Numerical solution of turbulent flow using DES model with mesh adaptation

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Introduction

Resolving turbulent flow is challenging but important problem for various applications. The standard approaches like the one based on Reynolds-averaged Navier–Stokes (RANS) equations and large eddy simulation (LES) have their disadvantages: RANS does not resolve all turbulent length scales properly and LES can become computationally very demanding, especially around walls where high resolution of computational grids is required. RANS-LES hybrid methods try to mitigate those disadvantages by employing a RANS model around walls and resolving detached eddies with LES.

Various such approaches can be utilized, e.g. detached eddy simulation (DES) and its delayed (DDES) and improved delayed (IDDES) variants. The thesis proposes new DDES and IDDES approaches based on Kok's X-LES model, named DX-LES and IDX-LES. Also, since the switching between RANS and LES mode is dependent on the grid spacing, this thesis proposes the use of adaptive mesh refinement (AMR) in combination with these hybrid methods.

Mathematical Model

The RANS equations, which can be found at [1], are the basis of the model used in the thesis. Several assumptions are made, e.g. Reynolds analogy of Fourier's law of heat conduction to obtain the turbulent heat flux, the fluid is compressible and behaves according to the ideal gas thermodynamic model, which also gives the equation of state. Furthermore, Boussinesq approximation is utilized and the dynamic viscosity μ is approximated through the use of Sutherland's Law. The RANS equations is closed not only by the equation of state, but also by the turbulence model, which is needed to obtain the eddy viscosity μ_T. All the turbulence models used in the thesis are based on two RANS models: Menter's Shear Stress Transport (SST) and Kok's Turbulent/Non-Turbulent (TNT), both described by equations where the turbulent kinetic energy k and the specific turbulence dissipation rate ω are the unknowns, multiplied by the fluid density ρ in their conservative forms.

The equations of the SST-based models can be written as

$$\frac{\partial \rho k}{\partial t} + \operatorname{div}(\rho k \boldsymbol{u}) = \operatorname{div}\left([\mu + \sigma_k \mu_T] \operatorname{grad} k\right) + P_k - \rho k \frac{\sqrt{k}}{L_T}, \tag{1a}$$

$$\frac{\partial \rho \omega}{\partial t} + \operatorname{div}(\rho \omega \boldsymbol{u}) = \operatorname{div}\left([\mu + \sigma_\omega \mu_T] \operatorname{grad} \omega\right) + P_\omega - \beta \rho \omega^2 + 2(1 - F_1)\mathrm{CD}, \tag{1b}$$

where L_T represents the model length scale (which is the only parameter that differs between the hyrid models), P_k and P_ω are the production terms, CD is the diffusion, S is the strain tensor norm, F_1 and F_2 are the SST blending functions and d_w is the distance to the nearest wall, the model coefficients are σ_k, σ_ω, β, C_ω. The eddy viscosity is

$$\mu_T^{(\mathrm{SST})} = \frac{a_1 \rho k}{\max\left(a_1 \omega, F_2 S\right)}, \quad a_1 = 0.31. \tag{2}$$

The equations for the TNT-based models can be written as

$$\frac{\partial \rho k}{\partial t} + \operatorname{div}(\rho k \boldsymbol{u}) = \operatorname{div}\left([\mu + \sigma_k \mu_T] \operatorname{grad} k\right) + P_k - \rho k \frac{\sqrt{k}}{L_T}, \tag{3a}$$

$$\frac{\partial \rho \omega}{\partial t} + \operatorname{div}(\rho \omega \boldsymbol{u}) = \operatorname{div}\left([\mu + \sigma_\omega \mu_T] \operatorname{grad} \omega\right) + P_\omega - \beta \rho \omega^2 + C_D, \tag{3b}$$

where, once again, L_T is the model length scale, P_k and P_ω are the production terms and C_D represents the cross-diffusion term and σ_k, σ_ω, σ_d, α_ω, β the model constants. Finally, the eddy viscosity for the TNT-based models is dependent on the model length scale:

$$\mu_T^{(\mathrm{TNT})} = \beta^* \rho \sqrt{k} L_T, \quad \beta^* = 0.09. \tag{4}$$

The model scales are given by:

$$l_{\mathrm{RANS}} = \frac{\sqrt{k}}{\beta^* \omega}, \quad l_{\mathrm{LES}} = C_{\mathrm{DES}} \Delta, \quad \hat{l}_{\mathrm{LES}} = C_{\mathrm{DES}} \hat{\Delta}, \tag{5}$$

where where Δ denotes a determined grid length, $\hat{\Delta} = \min\left\{C_w \max\left[d_w, \Delta\right], \Delta\right\}$ is used to introduce wall-modeled LES into the method, $C_w = 0.15$.

