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Teze disertační práce

Aeroacoustic Characteristics of Boundary-Layer Regimes
Aeroakustická charakteristika režimů mezní vrstvy

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Oponenti:

Teze byly rozeslány dny: ....

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Anotace

Aeroakustika mezní vrstvy je relativně nový studijní obor, který se zabývá mezní vrstvou jako zdrojem zvuku. Tato práce si klade za cíl rozšířit teorii aeroakustických charakteristik mezní vrstvy během počáteční fáze přechodu ze stavu laminární mezní vrstvy do stavu turbulentní mezní vrstvy. Nejprve jsou představeny základní teoretické základy dynamiky tekutin, poté následuje literární přehled o aeroakustice mezní vrstvy. Tato práce představuje nový teoretický přístup, metodu akustického zdroje, k řešení frekvenčních charakteristik mezní vrstvy v počáteční fázi přechodu. Navržený model je založen na teorii lineární nestability. Poslední část práce se zabývá experimentem, který je navržen s ohledem na aerodynamický tunel, který není primárně určen pro aeroakustický experiment. Experimentální výsledky získané na NACA 0012 při nulovém úhlu náběhu jsou porovnány s výsledky získanými z navržené metody řešení frekvenčních charakteristik. Teoretické a experimentální výsledky se velmi dobře shodují mezi sebou a s výsledky získanými z empirické metody dle Brookse, Popea a Marcoliniho.

Klíčová slova

laminární mezní vrstva, aeroakustika mezní vrstvy, Tollmienovy-Schlichtingovy vlny, lineární nestabilita, akustická zpětná vazba

Abstract

The aeroacoustics of the boundary layer is a relatively new field of study that considers the boundary layer as a source of sound. This thesis aims to extend the theory of the aeroacoustic characteristics of the boundary layer during the initial stage of the transition from a laminar to a turbulent boundary-layer state. The fundamental theoretical background of fluid dynamics is introduced first, followed by a literature survey on boundary-layer aeroacoustics. A novel theoretical approach is presented, an acoustic source method, for solving the frequency characteristics of the boundary layer in the initial phase of the transition. The proposed model is based on the theory of single-mode linear instability. The last part of the thesis describes experiments performed in a non-aeroacoustic wind tunnel facility. The experimental results obtained on a NACA 0012 airfoil at zero angle of attack are compared with the results obtained from the proposed method of solution of frequency characteristics. The theoretical and experimental results agree closely with each other and with the results obtained from the empirical model of Brooks, Pope, and Marcolini.

Keywords

laminar boundary layer, boundary-layer aeroacoustics, Tollmien–Schlichting waves, linear instability, acoustic feedback

Nomenclature

c	=	convective velocity (of sound wave or disturbance)
f	=	frequency
i	=	imaginary unit
k	=	streamwise wave number
L	=	characteristic length (e.g., chord length)
L_F	=	acoustic feedback length
N_{crit}	=	critical exponent of the amplification factor of the $e^{N_{\text{crit}}}$ method
p	=	pressure
Q	=	acoustic source term
Re	=	Reynolds number
Re_L	=	Reynolds number with chord length as characteristic length
St	=	Strouhal number
t	=	time
T_{ij}	=	Lighthill stress tensor
u, v	=	velocity
U, V	=	base velocity
x, y, z	=	Cartesian spatial coordinates
δ	=	conventional boundary-layer thickness
δ^*	=	boundary-layer displacement thickness
δ_{ij}	=	Kronecker delta
δ_L	=	boundary-layer length scale
ρ	=	density
τ_{ij}	=	viscous stress tensor
Φ	=	amplitude function
φ	=	phase shift
Ψ	=	stream function
ω	=	angular frequency
$\mathcal{A}$	=	amplification factor
$\mathcal{F}$	=	frequency model function

Indices

i, j	=	iterative indices
i	=	imaginary part of a complex number
r	=	real part of a complex number
RMS	=	root-mean-square value
TE	=	trailing edge
∞	=	freestream

Chapter 1
Introduction

Sound is one of the physical phenomena that can be perceived directly by human beings, although an observer does not necessarily need to be in close proximity to a source of sound. Usually, humans can readily distinguish between loud and quiet sounds, and between discrete high- and low-pitch tones in an emitted acoustic spectrum. Sound can be described as an acoustic wave that propagates through a medium. The acoustic wave causes a small perturbation in the local static pressure field. Such an acoustic wave can be generated, for example, by vibrating surfaces or by certain local changes in the pressure field. A solid object placed in a moving fluid (e.g., an airfoil in an air flow) produces changes in the pressure field, which can be propagated to the far field in the form of an acoustic wave.

The boundary layer is defined as a thin viscous layer around an object placed in a moving fluid [1]. One of the main goals of studying the boundary layer is to understand transitions from laminar to turbulent regimes. For these investigations, it is usually necessary to conduct multiple measurements. Intrusive measurement techniques affect the flow, so there can be no certainty that the transition to the turbulent boundary layer is not caused by the measurement device itself.

Aeroacoustics is a relatively new field of study that aims to understand both aerodynamically generated sound and the effects of sound on fluid flow. The foundation of modern aeroacoustics was laid by sir James Lighthill [2, 3], Samuel Newby Curle [4], and John Ffowcs Williams and D. L. Hawkings [5].

This thesis aims to describe the boundary layer as a source of aerodynamic noise, and to study the effects of the boundary-layer transition in the early stages on the acoustic field. This research extends the current aeroacoustic theory and experimentally validates the new theoretical findings, contributing to the field of fundamental research into the aeroacoustics of the boundary layer.

Since the boundary layer develops near walls in the flow of every viscous fluid, the aims of thesis are focused on the external flow around flat plates and surfaces with small curvature.

