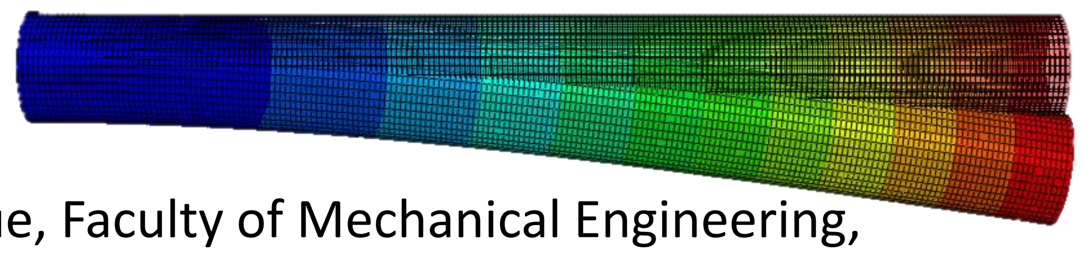
Design Methods of Composite Beam Analysis

Ing. Tereza Zámečníková, doc. Ing. Tomáš Mareš, Ph.D.

Czech Technical University in Prague, Faculty of Mechanical Engineering,
Department of Mechanics, Biomechanics and Mechatronics.
Technická 4, 166 07 Praha 6, Czech Republic



From: https://compotech.com/#1455652486566-980bcf13-e329

Analytical Calculation Methods

- Clasical Laminate Theory

$$C_{xy} = S_{xy}^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{xy}}{E_y} & \frac{1}{E_Y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \Rightarrow$$

$$U = U_{M_o} + U_\tau$$

- Summary of bending and shear energy

$$E_x \cdot J_y = \sum_{k=1}^{n} E_{x_k} \cdot J_{y_k}$$

- Equivalent tensile modulus

- ABD Matrices

$$\begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \vdots & \mathbf{B} \\ \cdots & \vdots & \cdots \\ \mathbf{B} & \vdots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}{}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix} \Rightarrow \sigma_1 = \frac{N_1}{t} = E \cdot \boldsymbol{\varepsilon}^{\circ}{}_1$$

- Equivalent tensile modulus

$$E_{eq} = \left(A_{11} - \begin{bmatrix} A_{12} & A_{13} \end{bmatrix} \cdot \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_{21} \\ A_{31} \end{bmatrix}\right) \cdot \frac{1}{t}$$

- Average values from Stiffness and Compliance matrix

$$EJ_{eq} = \frac{EJ_S + EJ_C}{2}$$

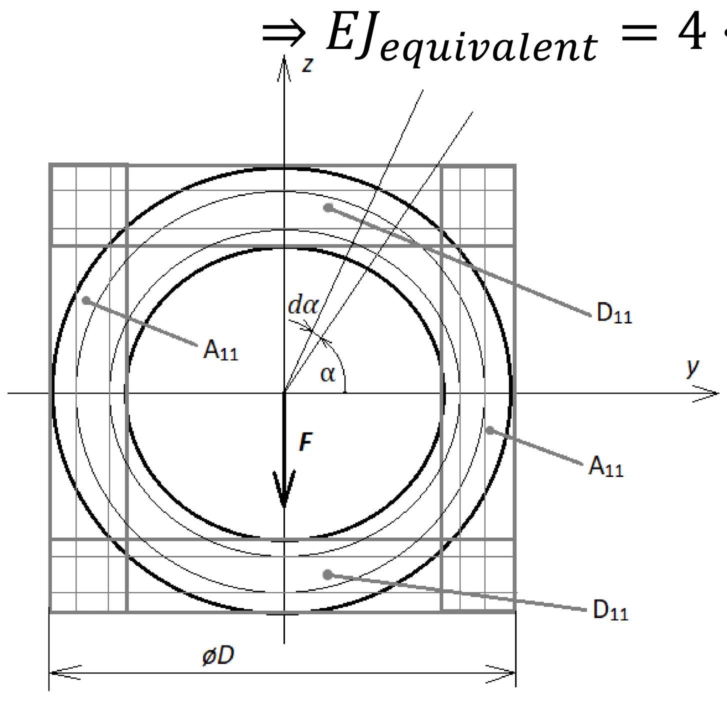
- Equivalent tensile modulus reached by arithmetical mean