The model length scales for the base RANS method and the hybrid DES (X-LES), DDES (DX-LES) and IDDES (IDX-LES) models are:

$$L_T^{(\mathrm{RANS})} = l_{\mathrm{RANS}}, \quad L_T^{(\mathrm{DES})} = \min\left(l_{\mathrm{RANS}}, l_{\mathrm{LES}}\right),$$
$$L_T^{(\mathrm{DDES})} = l_{\mathrm{RANS}} - f_d \max\left(0, l_{\mathrm{RANS}} - l_{\mathrm{LES}}\right),$$
$$L_T^{(\mathrm{IDDES})} = \tilde{f}_d \left(1 + f_e\right) l_{\mathrm{RANS}} + \left(1 - \tilde{f}_d\right) \hat{l}_{\mathrm{LES}},$$

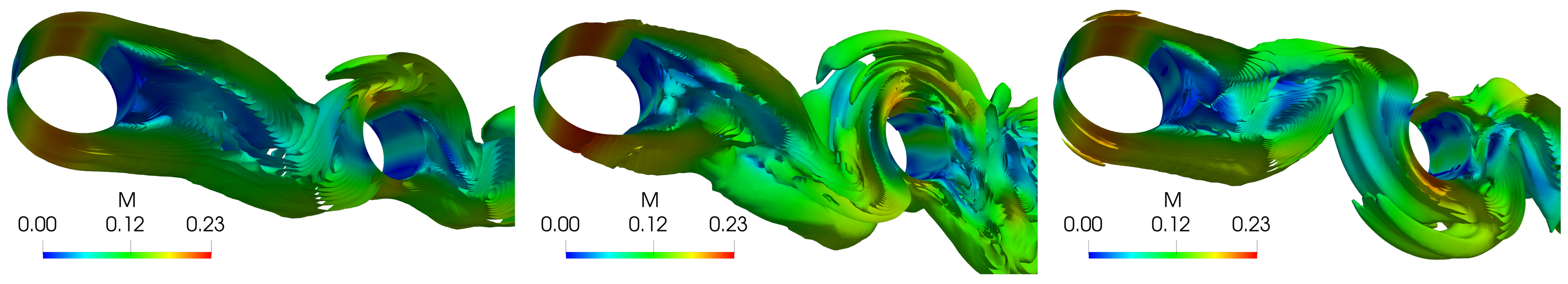
where f_d, $\tilde{f}_d$ and f_d are functions described in [2].

Numerical Methods

In-house software Orion was used for the computations in the thesis. It uses implicit formulation of finite volume method and utilizes parallel computing techniques. System given by RANS equations is solved separately from the Eq. (1) and (3). Convective fluxes of RANS system are approximated using the HLLC Riemann solver. Least square reconstruction with Barth-Jespersen limiter is then used to obtain convective fluxes. The derivatives of these fluxes required by the implicit scheme are then obtained using analytical relations. For the diffusive fluxes, the gradients of variables are computed from values in points of "diamond cell". Time discretization uses dual time stepping and local time stepping in the dual time. The resulting linear system is then solved using the GMRES method.

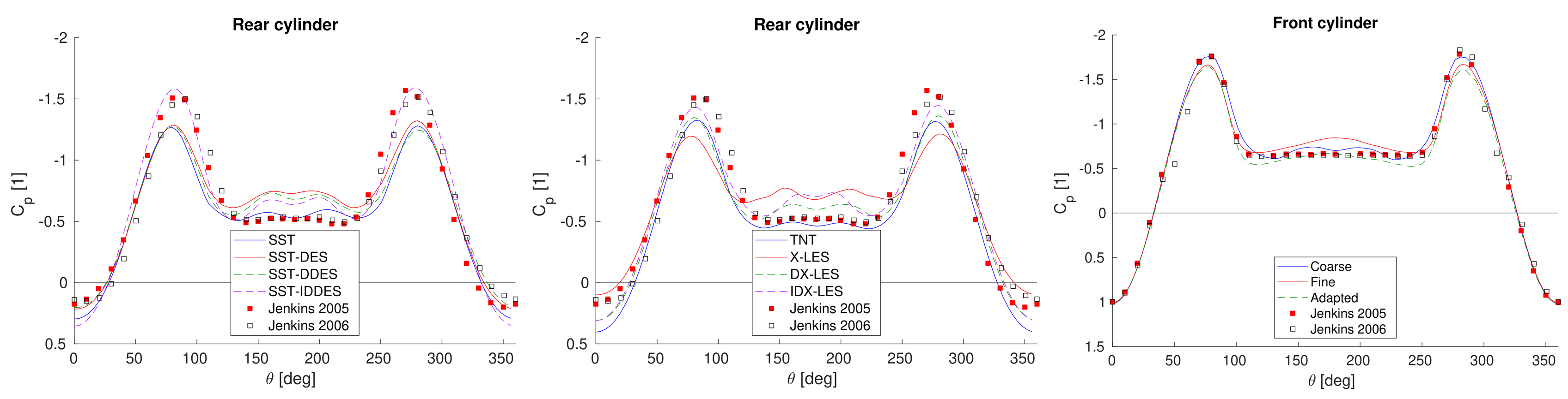
Results

The thesis presents various test cases, one of the being the tandem cylinder vortex shedding problem with experimental data from [3,4], measured for the purpose of aircraft landing gear development. The Reynolds number for this test case is 166,000. Contours of vorticity calculated by several different methods in Fig. 1 show symmetry for the TNT model in the direction parallel to the cylinder axes, as the RANS method did not properly resolve the fluctuations, unlike in the case of hybrid methods. A comparison of different methods with the experimental data for the pressure coefficient on the rear cylinder is then shown in Fig. 2 (a,b). The SST-IDDES and DX-LES methods are closest to the measured values.



