E is the energy consumption for the whole year (kWh),
k_i is the energy distribution coefficient for i-th hour of the year (–).

Sometimes, there is insufficient space or time to estimate all the 8760 values. Therefore, predefined energy distributions (TDD) could be used. These distributions are based on the statistical data from the Czech Electricity Market Operator (OTE). The OTE gathers statistical data about electricity national electricity consumption for each hour of the year. There are a total of eight distributions that could be used, which are named TDD1, TDD2, . . . , TDD8 [56].

Each distribution should be used for certain types of tariffs. For example, if someone has an electricity tariff of D25d, the user should use TDD5 as the values to create this diagram have been gathered from consumers mainly using this tariff. The table below shows the recommended distribution for each tariff.

Class	Consumption character	Tariffs
1	without thermal use of electricity	C01d, C02d, C03d
2	storage appliance, hybrid heating	C25d, C26d, C35d
3	direct heating system, heat pump	C45d, C55d, C56d
4	without thermal use of electricity	D01d, D02d, D61d
5	storage device	D25d, D26d
6	hybrid heating	D35d
7	direct heating system, heat pump	D45d, D55d, D56d
8	public lighting	C62d

Table 6.1: Recommended distribution for each tariff [18].

Sometimes, the user wants to define custom distribution but doesn't have data to estimate all 8760 values. Therefore, the user could define only 24 values for each hour denoted as $k_{h,1}, k_{h,2}, \ldots, k_{h,24}$, where $k_{h,i}$ is the hour energy distribution coefficient for i-th hour. The sum of these values should be equal to 1:

$$\sum_{i=1}^{24} k_{h,i} = 1. \tag{6.3}$$

Also, to estimate the month distribution through the year, we need month coefficients $k_{m,1}, k_{m,2}, \ldots, k_{m,12}$, where $k_{m,i}$ is the month energy distribution coefficient for i-th month. The sum of these values should also be equal to 1:

$$\sum_{i=1}^{12} k_{m,i} = 1. \tag{6.4}$$

Then the energy consumption in i-th hour in the day and j-th month in the year is:

$$E_{i,j} = E \cdot k_{h,i} \cdot k_{m,j}, \qquad \text{(kWh)} \tag{6.5}$$

where:
$E_{i,j}$ is the energy consumption for i-th hour in day and j-th month in year (kWh),
E is the energy consumption for the whole year (kWh),
$k_{h,i}$ is the hour energy distribution coefficient for i-th hour (–),
$k_{m,j}$ is the month energy distribution coefficient for j-th month (–).

On the energy distributions page in the app, there is a list of all energy distributions. There are three types of distributions. The first type is default distributions that are predefined and cannot be modified or deleted. The default distribution is, for example, the constant distribution, where the energy consumption is the same for each hour of the year. The second type is the TDD distribution. The final type is user-defined distributions that could be modified or deleted.

To create hour and month coefficients for TDD distribution, which is characterized by 8760 values $k_1, k_2, \ldots, k_{8760}$, we need to calculate these coefficients by averaging the values for each hour of the year:

$$k_{h,i} = \frac{\sum_{j=1}^{365} k_{i+24\cdot(j-1)}}{\sum_{j=1}^{8760} k_j}, \tag{6.6}$$

where:
$k_{h,i}$ is the hour energy distribution coefficient for i-th hour (–),
$k_{i+24\cdot(j-1)}$ is the i-th value of the distribution for j-th day of the year (–).

The estimation for month coefficients is done by averaging the values for each month of the year. The following formula does the estimation:

$$k_{m,i} = \frac{\sum_{j=1}^{30} k_{(i-1)\cdot 24\cdot 30+j}}{\sum_{j=1}^{8760} k_j}, \tag{6.7}$$

where:
$k_{m,i}$ is the month energy distribution coefficient for i-th month (–),
$k_{(i-1)\cdot 24\cdot 30+j}$ is the j-th value of the distribution for i-th month of the year (–).

The custom distribution is defined by the distribution's name, which must be unique for each distribution. If the user wants to name the distribution with a name that already exists, they will get the error message stating that the distribution with the same name already exists. The name uniques also implies default and TDD distributions, which means that the user cannot name the distributions as, for example, TDD3. The user then specifies the hour and month coefficients; therefore, they must fill 24 + 12 = 36 values.

6.3 Household

The essential instance of the application is the household. The user with the properties defines the household. The properties are:

- name
- type
- area (m^2)
- currency (CZK, EUR, USD)

The type of the household could be one of the following:

- apartment
- house
- business building

Along with the essential properties, the household consists of three main parts, which will be described in the following sections:

- appliances
- facilities
- settings

6.4 Appliances

The appliance is a device that consumes the energy source type to provide a service. The following properties are defined for each equipment:

- energy source type $e_{st} \in (1, 2, \ldots, n_{est})$ (denoted by the index of the energy source type)
- energy consumption e_c (kWh/year)
- maintenance costs c_m (CCY/year)
- count n (–)

6.4.1 Energy consumption

Three methods define the energy consumption:

- annual energy consumption
- input power
- cycles

Annual energy consumption

This method is used when the user knows the appliance's annual energy consumption. The energy consumption is defined as:

$$e_c = e_{ca}, \qquad \text{(kWh/year)} \qquad (6.8)$$

where:
e_{ca} is the annual energy consumption (kWh/year).

Input power

This method is used when the user knows the input power of the appliance and the maximum daily operation time. The energy consumption is defined as:

$$e_c = p_{int} \cdot t_{dmax} \cdot 365, \qquad \text{(kWh/year)} \quad (6.9)$$

where:
p_{int} is the total input power (kW),
t_{dmax} is the maximum daily operation time in hours per day.

Cycles

This method is used when the user knows the number of cycles per year and the energy consumption per cycle. The energy consumption is defined as:

$$e_c = e_{cc} \cdot n, \qquad \text{(kWh/year)} \quad (6.10)$$

where:
e_{cc} is the energy consumption per cycle (kWh),
n is the number of cycles (–).

6.4.2 Computed properties

The appliance also has several computed properties from the defined properties:

- total energy consumption e_{ct} (kWh/year)
- total maintenance costs c_{mt} (CCY/year)

The total energy consumption is calculated as:

$$e_{ct} = e_c \cdot n. \qquad \text{(kWh/year)} \quad (6.11)$$

The total maintenance costs are calculated as follows:

$$c_{mt} = c_m \cdot n. \qquad \text{(CCY/year)} \quad (6.12)$$

6.4.3 Appliances list

In the application, the appliances are defined by the user with the properties. Therefore, we will have a list of the appliances with the length $n_{ap} \in \mathbb{N}$. Furthermore, the property variable will be used with the index i to denote the i-th appliance.

6.4.4 Energy consumption vector

The energy consumption vector for appliances is defined as:

$$\vec{E}_{ca} = \begin{pmatrix} E_{ca,1} \\ E_{ca,2} \\ \vdots \\ E_{ca,n_{est}} \end{pmatrix}, \qquad \text{(kWh/year)} \quad (6.13)$$

where:
$E_{ca,i}$ is the energy consumption of the i-th energy source type (kWh/year).

The $E_{ca,i}$ is calculated as the sum of the total energy consumptions of the appliances that consume the i-th energy source type (the index of the energy source type is i). The formula for $E_{ca,i}$ is:

$$E_{ca,i} = \sum_{j=1}^{m} e_{ct,j} \cdot \delta_{e_{st,j},i}, \qquad \text{(kWh/year)} \quad (6.14)$$

where:
$e_{ct,j}$ is the total energy consumption of the j-th appliance (kWh/year),
$\delta_{e_{st,j},i}$ is the Kronecker delta (if the j-th appliance has the energy source type i, then $\delta_{e_{st,j},i} = 1$, otherwise $\delta_{e_{st,j},i} = 0$).

6.4.5 Maintenance vector

The maintenance vector for appliances is defined as:

$$\vec{C}_{ma} = \begin{pmatrix} C_{ma,1} \\ C_{ma,2} \\ \vdots \\ C_{ma,n_{est}} \end{pmatrix}, \qquad \text{(CCY/year)} \quad (6.15)$$

where:
$C_{ma,i}$ is the maintenance costs of the i-th energy source type (CCY/year).

The maintenance costs of the i-th energy source type are calculated as the sum of the total maintenance costs of the appliances that consume the i-th energy source type (the index of the energy source type is i). The formula for $C_{ma,i}$ is:

$$C_{ma,i} = \sum_{j=1}^{m} c_{mt,j} \cdot \delta_{e_{st,j},i}, \qquad \text{(CCY/year)} \quad (6.16)$$

where:
$c_{mt,j}$ is the total maintenance costs of the j-th appliance (CCY/year),
$\delta_{e_{st,j},i}$ is the Kronecker delta (if the j-th appliance has the energy source type i, then $\delta_{e_{st,j},i} = 1$, otherwise $\delta_{e_{st,j},i} = 0$).

6.5 Facilities

The facilities are additional devices that could be used in the household. The facility is either null, meaning that the household has no facilities, or the user with the properties defines it.

6.5.1 Photovoltaic system

A photovoltaic system is either null, which means that the household does not have a photovoltaic system, or the user with the properties defines it. The photovoltaic system consists of a list of photovoltaic modules with the length $n_{pvm} \in \mathbb{N}$. The properties of each photovoltaic module are:

- area a_{pvm} (m^2)
- azimuth β_{1pvm} ($^\circ$)
- slope β_{2pvm} ($^\circ$)
- efficiency η_{pvm} ($-$)
- efficiency escalation $\gamma_{\eta pvm}$ ($-$)

For the following calculations, the properties of the photovoltaic module will be used with index i to denote the i-th photovoltaic module. The properties of the photovoltaic system are:

- efficiency η_{pv} ($-$)
- efficiency escalation $\gamma_{\eta pv}$ ($-$)
- maintenance costs c_{mpv} (CCY/year)
- maintenance costs escalation coefficient δ_{cmpv} ($-$)

6.5.2 Battery

The battery is either null or the user with the properties defines it. The properties are:

- capacity c_b (kWh)
- power p_b (kW)
- self discharge sd_b ($-$/year)
- depth of discharge dod_b ($-$)
- efficiency η_b ($-$)
- efficiency escalation $\gamma_{\eta b}$ ($-$)
- maintenance costs c_{mb} (CCY/year)
- maintenance costs escalation coefficient δ_{cmb} ($-$)

6.6 Settings

The settings are the user-defined settings that are used for the calculations. Each settings part will be described in the following sections.

6.6.1 Energy distributions

For each energy source type, the user can assign the distribution. If the distribution is not assigned, then the constant distribution is used. Thus, we can define the energy distributions as a matrix as:

$$\mathbb{D}_e = \left(\vec{D}_{e,1}, \quad \vec{D}_{e,2}, \quad \ldots, \quad \vec{D}_{e,n_{est}}\right) = \\ = \begin{pmatrix} d_{e,1,1} & d_{e,2,1} & \ldots & d_{e,n_{est},1} \\ d_{1,2} & d_{e,2,2} & \ldots & d_{e,n_{est},2} \\ \vdots & \vdots & \ddots & \vdots \\ d_{e,1,8760} & d_{e,2,8760} & \ldots & d_{e,n_{est},8760} \end{pmatrix}, \qquad (-) \quad (6.17)$$

where:
$\vec{D}_{e,i}$ is the distribution of the i-th energy source type (–),
$d_{e,i,j}$ is the distribution value of the i-th energy source type in the j-th hour of the year (–).

The distribution is normalized, which means that the sum of the distribution vector is equal to 1:

$$\sum_{j=1}^{8760} d_{e,i,j} = 1, \ \forall i = 1, 2, \ldots, n_{est}. \qquad (6.18)$$

6.6.2 Energy consumption escalations

Each year, household consumption can increase. The relative increase is called the consumption escalation. The consumption escalation is the following vector:

$$\vec{\gamma}_{ec} = \begin{pmatrix} \gamma_{ec,1} \\ \gamma_{ec,2} \\ \vdots \\ \gamma_{ec,n_{est}} \end{pmatrix}, \qquad (-) \quad (6.19)$$

where
$\gamma_{ec,i}$ is the consumption escalation of the i-th energy source type.

6.6.3 Inflation

Inflation is a general price increase and a fall in money's purchasing value. The inflation is a vector with values continue properties, defined as:

$$\vec{\alpha} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{T-1} \end{pmatrix}, \qquad (-) \quad (6.20)$$

where:
α_i is the inflation in the i-th year.

6.6.4 Energy pricings

Energy prices are defined by fixed, variable, and selling prices. The fixed price is a price that is paid for the energy source type regardless of the amount of energy consumed. The variable price is a price that is paid for the energy source type for each unit of the energy consumed. The selling price is the price paid for the energy source type for each unit sold.

Fixed prices

The vector of the fixed prices is defined as:

$$\vec{P_f} = \begin{pmatrix} P_{f,1} \\ P_{f,2} \\ \vdots \\ P_{f,n_{est}} \end{pmatrix}, \qquad (\mathrm{CCY/year}) \quad (6.21)$$

where:
$P_{f,i}$ is the fixed price of the i-th energy source type (CCY/year).

Note: The fixed prices have a unit of the CCY / month in the app. Therefore, the fixed prices are multiplied by 12 to get the yearly fixed prices.

The escalation coefficient vector for the fixed prices is defined as:

$$\vec{\delta}_{pf} = \begin{pmatrix} \delta_{pf,1} \\ \delta_{pf,2} \\ \vdots \\ \delta_{pf,n_{est}} \end{pmatrix}, \qquad (-) \quad (6.22)$$

where:
$\delta_{pf,i}$ is the escalation coefficient of the i-th energy source type for the fixed prices (–).

Variable prices

The vector of the variable prices is defined as:

$$\vec{P}_v = \begin{pmatrix} P_{v,1} \\ P_{v,2} \\ \vdots \\ P_{v,n_{est}} \end{pmatrix}, \qquad (\text{CCY/kWh}) \quad (6.23)$$

where:
$P_{v,i}$ is the variable price of the i-th energy source type (CCY/kWh).

The escalation coefficient vector for the variable prices is defined as:

$$\vec{\delta}_{pv} = \begin{pmatrix} \delta_{pv,1} \\ \delta_{pv,2} \\ \vdots \\ \delta_{pv,n_{est}} \end{pmatrix}, \qquad (-) \quad (6.24)$$

where:
$\delta_{pv,i}$ is the escalation coefficient of the i-th energy source type for the variable prices (–).

Selling prices

The vector of the selling prices is defined as:

$$\vec{P}_s = \begin{pmatrix} P_{s,1} \\ P_{s,2} \\ \vdots \\ P_{s,n_{est}} \end{pmatrix}, \qquad (\text{CCY/kWh}) \quad (6.25)$$

where:
$P_{s,i}$ is the selling price of the i-th energy source type (CCY/kWh).

The escalation coefficient vector for the selling prices is defined as:

$$\vec{\delta}_{ps} = \begin{pmatrix} \delta_{ps,1} \\ \delta_{ps,2} \\ \vdots \\ \delta_{ps,n_{est}} \end{pmatrix}, \qquad (-) \quad (6.26)$$

where:
$\delta_{ps,i}$ is the escalation coefficient of the i-th energy source type for the selling prices (–).

6.6.5 Maintenance costs escalation coefficient

The maintenance costs escalation coefficient vector is defined as:

$$\vec{\delta}_{cm} = \begin{pmatrix} \delta_{cm,1} \\ \delta_{cm,2} \\ \vdots \\ \delta_{cm,n_{est}} \end{pmatrix}, \qquad (-) \quad (6.27)$$

where:
$\delta_{cm,i}$ is the escalation coefficient of the i-th energy source type for the maintenance costs $(-)$.

6.6.6 Financial settings

The user defines the financial settings. The properties are:

- discount rate r $(-)$
- tax rate τ $(-)$

6.6.7 Location

The user defines the location. The properties are:

- latitude θ $(^\circ)$
- longitude ϕ $(^\circ)$

6.7 Basic analysis

Basic analysis is an analysis of household energy consumption and costs. The analysis is based on the appliances, which the user created, and the settings. Some impacts could also have additional facilities like photovoltaic systems, which could lower electricity consumption and bring savings in energy costs. Electricity can also be sold with a photovoltaic system, creating additional revenues.

For basic analysis, we need to calculate the costs and revenues for each year. The costs and revenues are calculated for T years (from year 1 to year T). The final output of the basic analysis will be the total discounted cash flow of the household.

6.7.1 Energy prices

The energy prices are a set of matrices describing the prices across the years for each energy source type.

Fixed prices

The matrix of the fixed prices is:

$$\mathbb{P}_f = \mathrm{f}^{\vec{\delta}}(\vec{P}_f, \vec{\delta}_{pf}). \qquad \text{(CCY/year)} \quad (6.28)$$

Variable prices

The matrix of the variable prices is:

$$\mathbb{P}_v = \mathrm{f}^{\vec{\delta}}(\vec{P}_v, \vec{\delta}_{pv}). \qquad \text{(CCY/kWh)} \quad (6.29)$$

Selling prices

The matrix of the selling prices is:

$$\mathbb{P}_s = \mathrm{f}^{\vec{\delta}}(\vec{P}_s, \vec{\delta}_{ps}). \qquad \text{(CCY/kWh)} \quad (6.30)$$

6.7.2 Appliances

Consumption and maintenance of the appliances are defined by the two matrices described in the following sections.

Energy consumption

The matrix of the energy consumption is:

$$\mathbb{E}_{ca} = \mathrm{f}^{\vec{\gamma}}(\vec{E}_{ca}, \vec{\gamma}_{ec}). \qquad \text{(kWh)} \quad (6.31)$$

Maintenance costs

The matrix of the maintenance costs is:

$$\mathbb{C}_{ma} = \mathrm{f}^{\vec{\delta}}(\vec{C}_{ma}, \vec{\delta}_{cm}). \qquad \text{(CCY)} \quad (6.32)$$

6.7.3 Photovoltaic system

At the beginning of the evaluation of the photovoltaic system, we need to calculate the production. As aforementioned, we have the list of the photovoltaic modules with length n_{pvm}. For each module, we need to obtain an irradiance vector. Therefore, we have to obtain the total of n_{pvm} irradiance vectors, where each irradiance vector is defined as:

$$I\vec{R}_i = \begin{pmatrix} IR_{i,1} \\ IR_{i,2} \\ \vdots \\ IR_{i,8760} \end{pmatrix}, \qquad \text{(kW/m}^2\text{)} \quad (6.33)$$

where:
$IR_{i,j}$ is the irradiance of the i-th module in the j-th hour of the year (kW/m^2).

The irradiance vector is a function of θ (latitude), ϕ (longitude), β_1 (azimuth), β_2 (slope). The way the irradiance vector is obtained is described in the next chapter.

The efficiency vector for i-th module is defined as:

$$\vec{\eta}_{pvm,i} = \mathrm{f}^{\,\gamma}(\eta_{pvm,i}, \gamma_{\eta pvm,i}), \qquad (-) \quad (6.34)$$

where:
$\eta_{pvm,i}$ is the efficiency of the i-th module (–),
$\gamma_{\eta pvm,i}$ is the efficiency escalation of the i-th module (–).

The production matrix of i-th module is calculated as:

$$\mathbb{P}_{pvm,i} = a_{pvm,i} \cdot \vec{IR}_i \cdot \vec{\eta}^{\,T}_{pvm,i}, \qquad \text{(kWh)} \quad (6.35)$$

where:
$a_{pvm,i}$ is the area of the i-th module (m^2).

The efficiency vector for the whole photovoltaic system is defined as:

$$\vec{\eta}_{pv} = \mathrm{f}^{\,\gamma}(\eta_{pv}, \gamma_{\eta pv}). \qquad (-) \quad (6.36)$$

The total production matrix is calculated as follows:

$$\mathbb{P}_{pv} = \left(\sum_{i=1}^{n_{pvm}} \mathbb{P}_{pvm,i} \right) \circ \vec{\eta}^{\,T}_{pv}. \qquad \text{(kWh)} \quad (6.37)$$

The transposed efficiency vector is broadcasted as follows:

$$\vec{\eta}^{\,T}_{pv} = \begin{pmatrix} \eta_{pv,1} & \eta_{pv,2} & \cdots & \eta_{pv,T} \\ \eta_{pv,1} & \eta_{pv,2} & \cdots & \eta_{pv,T} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{pv,1} & \eta_{pv,2} & \cdots & \eta_{pv,T} \end{pmatrix}, \qquad (-) \quad (6.38)$$

Note: The number of rows is the number of hours in the year, which is 8760. The number of columns is the number of years, which is T.

To compare how much electricity is produced and used to cover the consumption, we need to calculate the difference between the production and consumption. Firstly, we need to calculate the matrix of electricity consumption as follows:

$$\mathbb{E}_{c1} = \vec{D}_1 \cdot \vec{E}^{\,T}_{ca1}, \qquad (6.39)$$

where:
$\vec{D}_1$ is the distribution vector of the first energy source type (electricity) (–),

$\vec{E}_{ca1}$ is the first row of the $\mathbb{E}_{ca}$ matrix, which represents the energy consumption of the electricity (kWh).

The electricity from the photovoltaic system, which is used directly, is calculated as follows:

$$\mathbb{E}_{pupv1} = \min(\mathbb{P}_{pv}, \mathbb{E}_{c1}), \tag{6.40}$$

where:
min function is element-wise minimum.

The electricity consumption after the PV system is calculated as:

$$\mathbb{E}_{c1apv} = \mathbb{E}_{c1} - \mathbb{E}_{pupv1}. \tag{6.41}$$

The electricity produced, which is not used directly, is calculated as:

$$\mathbb{E}_{p1apv} = \mathbb{P}_{pv} - \mathbb{E}_{pupv1}. \tag{6.42}$$

Finally, we can calculate the maintenance costs of the photovoltaic system as:

$$\vec{C}_{mpv} = \mathrm{f}^{\,\delta}(c_{mpv}, \delta_{cmpv}). \qquad \text{(CCY)} \quad (6.43)$$

6.7.4 Battery

Battery takes matrices $\mathbb{E}_{c1apv}$ and $\mathbb{E}_{p1apv}$ as inputs. From the production matrix $\mathbb{E}_{p1apv}$, the battery takes energy to charge itself. The output of the battery is a matrix with the same dimension as the input matrices $\mathbb{E}_{ccb1}$, which represents the electricity consumption covered by the battery and $\mathbb{E}_{pub1}$, which represents the electricity produced (not used before entering battery), which was used by the battery. From the battery output, we can calculate the final electricity produced, which was not used year as:

$$\mathbb{E}_{p1ab} = \mathbb{E}_{p1apv} - \mathbb{E}_{pub1}. \tag{6.44}$$

The final electricity consumption after the battery is calculated as:

$$\mathbb{E}_{c1ab} = \mathbb{E}_{c1apv} - \mathbb{E}_{ccb1}. \tag{6.45}$$

Finally, the maintenance costs of the battery are calculated as:

$$\vec{C}_{mb} = \mathrm{f}^{\,\delta}(c_{mb}, \delta_{cmb}). \qquad \text{(CCY)} \quad (6.46)$$

6.7.5 Energy sell

The electricity produced, which was unused and is represented by $\mathbb{E}_{p1ab}$ matrix, is sold to the grid with the selling price. The revenues from selling electricity are calculated as follows:

$$\vec{R}_{s,1} = \mathrm{broad}\,(1, 8760)^{T} \cdot \mathbb{E}_{p1ab}, \qquad \text{(CCY)} \quad (6.47)$$

where:
$\vec{P}_{s1}$ is the first row of the $\mathbb{P}_s$ matrix, which represents the selling price of the electricity.