Chapter 2
State of the Art

2.1 Aerodynamically Generated Sound

In 1952, Lighthill proposed the fundamental theory of aerodynamically generated noise [2, 3]. His theory is today known as *Lighthill's analogy.* A different approach was proposed by Goldstein [6] by linearizing the momentum and continuity equations. Powell [7] introduced the vortex sound theory, which was later extended by Howe [8]. In one of the first applications of Lighthill's analogy, Proudman [9], and later Lilley [10], derived a simple approach to obtain the acoustic power of isotropic turbulence.

Lighthill's equation is derived by subtracting the divergence of the momentum equation ([11, p. 307]) from the time derivative of the continuity equation ([11, p. 270]) and substituting the density with its fluctuations ([12, p. 75]):

$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{2.1}$$

Here, T_{ij} is the *Lighthill stress tensor*, defined as follows:

$$T_{ij} = \rho v_i v_j - \tau_{ij} + (p' - \rho' \cdot c_\infty^2) \cdot \delta_{ij} \tag{2.2}$$

where τ_{ij} is the *stress tensor* defined in [11, p. 307], ρ' is the density fluctuation, ρ is the density, p' is the pressure fluctuation, c_∞ is the speed of sound, and δ_{ij} is the Kronecker symbol.

Brooks, Pope, and Marcolini [13] conducted an extensive experimental study of the *self-noise* of the NACA 0012 airfoil. They investigated the noise generation and emission of this airfoil under different flow conditions, and described five basic *self-noise* mechanisms. For each of these noise generation mechanisms, they published measured (1/3 octave band) sound pressure levels (SPLs), a proposed scaling method, and a scaled SPL spectrum. Their scaling method is based on the boundary-layer parameters, such as the conventional and displacement thicknesses, and the empirical spectral shape function.

2.2 Boundary-Layer Noise

The typical representatives of objects around which a boundary layer develops are flat plates or thin airfoils. The airfoil self-noise [13] directly connected to the boundary layer can be separated into categories based on the mechanism:

- Laminar boundary-layer (instability) noise
- Separated flow noise
- Turbulent boundary-layer noise

Glegg and Devenport [12] proposed a method to evaluate the right-hand side of Lighthill's equation (Eq. (2.1)) for low Mach numbers, in which case the fluid is considered nearly incompressible. For homentropic flow of incompressible fluid, the definition of the Lighthill stress tensor (Eq. (2.2)) leads to a modified right-hand side of the Ligthill's wave equation [12, 14]:

$$Q(x_i,t) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} = \rho_\infty \frac{\partial^2 (v_i v_j)}{\partial x_i \partial x_j} \tag{2.3}$$

This equation is rather important because no further approximation is made, and this source term is related to the unsteady part of the Reynolds stress of the flow.

Using Reynolds' decomposition $v_i = V_i + v_i'$ and the boundary-layer flow assumptions—two-dimensional flow with $V_2 = 0$, and V_1 being a function of x_2 only—the source term for the boundary-layer noise is as follows:

$$Q(x_i,t) = 2\rho_\infty \frac{\partial V_1}{\partial x_2}\frac{\partial v_2'}{\partial x_1} + \rho_\infty \frac{\partial v_j'}{\partial x_i}\frac{\partial v_i'}{\partial x_j} \tag{2.4}$$

The first term is called the *mean shear–turbulence interaction term* and the second is the *mean turbulence–turbulence interaction term* [14].

2.2.1 Laminar Boundary-Layer Instability Noise

One of the mechanisms of boundary-layer noise that has been described is laminar boundary-layer instability noise. The first systematic experimental study was carried out by Paterson et al. [15] in 1973 on NACA 0012 and NACA 0018 airfoils. Based on their experimental investigation, they derived a law for the frequency of a discrete tone $f = 0.011 U_\infty^{1.5}/(L\nu)^{0.5}$, where U_∞ is the freestream velocity, L is the chord length of the airfoil, and ν is the kinematic viscosity. This is also related to the Strouhal number[1] of the vortex street, so they suggested that the noise was generated by the vortex wake behind the airfoil.

[1]The Strouhal number is given by $St = 2 \cdot f \cdot \delta / U_\infty$, based on the dimension of twice the thickness of the boundary layer at the trailing edge [15].

This generation mechanism was argued against by Tam [16], who in 1974 proposed a *self-excited feedback loop* formed by the acoustic field, the boundary layer, and the wake flow. Tam showed that the primary and secondary frequencies observed by Paterson et al. lie in the hydrodynamic instability region of the Blasius boundary-layer profile. The proposed noise source is located downstream in the wake, in the region where the instabilities of the boundary layer cause strong lateral vibration.

Arbey and Bataille [17] (in 1983) found discrepancies between Tam's feedback loop and Fink's theory [18], and proposed a new phase-loop condition for discrete frequencies.

Lowson et al. [19] obtained experimental data and discovered an occurrence of Tollmien–Schlichting waves in the boundary layer of NACA 0012 without vortex-shedding noise. Therefore, the presence of the Tollmien–Schlichting waves in the boundary layer is not a sufficient condition for vortex-shedding noise. After further study, they found a correlation between the existence of a laminar separation bubble and the occurrence of laminar vortex-shedding noise.

Nash et al. [20] conducted laser Doppler anemometry and acoustic measurements, which showed a high correlation between strongly amplified instabilities and the trailing-edge separated flow region on the pressure surface and the acoustic field. The same authors conducted deeper theoretical work in [21] and connected the frequency of vortex-shedding noise to the frequency of instabilities (i.e., Tollmien–Schlichting waves) on the pressure surface of the airfoil. They also identified an important role for the laminar separation bubble, which is needed to amplify the incoming Tollmien–Schlichting waves.