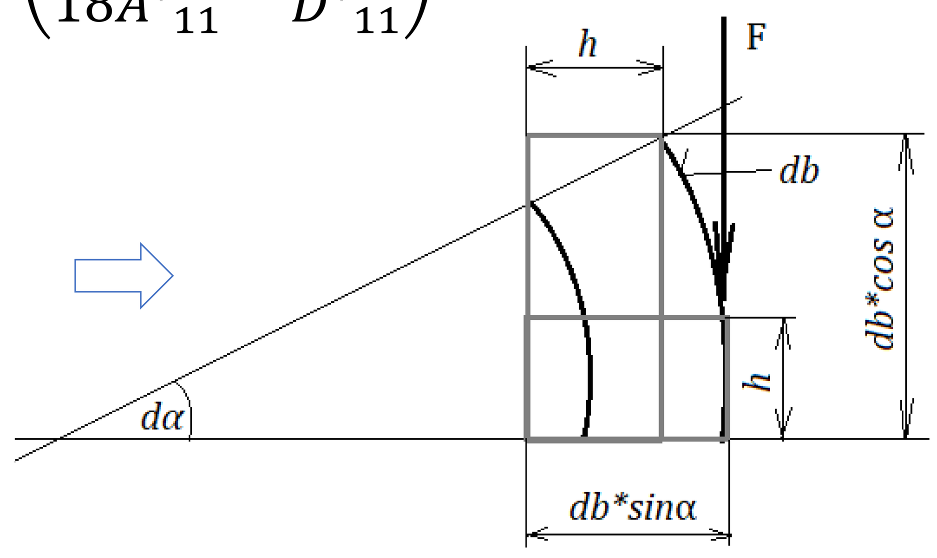
- A new calculation method

$$(EJ)_{equivalent} = EJ_{tensile} + EJ_{bending} = (EJ)_{A^*{}_{11}} + (EJ)_{D^*{}_{11}}$$

$$EJ_{equivalent} \cdot \frac{1}{4} = \int_0^D \int_0^{\frac{\pi}{2}} \frac{1}{h \cdot A^*{}_{11}} \cdot \frac{h \cdot (\cos\alpha)^3}{12} \, d\alpha \, db^3 + \int_0^D \int_0^{\frac{\pi}{2}} \frac{12}{h^3 \cdot D^*{}_{11}} \cdot \frac{h^3 \cdot (\sin\alpha)}{12} \, d\alpha \, db$$

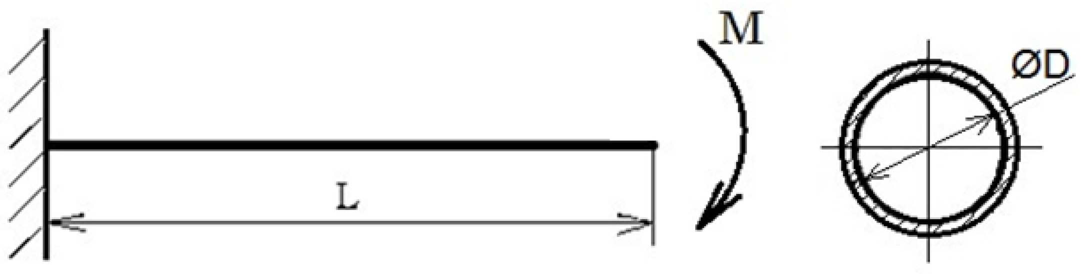
$$\Rightarrow EJ_{equivalent} = 4 \cdot \left(\frac{D^3}{18A^*{}_{11}} + \frac{D}{D^*{}_{11}}\right)$$