6.7.6 Summary

Energy consumption is calculated as:

$$\mathbb{E}_c = \mathbb{E}_{ca} - \begin{pmatrix} \vec{1}^T \cdot \mathbb{E}_{c1ab} \\ \mathbb{O} \end{pmatrix}, \qquad \text{(kWh)} \quad (6.48)$$

where:
$\mathbb{O}$ represents that only the first row of the matrix is not zero, the other rows are zero.

Total variable energy costs are calculated as follows:

$$\mathbb{C}_v = \mathbb{E}_c \circ \mathbb{P}_v, \qquad \text{(CCY)} \quad (6.49)$$

Total energy costs are calculated as:

$$\mathbb{C}_e = \mathbb{C}_v + \mathbb{P}_f, \qquad \text{(CCY)} \quad (6.50)$$

Total maintenance costs are calculated as follows:

$$\vec{C}_m = \langle \mathbb{C}_{ma}^T \mid 1 \rangle + \vec{C}_{mpv} + \vec{C}_{mb}. \qquad \text{(CCY)} \quad (6.51)$$

The vector of total household costs is calculated as follows:

$$\vec{C}_h = \langle \mathbb{C}_e^T \mid 1 \rangle + \vec{C}_m. \qquad \text{(CCY)} \quad (6.52)$$

The vector of total household revenues is calculated as follows:

$$\vec{R}_h = \vec{R}_{s,1}. \qquad \text{(CCY)} \quad (6.53)$$

6.7.7 Cash flow

The household cash flow vector is calculated as follows:

$$C\vec{F}_h = \vec{C}_h - \vec{R}_h, \qquad \text{(CCY)} \quad (6.54)$$

Note: The cash flow is the costs cash flow, which means that the positive values are costs and the negative values are revenues.

The household discounted cash flow is calculated as follows:

$$D\vec{C}F_h = C\vec{F}_h \circ \vec{\nabla}_{E\alpha} \qquad \text{(CCY)} \quad (6.55)$$

The total discounted cash flow is calculated as follows:

$$DCF_h = \langle D\vec{C}F_h \mid 1 \rangle \qquad \text{(CCY)} \quad (6.56)$$

The cumulated cash flow is calculated as follows:

$$C\vec{C}F_h = \mathbb{W} \cdot C\vec{F}_h \qquad \text{(CCY)} \quad (6.57)$$

Cumulated discounted cash flow is calculated as:

$$CD\vec{C}F_h = \mathbb{W} \cdot D\vec{C}F_h \qquad \text{(CCY)} \quad (6.58)$$

6.7.8 What if analysis

What if analysis is used to determine how the household cash flow would change according to certain variables. The what-if analysis plots the function of DCF (x), where x is the input variable. The types of input variables are:

- Discount rate r
- Tax rate τ
- Inflation coefficient k_α

For each input variable, the what-if analysis is calculated. The what-if analysis is calculated for the range of the input variable values.

6.8 Compare analysis

Compare analysis compares two households, where one is the base household and the other is mutated according to the user's input. The base is denoted with the number 0 and mutated with the number 1.

6.8.1 Mutations

The user defines the mutations. The app supports the following mutations:

- Add/Edit energy pricing for any energy source type.
- Remove appliances.
- Add new appliances.
- Add/Edit photovoltaic system.
- Add/Edit battery.

6.8.2 Analysis base

The analysis base defines basic information about the analysis. The basic properties are:

- Years (evaluation time T)
- Investment (CCY)
- Externalities

6.8.3 Externalities

The externalities represent the cash flow, which the household's cash flow cannot represent. For example, if we buy an appliance that saves us time, we need to define the externalities, which are:

- External revenues R_e (CCY/year)
- External revenues escalation coefficient δ_{re} (–)
- External costs C_e (CCY/year)
- External costs escalation coefficient δ_{ce} (–)

6.8.4 Cash flow

From the knowledge of basic analysis, the cash flow of the compared analysis is simple. First, we need to calculate the difference in cash flow between the base and the mutated household. The difference cash flow vector is calculated as:

$$\vec{CF}_{\Delta} = \vec{CF}_{h0} - \vec{CF}_{h1}, \qquad \text{(CCY)} \quad (6.59)$$

where:
$\vec{CF}_{h0}$ is the cash flow vector of the base household,
$\vec{CF}_{h1}$ is the cash flow vector of the mutated household.

Note: The calculated cash flow represents the savings and revenues of the mutated household.

The vector of the external revenues is calculated as follows:

$$\vec{R}_e = \mathrm{f}^{\vec{\delta}}(R_e, \delta_{re}). \qquad \text{(CCY)} \quad (6.60)$$

The vector of the external costs is calculated as follows:

$$\vec{C}_e = \mathrm{f}^{\vec{\delta}}(C_e, \delta_{ce}). \qquad \text{(CCY)} \quad (6.61)$$

Finally, the vector of the total cash flow is calculated as:

$$\vec{CF} = \vec{CF}_{\Delta} + \vec{R}_e - \vec{C}_e. \qquad \text{(CCY)} \quad (6.62)$$

The discounted cash flow is calculated as follows:

$$D\vec{C}F = \vec{CF} \circ \vec{\nabla}_{E\alpha}. \qquad \text{(CCY)} \quad (6.63)$$

The cumulative cash flow is calculated as follows:

$$C\vec{C}F = \mathbb{W} \cdot \vec{CF} - I. \qquad \text{(CCY)} \quad (6.64)$$

Note: The investment is implicitly broadcasted.

The cumulative discounted cash flow is calculated as follows:

$$CD\vec{C}F = \mathbb{W} \cdot D\vec{C}F - I. \qquad \text{(CCY)} \quad (6.65)$$

Note: The investment is implicitly broadcasted.

6.8.5 Net present value

NPV of the compare analysis is calculated as follows:

$$NPV = \langle \vec{CF} \mid \vec{\nabla}_{E\alpha} \rangle - I = \langle D\vec{CF} \mid 1 \rangle - I. \quad \text{(CCY)} \quad (6.66)$$

The IRR, PP, and DPP are calculated as described in the chapter on economic background.

6.8.6 What if analysis

What if the analysis determines how the NPV will change if certain input variables change. The what-if analysis plots the function of $NPV(x)$, where x is the input variable. The types of input variables are:

- Discount rate r
- Tax rate τ
- Inflation coefficient k_α
- Change in investment ΔI
- Change in external revenues ΔR_e
- Change in external costs ΔC_e

For each input variable, the what-if analysis is calculated. The what-if analysis is calculated for the range of the input variable values.

Chapter 7

Irradiance data

This chapter will describe how the irradiance data are obtained. As a brief introduction, the irradiance data is a vector defined as:

$$\vec{IR} = \begin{pmatrix} IR_1 \\ IR_2 \\ \vdots \\ IR_{8760} \end{pmatrix}, \qquad (\text{kW/m}^2) \quad (7.1)$$

where:
IR_i is the irradiance value at the i-th hour of the year (kW/m^2) for a common year with 8760 hours.

The values describe how much solar energy is received at a specific location, as well as photovoltaic panel azimuth and slope (inclination). According to this information, we can denote the vector $\vec{IR}$ as a function of the latitude, longitude, azimuth, and slope:

$$\vec{IR} = f\left(\theta, \phi, \beta_1, \beta_2\right), \qquad (\text{kW/m}^2) \quad (7.2)$$

where:
θ is the latitude (°), $\theta \in [-90, 90]$,
ϕ is the longitude (°), $\phi \in [-180, 180]$,
β_1 is the azimuth (°), $\beta_1 \in [-90, 90]$,
β_2 is the slope (°), $\beta_2 \in [0, 90]$.

There are two methods of obtaining the irradiance data. The first one uses the PVGIS API, which provides the irradiance data for a specific latitude, longitude, azimuth, and slope. The second one is an option for users without an internet connection, where the data are sampled from the PVGIS website on specific latitudes, longitudes, azimuths, and slopes. The data are then interpolated for the desired location, azimuth, and slope to get the irradiance data. Both methods use data from the Photovoltaic Geographical Information System (PVGIS), a solar radiation database [57].

7.1 PVGIS API

The PVGIS API is a straightforward way to obtain the irradiance data. The API provides an endpoint where parameters such as latitude, longitude, azimuth, and slope are passed into the URL as get parameters. The API returns the irradiance data in CSV format [58]. This method is suitable for users with an internet connection, as the data are obtained in real time when some analysis in the application is performed.

7.2 PVGIS Lite

The PVGIS Lite is just a conceived name for the second method of obtaining the irradiance data. The process starts with obtaining data on the specific locations with specific values of azimuth and slope. The obtained data are then interpolated to get the final irradiance values. This method is suitable for users without an internet connection, as the data are obtained in advance and stored in the application.

7.2.1 Points creation

The points on which the data are obtained could be created in many ways. The most common way is to create a grid of points, where the points are evenly distributed on the map. This method is simple, but the points are not evenly distributed on the Earth's surface as the Earth is not flat but a sphere.

The method used in this application is the Fibonacci sphere [59]. The examples of the points created by the Fibonacci sphere are shown in the following figure:

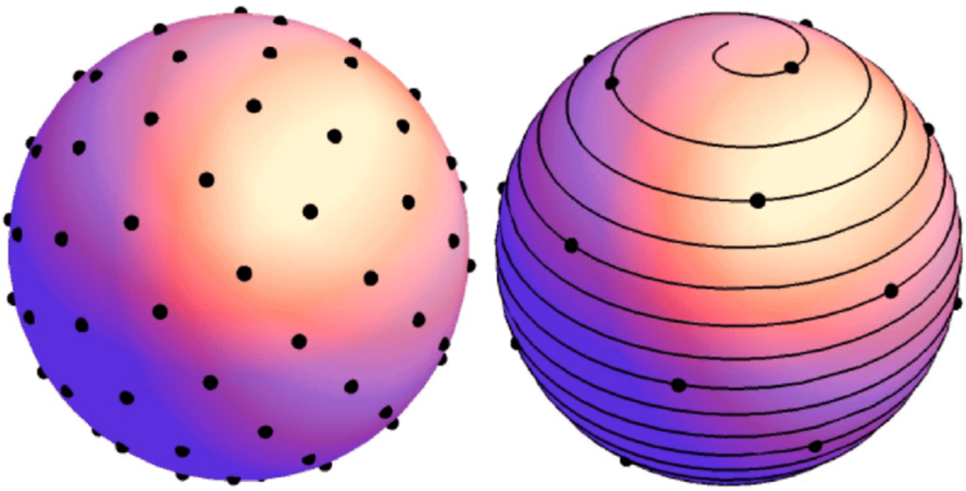

Figure 7.1: Fibonacci lattice [7].

For this application, 70 points were created. The points displayed on the map are shown in the following figure. The map shows the density of the points is higher on the equator and lower on the poles. The map is just a 2D projection of the 3D sphere. On the sphere, the points are more evenly distributed.

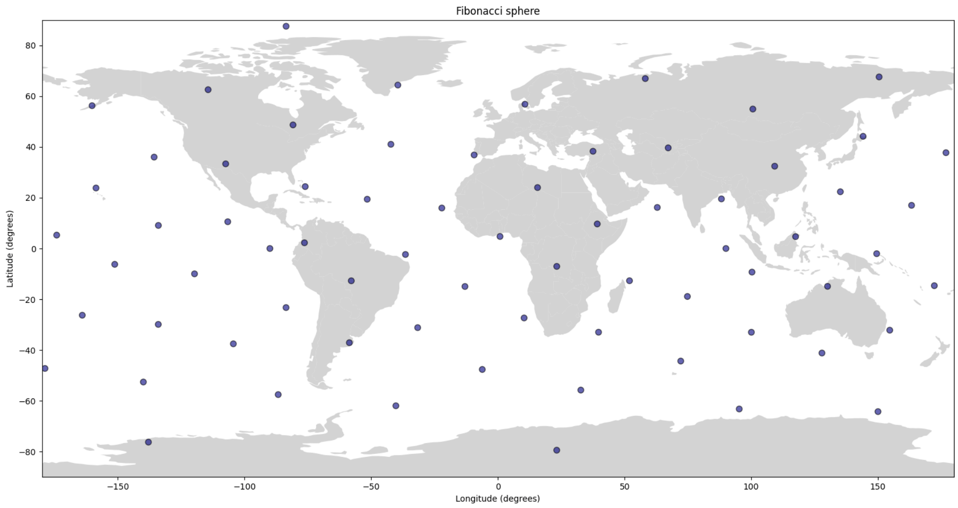

Figure 7.2: Fibonacci sphere points.

The PVGIS database provides data only for the land points. The points on the water are not helpful. Therefore, there is a need to filter out the points in the water. Fortunately, there is a python package called `global-land-mask` [60], by which we can quickly determine whether the point is on the land or the water. The filtered points on the land are shown in the following figure.

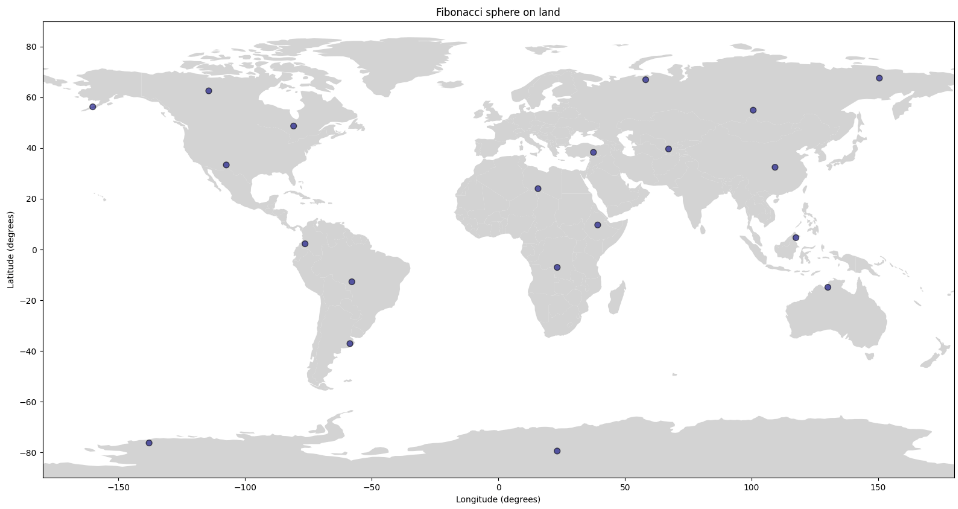

Figure 7.3: Fibonacci sphere on land points.

7.2.2 Obtain data

The data are obtained from the PVGIS API. A total of 8760 values are obtained for each point on the map, and each point on the map, the data are obtained for azimuths of -180, -90, 0, 90, 180, and slopes of 0, 30, 60, and 90. The data are then stored in the CSV file. Not every predefined point on the map has corresponding values in the PVGIS database. Therefore, data may not be obtained at some point and will be skipped. The points with the data are shown in the following figure.

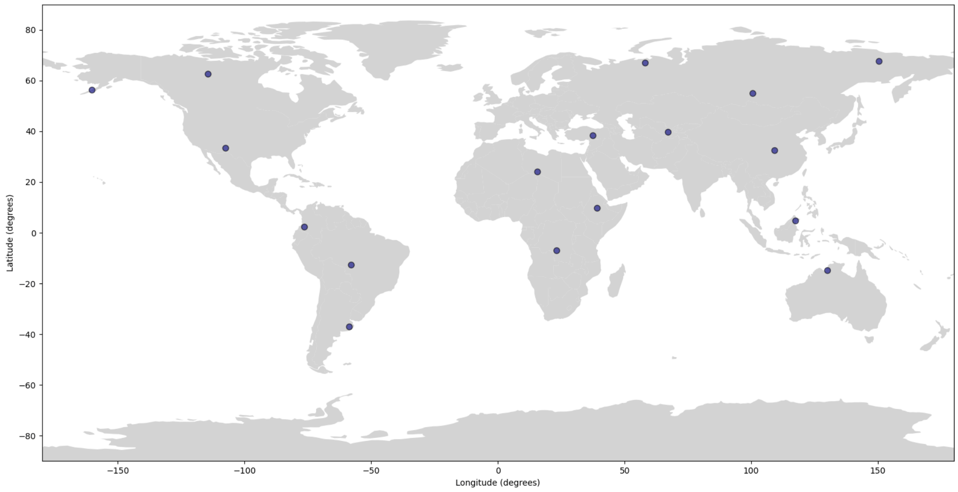

Figure 7.4: Map data points obtained from PVGIS.

7.2.3 Interpolation

The final part is the interpolation of the values. Let's consider that we need to get the irradiance vector for the point (θ, ϕ), with azimuth β_1 and slope β_2.

The database consists of a list of points:

$$\{(\theta_1, \phi_1), (\theta_2, \phi_2), \ldots (\theta_n, \phi_n)\}, \tag{7.3}$$

where:
θ_i is the latitude of the i-th point (°)
ϕ_i is the longitude of the i-th point (°)
n is the number of locations in the database.

Note: For yeach point (θ_i, ϕ_i), there are values for the azimuths and slopes.

Firstly, we must find the three closest points to the point (θ, ϕ). We will transform each point from spherical coordinates to Cartesian coordinates using the following formulas:

$$\begin{aligned} x &= \cos(\theta \cdot \frac{\pi}{180}) \cdot \cos(\phi \cdot \frac{\pi}{180}), \\ y &= \cos(\theta \cdot \frac{\pi}{180}) \cdot \sin(\phi \cdot \frac{\pi}{180}), \\ z &= \sin(\theta \cdot \frac{\pi}{180}). \end{aligned} \tag{7.4}$$

Note: The radius is considered as 1.

In the next step, we will take the first three points with the largest dot product with the point (x, y, z). The dot product is defined as:

$$\langle \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \rangle = x \cdot x_i + y \cdot y_i + z \cdot z_i. \tag{7.5}$$

The three closest points will be then denoted as:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}.$$

With the corresponding spherical coordinates:

$$\begin{pmatrix} \theta_1 \\ \phi_1 \end{pmatrix}, \begin{pmatrix} \theta_2 \\ \phi_2 \end{pmatrix}, \begin{pmatrix} \theta_3 \\ \phi_3 \end{pmatrix}.$$

As mentioned, each point has the corresponding irradiance values for the azimuths and slopes. The next step is to deal with the azimuths and slopes. For each location, we need the closest values for the azimuth, which is less or equal to the desired azimuth (down and up azimuth), such as these values are not equal (if they are equal, the division by zero will result). Then, we also need a down and up value for the slope with the same rules for down and up azimuth. The values will then be interpolated using linear interpolation. With this approach, we have a total of 12 irradiance vectors:

$$\begin{aligned} &\vec{IR}_{1,ad,sd}, \vec{IR}_{1,ad,su}, \vec{IR}_{1,au,sd}, \vec{IR}_{1,au,su}, \\ &\vec{IR}_{2,ad,sd}, \vec{IR}_{2,ad,su}, \vec{IR}_{2,au,sd}, \vec{IR}_{2,au,su}, \qquad (\mathrm{kW/m^2}) \\ &\vec{IR}_{3,ad,sd}, \vec{IR}_{3,ad,su}, \vec{IR}_{3,au,sd}, \vec{IR}_{3,au,su}, \end{aligned} \tag{7.6}$$

where:
$\vec{IR}_{i,ad,sd}$ is the irradiance vector for the i-th point, azimuth down and slope down $(\mathrm{kW/m^2})$,
$\vec{IR}_{i,ad,su}$ is the irradiance vector for the i-th point, azimuth down and slope up $(\mathrm{kW/m^2})$,
$\vec{IR}_{i,au,sd}$ is the irradiance vector for the i-th point, azimuth up and slope down $(\mathrm{kW/m^2})$,
$\vec{IR}_{i,au,su}$ is the irradiance vector for the i-th point, azimuth up and slope up $(\mathrm{kW/m^2})$.

Now, we can interpolate the values according to the slope:

$$\begin{aligned} \vec{IR}_{i,ad} &= \vec{IR}_{i,ad,sd} + \left(\vec{IR}_{i,ad,su} - \vec{IR}_{i,ad,sd}\right) \cdot \frac{\beta_2 - \beta_{2,sd}}{\beta_{2,su} - \beta_{2,sd}}, \\ \vec{IR}_{i,au} &= \vec{IR}_{i,au,sd} + \left(\vec{IR}_{i,au,su} - \vec{IR}_{i,au,sd}\right) \cdot \frac{\beta_2 - \beta_{2,sd}}{\beta_{2,su} - \beta_{2,sd}}, \end{aligned} \tag{7.7}$$

where:
$\vec{IR}_{i,ad}$ is the interpolated irradiance vector for the i-th point, azimuth down (kW/m^2),
$\vec{IR}_{i,au}$ is the interpolated irradiance vector for the i-th point, azimuth up (kW/m^2),
$\beta_{2,sd}$ is the slope down (°),
$\beta_{2,su}$ is the slope up (°).

The next step is to interpolate the values according to the azimuth:

$$\vec{IR}_i = \vec{IR}_{i,ad} + \left(\vec{IR}_{i,au} - \vec{IR}_{i,ad}\right) \cdot \frac{\beta_1 - \beta_{1,ad}}{\beta_{1,au} - \beta_{1,ad}}, \tag{7.8}$$

where:
$\vec{IR}_i$ is the interpolated irradiance vector for the i-th point (kW/m^2),
$\beta_{1,ad}$ is the azimuth down (°),
$\beta_{1,au}$ is the azimuth up (°).

Now, we have three irradiance vectors for the three closest points and desired values of azimuth and slope. The vectors are denoted as:

$$\vec{IR}_1, \vec{IR}_2, \vec{IR}_3.$$

With the corresponding cartesian coordinates:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}.$$

As a final step, we need to interpolate the three points. However, here, the points are on a sphere. Therefore, we will use interpolation on a sphere [61]. We need to find the irradiance vector for the point:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

We will calculate the following determinants:

$$\begin{aligned} det_1 &= \begin{vmatrix} x_3 & x & x_2 \\ y_3 & y & y_2 \\ z_3 & z & z_2 \end{vmatrix}, \\ det_2 &= \begin{vmatrix} x_1 & x & x_3 \\ y_1 & y & y_3 \\ z_1 & z & z_3 \end{vmatrix}, \\ det_3 &= \begin{vmatrix} x_2 & x & x_1 \\ y_2 & y & y_1 \\ z_2 & z & z_1 \end{vmatrix}. \end{aligned} \tag{7.9}$$

Using these determinants, we can calculate the weight coefficients for the interpolation. The weight coefficients are defined as:

$$c_1 = \frac{\|det_1\|}{\|det_1\| + \|det_2\| + \|det_3\|},$$
$$c_2 = \frac{\|det_2\|}{\|det_1\| + \|det_2\| + \|det_3\|}, \qquad (7.10)$$
$$c_3 = \frac{\|det_3\|}{\|det_1\| + \|det_2\| + \|det_3\|},$$

where:
$\|det_i\|$ is the absolute value of the i-th determinant.