Kingan and Pearse [22] used the XFOIL software, by M. Drela [23], to determine the *shape parameters*, and thus the *Falkner–Skan velocity profiles*, along the pressure side of an airfoil. These velocity profiles were used to solve the problem of linear stability (Orr–Sommerfeld equation) and find the modes of the Tollmien–Schlichting waves, which they connected with the frequency of the vortex-shedding noise.

Chong et al. [24] conducted experimental research on laminar boundary-layer instability noise. They found that the SPL of the instability noise may not be determined solely by Tollmien–Schlichting wave amplification. Chong et al. [25] also investigated the tonal noise generated by the airfoil if the trailing edge is serrated and found, significantly, that the serration reduces this noise.

Pröbsting et al. [26, 27] experimentally investigated tonal noise generation in 2014. Their research aimed to provide an experimental background for the DNS performed by Desquesnes et al. [28] using the particle image velocimetry (PIV) method. They concluded that multiple tones arise not only from phase modulation of fluctuations (as proposed by Desquesnes et al. [28]) but also from periodic modulation of the fluctuation amplitude. Pröbsting and Yarusevych [29] investigated the laminar separation bubble in more detail using two-component PIV measurements. They stated that amplified disturbances in the laminar separated bubble play a crucial role in the mechanism of the boundary-layer transition and that they can create *acoustic feedback*.

In 2019, Arcondoulis et al. [30] studied a dual acoustic feedback mechanism. Based on their findings, they proposed a variation of the original feedback model of Arbey and Bataille:

$$f_n(Re,\alpha) = \frac{c_r(Re,\alpha)}{L_{Sp+s}(Re,\alpha)} \left(n + \frac{1}{2}\right) \left(1 + \frac{c_r(Re,\alpha)}{c_\infty - U_\infty}\right)^{-1} \tag{2.5}$$

They used the distance from the boundary-layer separation point to the trailing edge L_S as the feedback length. They also proposed an alternative empirical feedback length based on the primary measured frequency.

Jaiswal et al. [31] experimentally investigated a controlled-diffusion airfoil using tomographic PIV and remote microphone probes. Based on the remote microphone probe measurement, they verified the existence of an acoustic feedback loop.

Laminar Boundary Layer—Instability Noise

The laminar boundary-layer vortex-shedding noise is directly connected to the instabilities in the laminar boundary layer (Tollmien–Schlichting waves). These instabilities are amplified by the laminar separation bubble. The boundary layer eventually reattaches to the surface, and as the instability moves past the trailing edge, it generates an omnidirectional noise source with a phase shift, which affects the point of boundary-layer separation and the boundary-layer instabilities.

However, the actual primary acoustic source may not be located directly at the trailing edge, but a short distance downstream [28]. The transition in the boundary layer to the turbulent state causes the acoustic feedback of the Tollmien–Schlichting waves to break.

Chapter 3
Objectives of the Thesis

This chapter summarizes key findings on the subject of boundary-layer aeroacoustics. Based on this literature survey, the following **hypothesis** is formulated:

> *In a laminar boundary-layer regime, the acoustic source is created by a velocity perturbation. If the perturbation is assumed to be of a single mode, the linear Orr–Sommerfeld equation can be used to estimate the acoustic source term of Lighthill's equation for the boundary layer to obtain the peak tone emitted by the laminar boundary layer.*

To validate this hypothesis, the following constituent **objectives** of the thesis are formulated:

- Description of the aeroacoustic footprint of a boundary layer experiencing linear instabilities in a laminar regime before the transition to a turbulent regime
  - Theoretical description of the aeroacoustic sources caused by linear instabilities in the boundary layer
  - Proposal of a frequency model for the prediction of laminar boundary-layer instability noise
- Experimental investigation of the boundary-layer instability noise
  - Design of an experiment to validate the obtained theoretical results
  - Further examination of the experimental data—using the experimental data to determine the acoustic feedback length
- Validation of the proposed model and experimental results
  - Validation of the proposed theoretical frequency model and experimental results on the NACA 0012 airfoil with the empirical model

These objectives can be divided into two parts, **the extension of the current theoretical findings** and **the setup of the experimental method**. The theoretical part of this thesis should bring new insight into the frequency of sound emitted by a laminar boundary layer with linear instabilities.

Methods for Achieving the Objectives

The theoretical description of the acoustic footprint of the boundary layer will be based on studying linear instabilities in the boundary layer, i.e., the spatial solution of the Orr-Sommerfeld equation. The solution of the Orr-Sommerfeld equation will be used with the source term of Lighthill's equation for the boundary layer to obtain the point acoustic source. These acoustic sources will then be evaluated to obtain the frequency footprint of the boundary layer. For the case study, the NACA 0012 airfoil is chosen, due to the availability of an empirical model of Brooks, Pope, and Marcolini [13]. The BPM empirical model of NACA 0012 is quite extensive and will be used for the validation and comparison of the proposed frequency model.

The experimental part will include measurement of acoustic pressure using built-in microphones on the NACA 0012 airfoil. These results will be used to confirm the theoretical frequency model and analyze the instability feedback length.

Chapter 4

Laminar Boundary-Layer Instability Noise

A laminar boundary layer without disturbances does not generate any noise, and even with a small perturbation (a perturbation that is damped and does not cause a transition to a turbulent boundary layer) in the flow field, the laminar boundary layer emits a very weak acoustic wave. These disturbances become more important when they grow in space, i.e., when they are spatially unstable. To analyze the stability of the disturbances, the velocity profile in the boundary layer is required. To estimate laminar boundary-layer velocity profile, the Falkner–Skan boundary layer velocity profile [32, 33] can be used. Kingan et al. [22] proposed a promising method based on solving the spatial instability problem of the Orr–Sommerfeld equation using Falkner–Skan velocity profiles based on knowledge of the shape factor of the boundary layer. The shape factor can be obtained experimentally or by using a computational approach, such as XFOIL. Kingan originally proposed this method in his thesis [34]. Another approach to prediction of boundary-layer instability noise is based on the empirical model of Brooks, Pope, and Marcolini [13] (the BPM model). The comparison of the BPM model to experimental results was presented in [A1].