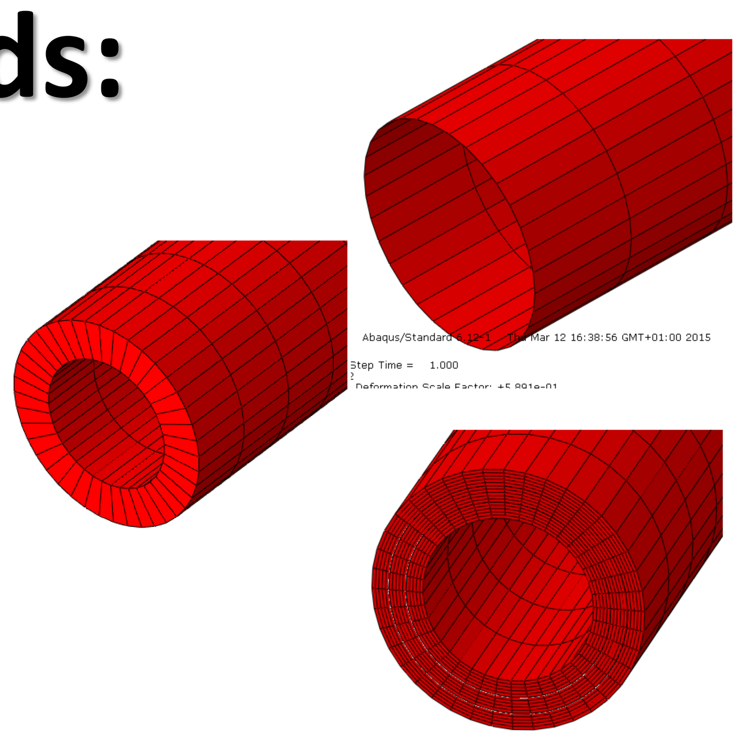




FE methods:

- Conventional shell
- Continuum shell (Solid shell)
- Volume model



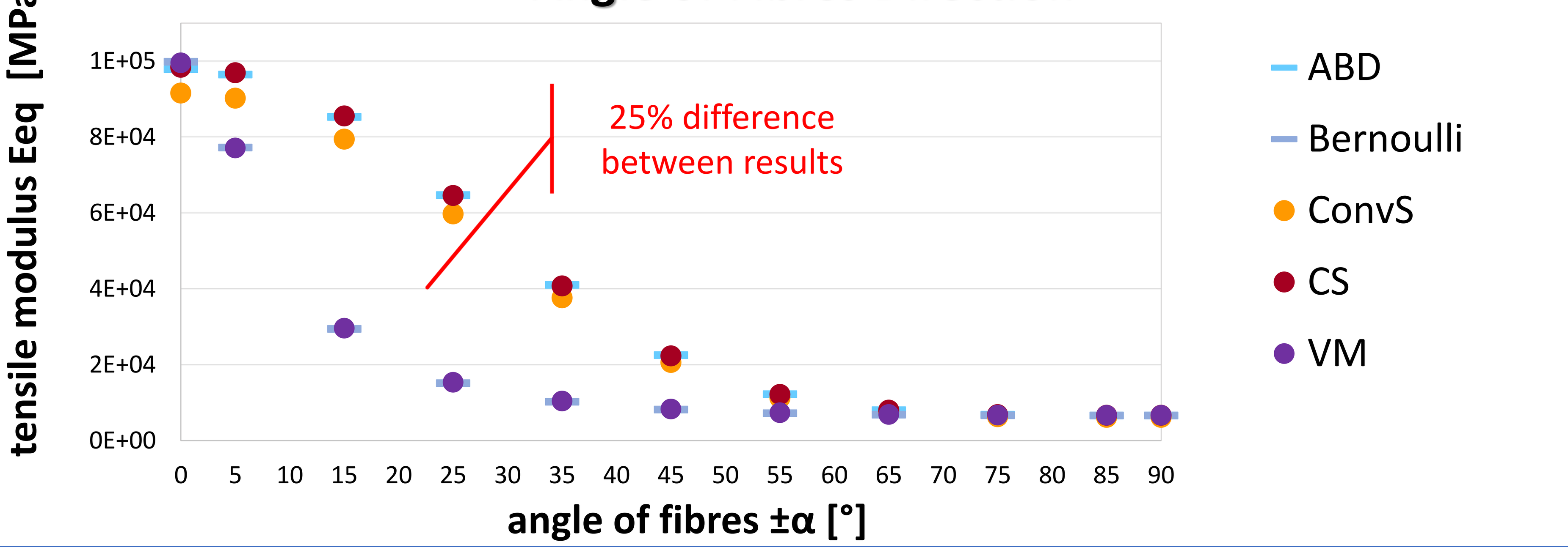


- Statistic method

$$E_i = \Pi = U - F \cdot u = -\frac{1}{2} F \cdot u = -\frac{1}{2} F \cdot \frac{Fl^3}{48(EJ)_i}$$

$$Z = \sum_i e^{-\beta E_i} \qquad \langle E \rangle = -\frac{\partial \ln Z}{\partial \beta} \qquad (EJ)_{efective} = -\frac{F^2 l^3}{96 \langle E \rangle}$$

Dependece of Eeq from the Load by a Momentum to the Angle of Fibres Direction



Tensile Modulus Eeq from Analytical and FEM Models

- Deflection from the momentum:

$$v_M = \frac{M \cdot l^2}{2E_{eq}(M)J_y} \Rightarrow E_{eq}(M) = \frac{M \cdot l^2}{2 \cdot v_M \cdot J_y}$$