The final interpolated irradiance vector is then defined as:

$$\vec{IR} = c_1 \cdot \vec{IR}_1 + c_2 \cdot \vec{IR}_2 + c_3 \cdot \vec{IR}_3. \qquad (\mathrm{kW/m^2}) \quad (7.11)$$

Using this approach, we can get the irradiance vector for any point on the Earth's surface, including any azimuth and slope. When we sum the interpolated irradiance vectors for demonstration, we can generate the scalar field for the yearly irradiance. The interpolated irradiance map is shown in the following figure:

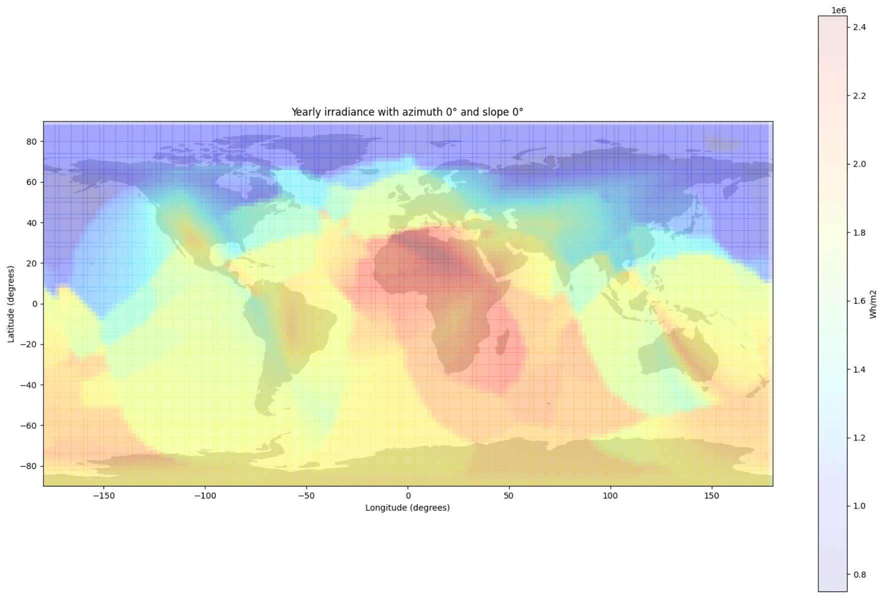

Figure 7.5: Interpolated irradiance map.

Also, we can present the irradiance depending on the azimuth and slope of the specific point, which is 50° latitude and 14° longitude. The point is concentrated in the middle of Europe in Czechia. The irradiance, depending on the azimuth and slope, is shown in the following figure:

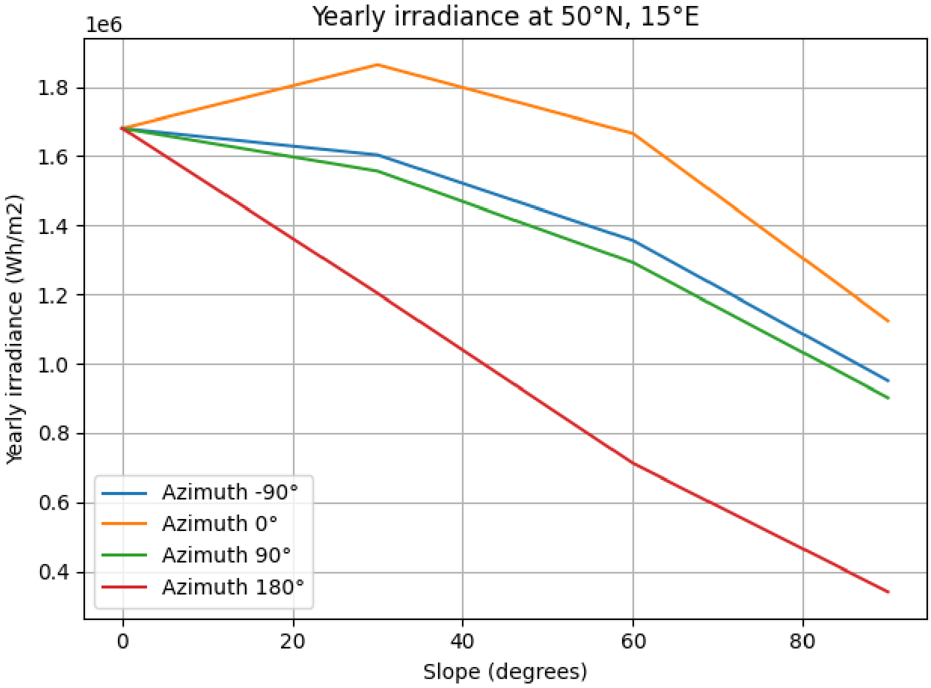

Figure 7.6: irradiance depending on the azimuth and slope.

The values are larger than the actual values from PVGIS because, using the Fibonacci lattice, there were no points in Europe. Therefore, the nerby points are used mainly from Africa and Asia in the interpolation. As the irradiance is larger in Africa and Asia, the interpolated values are larger than the actual values.

Chapter 8

Software development background

This chapter will describe the different approaches to mobile development, such as native development, hybrid development, and cross-platform development. We will also describe the most common cross-platform development frameworks like React Native, Xamarin, and Maui. Finally, the text will dive into the Flutter framework, as we used it to implement the application for energy evaluation.

8.1 Native development

Native app development refers to building applications for a specific operating system, such as Android or iOS. Native applications are written in the programming language specific to the platform [62].

In the past, when developers wanted to create an application for both Android and iOS, they had to write two separate applications in two different programming languages, which was expensive and time-consuming. This approach has high initial costs, and the costs for future maintenance are also high as the application needs to be updated twice.

The application for Android was generally written in Java, a programming language developed by Sun Microsystems in 1995 [63]. Currently, the Kotlin programming language is also widely used for Android development. Kotlin is an open-source programming language developed by JetBrains [64].

Similarly, the application for iOS was generally written in Objective-C, a programming language developed in the early 1980s [65]. In 2014, Swift programming language was released by Apple [66]. Swift is currently the primary programming language for iOS development. It has several enhancements over Objective-C, like memory management, whereas, in Objective-C, the developer had to manage the memory manually using pointers, while in Swift, the memory is managed automatically. Other enhancements include type inference, generic collections, more straightforward string manipulation, and many more [67].

8.2 Hybrid development

According to [68], hybrid mobile development apps generally refer to apps built using web technologies, such as HTML, CSS, and JavaScript. The core of the application is then encapsulated in a native application. By using plugins, the application can access native features of the device, such as camera, GPS, etc. The drawback could be in the final performance, but the performance is usually sufficient unless the application is very complex. The advantage of this approach is that web developers can develop the application and quickly deploy it to Android and iOS. This approach has low initial costs, and the costs for future maintenance are low as the application needs to be updated only once. The problem could be that the app store providers could reject the application, as they prefer native applications.

Recently, Apple published in their App Store Review Guidelines that the app should include features, content, and UI that elevate it beyond a repackaged website and should have some unique or "app-like" features. Otherwise, this app does not belong on the App Store [69]. This statement made it harder for hybrid development as Apple could reject the final app.

8.3 Cross-platform development

Cross-platform development refers to the process of creating software applications that are compatible with multiple hardware platforms. This approach enables a single application to function seamlessly on different operating systems like iOS, Android, and Windows. Common examples of cross-platform applications include web browsers, which offer consistent performance regardless of the computer or mobile device they are operated on. In mobile development, cross-platform development is often used to create applications for both Android and iOS using a single codebase. The most common cross-platform development frameworks are React Native, Flutter, Xamarin, and Maui [70].

The difference between hybrid and cross-platform development is that hybrid development uses web technologies, while cross-platform development uses specified technologies like React Native, Flutter, Xamarin, and Maui. Both approaches have a single codebase, whereas native development has one codebase for each platform [71].

8.3.1 React Native

React Native, developed by Facebook in 2015 as an open-source project, is a widely used cross-platform framework. It enables developers to build mobile apps using JavaScript. React Native allows developers to create applications both for Android and iOS using the same codebase. Often, React Native is compared with ReactJS (also developed by Facebook), which is a JavaScript library for building user interfaces for websites. ReactJS and React Native use a mixture of JavaScript and a particular markup language called JSX. However, the syntax differs as ReactJS uses HTML-like syntax and HTML elements, while React Native uses native mobile user interface components [70].

8.3.2 Xamarin

Xamarin is a cross-platform framework developed by Microsoft in 2011. It enables developers to build mobile apps using C#. Xamarin allows developers to create applications both for Android and iOS using the same codebase. Xamarin uses a single language, C#, compiled into native code for each platform. Xamarin also provides access to native APIs, which enables developers to create native user interfaces [72].

8.3.3 Maui

Maui is a cross-platform framework developed by Microsoft. It is a successor to Xamarin. Maui enables developers to build mobile apps using C#. Maui allows developers to create applications for multiple platforms using the same codebase [73].

8.4 Flutter

Flutter is a cross-platform and open-source framework developed by Google. It enables developers to build mobile apps using the Dart programming language. Flutter could develop applications for Android, iOS, Windows, macOS, Linux, and the web. Flutter also provides access to native APIs, which enables developers to create native user interfaces [74].

8.4.1 Widgets

Flutter uses widgets to build the user interface. Widgets describe their view, given their current configuration and state. Widgets are the building blocks of a Flutter app's user interface, and each widget is an immutable declaration of part of the user interface. Widgets are nested inside each other, and the child widget, which is nested inside the parent widget, receives context from the parent. The widget could add some additional information to the context, which could be used by the child widget, which will obtain the context afterward. When going up the widget tree, we will reach the root widget, the container that hosts the Flutter app. This root widget is either a MaterialApp or a CupertinoApp [8].

8.4.2 Anatomy of Flutter application

The flutter anatomy could be split into the framework, the engine, and the embedder. The framework is written in Dart and provides the building blocks for the application's user interface. The engine is written in C++ and offers low-level rendering support using Google's Skia graphics library. The embedder is written in C++ and provides the platform-specific code that communicates with the engine and the framework. The embedder also provides the platform-specific code that hosts the Flutter app. The embedder is responsible for starting the Flutter engine and running the Flutter app. The Flutter engine renders the user interface and communicates with the framework. The framework is responsible for building the user interface and communicating with the engine. The framework is also responsible for sharing with the embedder [8].

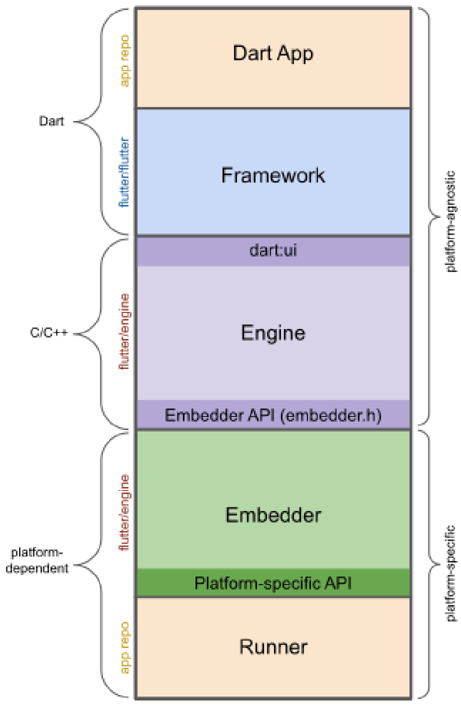

Figure 8.1: Anatomy of Flutter application [8].

8.4.3 Layout and rendering

The Flutter application has three types of trees: widget tree, element tree, and render tree. The widget tree is a tree of widgets that describe the user interface. During the build phase, the widget tree gets translated into an element tree. In the element tree, each element is for each corresponding widget. Finally, the render tree is created from the element tree. The render tree is a tree of render objects that describe the layout and painting [8].

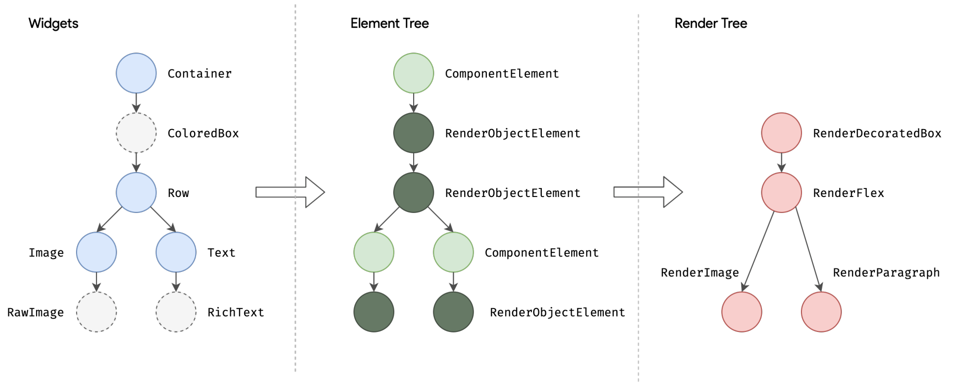

Figure 8.2: Flutter trees [8].

One of the most important concepts, which denotes how the widgets are placed and rendered, is: "Constraints go down. Sizes go up. The parent sets the position." [75]. This rule governs how the widget tree is actually rendered.

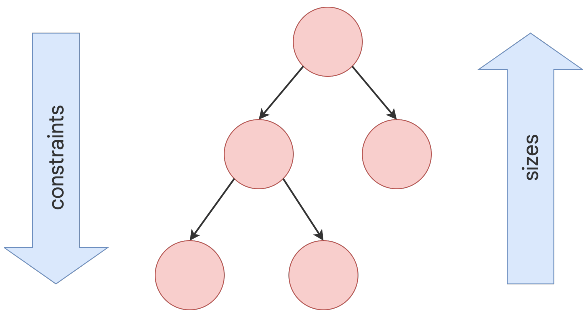

Figure 8.3: Flutter constraints and sizes [8].

Chapter 9

Implementation

This chapter provides an introduction to the app implementation. The development process and architecture will be described. At the end, the chapter will dive into testing.

9.1 Development

This section deals with a brief description of the development, including IDE and version control.

9.1.1 Integrated Development Environment

The app was developed using Visual Studio, a robust integrated development environment developed by Microsoft [76]. It is widely used for various programming languages, including Flutter development. It has a rich set of features, such as code completion, debugging tools, and integrated Git support, making it a preferred choice among developers.

9.1.2 Version Control System

As a Git system [77], the Sublime Merge was used, which, along with Git, provides a decent user interface [78]. For remote storage, GitHub was used [79]. It is a widely adopted version control platform that facilitates collaborative development and code management. It offers features such as pull requests, branching strategies, and issue tracking, making it ideal for individual and team-based projects.

9.2 Architecture

The following subsection will describe the core architecture and design patterns used in the app.

9.2.1 Bloc pattern

The Bloc makes it easy to separate the presentation layer from the business logic [9]. The following figure illustrates the basic concept of the Bloc pattern. The UI layer interacts with the Bloc using events, which interacts with the repository layer. The repository layer fetches data from the network or local storage and passes them to the Bloc. The Bloc then emits certain states like loading, error, and success with the fetched data. The Bloc pattern ensures that the business logic is decoupled from the UI, making the app more testable and maintainable.

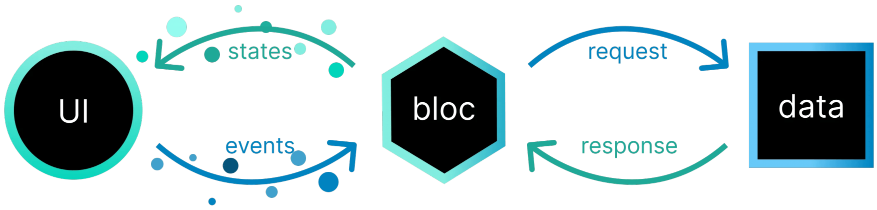

Figure 9.1: Bloc pattern [9].

In the app, the bloc pattern is widely used mainly to handle states of the pages, which requires obtaining data from the shared preferences (will be described later). The commonly used states are loading, error, and loaded, which the Bloc emits. The loaded state passes the data to the UI layer, which is displayed to the user.

9.2.2 Dependency injection

The dependency injection helps to provide services to the classes without creating them directly [80]. In the app, the package GetIt was used [81]. It is a simple service locator for Dart and Flutter projects. It allows developers to register services and retrieve them anywhere in the app. In the app, the services mainly for fetching data from the shared preferences are registered as factory services, meaning that every time the service is requested, a new service instance is created.

9.2.3 Freezed package

The Freezed package was used to create immutable classes [82]. With build runner [83], the Freezed package generates the necessary code for the classes. The generated code includes the equality and hashcode methods, which are essential for comparing objects, along with the copyWith method, which helps create a new class instance with the updated values. The Freezed package also generates the fromJson and toJson methods used to serialize and deserialize objects. The Freezed package ensures that the classes are immutable, making them easier to reason with and less prone to bugs.

9.2.4 Database

The shared preferences package was used to store the data locally [84]. It is a simple key-value store that allows primitive data types such as integers, strings, and booleans to be stored. The data stored in the app are serialized to JSON format before being stored in the shared preferences. For example, the household detail is stored as a JSON string in the shared preferences with the key of the household ID.

9.2.5 Localization

The app supports English and Czech languages. The localization preferences are saved in the settings and stored in the shared preferences. When the app is loaded, the settings object is loaded from the shared preferences and provided by the Bloc. From that, language is passed to the app.

9.2.6 Themes

The app supports light and dark themes. The theme preferences are saved in the settings and stored in the shared preferences. When the app is loaded, the settings object is loaded from the shared preferences provided by the Bloc. From that, the theme is passed to the app.

9.3 Testing

The testing is an integral part of the development process. The following subsections will describe the types of tests used in the app.

9.3.1 Unit tests

Unit tests are used to test the app's business logic, mainly mathematical calculations, as they are the app's core. It helps to ensure that the calculations are correct and that the app behaves as expected.

9.3.2 Integration tests

There are basic integration tests for the app. The integration tests are created to test fundamental interactions in the app, like accessing pages and checking if the data are displayed correctly. The integration tests are used to ensure that the app behaves as expected when the user interacts with it.

Chapter 10
Application User Interface

This chapter deals with the application user interface. At the beginning, the logo and basic widgets of the application will be described. After that, the screenshots of the app will be presented.

10.1 Logo

The app's logo is simply constructed with three equal hexagons in green. The hexagons are placed in a triangle shape. The logo is displayed in the following figure.

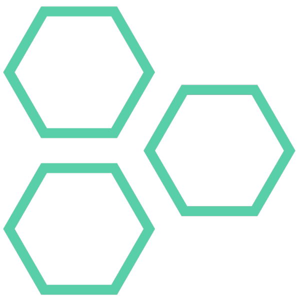

Figure 10.1: Logo

10.2 Loading widgets

There are several loading widgets in the app. In this section, each loading widget will be described and presented in a screenshot.

10.2.1 Splash screen

The splash screen is displayed when the app is loading. The splash screen is displayed for a few seconds, and then the app is redirected to the home page. On the splash screen, the logo is steady in the middle.

Figure 10.2: Splash screen.

10.2.2 Screen loading

The screen loading is displayed when some content is loading. The logo is circling during the loading. The screen loading is displayed in the following figure.

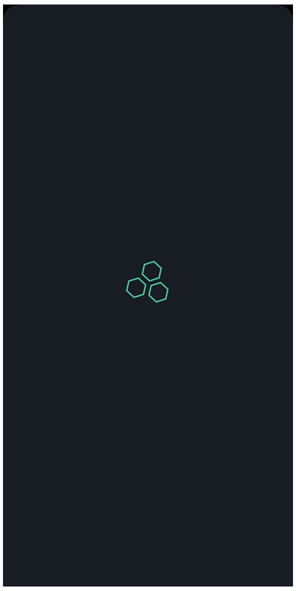

Figure 10.3: Screen loading.

10.2.3 Partial loading

Partial loading is used when some content is already loaded but some content is still loading. The partial loading is displayed in the following figure.

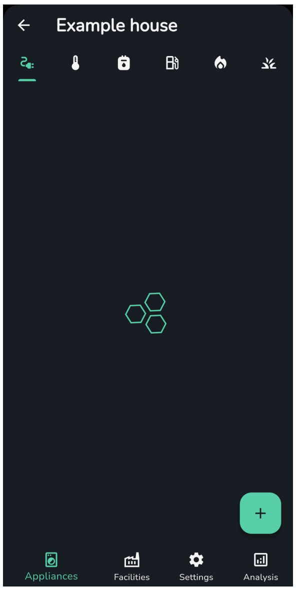

Figure 10.4: Partial loading.

10.2.4 Loading dialog

The loading dialog is displayed chiefly after the user clicks the button and the content is loading. The loading dialog is displayed in the following figure.

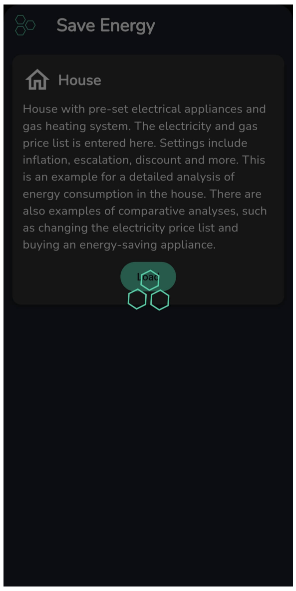

Figure 10.5: Loading dialog.

10.2.5 Progress loading

Progress loading is used when the time to load the content is estimated to be higher. The progress loading is displayed in the following figure.

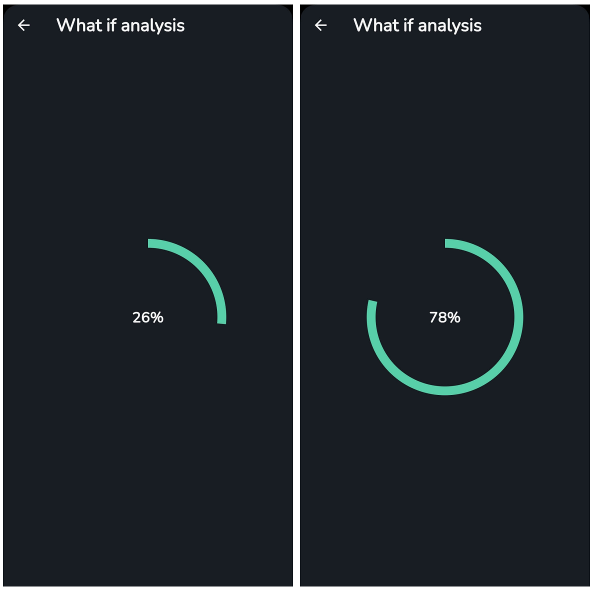

Figure 10.6: Progress loading.

10.3 Main navigation

The main navigation is located in the left drawer. It contains links to the main pages of the application. The following figure shows the main navigation of the application.

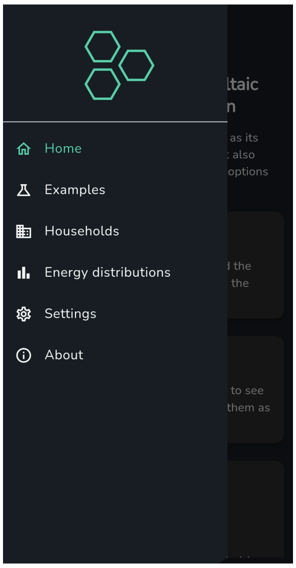

Figure 10.7: Main navigation.