To utilize the general approach to the boundary-layer instability noise, first, the velocity profiles along a flat plate or airfoil need to be obtained. With the knowledge of the velocity profile (or the parameters of the Falkner–Skan velocity profile—the shape factor H, the displacement thickness δ^*, and the velocity outside the boundary layer U_e) the spatial stability can be solved at a given point of the flat plate (or airfoil). This method to solve the spatial stability problem is designed using the Chebyshev collocation method [35, 36, 37]. The solution of Orr–Sommerfeld equation is used to obtain the most unstable mode (i.e., selecting the mode of the Tollmien–Schlichting wave) or the least stable mode of velocity perturbation. The method of normalizing the amplitude function is described to avoid ambiguity in the solution of the complex amplitude function.

4.1 Spatial Instability in the Boundary Layer of an Airfoil

For airfoils, the widely used and validated software package XFOIL is available [23]. This software uses a panel method with a coupled boundary-layer model [38] to obtain the characteristics of the airfoil and the parameters of the boundary layer (i.e, velocity outside boundary layer, displacement thickness, and shape factor). For validation and demonstration purposes, the well-known NACA 0012 airfoil will be used.

4.1.1 Linear Perturbation

It is assumed that the disturbance moves slowly along the parallel axis, so that some derivatives can be neglected. The perturbation stream function ([39, p. 428]) modified for a non-constant wave number is

$$\Psi(x, y, t) = \Phi \cdot e^{i\left(\int_{x_0}^{x} k \cdot dx - \omega t\right)} \tag{4.1}$$

where Φ is the local amplitude function, k is the complex wave number, and ω is the angular velocity, which is related to the frequency f by

$$\omega = 2 \cdot \pi \cdot f \tag{4.2}$$

4.2 Noise Source in a Laminar Boundary Layer

The noise source in the boundary layer itself can be described using Eq. (2.4). Jordinson [40] evaluated the Reynolds stress in terms of amplitude functions of the perturbation stream function. The value of the acoustic source term Q can be evaluated in a similar manner. According to Schlichting and Gersten [39, p. 428], only the real part Ψ_r of the perturbation stream function $\Psi(x, y) = \Psi_r(x, y) + i \cdot \Psi_i(x, y)$ has a physical meaning. Substituting the perturbation velocities u' and v' as functions of the perturbation stream function, Eq. (4.1), into Eq. (2.4) leads to the following equation:

$$Q(x, y, t) = -2\rho_\infty \frac{\partial U}{\partial y} \frac{\partial^2 \Psi_r}{\partial x^2} + 2\rho_\infty \left[\left(\frac{\partial^2 \Psi_r}{\partial x \partial y} \right)^2 - \frac{\partial^2 \Psi_r}{\partial x^2} \frac{\partial^2 \Psi_r}{\partial y^2} \right] \tag{4.3}$$

The complex amplitude function is $\Phi(x, y) = \Phi_r(x, y) + i \cdot \Phi_i(x, y)$. For the spatial stability problem, the components of the complex wave number are $k(x) = k_r(x) + i \cdot k_i(x)$.

This first major simplification relies on the assumption that the changes in the amplitude function $\Phi(x, y)$ and wave number $k(x)$ with the coordinate x are negligible, and therefore their partial derivatives with respect to x are zero. Then, the second derivatives of the stream function are as follows:

$$\frac{\partial^2 \Psi_r}{\partial x \partial y} = \mathcal{A} \cdot \left[\left(-k_r \cdot \frac{\partial \Phi_i}{\partial y} - k_i \cdot \frac{\partial \Phi_r}{\partial y} \right) \cos\left(\omega t - \varphi(x)\right) \right] + \mathcal{A} \cdot \left[\left(k_r \cdot \frac{\partial \Phi_r}{\partial y} - k_i \cdot \frac{\partial \Phi_i}{\partial y} \right) \sin\left(\omega t - \varphi(x)\right) \right] \quad (4.4a)$$

$$\frac{\partial^2 \Psi_r}{\partial x^2} = \mathcal{A} \cdot \left[\left(k_r^2 \Phi_r - k_i^2 \Phi_r - 2k_i k_r \Phi_i \right) \cos\left(\omega t - \varphi(x)\right) \right] + \mathcal{A} \cdot \left[\left(k_r^2 \Phi_i - k_i^2 \Phi_i + 2k_i k_r \Phi_r \right) \sin\left(\omega t - \varphi(x)\right) \right] \quad (4.4b)$$

$$\frac{\partial^2 \Psi_r}{\partial y^2} = \mathcal{A} \cdot \left[\left(\frac{\partial^2 \Phi_r}{\partial y^2} \right) \cos\left(\omega t - \varphi(x)\right) \right] + \mathcal{A} \cdot \left[\left(\frac{\partial^2 \Phi_i}{\partial y^2} \right) \sin\left(\omega t - \varphi(x)\right) \right] \quad (4.4c)$$