- Bernoulli's and Timoshenko's theorem is used

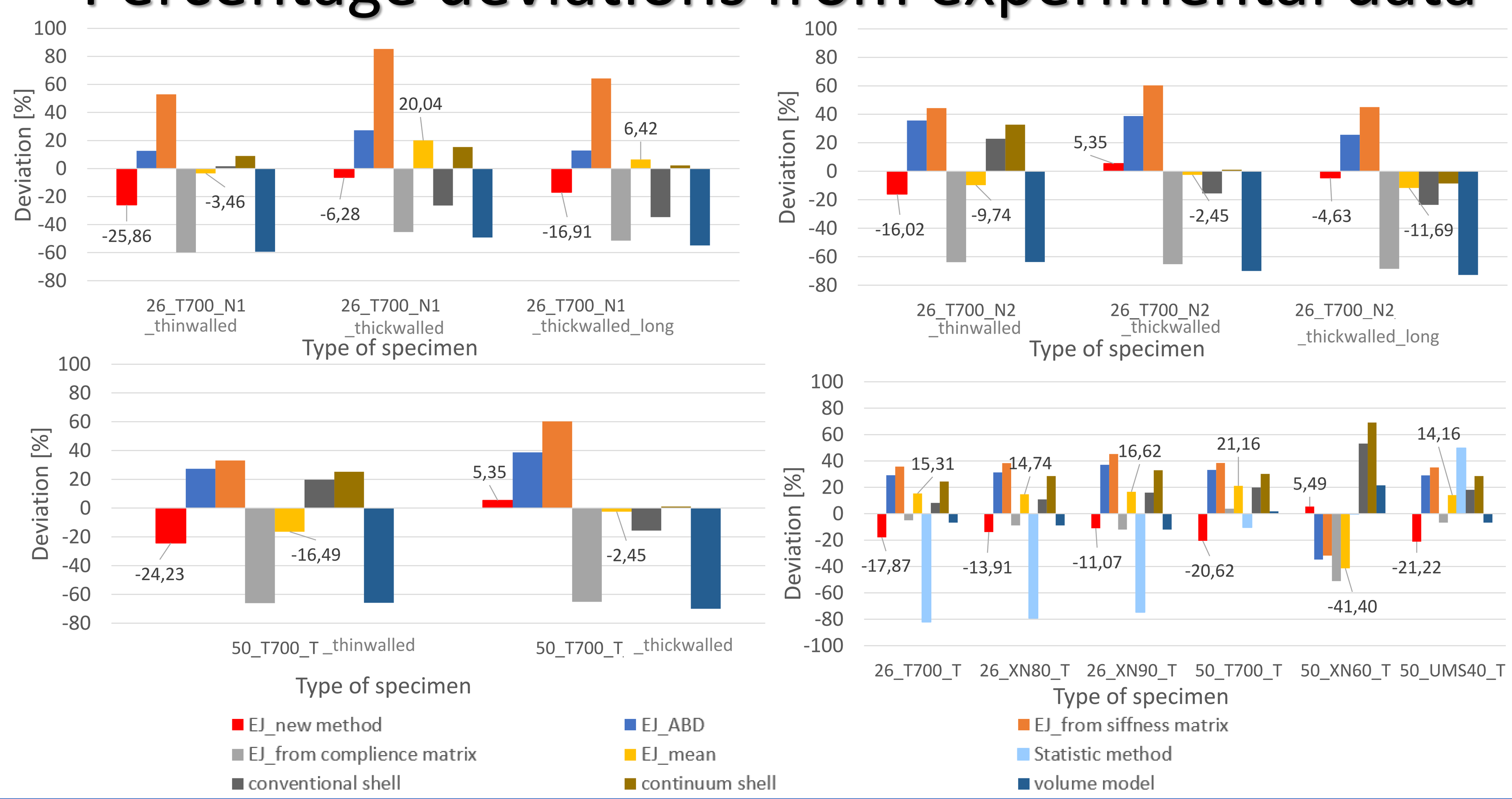
$$U = U_{M_O} + U_\tau = \int_0^L \frac{M_O^2(x)}{2 \cdot E \cdot J_z(x)} dx + \int_0^L \frac{\beta \cdot T^2(x)}{2 \cdot G \cdot A(x)} dx \Rightarrow v_F = \frac{Fl^3}{3E_{eq}J} + \frac{\beta Fl}{GA}$$

Experiment

- Three point bending test was made on Tira 2300 testing machine
- Layups of composite beams:
  - Longitudinal [90°, 0°], Diagonal [90°, ±45°], Diagonal 2 [90°, ±20°], Typical [90°, 0°, ±30°]
- Speciemens geometry and materials:
  - ID 26mm and ID 50 mm made of fibres T700, XN60, XN80, XN90, UMS 40 and epoxy matrix
- Supports of 200mm span for ID26 and supports of 400mm span for ID50 were used for short deflection test
- Supports of 400mm span for ID26 and supports of 600mm span for ID50 were used for long deflection test
- The beam was loaded by force over the strap

Results

- Percentage deviations from experimental data



Conclusion

In most cases, the results of the new stiffness matrix and compliance matrix average value approaches and the geometry-based method showed good agreement with the experimental results. With a deviation of less than 20% from the experimental results for thick-walled pipes and a deviation between 20% and 25% from the experimental results for thin-walled tubes from the base series of samples. For these thin-walled tubes, greater inaccuracies involving local deformations in the load members during the experiment are assumed. For an additional series of thick-walled specimens, the values are even closer to the experimental data with a maximum deviation of 17% from the experimental value. The results of both methods are much closer to experimental data than other analytical methods and FEM models. Since the new approach with a geometric approach predicts a lower stiffness than the experiment, the results of this method are appropriately conservative and can be evaluated as safe. Both methods can be recommended for preliminary calculations in new designs of mechanical systems with tubular composite beams.