At the top, the app logo is located. Below the logo are links to the main pages of the application. The links are as follows:

- Home page
- Examples page
- Households page
- Energy distributions page
- Settings page
- About page

10.3.1 Home page

The following figures are from the home page. The left is from the top of the home page, and the right is after scrolling down. On the home page, a brief description of the app is located. After that, several cards containing information about help, examples, and getting started are displayed. When users click on the examples card, they will be redirected to the examples page directly. The Getting Started section presents the steps users should follow to use the app and obtain the results.

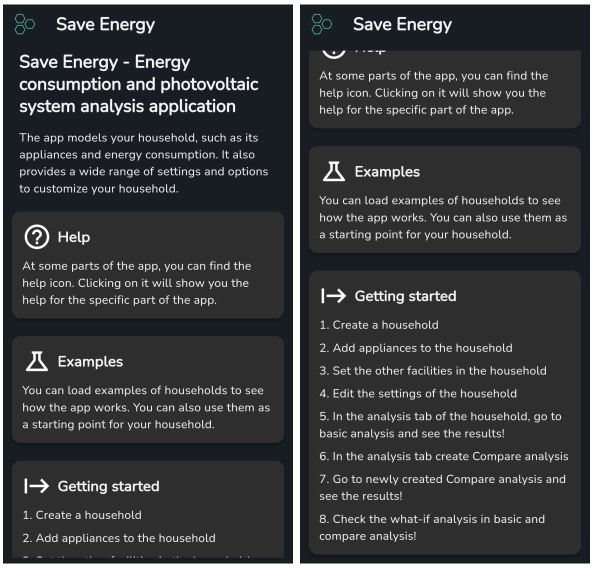

Figure 10.8: Home page.

10.3.2 Examples page

On the examples page, there is an option to load the example house, creating the household with predefined appliances and settings. This example house will be assessed in more detail in the final chapter. On the left figure below, the example page is displayed with the card for loading the example house. The dialog for loading the example house is displayed on the right side.

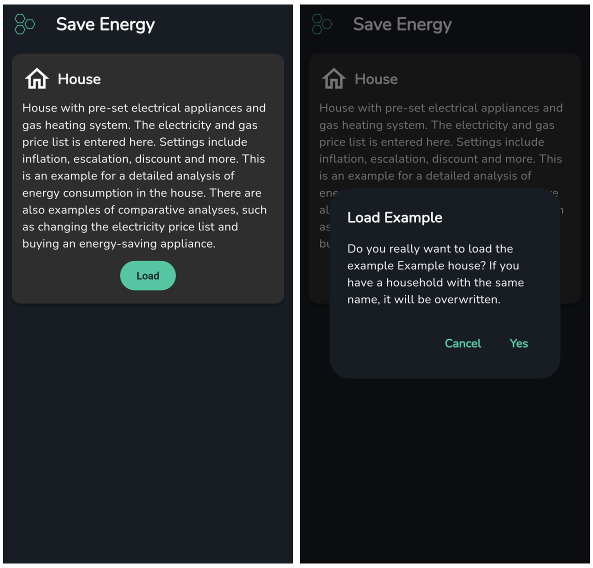

Figure 10.9: Examples page.

10.3.3 Households page

The households page is displayed on the following figure on the left side. There is a scrollable list on the page in case there are multiple households, which will overflow the page. Each card displays basic household properties such as name, type, area, and currency. The options for edit and delete are presented on each card. The delete dialog will be displayed when the user clicks the delete button, as shown on the right of the figure below.

There is an option to create a new household by clicking the floating action button (located at the bottom right on the left figure). The Create Household page is displayed in the middle figure below. The user can set the household name, type, area, and currency. The type is selected by clicking on the icons at the top of the page. The name and area are input fields. The currency is selected from the dropdown list.

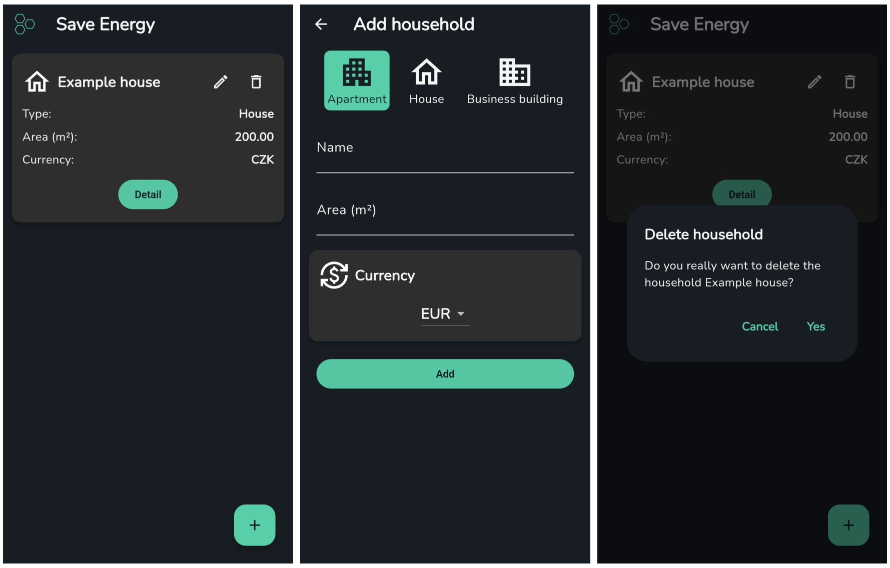

Figure 10.10: Household pages.

10.3.4 Energy distributions page

The energy distributions page displays the list of energy distributions (right screenshot). The energy distributions are defaults (constant at the top), tdds (TDD1 to TDD8), and could also be custom-created by the user. By clicking on the floating button in the bottom right, the user will be redirected to the creation page. On the right side, the TDD1 detail is displayed. There are two charts. The first is the yearly distribution, representing the distribution over the year, where the first month corresponds to January. The second chart is the daily distribution representing the distribution over the day with 24 values for each hour. For both distributions, it applies that the sum of the values across the domain equals 1.

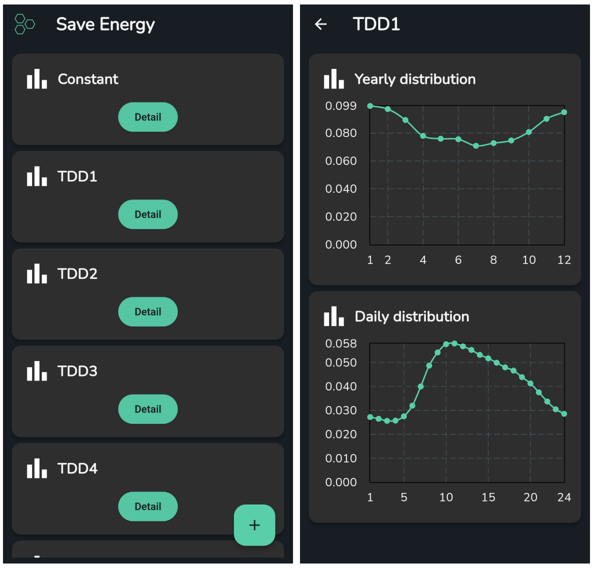

Figure 10.11: Energy distribution pages.

The creation page offers to set the name of the energy distribution and the values for the yearly and daily distribution. The values are input fields where the user can set the values. The value count is 12 for the yearly distribution and 24 for the daily distribution. The app validates these counts, and the user is informed of the error. The sum of the values does not have to be 1, but the app will normalize the values to sum to 1. The figures below show the creation page, where on the left side, the first part of the page is displayed, and on the right side, the page's content after scrolling down is displayed.

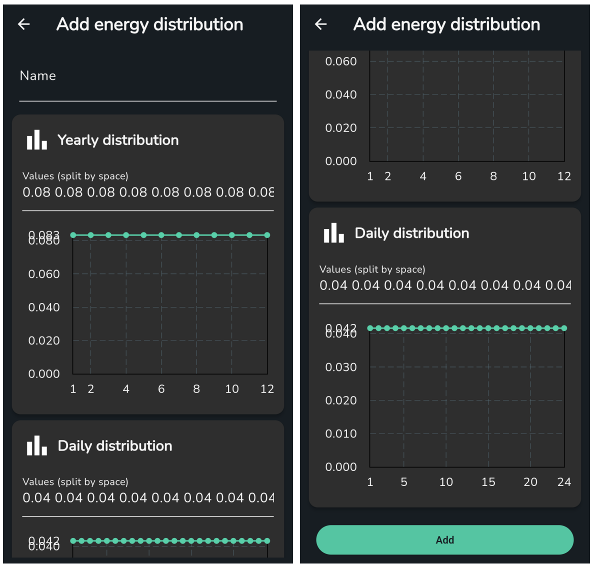

Figure 10.12: Energy distribution creation page.

10.3.5 Settings page

The settings page allows the user to change the app's color theme. The user can choose between system, light, and dark themes. The system theme is the default theme, which follows the system settings. The language options are English and Czech. Finally, there are two options for the irradiance database: the official PVGIS and custom-created PVGIS Lite (described in previous chapters).

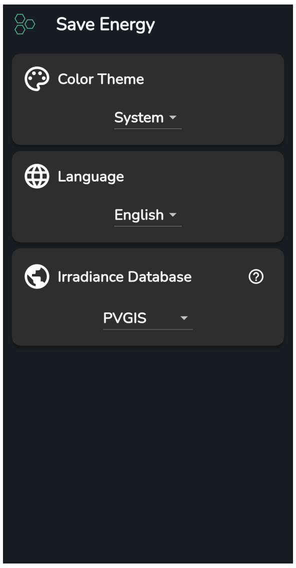

Figure 10.13: Settings page.

10.3.6 About page

The About page contains information about the app, and the name and version are displayed.

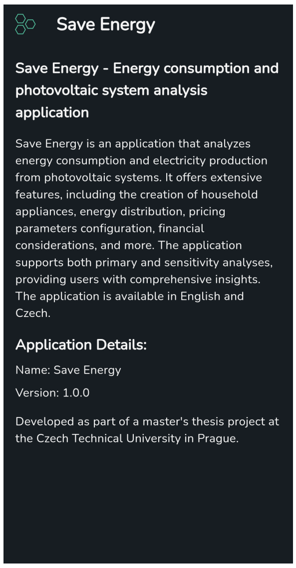

Figure 10.14: About page.

10.4 Household detail

The following section will present the content of household details. This content could be reached by clicking the household detail button on the households page. The household presented is the created example house, which was already shown on the households page. The household detail page is divided into several tabs. The tabs are switched by the bottom navigation bar on the page. The tabs are as follows:

- Appliances
- Facilities
- Settings
- Analysis

10.4.1 Appliances tab

The appliances tab is the first tab when entering the household details. At the top, the tab for each energy source type is located to switch the appliances. The appliances are displayed in the expansion list. The appliance detail will be shown when a user clicks the arrow on the right of the card. The following figures display the appliances tab on the left side and the same tab on the right after expanding the appliance detail. Each appliance has edit and delete buttons when the appliance detail is expanded. The floating button in the bottom right is for creating a new appliance.

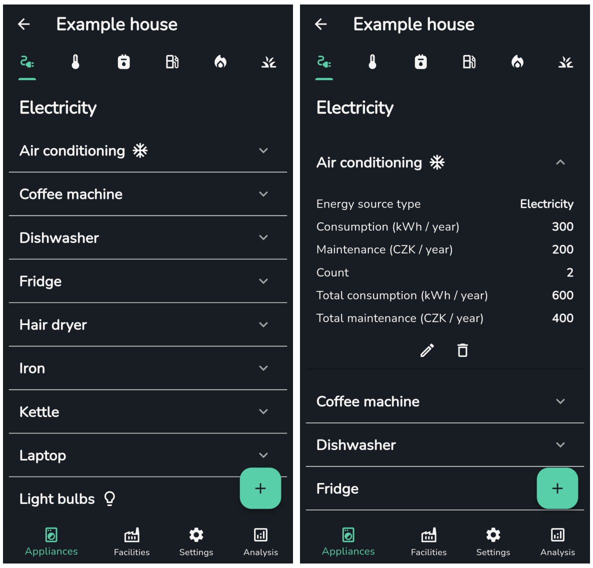

Figure 10.15: Appliances tab.

The appliance creation page is displayed in the following figure. The user can set the appliance energy source type by clicking on the icons at the top of the page. Then, the name has to be written in the text field. After that, the consumption type could be selected. The consumption type is how the consumption is calculated. The user can choose between the following options:

- Annual consumption - the consumption is set for the whole year.
- Input power - the input power and daily max load is set.
- Cycles - the number of cycles and consumption per cycle is set.

For each option, a different form is presented. After, the maintenance and appliance count can be set in the input fields. Finally, the user can select several appliance purposes by clicking the checkboxes. The purposes are as follows:

- Heating
- Heating water
- Lighting
- Cooling
- Ventilation

There is also an option to choose an appliance from the predefined appliances. Each energy source type has several predefined appliances with predefined values for consumption, maintenance, and count.

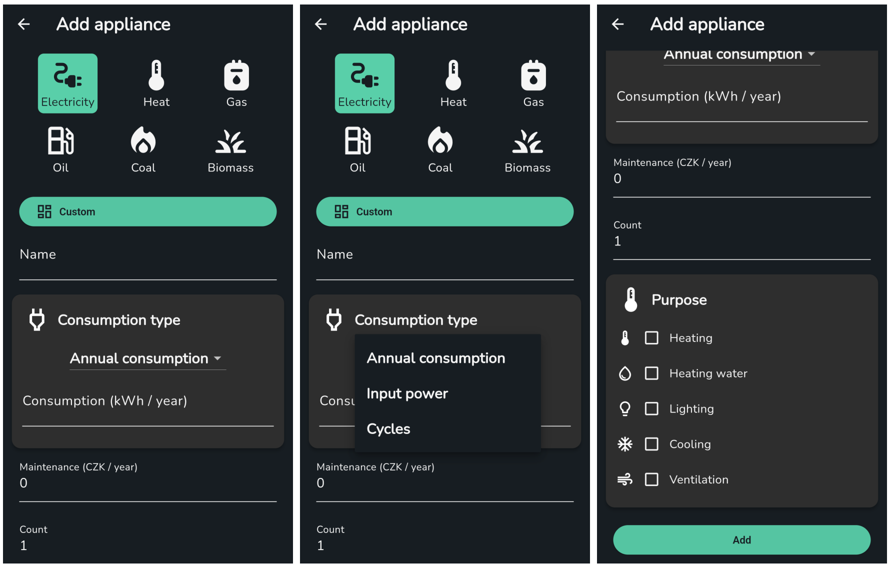

Figure 10.16: Appliance creation page.

10.4.2 Facilities tab

The user can set the household facilities on the facilities tab presented in the following figure. The facilities are the photovoltaic system and battery. The photovoltaic system and battery could also be edited if the user wants to change some values, not the whole system. The user can also delete the facility by clicking on the delete button.

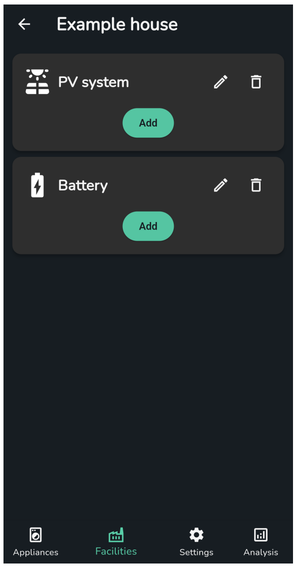

Figure 10.17: Facilities tab.

Photovoltaic system page

The photovoltaic system page is displayed in the following figure. The photovoltaic system consists of a list of modules. The creation page for the module is displayed on the right side of the figure. The area, azimuth, slope, efficiency, and efficiency escalation could be set for each module. On the left side, the creation page for the photovoltaic system page is displayed. For the photovoltaic system, the user can set the efficiency, efficiency escalation, maintenance, and maintenance escalation coefficient.

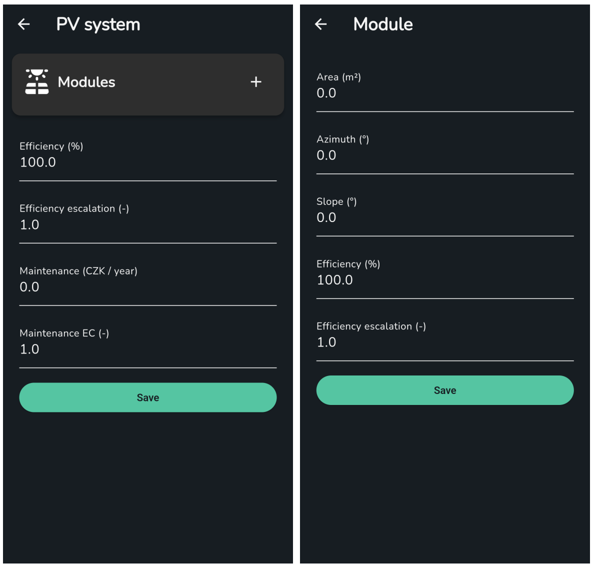

Figure 10.18: Photovoltaic system creation page.

Battery page

The battery properties, like capacity, power, etc., could be set on the battery creation page in the figure below.

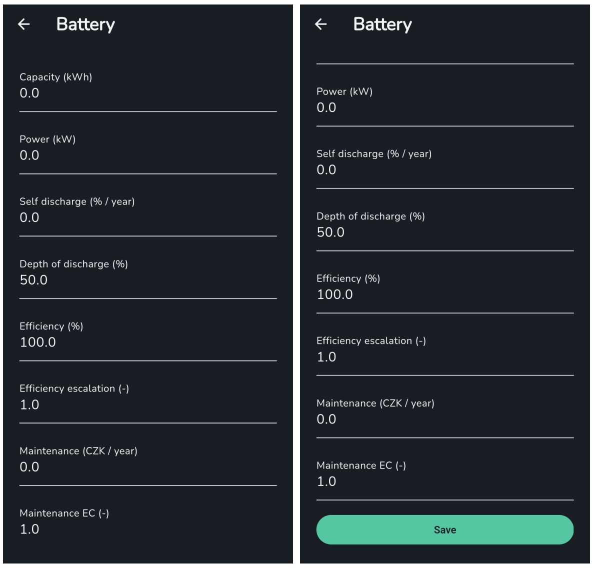

Figure 10.19: Battery creation page.

10.4.3 Settings tab

The settings tab consists of several settings cards, each for a specific part of the settings. The following figure shows the settings tab. The figure is divided into three parts, each at a particular vertical position. The parts to the right side are the continuation of the page when scrolling down. Each settings part will be described in the following subsections. Each part is displayed on a separate card in the settings tab. The creation page for particular settings will be displayed if the user clicks on the edit icon on the particular card. The settings page will be presented and described for each part in the following subsections.

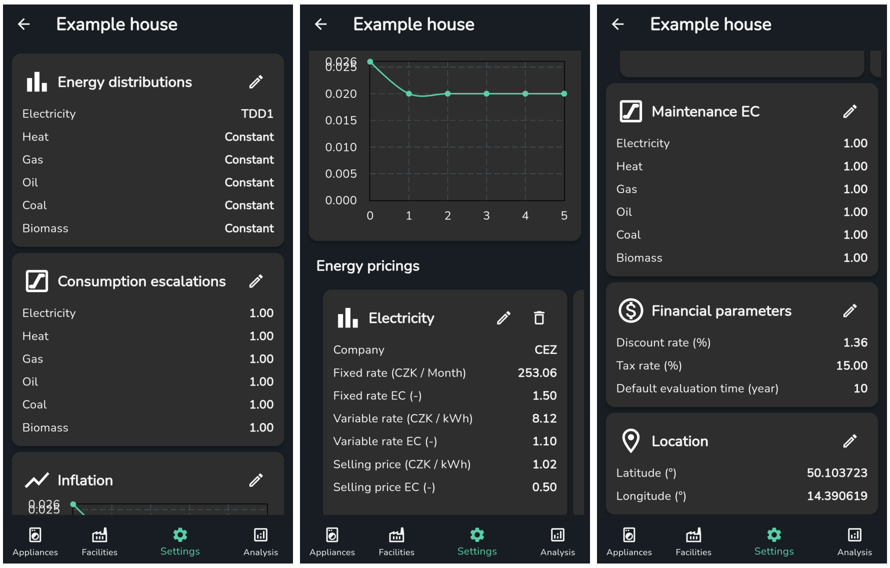

Figure 10.20: Settings tab.

Energy distributions

The first settings part is energy distributions. The user can assign the energy distribution from the default, tdd, and custom distributions to each energy source type.

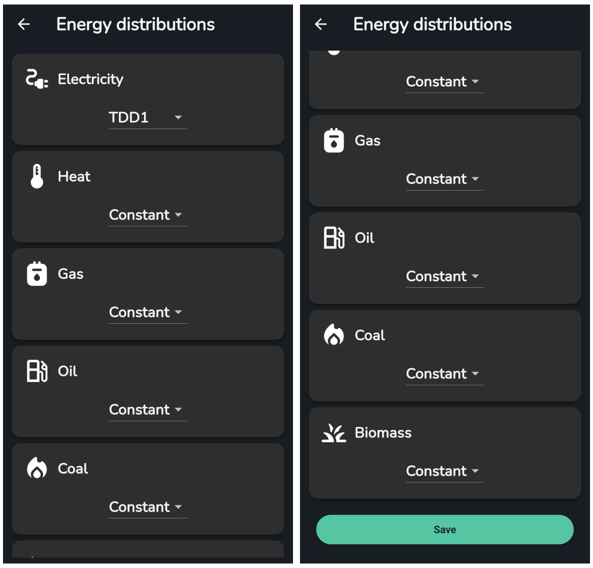

Figure 10.21: Energy distributions settings page.

Consumption escalations

The page for settings consumption escalations is simple, with the text field for the value for each energy source type.

Figure 10.22: Consumption escalations settings page.

Inflation

The inflation settings page has a text field for the values, where the user can set the first n values for the first n years. Then, there is a dropdown to set the type of how the values will continue. There is also an option to select predefined inflations.

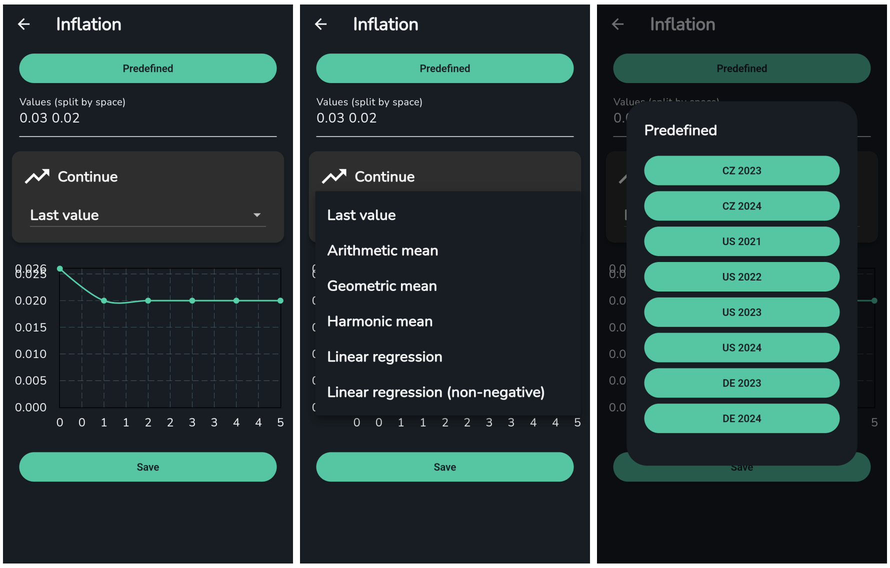

Figure 10.23: Inflation settings page.

