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REPORT OF A THESIS ADVISOR

Master's thesis title: "Extended theories of gravity and their cosmological and astrophysical applications"
Candidate: Kamil Mudruňka
Institution: Department of Physics, Faculty of Nuclear Science and Physical Engineering, Czech Technical University in Prague, Břehová 7, 115 19
Assessment: A (výborně)

It is a pleasure to report on Master's Thesis of Kamil Mudruňka. Mr. Mudruňka worked under my supervision throughout the period September, 2023 – May, 2024. During that time he focused his attention on the understanding of classical and quantum aspects of extended theories of gravity with a particular focus on Weyl conformal gravity.

Extended gravity theories have gained momentum in the last 20 years in conjunction with new high-precision astronomical observations and novel developments in Quantum Gravity. Particularly the concept of Weyl's conformal gravity was originally introduced in a series of papers by H. Weyl in 1918-20, even though the incentives can be traced back to A. Einstens' 1915 General Relativity. Broadly speaking, Weyl's conformal gravity is a pure metric theory that possesses general coordinate invariance, which augments standard General Relativity with the additional Weyl scale symmetry. Weyl gravity is due to its power-counting renormalizability and asymptotic freedom considered currently as one of the serious contenders for consistent Quantum Gravity.

In his Thesis was Mr. Mudruňka inspired by a number works on extended theories of gravity [1,2,3] and namely by papers of P.D. Mannheim *at al.,* [3,4,5] on Weyl conformal gravity and its ensuing applications. He approached the

subject of his Thesis with a wide knowledge of prerequisite mathematical concepts. The material for the thesis also undoubtedly benefited from the expertise Mr Mudruňka acquired during his two semesters at Leiden University, Holland, and Tohoku University in Sendai, Japan.

The Thesis by Mr. Mudruňka is certainly exceptional both in terms of its length (109 pages) and in its technical aspects (both analytical and numerical). Though the work itself is quite technical, it is well structured and the basic concepts are relatively easy to grasp. Chapters 1 and 2 provide the conceptual and technical background necessary to understand the rationale behind extending gravity beyond Einstein theory. In particular, Chapter 2 focuses on classical Weyl conformal gravity and it cosmological implications. In Chapter 3 Mr. Mudruňka derives and discusses the most important class of vacuum Petrov type D solutions in Weyl conformal gravity, namely spherically symmetric metrics. Here he used, in a rather innovative way, the Newman–Penrose formalism, which is a particular form of the tetrade formalism that allows for various technical simplifications. Apart from the well known black hole solutions, Mr. Mudruňka found a new exotic wormhole solution that asymptotically approaches the hyperbolic case of the FLRW metric. This is important original result that we hope to publish shortly. Chapters 4 and 5 are devoted to exploring the stability and robustness of some of the explicit solutions in Weyl conformal gravity with respect to small perturbations. Perturbations are considered both classical (Chapter 4) and due to gravitating quantum matter (Chapter 5). In particular, black hole horizon corrections in conformal gravity due to different quantum matter fields (Chapter 5) again represent new results worthy of publication. Finally, Chapter 6 briefly discusses existing approaches to the quantization of conformal gravity. The Thesis ends with six appendices which: a) give necessary technical background that is needed in the main body of the text and b) provide codes in Python for computations of relevant tensor quantities.

All in all, Master's Thesis of Mr. Mudruňka has in my opinion a very high quality. It offers a thorough and in many respects original selection and corresponding discussion of topics that are indispensable for a full-fledged research in the field of quantum gravity. The way in which the assigned problems have been dealt with demonstrates the candidate's astounding potential for independent creative work. Key aspects of both classical and quantum Weyl conformal gravity, and ensuing cosmological and astrophysical implications are worked out logically and clearly. Last but not least,

higher-derivative gravity is conceptually and numerically demanding endeavor within the steadily growing field of Quantum Gravity. I am sure that expertise gained by Mr. Mudruňka will be beneficial to him in the years to come.

[**1**] V. Faraoni and S. Capozziello, *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*, (Springer, Dordrecht, 2011).

[**2**] A. Kehagias, A. Moradinezhad Dizgah, and A. Riotto, *Remarks on the Starobinsky model of inflation and its descendants*, Phys. Rev. D **89** (2014) 043527.

[**3**] S. Alexander and N. Yunes, *Chern–simons modified general relativity*, Physics Reports **480** (2009) 1.

[**4**] P.D. Mannheim and D Kazanas, *Exact vacuum solution to conformal Weyl gravity and galactic rotation curves*, Astrophysical Journal **342** (1989) 635.

[**5**] P. D. Mannheim, *Comprehensive Solution to the Cosmological Constant, Zero- Point Energy, and Quantum Gravity Problems*, Gen. Rel. Grav. **43** (2011, 703.

[**6**] P.D. Mannheim, *Is dark matter fact or fantasy? — Clues from the data*, International Journal of Modern Physics D **28** (2019) 1944022.

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