Here, two substitutions have been made for the phase change $\varphi(x)$ of the perturbation wave and the amplification factor $\mathcal{A}$:

$$\varphi(x) = \int_{x_0}^{x} k_r(x) dx \quad (4.5)$$

$$\mathcal{A}(x) = e^{-\int_{x_0}^{x} k_i(x) dx} \quad (4.6)$$

where x_0 is the position of the first instability. To obtain a time-dependent value of the simplified source term $Q(x, y, t)$, the equations (4.4) can be substituted into Eq. (4.3). The time-dependent source term can be used to obtain information on the acoustic pressure in the far field (for example, using the nonfield method of Kulish et al. [A2]). However, such a solution also requires taking into account sound reflection on the surface and scattering at the trailing edge, which would require further computational models to be established. The root-mean-square value of $Q(x, y, t)$ for one period corresponding to a given frequency can be found:

$$Q(x, y)_{\mathrm{RMS}} = \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \left(Q(x, y, t)\right)^2 dt} \quad (4.7)$$

By substituting the derivatives of the stream function into Eq. (4.7), a full expression of the point source strength is obtained. There is no analytical solution for this integral in the general form, so the solution must be obtained numerically for each point in the discretized boundary layer.

In Fig. 4.1, results are presented for the solution of the acoustic source field defined by Eq. (4.7) under one of the specified boundary conditions. The boundary-layer length scale at the trailing edge is used as the length scale for this figure. The results show, as expected, that the most important acoustic source is near the trailing edge. The displacement (red curve) and conventional (green curve) boundary-layer thicknesses are also shown. The displacement thickness is very similar to the position of the maximum value of the acoustic source (Q_{RMS}) for each span station, while the conventional thickness behaves as an envelope curve for the location of the significant part of the acoustic source field.

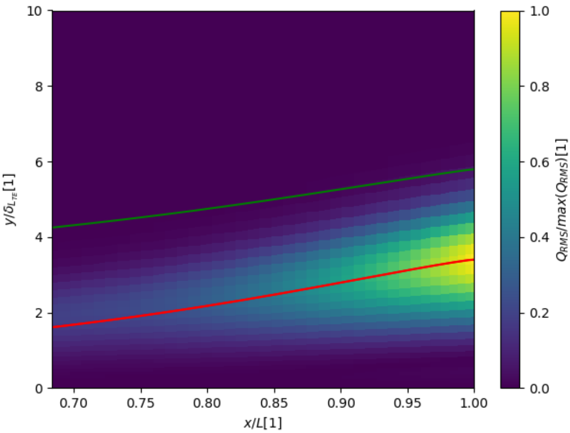

Figure 4.1: Field of acoustic sources in the boundary layer $Q(x, y)_{\mathrm{RMS}}$ of NACA 0012 with 6.62 m/s freestream velocity and perturbation frequency 293 Hz. The red curve is the local boundary-layer displacement thickness ($\delta^*/\delta_{L_{\mathrm{TE}}}$) and the green curve is the local conventional boundary-layer thickness ($\delta/\delta_{L_{\mathrm{TE}}}$). The displacement and conventional thicknesses are both normalized to the boundary-layer length scale at the trailing edge ($\delta_{L_{\mathrm{TE}}}$).

4.2.1 Tonal Noise Frequency Model

To find the peak frequency of boundary-layer instability noise, it is necessary to evaluate Eq. (4.7) for a range of frequencies. For the given boundary-layer conditions and constant frequency, the maximum value of the acoustic source term in the boundary layer can be found:

$$\max_x \left(\sum_y Q(x,y)_{\mathrm{RMS}} \Delta y \right)_\omega = \mathcal{F}(\omega) \tag{4.8}$$

The peak frequency of the boundary layer instability is the frequency for which the expression in (4.8) is maximized. This model is derived without any assumptions regarding refraction, reflection, or scattering of the sound wave due to the presence of the solid boundary. Hence, the magnitudes of the RMS values from the source-term model are not directly comparable to the measured spectrum. It is also important to note that the function $\mathcal{F}$ has no physical meaning and is only used to predict the frequency with the strongest source terms.

Fig. 4.2 compares the method of maximal amplification of Tollmien–Schlichting waves at the trailing edge (i.e., the maximum of $\mathcal{A}(x_{\mathrm{TE}})$) and the proposed method (Eq. (4.8)) based on the acoustic sources. The proposed method predicts a higher dominant frequency.

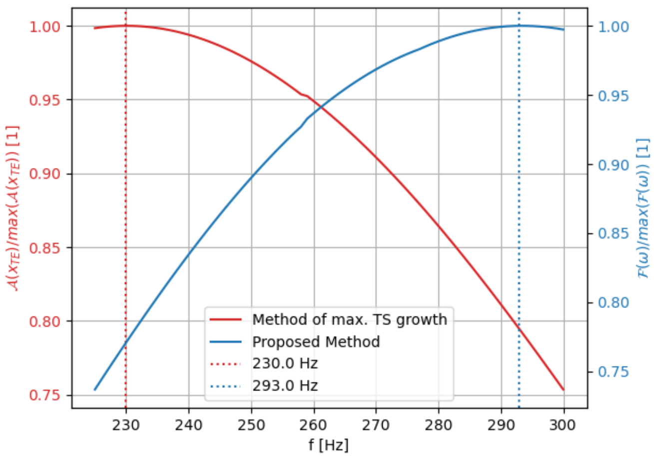

Figure 4.2: Comparison of the proposed tonal model with the model of the maximal growth of Tollmien–Schlichting waves for velocity 6.62 m/s (Reynolds number $Re_L = 44143$).

