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Report PhD thesis Jana Lepsǒvá

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The PhD thesis of Jana Lepsǒvá, submitted February 2024, titled

Substitutive structures in combinatorics, number theory, and discrete geometry

is dedicated to several topics in the general area of word combinatorics and aperiodic substitutions. A common thread of those topics is the study of the connection between aperiodic structures and numeration systems. It is clear for decades that there is a strong connection, and most experts will agree that there is still much more to explore than already known. This is carried out here by focussing on particular questions around the intersection of these fields.

More precisely, the thesis is based on five scientific papers coauthored by Mrs. Lepsǒvá (and contains material for further papers). Each of the five papers (I to V) corresponds to a chapter (3 to 7) in the thesis, and each treats a quite distinct aspect of the general topic. In particular, the explicit results are, with respect to the enumeration of the chapters:

(3) a generalization of the two's complement notation for integers [Knu69] to numeration systems based on simple Parry numbers, thus including a Fibonacci complement notation;

(4) a formula for the (finite and asymptotic) critical exponent for Arnoux-Rauzy words — certain (bi-)infinite words arising from particularly nice cut-and-project sets — together with the result that the minimal value among those is achieved by d-bonacci words;

(5) a faithful representation of the monoid of Sturmian morphisms by 3×3 matrices, in contrast to the well-known non-faithful representation by 2×2 matrices;

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(6) a generalization of numeration systems based on substitutions to non-negative integers, along the lines of the numeration systems in [DT89], recovering the Fibonacci complement notation;

(7) a description of a particular aperiodic Wang tiling by a deterministic automaton.

The thesis is written in a clear and precise style. All definitions and concepts are explained pretty well, and these explanations are supported by reasonable chosen examples and illustrations. The thesis shows a broad and firm knowledge of all the distinct concepts from mathematics and theoretical computer science needed here, as well as a certain maturity in their presentation.

All results are obtained by proper mathematical proofs. The proofs I probed are all correct and understandable. This part of the work is supported by numerical computations. (By the way, as an advise to Mrs. Lepsŏvá: if you check finitely many cases exhaustively by computer, please call it "numerical computations", or "exhaustive computer search", this sounds better — less random — than "computer experiments".)

The topics span a broad range within the field of word combinatorics and aperiodic order. The results are clearly state-of-the-art, which is reflected by the fact that they are already published (or accepted) in pretty decent scientific journals — in one or two cases, in renowned mathematical journals.

Overall I consider this work quite broad and substantial; it is more than sufficient for a PhD thesis. So it is a pleasure to recommend this PhD thesis for presentation and defense.

Best regards,

Dr. habil. Dirk Frettlöh