Energy pricings

The energy pricing settings page allows users to set the company name and fixed, variable, and selling prices. For each price, there is an option to set the escalation coefficient. The energy pricing could be assigned to every energy source type.

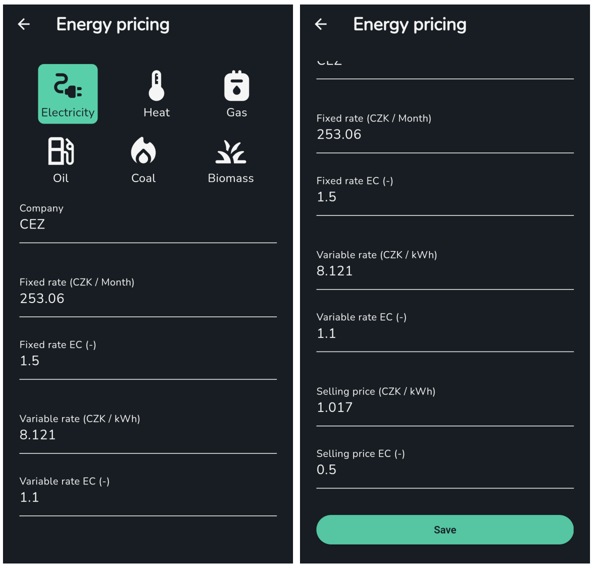

Figure 10.24: Electricity pricing settings page.

Maintenance escalation coefficients

The maintenance escalation coefficients page is similar to the energy consumption settings page. Similarly, there are text fields for the values for each energy source type, where the user can assess escalation coefficients.

Figure 10.25: Maintenance escalation coefficients settings page.

Financial parameters

The financial parameters settings page allows users to assign the discount rate, tax rate, and default evaluation time. The text field assigns each variable.

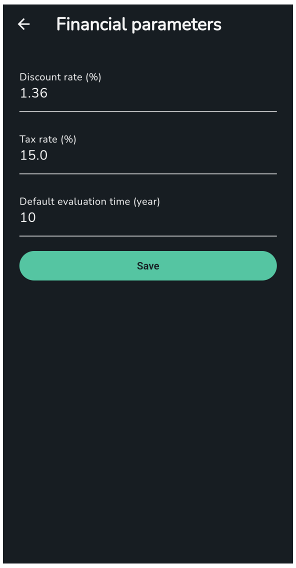

Figure 10.26: Financial parameters settings page.

Location

The location settings page consists of two text fields for latitude and longitude. There is also an option to set the location as the current location. Therefore, the user can use GPS to set the current location or set the location manually easily.

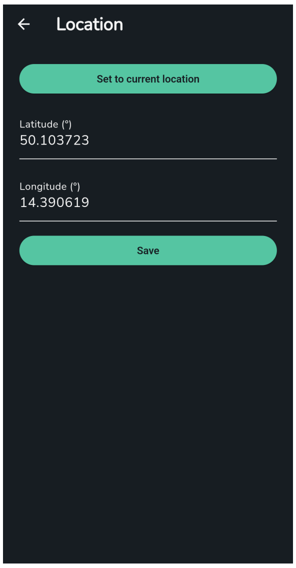

Figure 10.27: Location settings page.

10.4.4 Analysis tab

Finally, the analysis tab consists of two cards. The first is for basic analysis, and the second is for comparison analyses. The basic analysis is at the top of the page with a button to start the analysis. The compare analysis card consists of a list of buttons for each compare analysis. Each compare analysis has a button to start the analysis along with edit and delete buttons. Also, the add button at the top right corner of the compare analysis card redirects the user to the compare analysis creation page.

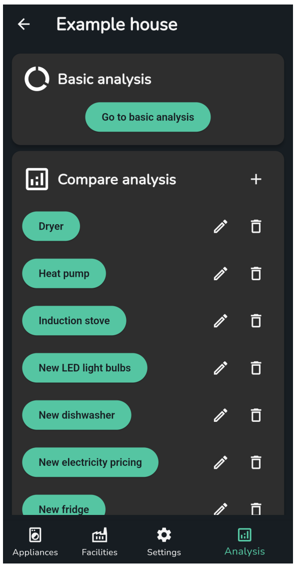

Figure 10.28: Analysis tab.

Basic analysis

The basic analysis page is split into several parts. Each part has a title with expandable content. The card with information about the part is displayed when the content is expanded. The user could also display the what-if analysis by clicking the button at the bottom of the page.

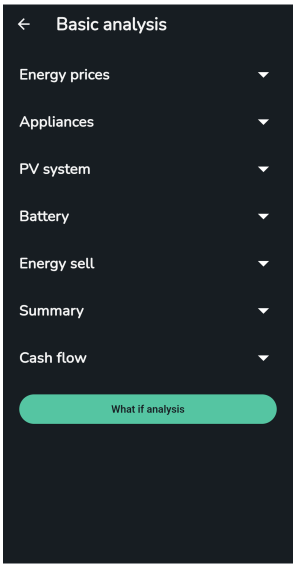

Figure 10.29: Basic analysis page.

Compare analysis

The financial results card is presented at the top on the compare analysis page. The card contains the financial results for the comparison analysis, such as NPV, IRR, PP, etc. Below that card, the user could see the basic analysis for the new household created by the compared analysis by clicking on the button below the financial results card. Then, there is a button that shows externalities. This button will show predictions for the future years of the externalities. The set value and escalation coefficient of each externality resolve this. Then, the user can see the cash flow prediction graphs by clicking the cash flow button. Finally, there is an option to see the what-if analysis by clicking the button at the bottom of the page.

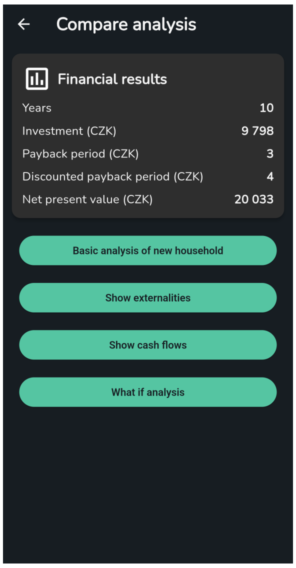

Figure 10.30: Compare analysis page.

The following figure presents the compare analysis creation page, where the user can set the household's mutations and the analysis's basic properties. At the top of the page, the text field for the name is presented, and right after the card for analysis, basic data with an edit button is displayed. The edit button will redirect the user to the edit page, where the user can adjust values for the evaluation years, investment, externalities such as external revenues and costs, and the escalation coefficients for revenues and costs.

After that, there is a carousel for creating new energy pricing for each energy source type. If the pricing is not created, the household will use the energy pricing from the original household, which is defined in the settings tab. If the pricing is set, the original pricing will be replaced by the new one. The edit page for the pricing is the same as the original one.

The next part is for removing and adding new appliances. There are two cards presented. On the first one, the user can add appliances that will be removed from the original one and count how many will be removed. The second card is for adding new appliances. The creation page for the new appliance is the same as for the original one.

Finally, the user can set the photovoltaic system and battery for the new household. The original one will be used if the photovoltaic system is not set. The battery works the same way. The user can set the new battery or use the original one.

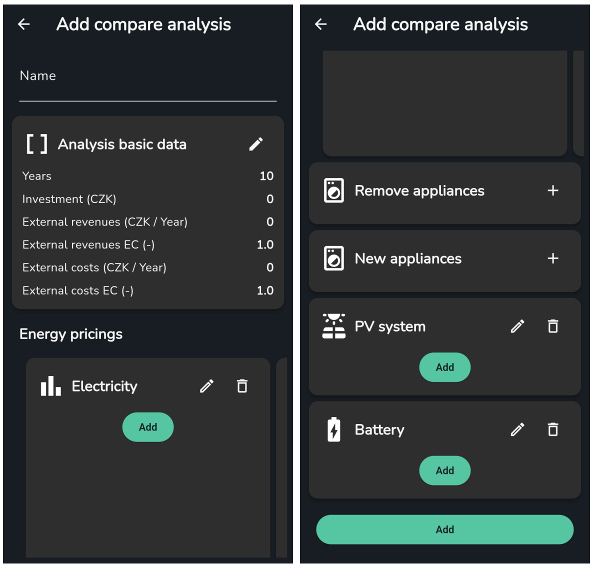

Figure 10.31: Compare analysis creation page.

Chapter 11
Example house

This chapter is focused on showing an example of how the app could be used and what results it gives. The example sample is a family house with an area of 200 m^2 located in Czechia. The house consists of several electric appliances, which will be thoroughly assessed, and a gas heating system to heat the house and water. The first two sections will analyze the current situation, including setting up the appliances and settings in the app. Then, the following section will focus on the basic analysis of the house, which provides insights into house consumption and energy costs. Then, sections will evaluate specific options for increasing the house's energy efficiency. These options will be calculated chiefly separately, as no combination of those options will be considered. For each analysis, the economic evaluation table will provide values for the PP, DPP, NPV, and IRR. Also, the what-if analyses will be presented as screenshots from the app.

11.1 Appliances

The following table provides the list of electric appliances with their consumption, maintenance costs, and count. The unit specification is under the table. Each appliance consumption was calculated based on annual consumption, input power, or cycles. The following subsections will deal with consumption estimation and divide this complete table into three separate tables based on the consumption estimation method. In each subsection, the crucial appliances will be described in more detail as they will be virtually replaced with new ones in the analysis parts.

Name	Consumption	Maintenance	Count
Dishwasher	266	200	1
Washing machine	162	200	1
Air conditioning	300	200	2
Iron	234	50	1
Light bulbs	29	0	10
TV	138	0	1
Oven	365	100	1
Stove	604	0	1
Fridge	254	0	1
Wifi router	105	0	1
Kettle	80	0	1
Coffee machine	52	0	1
Hair dryer	83	0	1
Microwave	43	0	1
Monitor	16	0	2
Laptop	292	0	2
Printer	9	0	1
Vacuum cleaner	47	100	1

Table 11.1: House electricity appliances.

Table units:

- Consumption: kWh/year
- Maintenance: CZK/year
- Count: –

The following table deals with the same list of appliances as the previous table but provides total consumption and maintenance costs. These values are simply calculated from values in the previous table multiplied by the count of each appliance.

Name	Total Consumption	Total Maintenance
Dishwasher	266	200
Washing machine	162	200
Air conditioning	600	400
Iron	234	50
Light bulbs	292	0
TV	138	0
Oven	365	100
Stove	604	0
Fridge	254	0
Wifi router	105	0
Kettle	80	0
Coffee machine	52	0
Hair dryer	83	0
Microwave	43	0
Monitor	32	0
Laptop	584	0
Printer	9	0
Vacuum cleaner	47	100

Table 11.2: Total consumption and maintenance of house electricity appliances.

Table units:

- Total Consumption: kWh/year
- Total Maintenance: CZK/year

The following two tables have the same architecture as the previous two for electric appliances. These two tables deal with the gas appliances. The only gas appliance is the heating system.

Name	Consumption	Maintenance	Count
Heating system	11 500	1 000	1

Table 11.3: House gas appliances.

Table units:

- Consumption: kWh/year
- Maintenance: CZK/year
- Count: –

Name	Total Consumption	Total Maintenance
Heating system	11 500	1 000

Table 11.4: Total consumption and maintenance of house gas appliances.

Table units:

- Total Consumption: kWh/year
- Total Maintenance: CZK/year

11.1.1 Based on annual consumption

The following table provides the list of electric appliances with the consumption based on the annual consumption. The unit specification is under the table. The consumption of each appliance was calculated based on the annual consumption. This method is the simplest one, as the annual consumption directly equals the consumption of the appliance. However, it can only be used on certain types of appliances.

Name	Annual Consumption
Air conditioning	300
Fridge	254

Table 11.5: House electricity appliances based on annual consumption.

Table units:

- Annual Consumption: kWh/year

Fridge

The fridge presented in the house is SAMSUNG RB33B610ESA/EF with 254 kWh/year consumption [10]. It is a combination of fridge and freezer. The freezer's capacity is 114 liters, and the fridge's capacity is 230 liters. The energy category is E. The image with the label is presented below.

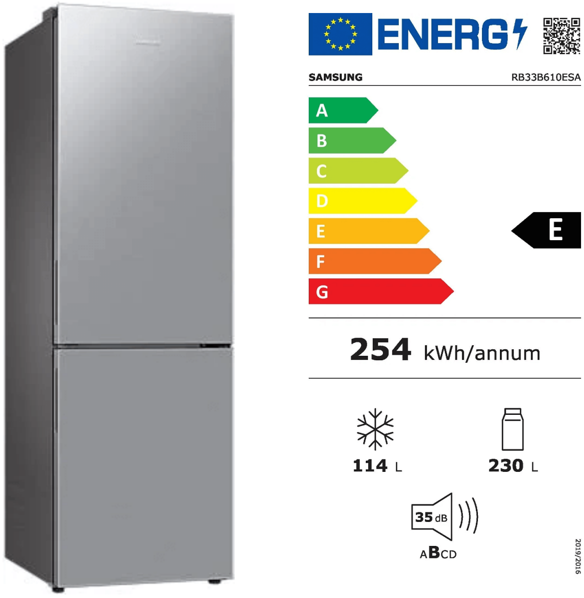

Figure 11.1: Fridge [10].

11.1.2 Based on input power

The following table provides the list of electric appliances with the consumption based on the input power and daily max load. The unit specification is under the table. The consumption of each appliance was calculated based on the input power and daily max load. This method is more complex as it requires knowledge of the appliance's daily maximum load. The annual consumption is calculated as the product of the input power, daily max load, and number of days in a year.

Name	Input Power	Daily Max Load
Iron	3.000	0.214
Light bulbs	0.040	2.000
TV	0.095	4.000
Oven	3.500	0.286
Stove	5.800	0.286
Wifi router	0.012	24.000
Kettle	2.200	0.100
Coffee machine	1.450	0.100
Hair dryer	2.300	0.100
Microwave	1.200	0.100
Monitor	0.015	3.000
Laptop	0.200	4.000
Printer	0.250	0.100
Vacuum cleaner	0.650	0.200

Table 11.6: House electricity appliances based on input power and daily max load.

Table units:

- Input Power: kW
- Daily Max Load: hour

Light bulbs

The house has old light bulbs with a power of 40 W. The daily max load is derived from the average length of lighting per day in the United States (US) as 2 hours per day [85]. As this light has large power, the category on the energy label is G, which is the worst category. These light bulbs are currently banned in the European Union (EU). Therefore, they are being sold as industrial light bulbs [86].

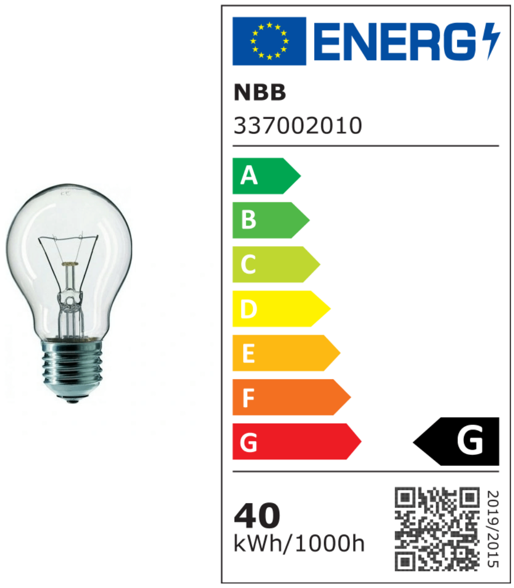

Figure 11.2: Light bulb.

Stove

The stove is a glass-ceramic ETA 679190000 with four cooking zones [3]. The first zone has the power of 1.8 kW, and the second and third zone has the power of 1.2 kW. The fourth zone has adjustable power from 1 to 2.2 kW[3]. The daily max load is 1 hour per day.

11.1.3 Based on cycles

The following table provides the list of electric appliances with the consumption based on the cycles. The unit specification is under the table. The consumption of each appliance was calculated based on the cycles. This method is complex as it requires knowledge of the cycles of the appliance. The annual consumption is calculated as the product of the consumption per cycle and the number of cycles in a year.

Name	Cycles	Consumption
Dishwasher	280	0.950
Washing machine	220	0.740

Table 11.7: House electricity appliances based on cycles.

Table units:

- Cycles: –
- Consumption per Cycle: kWh/ cycle

Dishwasher

The dishwasher is WHIRLPOOL WBC 3C26 X, which consumes 95 kWh/ 100 cycles [10]. It could fit 14 packs of dishes. Water consumption is 9.5 liters per cycle. According to the energy label, the dishwasher is in the category E. The number of cycles has been set to 280 per year [87].

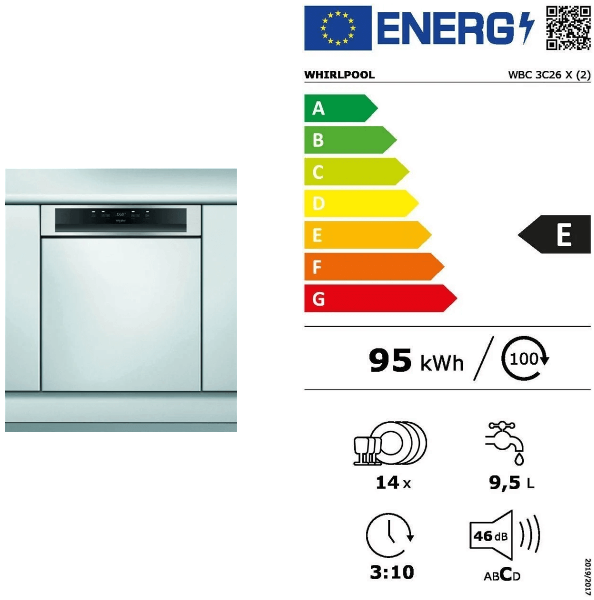

Figure 11.3: Dishwasher [10].

Washing machine

The washing machine is AMICA PPF 61002 EW. It has a consumption of 74 kWh/ 100 cycles [10]. The washing machine has a 6 kg capacity. It consumes 43 liters of water per cycle. The energy label is in the category E. The number of cycles has been set to 202 per year [88].

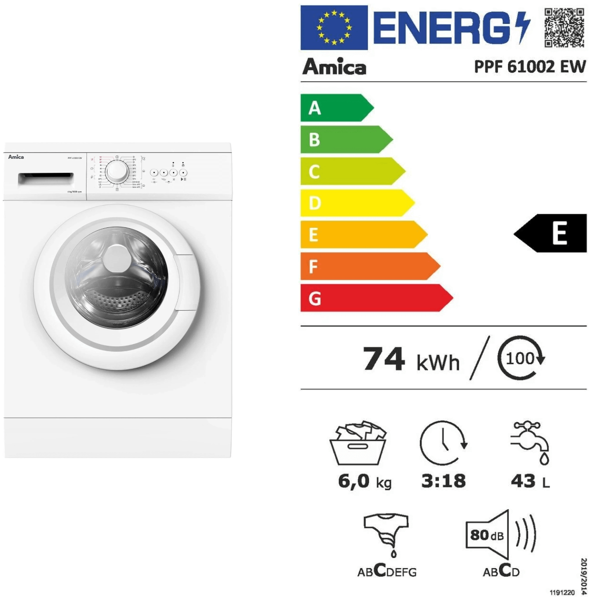

Figure 11.4: Washing machine [10].

11.2 Settings

The following subsections will describe the house settings. The settings include energy distributions, escalations, escalation coefficients, inflation, financial parameters, etc.

11.2.1 Energy distributions

The electricity has the distribution of TDD4, which is commonly used for tarif D02d, where no thermal usage of electricity is presented. The TDD4 is not precise, but it is the standard distribution in Czechia and could be used as a rough estimate. The distribution is only used when analyzing the photovoltaic system. When we perform basic equipment analysis, the distribution is unnecessary. The constant distribution is assigned to the other energy source types.

Electricity	TDD4
Heat	Constant
Gas	Constant
Oil	Constant
Coal	Constant
Biomass	Constant

Table 11.8: House energy distributions.

11.2.2 Consumption escalations

All consumption escalations are set to 1, meaning the consumption will stay the same over the years. There will be no increase or decrease in consumption over the years.

Electricity	1.00
Heat	1.00
Gas	1.00
Oil	1.00
Coal	1.00
Biomass	1.00

Table 11.9: House consumption escalations.

11.2.3 Inflation

The inflation is set according to the Czech National Bank forecast. Inflation in 2024 (year 0) is 2.6 %. Then it will be 2 % as this is the goal of the Czech National Bank [11]. The inflation forecast is presented in the figure below.

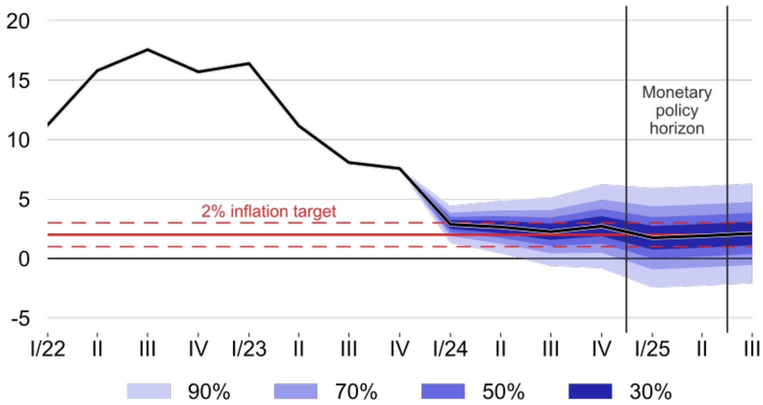

Figure 11.5: Czech National Bank inflation forecast [11].

11.2.4 Energy pricings

The energy pricing used in this household is for electricity and gas. In the following subsections, both prices will be described with their components.

Electricity

Electricity pricing is from the ČEZ company. The pricing is for the D02d tarif and 3x25A circuit breaker. The following table presents each component of the pricing, where the values are with Value Added Tax (VAT).

Component	Value
Tarif	D02d
Circuit breaker	3 x 25 A
Fixed payment for reserved power	248.05 CZK/month
OTE activity	5.01 CZK/month
Total fixed price	253.06 CZK/month
Price for energy	4.7916 CZK/kWh
Price for distribution	2.43895 CZK/kWh
Electricity tax	0.03424 CZK/kWh
Price for system services	0.25751 CZK/kWh
POZE	0.59895 CZK/kWh
Total variable price	8.12125 CZK/kWh

Table 11.10: Electricity pricing components [19].

The values for fixed and variable prices will be rounded to three decimal places. Before we present the final table, we need to consider the selling price. The selling price is also based on the ČEZ company pricing, where the selling price from the photovoltaic system is calculated as follows:

$$Price = c\,OTE_{DT} \cdot CNB\ rate_{CZK/EUR} - 500 \text{ CZK}, \tag{11.1}$$

where:
$Price$ is the selling price from the photovoltaic system (CZK/MWh),
$c\,OTE_{DT}$ is the price of day market organized by OTE (EUR/MWh),
$CNB\ rate_{CZK/EUR}$ is the exchange rate CZK/EUR declared by the Czech National Bank [89].