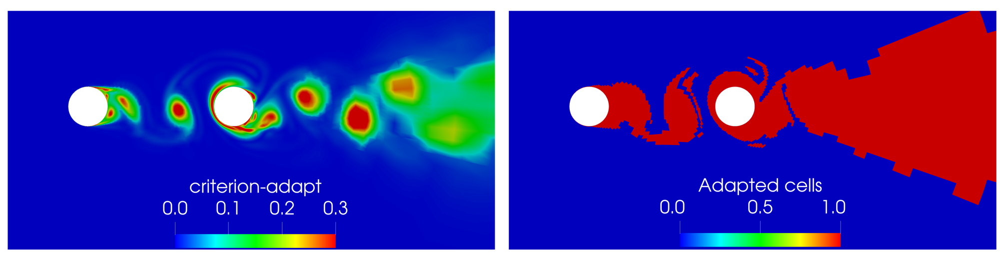
(a) *TNT* (b) *X-LES* (c) *DX-LES*

Figure 1: *Contours of vorticity colored by calculated Mach number obtained by various models.*



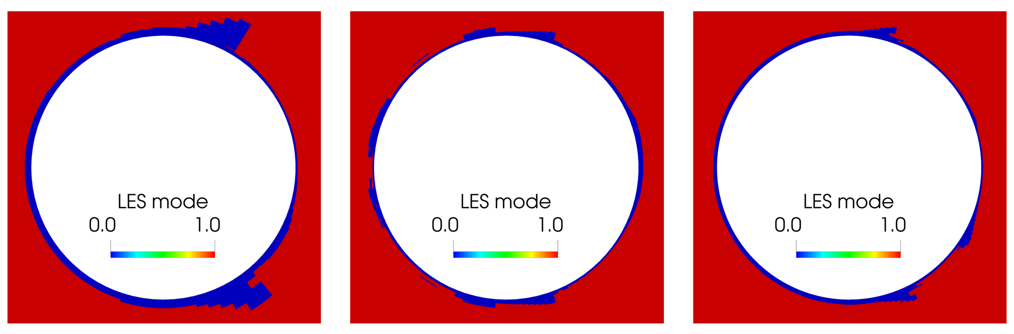
(a) *SST-based methods* (b) *TNT-based methods* (c) *Influence of AMR on SST-DES*

Figure 2: *Average pressure coefficient calculated by various methods on the rear (a,b) and front (c) cylinder.*



(a) *Criterion values* (b) *Adapted cells*

Figure 3: *Evaluation of the entropy criterion.*



(a) *Coarse grid* (b) *Adapted grid* (c) *Fine grid*

Figure 4: *RANS-LES switching around the front cylinder on different computational meshes using the SST-DES model: values of 0 and 1 indicate pure RANS or LES mode, respectively.*

The results with AMR on the tandem cylinder case were done on three different meshes: rough, fine (with approximately four times as many cells as the rough grid) and adapted (with approximately half of the cells as the fine grid at any given time). AMR switched based on values of deviation from a reference entropy, shown in Fig. 3. The results with AMR were close to the fine mesh, as seen in Fig. 2 (c). AMR also visibly affected RANS-LES switching, as demonstrated by Fig. 4.

Conclusions

The data show that a hybrid RANS-LES approach can bring significant improvement in results compared to the base RANS methods, with minimal additional costs. The delayed models, including the newly proposed DX-LES, also appear to be noticeably closer to the experimental data.

The adaptive mesh refinement also affected the computation with the hybrid method on the tandem cylinder problem, both in RANS-LES switching and in the quality of the results. The results on the adapted grid were in fact very close to the results obtained on the computational mesh with high resolution (and approximately twice as many cells) that would have been produced by fully adapting all cells at the start of the computation.

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References

[1] R. Christopher. Implementing Turbulence Models into the Compressible RANS Equations. https://turbmodels.larc.nasa.gov/implementrans.html, accessed 2023.

[2] M. Gritskevich, A. Garbaruk, J. Schütze, and F. Menter. Development of DDES and IDDES Formulations for the k-ω Shear Stress Transport Model. *Flow, Turbulence and Combustion*, 2012.

[3] L. Jenkins, M. Khorrami, M. Choudhari, and C. McGinley. Characterization of Unsteady Flow Structures Around Tandem Cylinders for Component Interaction Studies in Airframe Noise. In *11th AIAA/CEAS Aeroacoustics Conference*, 2005.

[4] L. Jenkins, D. Neuhart, C. McGinley, M. Khorrami, and M. Choudhari. Measurements of Unsteady Wake Interference Between Tandem Cylinders. In *36th AIAA Fluid Dynamics Conference and Exhibit*, 2006.