4.2.2 Summary

The source model for the noise in the boundary layer based on the perturbation stream function are proposed in Eq. (4.7). This equation only describes the source of noise, without any acoustic effects caused by the presence of the solid wall. This acoustic source model is used to predict the dominant acoustic frequency of the boundary layer caused by linear instabilities. Although the aerodynamically generated noise could be connected to the Tollmien–Schlichting waves, this does not necessarily mean that the frequency of the dominant sound source is the same as the frequency of the most amplified Tollmien–Schlichting waves. As Eq. (4.7) shows, the noise source depends not only on the amplification factor of the Tollmien–Schlichting waves, but also on their wave number and the magnitude of the perturbation. In the proposed model, the magnitude of the perturbation is normalized to its maximum absolute value. This magnitude of perturbation is influenced by the outer flow (i.e., by the turbulence intensity of the freestream) and cannot be expressed directly. Due to this, the proposed model cannot be used directly to evaluate the acoustic pressure in the far field (i.e., the sound pressure level) and evaluate whether the emitted sound is audible.

This model of the acoustic sources incorporates only the acoustic source above the surface. An acoustic source behind the trailing edge can be more powerful than those in the proposed boundary-layer model. An acoustic source in the wake is expected to be connected with acoustic sources in the boundary layer by frequency; i.e. the sources in the boundary layer are the origin of the acoustic source in the wake (for the laminar boundary-layer instability noise).

4.3 Acoustic Feedback

The empirical method for acoustic feedback can be based on the frequency difference because, based on the Tam feedback model (which is the foundation for the other feedback models), the difference between frequencies is the same for a given velocity. This frequency difference can be described using Eq. (2.5) and the definition of $\Delta f = f_{n+1} - f_n$:

$$\Delta f = \frac{c_r}{L_F} \left(1 + \frac{c_r(Re, \alpha)}{c_\infty - U} \right)^{-1} \tag{4.9}$$

The feedback length is still not well defined. This equation is used in Section 6.2 in an inverted way to obtain the feedback length based on the measured frequency difference.

Chapter 5
Experiment

The experimental validation of the theoretical findings of the aeroacoustic (frequency) signature of the boundary layer is carried out on the well-known NACA 0012 airfoil. This choice of airfoil is based on the possibility of validating the simulated solution and eventually extending the set of measured data for future use.

Aeroacoustic measurements are usually conducted in a special wind tunnel. However, for the required wind speed (up to 15 m/s, which is based on the Reynolds number for chord length 0.1 m), such a facility is not available in the Department of Fluid Dynamics and Thermodynamics at FME CTU in Prague, so a *substitute measurement method* is proposed in this chapter.

The main goal is to obtain the frequency footprint of one NACA 0012 airfoil for different freestream speeds. These measurements should be sufficient to obtain a comparison with simulated data and with the empirical model of Brooks, Pope, and Marcolini [13]. The chord length of the designed test model is 100 mm and the span is 390 mm. The airfoil has end plates to minimize the velocity in the third direction.

5.1 Wind Tunnel

Measurements are conducted in the low-speed wind tunnel at the Department of Fluid Dynamics and Thermodynamics at FME CTU in Prague. This wind tunnel has no special modification for acoustic measurement. An analysis of the possibility of some limited acoustic measurement was published in 2019 by Suchý [A3]. This analysis shows higher low-frequency background noise in the wind tunnel with an empty test section; however, by using the method of spectral subtraction [41]. Another problem is the presence of background noise at 4 kHz and its harmonic frequencies [A3].

5.2 Measurement

The measurement of static pressure fluctuations was performed using electret microphones. Due to the background noise in the wind tunnel, a modified measurement approach was established based on measuring the acoustic pressure on the airfoil surface, similar to pinhole measurement of the fluctuating turbulence pressure [12]. A small electret microphone is placed directly below the surface of the airfoil, which has a hole matching the diameter of the hole in the microphone capsule. The pinhole diameter is 0.9 mm, which should yield a lower frequency resolution for low-wavelength waves, so the microphone should primarily capture the acoustic noise. In the proposed measurement method, three microphones are placed in the surface of an airfoil test segment. These microphones are placed in the top surface of the airfoil at 55 %, 68 %, and 80 % of the chord length from the leading edge.

To verify that the captured frequency spectrum is of acoustic origin, two reference microphones are placed outside the airflow. The positioning of the microphones is also important because, in some locations, a dominant frequency could be distorted by the effects created on the other side of an airfoil. This is similar, for example, to vortex shedding behind a cylinder, for which there is a dominant frequency twice as high in the centerline downstream than elsewhere [A4].

The main data acquisition (DAQ) unit is NI cDAQ-9174, which provides slots for 4 NI DAQ modules. The raw electrical signal acquired is the voltage (from microphone probes), so the first two modules are chosen to capture the voltage with a high sampling rate per channel (NI 9250 and NI 9230). The third slot is used for a module (NI 9203) dedicated to recording the current electrical signal from the pressure transducer of a Pitot-static tube. The Pitot-static tube is used to calibrate the airspeed in the wind tunnel.

5.3 Conclusion

The measurement method proposed and used in this thesis mainly provides information on the peak/main tones in the sound emitted by an airfoil placed in the wind tunnel test section. This method was designed to be a substitute method for the measurement method in a special aeroacoustic facility; therefore, very accurate results are not expected. However, the main results of this measurement method are information about the dominant frequency, not its magnitude (the magnitude is highly contaminated by the sound produced by the running wind tunnel), so these results can be compared with those obtained from the theoretical approach described in Chapter 4. This comparison will be presented in the following chapter.

The obtained spectra can also be used to determine the length scale of the acoustic feedback, which could be considered as a minor result of this thesis. The determination of the length scale will also be addressed in the following chapter, which presents and discusses the obtained results.

Chapter 6

Results and Discussion

6.1 Dominant Frequency of NACA 0012

Dominant frequencies of the laminar boundary-layer instability were obtained using a novel method of acoustic sources (Section 4.2.1), the method of maximum instability amplification factor [22], and experimental measurements (Section 5). These results are compared to the empirical model proposed by Brooks, Pope, and Marcolini (the BPM model) [13].