For the 12th of April 2024, the day market price organized by OTE is 59.9 EUR/MWh (Base load) [90]. For the same day the exchange rate CZK/EUR is 25.33 [91]. The selling price is then calculated as follows:

$$Price = 59.9 \cdot 25.33 - 500 = 1017.267 \text{ CZK/MWh} \approx 1.017 \text{ CZK/kWh}.$$

The final table with the electricity pricing is presented below. The escalation coefficient for a fixed price is set to 1.5, which should take into account a more significant increase in the fixed price. The escalation coefficient for the variable price is set to 1.1. The escalation coefficient for the selling price is set to 0.5, which should consider that the selling price will increase slower than other components.

Company	CEZ
Fixed price (CZK/month)	253.06
Fixed price EC (–)	1.50
Variable price (CZK/kWh)	8.12
Variable price EC (–)	1.10
Selling price (CZK/kWh)	1.02
Selling price EC (–)	0.50

Table 11.11: House electricity pricing.

Gas

The gas price is from the GasNet company. The following table presents each pricing component, where the values are with VAT.

Component	**Value**
Tarif according to annual consumption	from 7.56 to 15 kWh/year
Fixed payment for commercial part	154.88 CZK/month
Fixed payment for distribution part	198.22 CZK/month
Total fixed price	353.1 CZK/month
Price for energy	2.1659 CZK/kWh
Price for distribution	0.39406 CZK/kWh
Total variable price	2.56 CZK/kWh

Table 11.12: Gas pricing components.

The final values are presented in the table below. The selling price is not considered as no source will produce gas in the house. The escalation coefficient for a fixed price is set to 1.3, and for the variable price, it is set to 1.2, as we expect the gas variable prices to increase faster than the electricity prices.

Company	GasNet
Fixed price (CZK/month)	353.10
Fixed price EC (–)	1.30
Variable price (CZK/kWh)	2.56
Variable price EC (–)	1.20
Selling price (CZK/kWh)	0.00
Selling price EC (–)	1.00

Table 11.13: House gas prices.

11.2.5 Maintenance escalation coefficients

The maintenance escalation coefficients are set to 1, meaning the maintenance costs will stay the same over the years.

Electricity	1.00
Heat	1.00
Gas	1.00
Oil	1.00
Coal	1.00
Biomass	1.00

Table 11.14: House maintenance escalation coefficients.

11.2.6 Financial parameters

The nominal discount rate is 4 %, the interest rate on a savings account in mBank [92]. The value is nominal as the inflation is already integrated into the rate. According to the Czech National Bank, the inflation is forecast 2.6 % [11]. We can, therefore, calculate the real discount rate as follows:

$$r_r = \frac{1 + 0.04}{1 + 0.026} - 1 = 0.013 \approx 1.36\ \%.$$

The tax rate in Czechia is 15 % [93]. The default evaluation time is set to 10 years, which will be used to determine how long the basic analysis will be evaluated. All parameters together are presented in the table below.

Discount rate (%)	1.36
Tax rate (%)	15.00
Default evaluation time (years)	10

Table 11.15: House financial parameters.

11.2.7 Location parameters

Location parameters are set to the Czechia. The values for latitude and longitude are in the table below.

Latitude	50.103723
Longitude	14.390619

Table 11.16: House location parameters.

11.3 Basic analysis

In the beginning, the consumption and cost analysis is done on all energy source types together. Then, the analysis is divided into electricity and gas. The values in the tables are for the first year, and then they get escalated.

11.3.1 Appliances

The following table provides the consumption for all appliances for all energy source types. We can calculate the energy costs using the consumption and fixed and variable prices. The maintenance costs are also calculated. The total costs are the sum of the energy and maintenance costs.

Consumption (kWh)	15 457
Fixed costs (CZK)	7 273
Variable costs (CZK)	61 579
Energy costs (CZK)	68 852
Maintenance (CZK)	2 050
Total costs (CZK)	70 902

Table 11.17: Basic analysis of house appliances.

Electricity

The following table is similar to the above, but it focuses only on the electricity appliances. The values are the same as in the previous table, but only for the electricity appliances.

Consumption (kWh)	3 957
Fixed costs (CZK)	3 036
Variable costs (CZK)	32 139
Energy costs (CZK)	35 175
Maintenance (CZK)	1 050
Total costs (CZK)	36 225

Table 11.18: Basic analysis of house electricity appliances.

Gas

The following table presents the basic analysis of the gas appliances. The values are the same as in the first table, but only for the gas appliances.

Consumption (kWh)	11 500
Fixed costs (CZK)	4 237
Variable costs (CZK)	29 440
Energy costs (CZK)	33 677
Maintenance (CZK)	1 000
Total costs (CZK)	34 677

Table 11.19: Basic analysis of house gas appliances.

11.3.2 Cash flow

The house evaluation is done over 10 years. The cash flow is presented in the table below. The cash flow is the costs cash flow, where the costs have a positive sign and revenues have a negative sign. The total cash flow and total discounted cash flow are sums of the cash flow and discounted cash flow for 10 years of operation.

Years	10
Total cash flow (CZK)	793 330
Total discounted cash flow (CZK)	655 431

Table 11.20: Basic analysis of house cash flow.

11.3.3 What if analysis

The what-if analysis is done for the discount rate, tax rate, and inflation coefficient as independent variables (horizontal axis). The dependent variable for each analysis is the total discounted cash flow. When the discount rate rises, the total discounted cash flow decreases, as the higher discount rate means that the future cash flows are less valuable. The tax rate does not affect the total discounted cash flow, as there are no sales from energy production. The inflation coefficient also affects the total discounted cash flow. When it is set to 1, the total discounted cash flow is the same as the value in the table above. As it increases, the total discounted cash flow decreases. This is not so intuitive as the higher inflation will increase the costs. Therefore, the total discounted cash flow will be higher. However, inflation also affects the discount rate (calculating the nominal discount rate), decreasing the total discounted cash flow.

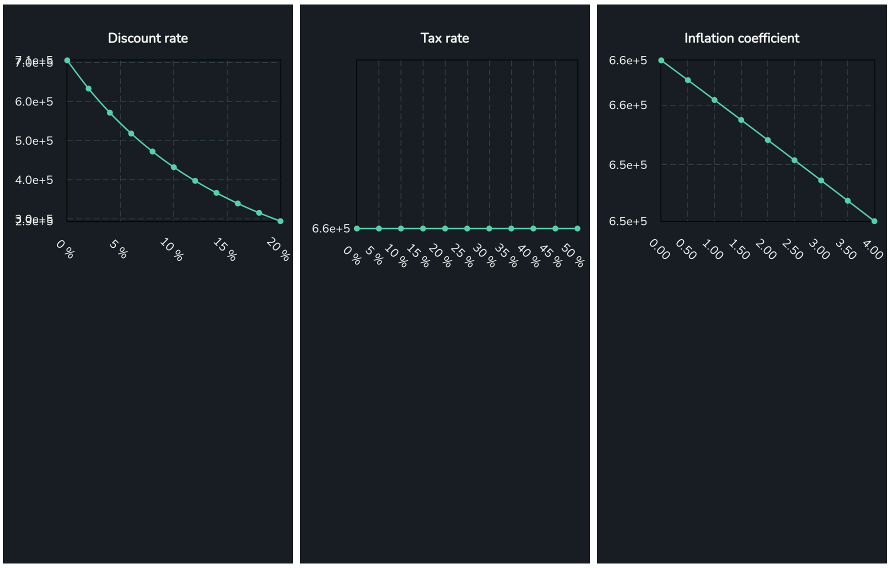

Figure 11.6: What if analysis of the house.

Note: The values on the y-axis are the total discounted cash flow in CZK.

11.4 New electricity pricing

The new electricity pricing is the first example of the compare analysis. The current pricing is from the ČEZ company, and the new one is from the MND company [20]. This company is not a distributor, as the distributor will still be ČEZ. The company is only selling the energy. Each component of the new electricity pricing is presented in the table below with the VAT. The tariff is D02d, and the circuit breaker is 3 x 25A, the same as in the current pricing. The fixed and variable prices are calculated as the sum of the components. The new pricing is then compared with the current pricing.

Component	Value
Tarif	D02d
Circuit breaker	3 x 25 A
Fixed payment for reserved power	248.05 CZK/month
OTE activity	5.01 CZK/month
Total fixed price	253.06 CZK/month
Price for energy	3.618 CZK/kWh
Price for distribution	2.43895 CZK/kWh
Electricity tax	0.03424 CZK/kWh
Price for system services	0.25751 CZK/kWh
POZE	0.59895 CZK/kWh
Total variable price	6.49765 CZK/kWh

Table 11.21: Electricity pricing components [20].

According to the table, the fixed energy price is the same as the current pricing, but the variable price is lower. The selling price will stay the same, and all escalation coefficients will stay the same. Therefore, the new electricity pricing will only have a variable price of 6.498 (value is rounded).

The results of the new electricity pricing are presented in the table below. The NPV is 59 118 CZK, which means that by changing the electricity pricing, the house will save 59 118 CZK in 10 years. The payback and discounted payback periods are 0 years because no investment is needed to change the pricing. The IRR is high, but mathematically, it goes to infinity as there are no negative cash flows, and investment is zero. The huge value resulted from the limited number of steps in IRR calculation, which is done using Newthon's method.

Years	10
Investment (CZK)	0
Payback period (years)	0
Discounted payback period (years)	0
Net present value (CZK)	59 118
Internal rate of return (%)	12998.38

Table 11.22: New electricity pricing results.

The what-if analysis is presented in the figure below. When increasing the discount rate, the NPV goes down rapidly, whereas for 20 %, the NPV goes down to the value of around 27 000 CZK, which is less than half of the original value. The inflation coefficient also negatively affects the NPV.

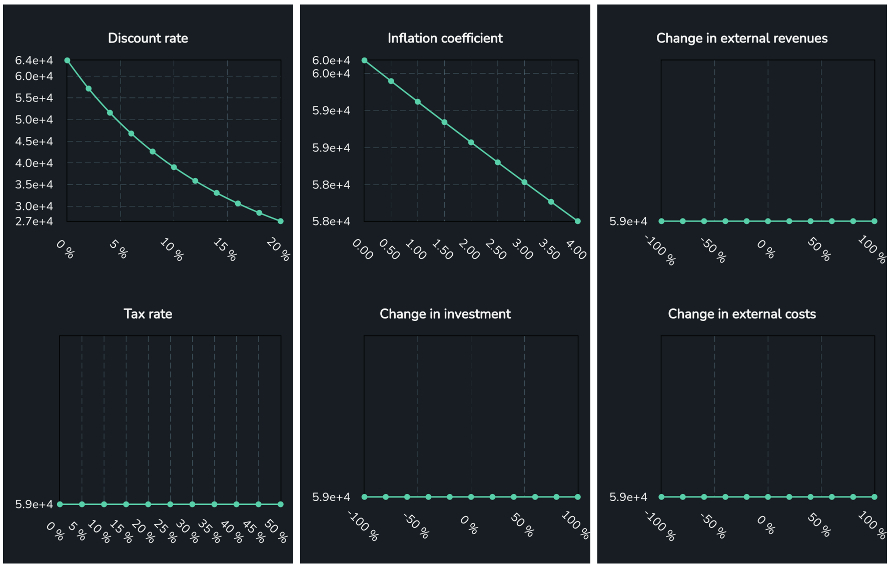

Figure 11.7: What if analysis of new electricity pricing.

Note: The values on the y-axis are the NPV in CZK.

11.5 Switching off wifi router at night

When switching off the wifi router at night, the appliance is virtually removed from the appliances. Then, a new wifi router appliance is added with the same input power, but the daily max load is changed from 24 to 16 hours per day. This results in electricity consumption savings. The investment is zero as no investment is needed to switch off the wifi router at night. Externalities are not presented. The results are presented in the table below. The IRR makes no sense as no investment is needed. Mathematically, the IRR goes to infinity, as in the previous example.

Years	10
Investment (CZK)	0
Payback period (years)	0
Discounted payback period (years)	0
Net present value (CZK)	7 857
Internal rate of return (%)	3250.20

Table 11.23: Switching off wifi router at night results.

The what-if analysis is presented below. The discount rate affects the NPV, where the NPV decreases with an increasing discount rate. The inflation coefficient also affects the NPV, where the NPV decreases with an increasing inflation coefficient. Other parameters do not affect the NPV.

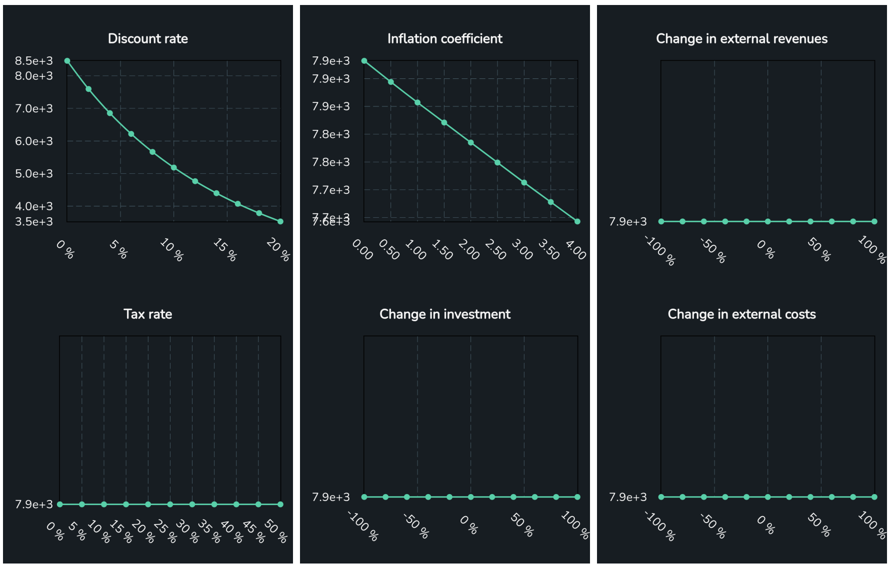

Figure 11.8: What if analysis of switching off wifi router at night.

Note: The values on the y-axis are the NPV in CZK.

11.6 New fridge

The new fridge is GORENJE NRC69BSXL5 with an annual consumption of 132 kWh [10]. The fridge and freezer have capacities similar to those of the current one. The main difference is in annual consumption, which is lower than in the previous one. The energy category is B rather than E. The investment is calculated as a sum of the price of the fridge and the installation price. The new fridge price is 17 990 CZK, and the installation price is 699 CZK. Therefore, the total investment is 18 689 CZK. The lifetime is estimated as the standard lifetime of the fridge [94].

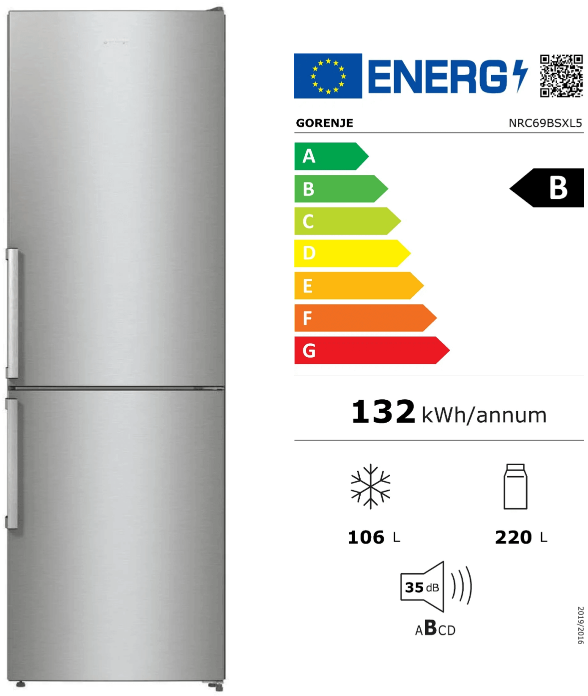

Figure 11.9: New fridge [10].

The NPV result is -7 037 CZK, which means that the appliance replacement is not economically effective. The payback period is -1, meaning the investment will never be returned. The real IRR is -4.99 %. The results are presented in the table below.

Years	13
Investment (CZK)	18 689
Payback period (years)	-1
Discounted payback period (years)	-1
Net present value (CZK)	-7 037
Internal rate of return (%)	-4.99

Table 11.24: New fridge results.

However, here the estimation is that we have already bought the old one, which has a long life long enough to cover the new lifetime, and we are only considering buying the new one. If, for example, the old one is at the end of a lifetime, and we need to buy a new one and consider two options, if the same or the more energy-effective, the situation will be different. The old one costs 9390 CZK [10]. This means that the difference in investment between new and old fridges is (not taking installation costs as they are the same for both fridges):

$$17990 - 9390 = 8600 \text{ CZK}.$$

The investment will be, therefore, 8 600 CZK. The NPV will be then:

$$-7037 + (18689 - 8600) = 3052 \text{ CZK},$$

This means that the situation will be different and that energy-efficient fridges will be more economically effective. Therefore, we can state that the NPV will be between -7 037 and 3 052 CZK based on the lifetime of the old fridge.

The what-if analysis is presented below. When the discount rate increases, the NPV decreases as expected. Here, the NPV is sensitive to the change of investment, where the NPV is around 12 000 CZK for 0 investment and roughly -25 000 CZK for double investment.

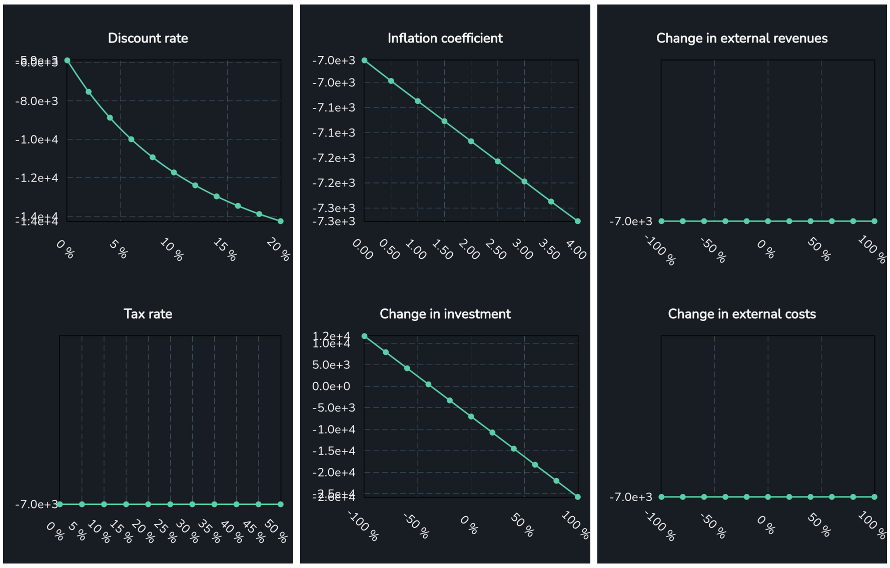

Figure 11.10: What if analysis of new fridge.

Note: The values on the y-axis are the NPV in CZK.

11.7 New dishwasher

The new dishwasher analyzed in this section is WHIRLPOOL WIO 3O540 PELG, which has properties similar to the current one. The annual consumption is less, with a value of 64 kWh [10]. As the capacity is the same as the current one, thus the number of cycles per year will be the same. The price of the dishwasher is 11 806 CZK, and the installation price is 1 758 CZK. Therefore, the total investment is 13 564 CZK. The lifetime is estimated as a standard lifetime of the dishwasher [95].

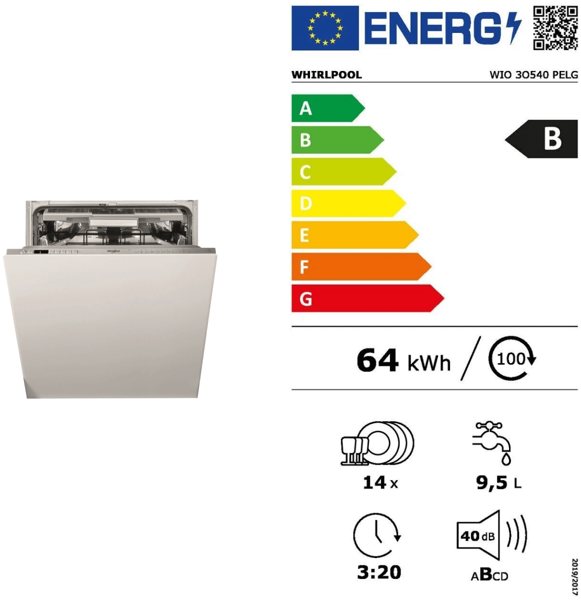

Figure 11.11: New dishwasher [10].

The NPV result is negative, which means that the appliance replacement is not economically effective. The payback period is -1, meaning the investment will never be returned. Nevertheless, again, we are considering that the old one has an entire lifetime, just like the new one, and we are making this silly replacement. The old dishwasher costs 12 990 CZK [10], which is more than the new one. Therefore if we consider that we need to decide between buying the current one again or buying the new one, the new one will be better.

Years	12
Investment (CZK)	13 564
Payback period (years)	-1
Discounted payback period (years)	-1
Net present value (CZK)	-5 867
Internal rate of return (%)	-6.67

Table 11.25: New dishwasher results.

The what-if analysis is presented below. The NPV dependency is similar to the previous example, with negative dependency on the discount rate, inflation coefficient, and change of investment.

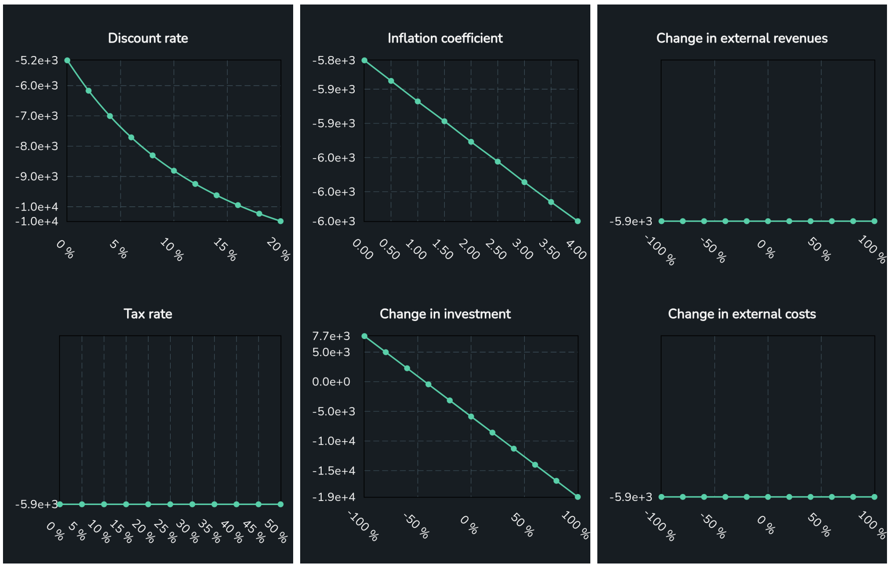

Figure 11.12: What if analysis of new dishwasher.

Note: The values on the y-axis are the NPV in CZK.

11.8 New washing machine

The analysis of the new washing machine is similar to the analysis of the new dishwasher because both appliances have consumption estimated based on cycles and consumption per cycle. Here, the new washing machine is a TOSHIBA TW-BL70A2CZ(SS) with similar properties and capacity as the old one, with a difference in annual consumption of 49 kWh [10].

