Thesis director review on the thesis
Substitutive structures in combinatorics, number theory, and discrete geometry
by Ing. Jana Lepšová

Let me start my review in a bit long-winded way. I met Jana Lepšová for the first time when she was in her first year of our faculty and I taught her Linear Algebra A. Let me explain that at our faculty we have 3 levels of mathematics, A, B, and C, where the level A is appropriate for future mathematicians, and is therefore quite demanding. She was one of the best students and passed difficult exams with excellent results. Moreover, she was working hard continuously and she was the best solver of the tricky problems that I was assigning to students as homework during the whole course. She lodged at that time in my mind and I was not surprised at all when I repeatedly heard my colleagues praise Jana for her results in their courses. I am sure that any teacher who came in touch with Jana remembers her as one of the best, most enthusiastic and hard working students.

During her bachelor and master studies, she was working in the domain of Mathematical Modelling, in particular on Reaction-Diffusion Equations in Electrocardiology. At that time she became experienced in programming. I am pleased that our paths crossed again before the end of her master studies. She decided to change the domain of her studies and to move from Applied Mathematics to Theoretical Informatics. She came to realize that during her bachelor and master studies, she liked the most subjects like Linear Algebra, Graph Theory, Number Theory, Discrete Mathematics, and she was always an enthusiastic solver of open problems. As I myself did my Ph.D. studies "en cotutelle", I deeply encouraged Jana in her decision to do the same.

Jana met my expectations. During her Ph.D. studies under the supervision of Sébastian Labbé from LaBRI (Laboratoire bordealais de recherche en informatique) of University of Bordeaux, and of myself, she managed to get familiar with three related but distinct mathematical domains: combinatorics on words, number theory and discrete geometry.

I would like to underline at this point that Sébastian Labbé played a more important role than I did during Jana's Ph.D. This can be explained quite naturally. Sébastien is an ethusiastic and excellent researcher and a good teacher of both mathematics and programmimg. During the first year of her Ph.D. studies, Jana became deeply involved in nice problems in discrete geometry and

numeration systems and she continued working on this topic during two years. Then we managed to find some interesting results in combinatorics on words together, too. But I want to use this place to express my gratitude to Sébastien for his ability to make Jana excited for mathematics and to teach her new deep mathematical concepts in a very short time.

Let me now summarize the content of the thesis (which is a self-contained carefully written text, not only a list of papers). Each chapter is based on results, where Jana is a co-author, that have been already published (4 papers) or are under review (1 paper). In each chapter, Jana adds some new results made on her own, in several cases answering open problems.

Concerning combinatorics on words, we investigated repetitions in infinite words, which is an extensively studied topic. The minimum repetition rate of factors is expressed by the critical exponent, and the minimum repetition rate of factors with increasing length is measured by asymptotic critical exponent. We found a formula to determine both the critical and asymptotic critical exponent of regular Arnoux–Rauzy words. With the help of this formula, we proved that the minimal critical and minimal asymptotic critical exponent among regular d-ary Arnoux–Rauzy words is attained by the d-bonacci word.

With Edita Pelantová and Štěpán Starosta, a faithful representation of the special Sturmian monoid by 3×3 matrices was presented. It enabled to tackle the question of the square roots of fixed points of morphisms in the special Sturmian monoid.

With Sébastien Labbé, Jana studied positional numeration systems for both nonnegative and negative integers: they defined an analogue of the two's complement notation for $\mathbb{Z}$ based on the sequence of Fibonacci numbers and they studied its properties with respect to addition. They recovered this positional numeration system in another context of numeration systems which describe fixed and periodic points of substitutions as automatic sequences. Jana and Sébastien studied also discrete geometry, more precisely Wang tilings. Using the Fibonacci complement numeration system extended to $\mathbb{Z}^2$, they characterized a particular tiling of the plane as an automatic sequence.

During her Ph.D. studies, Jana managed to be co-author of 5 papers (4 of them already published). She presented her results with great success

- at several conferences:
    - WORDS 2021 – online, presentation: A numeration system for Fibonacci-like Wang shifts
    - Journées SDA2 2021 in Caen – presentation: A numeration system for Fibonacci-like Wang shifts
    - Journées du GDR-IM 2022 in Lille – poster
    - 18e Journées Montoises d'Informatique Théorique 2022 in Prague – presentation: A Fibonacci's complement numeration system
    - WORDS 2023 in Umea, Sweden – presentation: Critical exponent of Arnoux-Rauzy sequences
    - Journées de combinatoire de Bordeaux 2024 – presentation: Dumont-Thomas numeration systems for $\mathbb{Z}$ (invited speaker)

- at seminars:
  - seminar of the group Combinatoire et interactions, 2021 – presentation: A numeration system for Fibonacci-like Wang shifts
  - online seminar One World Numeration, 2023 – presentation: Dumont-Thomas numeration systems for $\mathbb{Z}$
  - Doctoral days 2020, 2021, 2022, 2023
  - seminar of the group TIGR Combinatorics on Words, 2020, 2021, 2022, 2023

Last but not least, Jana is a multitalented person. Besides her great results in mathematics, she has been studying in distance Conservatoire in Pardubice (conducting) since 2019. During student projects or student competitions, she conducted the Chamber Philharmonic Orchestra in Pardubice several times and also the Symphonic Orchestra of Karlovy Vary and the Symphonic Orchestra of Conservatoire of Pardubice.

To summarize,

I consider this thesis to be excellent and even outstanding
(both for its scientific content and its high standard of writing).

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