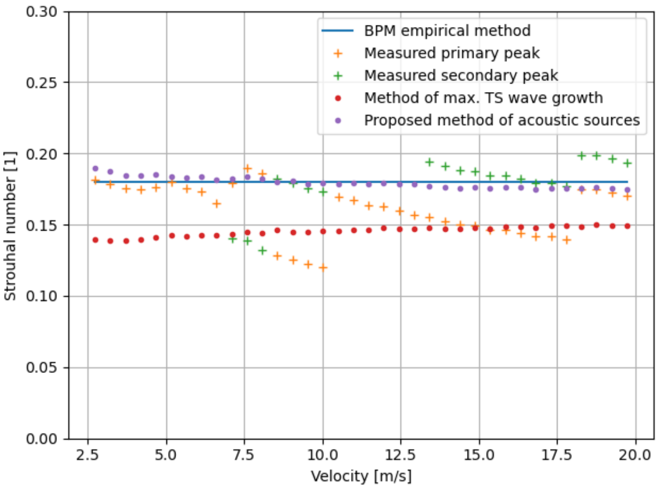

Figure 6.1: Measured, empirical, and simulation peak Strouhal number for velocities below the predicted transition in the laminar separation bubble. The characteristic length used to define the Strouhal number is based on the conventional boundary-layer thickness.

Fig. 6.1 resents one of the major results of this thesis. Peak non-dimensional frequencies of the NACA 0012 laminar boundary-layer instability noise are plotted, obtained using four different methods (of which two are newly proposed in this thesis). The frequencies are non-dimensionaled to Strouhal number based on the conventional boundary-layer thickness as the characteristic length.

The results from the proposed method of acoustic sources match the frequencies based on the BPM empirical method better than the method of maximal amplification of Tollmien–Schlichting waves. The experimentally obtained tones are also plotted in Fig. 6.1.

The experimentally obtained tones (primary and secondary) in Fig. 6.1 have a similar structure to the ladder structure described by Arbey and Bataille [17]. Either the primary or secondary peak frequency is, in most cases, close to the constant Strouhal number of 0.18 predicted by the empirical model. The mean value of the predicted Strouhal number by the developed method of acoustic sources is 0.1798, by the measured primary peak is 0.1662, and by the method of maximal growth of Tollmien–Schlichting wave is 0.1456.

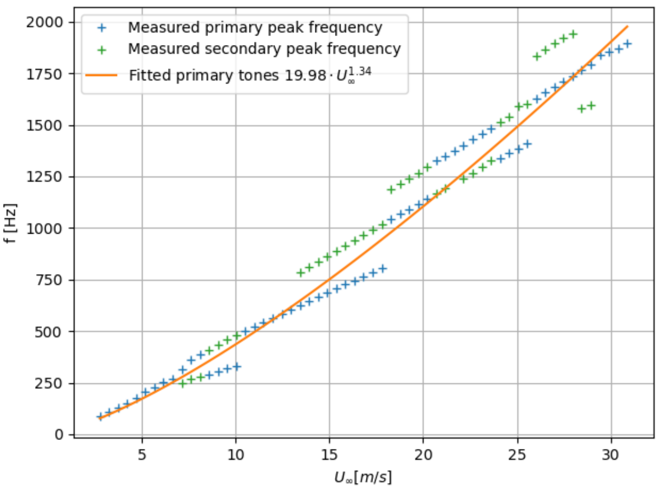

Figure 6.2: Primary and secondary tones in all measured points

The measurement was carried out for even higher velocities (up to 30.89 $\mathrm{m \cdot s^{-1}}$). However, there is no method available to treat linear instabilities in the turbulent boundary layer (even when assuming that only the linear instabilities that originated in the laminar state are present), so there are no computational results for comparison. In Fig. 6.2, measured peak frequencies are shown for the entire velocity range investigated, and the results are fitted to the function $a \cdot U_\infty^b$; this function is based on the findings of Paterson et al. [15]. Fig. 6.2 also shows the ladder-type evolution of tonal peaks in the measured spectrum, similar to the findings of Arbey and Bataille [17].

6.2 Acoustic Feedback

Equation (4.9) can be used to find an empirical value for the feedback length based on the measured Δf and the convective velocity c_r obtained as an average value between the point of the first occurrence of the unstable linear perturbation and the trailing edge.

In Fig. 6.3, empirical values of the feedback length are presented based on the method proposed in Section 4.3. For comparison, the distance from the first occurrence of an instability in the boundary layer to the trailing edge is shown with blue dots. Although XFOIL predicts (with $N_{\mathrm{crit}} = 11$) the transition to the turbulent boundary layer for velocities above $20\,\mathrm{m\cdot s^{-1}}$, it should be assumed that the transition in the experiment occurs at a lower velocity. It can be theorized that the change in feedback length around $18\,\mathrm{m\cdot s^{-1}}$ could be a footprint of the boundary-layer transition.

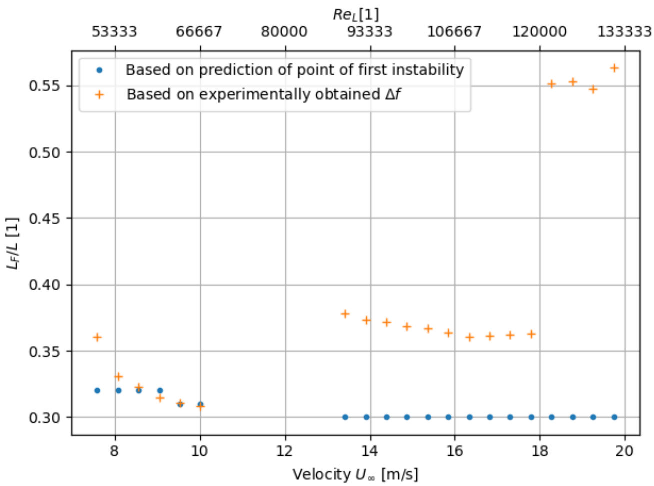

Figure 6.3: Feedback length based on experimentally obtained Δf and the theoretical feedback length, which is equal to the distance from the first occurrence of instability to the trailing edge.