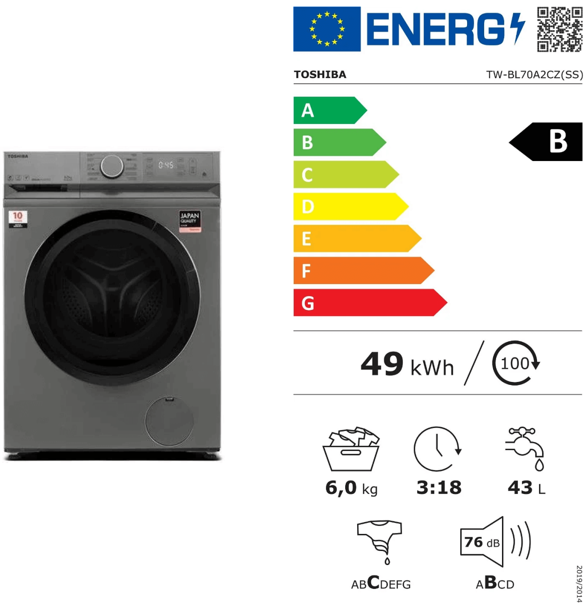

Figure 11.13: New washing machine [10].

The price of the new washing machine is 7 719 CZK with an installation price of 699 CZK. The total investment is 8 418 CZK. The lifetime is estimated as a standard lifetime of the washing machine as 10 years [96]. The NPV is negative, but as in the previous examples, the consideration is simplified to buying the new one. The price of the old washing machine is 5 999 CZK [10]. Therefore, if we consider the decision between buying the new one or the old one, we need to subtract this value and the installation price from the investment.

Years	10
Investment (CZK)	8 418
Payback period (years)	-1
Discounted payback period (years)	-1
Net present value (CZK)	-4 306
Internal rate of return (%)	-10.21

Table 11.26: New washing machine results.

The what-if analysis is presented below. The NPV dependency is similar to the previous example, with negative dependency on the discount rate, inflation coefficient, and change of investment.

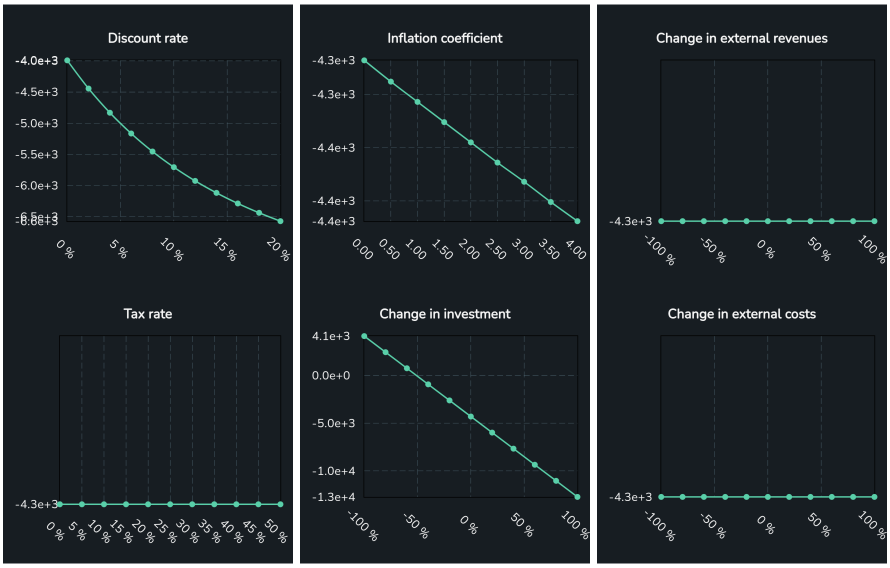

Figure 11.14: What if analysis of new washing machine.

Note: The values on the y-axis are the NPV in CZK.

11.9 LED light bulbs

This analysis considers that all light bulbs will be replaced with new LED light bulbs with a power of 7 W and a lifetime of 25 000 hours [3].

Figure 11.15: LED light bulbs [3].

The price for a pack with three light bulbs is 199 CZK. Therefore, for ten light bulbs, we can estimate the price for one light bulb by simple division. Therefore, the price for ten light bulbs is 663 CZK, which is the initial investment. The lifetime is estimated at 25 000 hours divided by daily max load (2 hours) and the number of days in year (365), which results in roughly 30 years. The NPV, therefore, is over 40 000 CZK for 30 years of operation. The payback period and discounted payback period are both one year.

Years	30
Investment (CZK)	663
Payback period (years)	1
Discounted payback period (years)	1
Net present value (CZK)	47 634
Internal rate of return (%)	288.27

Table 11.27: LED light bulbs results.

The NPV is sensitive to the change in discount rate, where the NPV decreases with the increasing discount rate. The inflation coefficient positively affects investment when it increases. The change in investment is affecting the NPV in a negative way, but not so much.

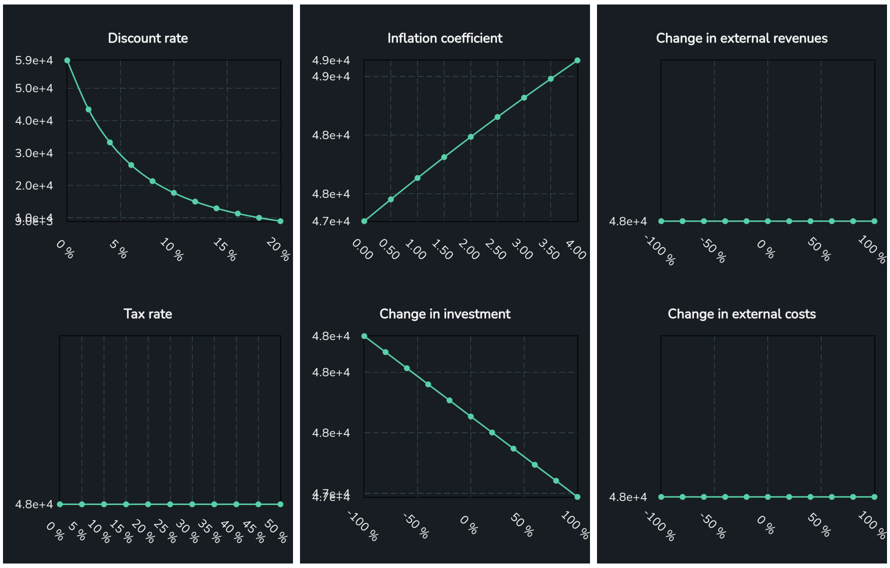

Figure 11.16: What if analysis of LED light bulbs.

Note: The values on the y-axis are the NPV in CZK.

11.10 Induction stove

The new induction stove is BOSCH PIE651FC1E Serie 6 [10]. The annual consumption is estimated based on input power and daily max load. The input power of old induction stove for all four hotplates is 5.8 kW with daily max load of 1 hour. According to [97], the efficiency of the old flass-ceramis stove is 70 %, and the efficiency of the new induction stove is 90 %. Therefore, we can calculate the new input power as follows:

$$5.8 \cdot \frac{0.7}{0.9} \approx 4.51 \text{ kW.}$$

To be more precise, we can also recalculate the daily max load. The approach will be on the basis of time to boil 2 liters of water, which is 9 minutes for a glass-ceramic stove and 5 minutes for an induction stove. The daily max load will be then:

$$0.286 \cdot \frac{5}{9} \approx 0.159 \text{ hours.}$$

The price of the new induction stove is 8 654 CZK, and the cost of assembly is 1 899 CZK. This results in a total investment of 10 553 CZK. The lifetime is estimated as a standard lifetime of the induction stove [98].

The NPV is positive, with a value of around 15 000 CZK for 10 years. The payback period and discounted payback period are 4 years. The IRR is 22.69 %.

Years	10
Investment (CZK)	10 553
Payback period (years)	4
Discounted payback period (years)	4
Net present value (CZK)	15 122
Internal rate of return (%)	22.69

Table 11.28: Induction stove results.

The what-if analysis is presented in the figure below. The discount rate and change in investment are affecting the NPV as predicted. The inflation coefficient has a negative affection on the NPV.

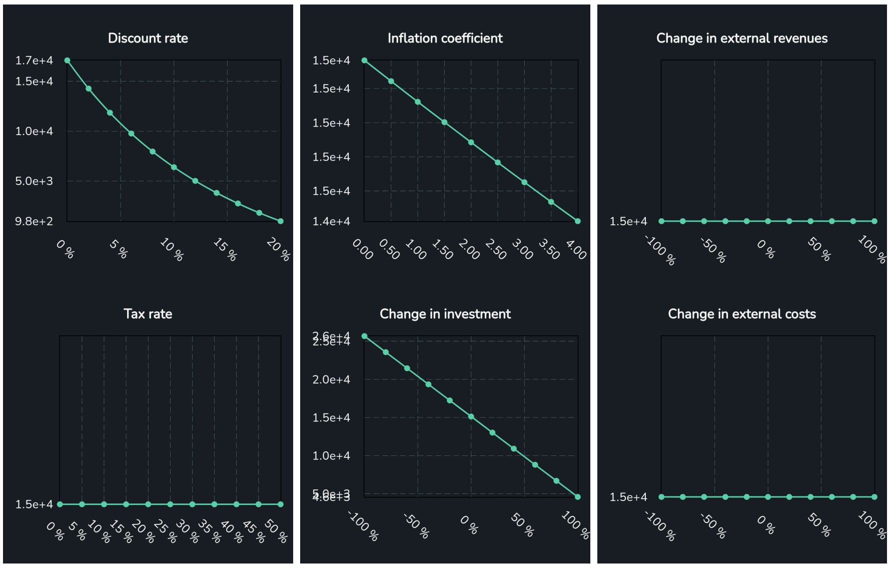

Figure 11.17: What if analysis of induction stove.

Note: The values on the y-axis are the NPV in CZK.

11.11 Dryer

The dryer is BEKO DPS 7405 G B5 with a consumption per cycle of 1.68 and a similar capacity of 7 kg that could handle the pack of cloats from the washing machine [10]. The number of cycles is the same as for the washing machine.

Figure 11.18: Dryer [10].

This analysis considers the external revenues as if the clothes are put into the dryer. They don't have to be hung. Therefore, the saved time is considered revenue. For estimation, the time saved per cycle is 0.15 hours, which is 9 minutes, and the time costs are 200 CZK per hour. The escalation coefficient is 1. The dryer costs 9 099 CZK. With the assembly of 699 CZK, the total investment is 9 798 CZK. The lifetime is estimated as a standard lifetime of the dryer [99]. The maintenance costs for a dryer are estimated at 300 CZK per year. The results are shown in the following table.

Years	10
Investment (CZK)	9 798
Payback period (years)	3
Discounted payback period (years)	4
Net present value (CZK)	20 033
Internal rate of return (%)	30.52

Table 11.29: Dryer results.

The discount rate is negatively affecting the NPV at a slightly higher rate. The inflation coefficient also affects the NPV negatively. The change in investment is also affecting negatively, where the 100 % increase in investment is decreasing the NPV by half. Here, the change in external revenues positively affects the final NPV as the externality is presented.

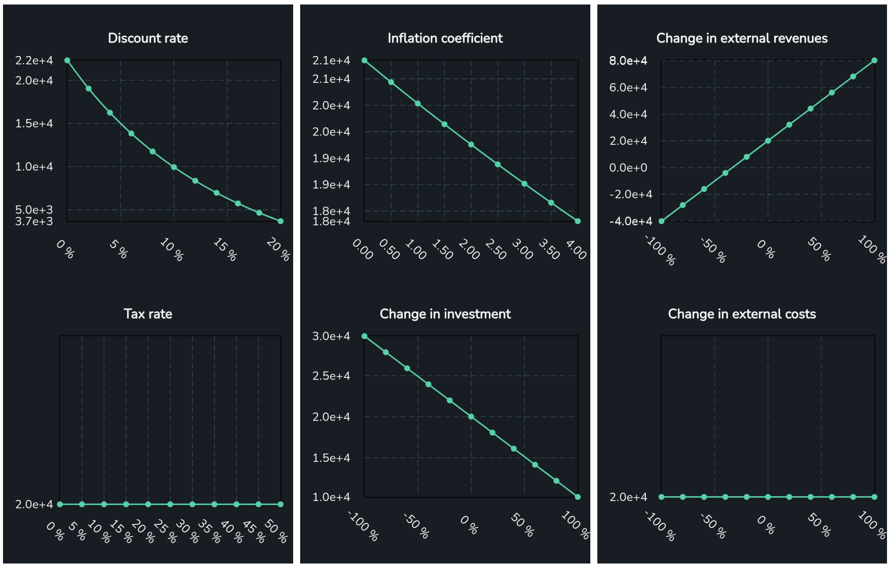

Figure 11.19: What if analysis of dryer.

Note: The values on the y-axis are the NPV in CZK.

11.12 Heat pump

The heat pump is NORDline N6B - 8,25 kW with the price and assembly costs 164 900 CZK [100]. As the heat pump sometimes requires additional construction work, the investment was estimated as 200 000 CZK.

Figure 11.20: Heat pump.

In case of a household mutation, the heat pump will be used for heating, heating water, and cooling. Therefore, the air conditioning and gas heating system will be removed and replaced with the new appliance heat pump with electricity energy source type. The heat pump efficiency is estimated as 3.5 (350 %) [101]. On the other hand, the efficiency of the gas heating system is estimated as 85 % [102]. Therefore, the annual electricity consumption to cover the gas heating system is calculated as follows:

$$11500 \cdot \frac{0.85}{3.5} \approx 2793 \text{ kWh}.$$

The air conditioning also has to be resolved. For simplification, the efficiency of air conditioning will be considered as 100 %. The annual electricity consumption from air conditioning is calculated as follows:

$$600 \cdot \frac{1}{3.5} \approx 171 \text{ kWh}.$$

The total annual consumption is then:

$$2793 + 171 = 2964 \text{ kWh}.$$

The lifetime is estimated at 20 years [103]. The maintenance costs are estimated at 1000 CZK per year. The results are presented in the table below.

Years	20
Investment (CZK)	200 000
Payback period (years)	16
Discounted payback period (years)	-1
Net present value (CZK)	-5 545
Internal rate of return (%)	1.08

Table 11.30: Dryer results.

The what-if analysis results are presented in the figure below. The discount rate affects the NPV in a negative way, where the NPV decreases with the increasing discount rate. The inflation coefficient has a positive effect on the NPV with its increase. The change in investment has a significant affection on the NPV.

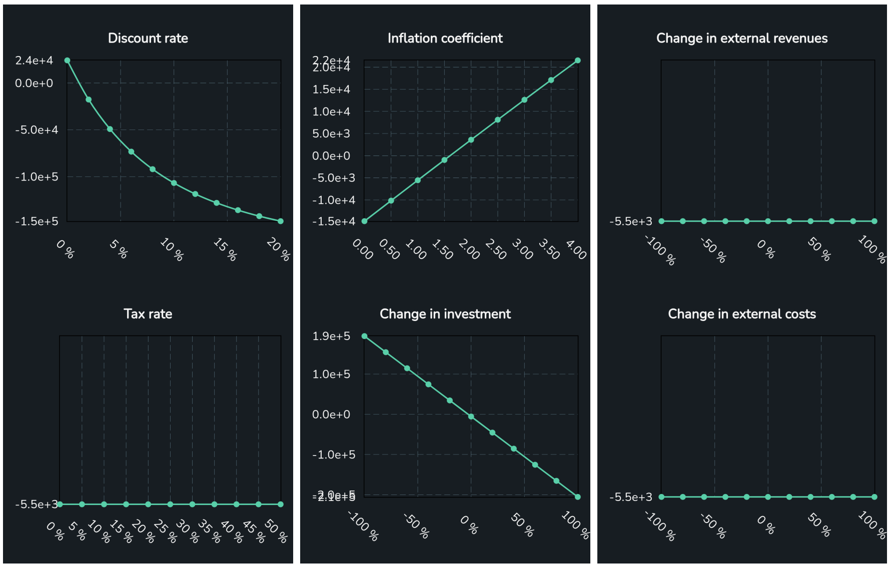

Figure 11.21: What if analysis of heat pump.

Note: The values on the y-axis are the NPV in CZK.

Chapter 12

Conclusion

Developing an application designed to conduct comprehensive technical and economic evaluations of household tasks presents a complex challenge that demands increased attention to various critical components. This endeavor requires thoughtful consideration, encompassing mathematical formulations, technical and economic considerations, software implementation, and user interface design.

Beginning with mathematical formulations, the foundation of the application's functionality lies in formulating algorithms that accurately assess and analyze household tasks. These algorithms must be robust and adaptable, accommodating diverse scenarios and variables inherent to household activities.

Moreover, through optimized task management strategies, household technical and economic impact must be integrated into the model, reflecting factors such as energy consumption, cost analysis, and potential savings. Incorporating economic variables adds depth to the application's analytical capabilities, empowering users to make informed decisions regarding resource allocation and efficiency improvements within their households.

Furthermore, translating these technical and economic analyses into a user-friendly interface is crucial to the application's development. The user interface serves as the primary point of interaction between the application and its users, shaping their experience and facilitating intuitive navigation through complex data and functionalities. Design considerations encompass visual aesthetics, usability principles, and accessibility standards to ensure that the interface effectively communicates information and empowers users to leverage the application's capabilities.

The thesis commences by elucidating energy audits, detailing their methodology and execution. Subsequently, it explores the certification of building energy efficiency and labeling appliances for energy consumption. The narrative then transitions to assessing existing mobile apps available on the Play Store, scrutinizing their functionality and user interfaces.

The thesis then delves into the mathematical and economic groundwork for developing the application's core mathematical model. These sections cover fundamental mathematical operations, extending to intricate financial calculations, focusing on matrices, vectors, real and nominal values, inflation, cash flow discounting, and key financial metrics like PP, DPP, NPV, and IRR.

A pivotal chapter then proceeds to construct the mathematical model of the application, leveraging insights from preceding chapters. This detailed model lays the foundation for the subsequent software implementation. Amidst these sections, a brief chapter elucidates the acquisition of irradiance data crucial for analyzing photovoltaic panels. It outlines two methods: one utilizing the PVGIS API and another explicitly crafted for the thesis, involving data interpolation from locations.

The second segment of the thesis elaborates on the actual software development process, detailing the technologies employed. The application uses the Flutter framework, utilizing Dart programming language for cross-platform compatibility across Android, iOS, web, and desktop environments. Employing the BLoC pattern for state management, the application seamlessly integrates business logic with the user interface. Additional packages like Freezed and GetIt enhanced the development workflow. Noteworthy features include support for dark and light themes, multilingual support (Czech and English), and customizable preferences without necessitating user authentication.

The final chapter showcases the application's practical utility by analyzing a sample house in Czechia. This involves setting up appliances, estimating consumption, adjusting for inflation and pricing variations, and presenting analyses and what-if scenarios via graphs and tables, highlighting key financial metrics such as PP, DPP, NPV, and IRR. The table below summarizes key financial metrics for sample house analyses.

Name	PP	DPP	NPV (CZK)	IRR (%)
New electricity pricing	0	0	59 118	12998.38
Switching off wifi router at night	0	0	7 857	3250.20
New fridge	-1	-1	-7 037	-4.99
New dishwasher	-1	-1	-5 867	-6.67
New washing machine	-1	-1	-4 306	-10.21
New LED light bulbs	1	1	47 634	288.27
Induction stove	4	4	15 122	22.69
Dryer	3	4	20 033	30.52
Heat pump	16	-1	-5 545	1.08

Table 12.1: Example house analysis summary.

The development process for the application proved to be a lengthy endeavor, encompassing intricate considerations regarding the mathematical core, data acquisition, and presentation to the end-user. Iterative refinement was essential, with numerous parts undergoing rewriting to align with the envisioned functionalities. A key challenge lies in establishing the robust mathematical foundation upon which the application operates. Unlike traditional spreadsheet models, where bugs can be promptly identified through direct observation, coding necessitated a meticulously crafted mathematical model to ensure accuracy.

Navigating the constraints of mobile space posed additional challenges, particularly concerning the visualization of line charts. Adjustments, such as rotating axis labels and relocating titles, were often required to optimize display within the limited screen on mobile devices. Despite these challenges, leveraging a programming language facilitated the creation of unit tests, providing a crucial advantage in validating mathematical operations and model components. This capability benefits complex models, offering assurance during development and future maintenance.

To summarize, ongoing efforts are crucial to address potential bugs and optimize feature performance post-release. Continuous updates will be pivotal in refining the application's functionality and user experience. A website under the domain "save-energy.cz" was established to complement the application, offering additional resources and information. The QR code below directs users to this website, facilitating access to comprehensive details about the application and related services.

Figure 12.1: Application web QR code.

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Appendix A

Physics fundamentals

The upcoming appendix introduces fundamental physics principles, initially focusing on the laws of thermodynamics and heat transfer mechanisms. Subsequently, the appendix delves into the explanation of Fourier's law of heat conduction and the heat equation. Finally, material properties are introduced. This appendix aims to equip readers with the indispensable foundational knowledge necessary for understanding the energy evaluation applications discussed in subsequent work.

A.1 Laws of thermodynamics

The laws of thermodynamics are fundamental principles that govern the energy behavior in a system. The laws of thermodynamics are essential for understanding the energy transfer and conversion processes.

A.1.1 First law of thermodynamics

According to [104], energy can neither be created nor destroyed in a system of constant mass, but it can be converted from one form to another. For example, a mass of gas contained within a cylinder fitted with frictionless piston at constant temperature and heat q supplied to the system. By absorbing this heat, the gas performs the amount of work w by expansion. The following equation gives the first law of thermodynamics:

$$\Delta U = q - w, \qquad \text{(J)} \quad \text{(A.1)}$$

where:
ΔU is the change in internal energy (J),
q is the heat added to the system (J),
w is the work done by the system (J) [104].

A.1.2 Second law of thermodynamics

The second law of thermodynamics states that the total entropy of an isolated system can never decrease over time [105, p. 585]. The entropy is a measure of the disorder or randomness of a system defined as:

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}, \qquad \text{(J/K)} \quad \text{(A.2)}$$

where:
ΔS is the change in entropy (J/K),
S_f is the final entropy (J/K),
S_i is the initial entropy (J/K),
i is the initial state,
f is the final state,
dQ is the heat transferred to or from the system (J),
T is the temperature of the system (K) [105, p. 585].

A.1.3 Third law of thermodynamics

According to [106], the entropy of an isolated system approaches a constant value as the system's temperature approaches absolute zero (0 K), the coldest temperature possible. This temperature cannot be reached in practice.

A.2 Heat transfer mechanisms

Heat transfer mechanisms govern the transfer of heat from one place to another. It can be defined as a movement of heat across the system's border due to a temperature difference between the system and its surroundings [107]. There are three main mechanisms of heat transfer: conduction, convection, and radiation.

A.2.1 Conduction

According to [108], conduction is a movement of kinetic energy in materials from a higher temperature to a lower temperature through a substance. It results from direct contact with those materials. The molecules will give their energy to adjacent molecules until an equilibrium is reached. Fourier's law of heat conduction describes the heat transfer by conduction. According to [107], the examples of heat conduction are the following:

- Ironing of clothes, where the heat is conducted from the iron to the clothes.
- Transferring heat from hands to ice cube.

A.2.2 Convection

Convection is the transfer of heat via a fluid (liquid or gas) by the movement of the fluid itself [109]. When the fluid moves from one place to another, it carries heat. According to [109], the examples of convection are following:

- Heating a room where the warm air rises and circulates around the room.
- Boiling water on a stove, where the hot water rises.

A.2.3 Radiation

Radiation is the transfer of heat through electromagnetic waves, which does not require a medium to transfer heat According to [107]. According to [107], the examples of radiation are following:

- UV rays coming from the sun.
- Microwave oven.

A.3 Fourier's law of heat conduction

Fourier's law is an essential principle that describes the heat transfer by conduction [110]. It states that the heat flux is proportional to the negative temperature gradient, meaning if we have two points with different temperatures, the heat will flow from the hotter point to the colder point [110]. The following equation gives the differential form of Fourier's law:

$$\vec{q} = -k \cdot \vec{\nabla} T, \qquad (\mathrm{W/m^2}) \quad \text{(A.3)}$$

where:
$\vec{q}$ is the heat flux ($\mathrm{W/m^2}$),
k is the thermal conductivity ($\mathrm{W/(m \cdot K)}$),
$\vec{\nabla} T$ is the temperature gradient (K/m) [110].

The following equation gives the one-dimensional form of Fourier's law:

$$q = -k \cdot \frac{dT}{dx}, \qquad (\mathrm{W/m^2}) \quad \text{(A.4)}$$

where:
q is the heat flux ($\mathrm{W/m^2}$),
k is the thermal conductivity ($\mathrm{W/(m \cdot K)}$),
$\frac{dT}{dx}$ is the temperature gradient (K/m) [110].

A.4 Heat equation

The heat equation is a partial differential equation describing the heat distribution in a given region over time [111]. The heat equation is derived from the first law of thermodynamics and Fourier's law of heat conduction [111]. The following equation gives the heat equation:

$$\frac{\partial T}{\partial t} = \alpha \cdot \nabla^2 T, \qquad (\mathrm{K/s}) \quad \text{(A.5)}$$

where:
$\frac{\partial T}{\partial t}$ is the rate of change of temperature concerning time (K/s),
α is the thermal diffusivity (m^2/s),
∇^2 is the Laplacian operator [112],
T is the temperature (K),
$\nabla^2 T$ is the Laplacian of temperature (K/m^2) [111].

A.5 Material properties

Material properties describe how well material stores and conducts heat. The most important material properties are heat capacity, thermal conductivity, thermal diffusivity, thermal resistance, and heat transfer coefficient.

A.5.1 Heat capacity

Heat capacity is a material property that describes the amount of heat required to raise the temperature of a unit mass of a material by one degree Celsius (Kelvine) [113]. The heat capacity is represented by the symbol c, with units of $J/(kg \cdot K)$ [113].

A.5.2 Thermal conductivity

Thermal conductivity is a material property that describes the ability of a material to conduct heat. The higher the thermal conductivity, the better the material conducts heat [114]. In Fourier's law, the thermal conductivity is represented by the symbol k. However, the λ symbol is commonly used to represent the thermal conductivity. The unit of thermal conductivity is $W/(m \cdot K)$ or in SI units $J/(s \cdot m \cdot K)$ [114].

Usually, when buying a material, the manufacturer provides thermal conductivity. The thermal conductivity of some common materials is shown in the following table.

Material	Thermal conductivity at 25 °C ($W/(m \cdot K)$)
Air	0.024
Wood (oak)	0.17
Steel	43
Aluminum	205
Water	0.58
Concrete	1.7

Table A.1: Thermal conductivity of common materials [21].

A.5.3 Thermal diffusivity

Thermal diffusivity is a material property that describes the ability of a material to conduct heat relative to its ability to store heat [115]. The thermal diffusivity is calculated using the following equation:

$$\alpha = \frac{\lambda}{\rho \cdot c_p}, \qquad (\mathrm{m}^2/\mathrm{s}) \quad (A.6)$$

where:
α is the thermal diffusivity (m^2/s),
λ is the thermal conductivity ($\mathrm{W}/(\mathrm{m}\cdot\mathrm{K})$),
ρ is the density (kg/m^3),
c_p is the specific heat capacity ($\mathrm{J}/(\mathrm{kg}\cdot\mathrm{K})$) [115].

A.5.4 Thermal resistance

Thermal resistance describes the ability of a material to resist the flow of heat [116]. The thermal resistance is calculated using the following equation:

$$R = \frac{d}{\lambda}, \qquad (\mathrm{m}^2\cdot\mathrm{K}/\mathrm{W}) \quad (A.7)$$

where:
R is the thermal resistance ($\mathrm{m}^2\cdot\mathrm{K}/\mathrm{W}$),
d is the thickness of the material (m),
λ is the thermal conductivity ($\mathrm{W}/(\mathrm{m}\cdot\mathrm{K})$) [116].

A.5.5 Heat transfer coefficient

The average heat transfer coefficient is usually calculated for the building to determine the heat loss. The heating system can be designed according to the calculated average heat transfer coefficient. The average heat transfer coefficient of the building envelope determines the average heat losses per meter square of the building envelope when the temperature difference between the inside and outside of the building is 1 K [117]. The average heat transfer coefficient is calculated using the following equation:

$$U_m = \frac{1}{A}\sum_i U_i \cdot A_i, \qquad (\mathrm{W}/(\mathrm{m}^2\cdot\mathrm{K})) \quad (A.8)$$

where:
U_m is the average heat transfer coefficient ($\mathrm{W}/(\mathrm{m}^2\cdot\mathrm{K})$),
A is the total area of the building envelope (m^2),
U_i is the heat transfer coefficient of the i-th part of the building envelope ($\mathrm{W}/(\mathrm{m}^2\cdot\mathrm{K})$),
A_i is the area of the i-th part of the building envelope (m^2) [117].

The partial heat transfer coefficient is calculated using the following equation:

$$U_i = \frac{1}{R_i}, \qquad (\mathrm{W}/(\mathrm{m}^2\cdot\mathrm{K})) \quad (A.9)$$

where:
R_i is the thermal resistance of the i-th part of the building envelope ($m^2 \cdot K/W$) [117].

Appendix B

Mathematical background

This appendix describes the mathematical background used in the work. The following sections describe the basic operations with vectors and matrices, the Kronecker delta, sequences, linear regression, degrees, radians, spherical coordinates, and Newton's method.

B.1 Vectors

The following section describes the basic operations with vectors, including addition, subtraction, and scalar multiplication.

B.1.1 Addition

If we have two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, we can add them by adding the corresponding elements of each vector. According to [118], the following formula does this.

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{pmatrix} \in \mathbb{R}^n. \tag{B.1}$$

B.1.2 Subtarction

If we have two vectors $\vec{a}$ and $\vec{b}$ of the same dimension n, we can subtract them by subtracting the corresponding elements of each vector. According to [119], the following formula does this:

$$\vec{a} - \vec{b} = \begin{pmatrix} a_1 - b_1 \\ \vdots \\ a_n - b_n \end{pmatrix} \in \mathbb{R}^n. \tag{B.2}$$

B.1.3 Scalar multiplication

If we have a vector $\vec{a}$ of dimension n and a scalar $k \in \mathbb{R}$, we can multiply the vector by the scalar by multiplying each element of the vector by the scalar.

According to [120], the following formula does this:

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ \vdots \\ k \cdot a_n \end{pmatrix} \in \mathbb{R}^n. \tag{B.3}$$

B.2 Matrices

The following section describes the basic operations with matrices, including addition, subtraction, multiplication, scalar multiplication, vector composition, and transposition.

B.2.1 Addition

The addition between two matrixes $\mathbb{A}$ and $\mathbb{B}$ of the same dimensions $n \times m$ is done by adding the corresponding elements of each matrix. The following formula does this [121]:

$$\begin{aligned} \mathbb{A} + \mathbb{B} &= \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{pmatrix} = \\ &= \begin{pmatrix} a_{11} + b_{11} & \cdots & a_{1m} + b_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} + b_{n1} & \cdots & a_{nm} + b_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \end{aligned} \tag{B.4}$$

B.2.2 Subtraction

The subtraction between two matrixes $\mathbb{A}$ and $\mathbb{B}$ of the same dimensions $n \times m$ is done by subtracting the corresponding elements of each matrix. The following formula does this [122]:

$$\begin{aligned} \mathbb{A} - \mathbb{B} &= \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} - \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{pmatrix} = \\ &= \begin{pmatrix} a_{11} - b_{11} & \cdots & a_{1m} - b_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} - b_{n1} & \cdots & a_{nm} - b_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \end{aligned} \tag{B.5}$$

B.2.3 Multiplication

The multiplication between two matrixes $\mathbb{A} \in \mathbb{R}^{n \times m}$ and $\mathbb{C} \in \mathbb{R}^{m \times p}$ is done by multiplying the rows of the first matrix by the columns of the second matrix. The following formula does this [123]:

$$\mathbb{A} \cdot \mathbb{C} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \cdot \begin{pmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mp} \end{pmatrix} =$$
$$= \begin{pmatrix} a_{11} \cdot c_{11} + \cdots + a_{1m} \cdot c_{m1} & \cdots & a_{11} \cdot c_{1p} + \cdots + a_{1m} \cdot c_{mp} \\ \vdots & \ddots & \vdots \\ a_{n1} \cdot c_{11} + \cdots + a_{nm} \cdot c_{m1} & \cdots & a_{n1} \cdot c_{1p} + \cdots + a_{nm} \cdot c_{mp} \end{pmatrix} \in \mathbb{R}^{n \times p}. \tag{B.6}$$

B.2.4 Scalar multiplication

The scalar multiplication of a matrix $\mathbb{A} \in \mathbb{R}^{n \times m}$ by a scalar k is done by multiplying each element of the matrix by the scalar. The following formula does this [124]:

$$\begin{aligned} k \cdot \mathbb{A} = k \cdot \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} = \\ = \begin{pmatrix} k \cdot a_{11} & \cdots & k \cdot a_{1m} \\ \vdots & \ddots & \vdots \\ k \cdot a_{n1} & \cdots & k \cdot a_{nm} \end{pmatrix} \in \mathbb{R}^{n \times m}. \end{aligned} \tag{B.7}$$

B.2.5 Vector composition

The vector composition is a special case of matrix multiplication, where the second matrix is a vector. The vector composition between a matrix $\mathbb{A} \in \mathbb{R}^{n \times m}$ and a vector $\vec{b} \in \mathbb{R}^m$ is done by multiplying the rows of the matrix by the vector. The following formula does this [125]:

$$\begin{aligned} \mathbb{A} \cdot \vec{b} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} = \\ = \begin{pmatrix} a_{11} \cdot b_1 + \cdots + a_{1m} \cdot b_m \\ \vdots \\ a_{n1} \cdot b_1 + \cdots + a_{nm} \cdot b_m \end{pmatrix} \in \mathbb{R}^n. \end{aligned} \tag{B.8}$$

B.2.6 Transposition

The transposition of a matrix $\mathbb{A} \in \mathbb{R}^{n \times m}$ is done by switching the rows with the columns. The following formula does this [126]:

$$\mathbb{A}^T = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}^T = \begin{pmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & \ddots & \vdots \\ a_{1m} & \cdots & a_{nm} \end{pmatrix} \in \mathbb{R}^{m \times n}. \tag{B.9}$$

B.3 Kronecker delta

The Kronecker delta is a discrete version of the delta function defined by the following formula [127]:

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases} \tag{B.10}$$

B.4 Sequence

The sequence is a list of numbers in a specific order. The sequence can be finite or infinite [128]. The following formula defines the sequence [128]:

$$(a_k)_{k=1}^{n} = (a_1, a_2, \ldots, a_n). \tag{B.11}$$

B.5 Linear regression

Suppose we have a vector $\vec{x}$ of dimension n of independent variables and a vector $\vec{y}$ of dimension n of dependent variables. We want to calculate the linear regression coefficients β_0 and β_1 for the linear regression model $y = \beta_0 + \beta_1 \cdot x$, such that the sum of the squared residuals (errors) is minimized. The residuals and square residuals are shown in the following figures:

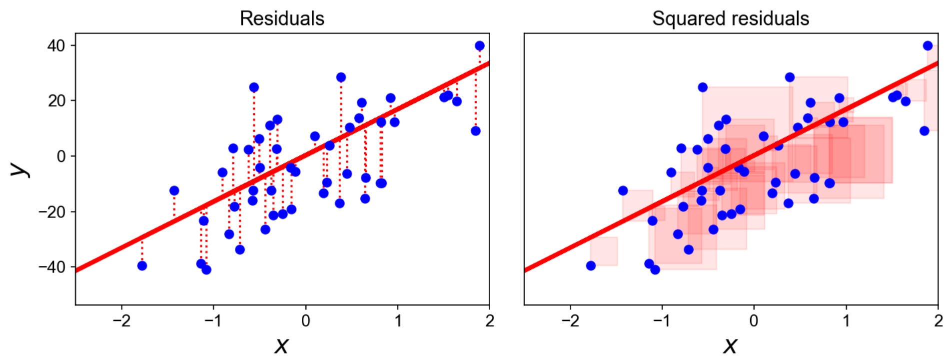

Figure B.1: Residuals and square residuals [12].

This method is called the ordinary least squares (OLS), and the problem is to find the coefficients β_0 and β_1 that minimize the following function:

$$f(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 \cdot x_i))^2. \tag{B.12}$$

For minimization, we can set the partial derivatives of the function $f(\beta_0, \beta_1)$ concerning the coefficients β_0 and β_1 to zero. The partial derivatives are shown in the following formulas:

$$\frac{\partial f}{\partial \beta_0} = 2\sum_{i=1}^{n}(y_i - (\beta_0 + \beta_1 \cdot x_i)) = 0, \tag{B.13}$$

$$\frac{\partial f}{\partial \beta_1} = 2\sum_{i=1}^{n} x_i \cdot (y_i - (\beta_0 + \beta_1 \cdot x_i)) = 0. \tag{B.14}$$

The solution of the system of equations is shown in the following formulas [12]:

$$\begin{aligned} \beta_1 &= \frac{\sum_{i=1}^{n}(x_i - \overline{x}) \cdot (y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}, \\ \beta_0 &= \overline{y} - \beta_1 \cdot \overline{x}. \end{aligned} \tag{B.15}$$

Here the $\overline{x}$ and $\overline{y}$ are the arithmetic mean values of the vectors $\vec{x}$ and $\vec{y}$ and are defined by the following formulas:

$$\begin{aligned} \overline{x} &= \frac{1}{n}\sum_{i=1}^{n} x_i, \\ \overline{y} &= \frac{1}{n}\sum_{i=1}^{n} y_i. \end{aligned} \tag{B.16}$$

B.6 Degrees and radians

The degrees and radians are two units used to measure the angles. The degrees are used in everyday life, but the radians are used in mathematics. The following formulas do the conversion between the degrees and radians [129]:

$$\text{degrees} = \text{radians} \cdot \frac{180}{\pi}. \tag{B.17}$$

B.7 Spherical coordinates

According to [130], the spherical coordinates are a system of coordinates used to locate a point in space by using the distance from the origin and two angles. The following formula defines the conversion between the spherical coordinates and the Cartesian coordinates:

$$\begin{aligned} x &= r \cdot \sin(\theta) \cdot \cos(\phi), \\ y &= r \cdot \sin(\theta) \cdot \sin(\phi), \\ z &= r \cdot \cos(\theta). \end{aligned} \tag{B.18}$$

B.8 Newton's method

Newton's method is an iterative method for finding the roots of the function. The following formula defines Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{B.19}$$

where:
x_n is the n-th approximation of the root,
x_{n+1} is the $(n+1)$-th approximation of the root,
$f(x_n)$ is the value of the function at the n-th approximation,
$f'(x_n)$ is the value of the function's derivative at the n-th approximation [131].

If the derivative is not known and must be computed numerically in each point as:

$$f'(x_n) = \frac{f(x_n + h) - f(x_n)}{h}, \tag{B.20}$$

where:
h is the step size.

Then the, Newton's method can be rewritten as:

$$x_{n+1} = x_n - \frac{f(x_n) \cdot h}{f(x_n + h) - f(x_n)}. \tag{B.21}$$

Appendix C

Application assets

C.1 Play store

C.1.1 Feature graphic

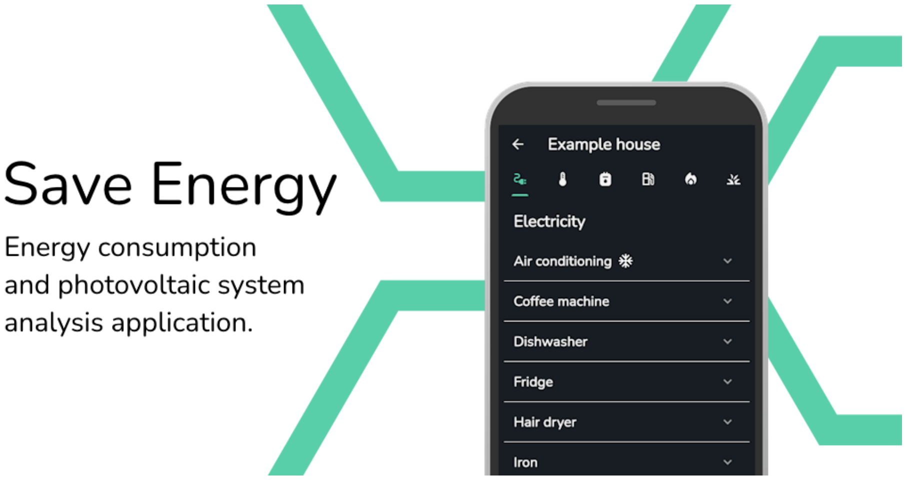

Figure C.1: Play store feature graphic.

C.1.2 Phone screenshots

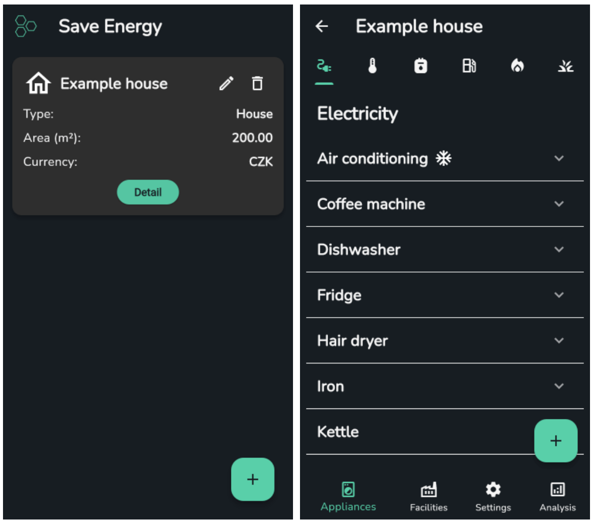

Figure C.2: Play store phone screenshots.

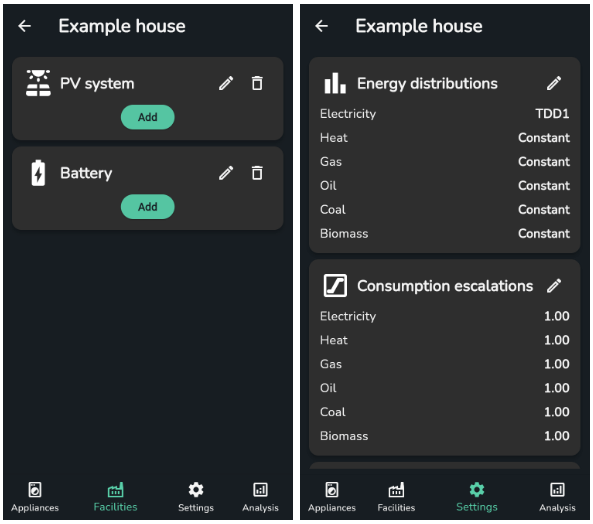

Figure C.3: Play store phone screenshots.

C.1.3 Tablet screenshots

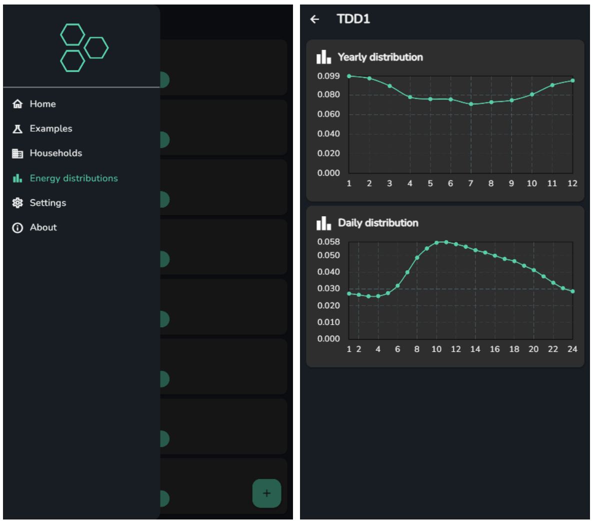

Figure C.4: Play store tablet screenshots.

C.2 Other assets

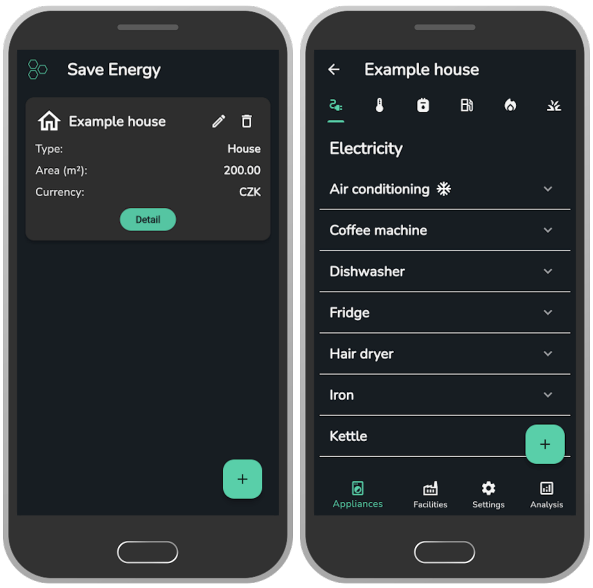

Figure C.5: Phone graphic.

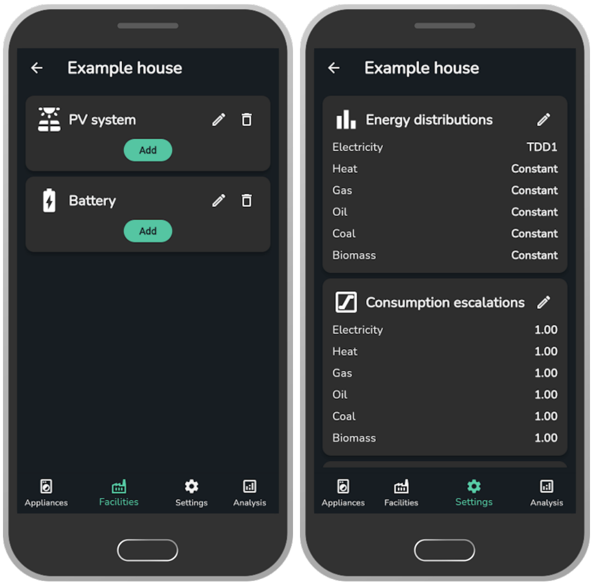

Figure C.6: Phone graphic.

Appendix D

Content of Enclosed CD

- **save_energy** - the source code of the application.
- **pvgis** - the source code for creating the PVGIS Lite database.