Chapter 7
Conclusions

This thesis is focused on fundamental research into the aeroacoustic characteristics of boundary-layer regimes. The novel theoretical model is based on the linear instability theory. The solution of the Orr–Sommerfeld equation is used to determine the acoustic source field of one mode within the boundary layer. Then, a method of comparison of acoustic fields for different frequencies is developed. The proposed model is compared with the solution of the method of maximal amplification of Tollmien–Schlichting waves.

For the experimental validation of the proposed theoretical model, a measurement method was established. Since it was not possible to use a specialized aeroacoustic facility (i.e., a wind tunnel with an anechoic chamber as a test section), a substitute method was proposed. This was based on measurements with electret microphones built into the surface of a test segment of a NACA 0012 airfoil. This airfoil was chosen as it offered the strongest prospects of comparing the theoretical and experimental results with the empirical model of Brooks, Pope, and Marcolini [13].

The experimental investigation showed that it is possible to obtain reliable results for the tonal noise even though there is a stronger background noise in the running wind tunnel. For the reference measurement, two microphones were placed outside the open test section of the wind tunnel. Since some of the measured tones were also detectable by the reference microphones, they were also used to confirm the measured main tones.

The proposed model of acoustic sources was successfully validated against the obtained experimental data. The proposed model was compared with the BPM empirical method. When using the conventional boundary-layer thickness, the Strouhal number of the laminar boundary-layer instability noise is 0.18, whereas when using the displacement thickness, the Strouhal number of the dominant tone is 0.1.

The experimental results were also used to analyze the acoustic feedback length. It was possible to obtain this quantity; however, it is rather difficult to compare these results. The feedback length (and the acoustic feedback overall) could be investigated in more detail in some future work using a different experimental approach in an aeroacoustic facility.

7.1 Application of the Thesis Results

The main result of this thesis is an acoustic source model of the boundary-layer instability noise. This noise is essentially tonal noise, which is quite annoying for humans. The model could be used as the core of a computational software package for boundary-layer noise. The model could also be extended to an airfoil under an arbitrary angle of attack and then used as a part of a more complex computational approach to predict the noise of an arbitrary airfoil. This would bring the benefit of predicting the tonal noise, which could be used in an optimization process to avoid undesirable tones.

7.2 Brief Summary of the Thesis Objectives

Recalling the objectives of the thesis in Section 3, the following hypothesis was stated:

> *In a laminar boundary-layer regime, the acoustic source is created by a velocity perturbation. If the perturbation is assumed to be of a single mode, the linear Orr–Sommerfeld equation can be used to estimate the acoustic source term of Lighthill's equation for the boundary layer to obtain the peak tone emitted by the laminar boundary layer.*

This hypothesis was confirmed by successfully achieving the constituent thesis objectives:

- *Description of the aeroacoustic footprint of a boundary layer experiencing linear instabilities in a laminar regime before the transition to a turbulent regime*
  - *Theoretical description of the aeroacoustic sources caused by linear instabilities in the boundary layer*
    A theoretical model of point acoustic sources (Eq. (4.3) and Eq. (4.7)) has been established.
  - *Proposal of a frequency model for the prediction of laminar boundary-layer instability noise*
    A tonal noise model has been proposed in Section 4.2.1.
- *Experimental investigation of the boundary-layer instability noise*
  - *Design of an experiment to validate the obtained theoretical results*
    The proposed experimental approach to obtaining tonal noise frequencies in a non-aeroacoustic facility is described in Section 5.

  - *Further examination of the experimental data—the using the experimental data to determine the acoustic feedback length*
    The experimental data were studied further. The ladder structure (in Fig. 6.2) is observed, and these results were utilized to obtain the acoustic feedback length, which was compared to the distance from the first occurrence of an instability to the trailing edge in Section 6.2.

- *Validation of the proposed model and experimental results*

  - *Validation of the proposed theoretical frequency model and experimental results on the NACA 0012 airfoil with the empirical model*
    An overall comparison of the proposed theoretical model with experimentally obtained data is presented in Section 6.1 together with a comparison to the BPM empirical model.

The initial hypothesis is confirmed. It is possible to use the linear Orr–Sommerfeld equation to estimate the source of Lighthill's equation to estimate the peak frequency of the laminar boundary-layer instability noise.

The most important contribution of this thesis is the validated model of the acoustic sources of the laminar boundary layer instability noise, formulated in Section 4.2.

7.3 Future Work

The proposed model could be extended with a (perhaps empirical) model of the behavior of the linear instabilities when they reach the part of the boundary layer in the turbulent state. The impact of the linear instability noise on the overall noise in the presence of turbulent boundary-layer noise could be investigated.

Another extension of the proposed model of acoustic sources could be based on estimating the magnitude of the initial velocity perturbation. With this estimation and by taking into account the presence of the solid surface, it should be possible to evaluate the emitted sound in the far field and determine whether it is audible.

Other future work could focus on acoustic feedback. It was mentioned at the end of the previous chapter that acoustic feedback has been theoretically proposed, but experimental data are lacking. Therefore, a new set of experimental data would be useful. The possible connection between the state of the boundary layer and the acoustic feedback length could also be explored.

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