Low frequency noise investigation of pT-level magnetic sensors by cross-spectral method

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Abstract— **We present a simple method to estimate the noise of magnetic sensors running in the Earth's field range by establishing the cross-power spectrum density during ambient field operation and performing spectral subtraction. This method has advantages to the usual subtraction of two sensors outputs, mainly in requirements for synchronization of the sample rate and gain calibration. With this method, verified in simulation and measurements with AMR magnetometers, we could use a fluxgate as a second sensor in order to estimate the low-frequency noise of an HTS SQUID in the ambient field.**

Keywords—magnetic sensor; noise; SQUID; fluxgate; AMR; correlation

I. INTRODUCTION

Estimating the ultralow frequency (ULF, 0.01-1 Hz) noise levels of magnetic sensors in a laboratory is not a straightforward task once the expected noise levels of the sensor are in the order of pT. The first option is to use a magnetic shielding cylinder ("zero-field cylinder") [1]. An even better option is a shielded room with large dimensions and "active shielding". The state-of-the-art "BMSR-2" with 8 layers states about 1×10^4 to 1×10^5 shielding factor between 0.01 and 1 Hz [2]; however, such an establishment is out of reach of a typical laboratory. We have estimated the shielding factor of a compact 6-layer magnetic shielding cylinder (length 0.75 m, inner diameter 17 cm) available at the CTU as approx. $1\text{x}10^4$ between 10 and 100 mHz and 2×10^4 at 20 Hz [3]. For a typical laboratory noise of 10 nT/√Hz @ 1 Hz, the transverse shield attenuation would yield about 1 pT/√Hz "residual" noise at 100 mHz.

The problem with finite shielding factors can be mitigated by doing the noise measurements with the shield in a low noise environment. However, for the HTS DC SQUID (High-temperature-superconductor Superconducting Quantum Interference Device) magnetometer, the noise obtained in a zero-field environment might be much smaller than when exposed to the Earth's magnetic field (~20 - 60 µT) during geomagnetic observations [4]. Although the SQUID sensor design can be optimized to reduce the effect of "flux trapping" and subsequent ULF noise due to exposure to large magnetic fields [5], the researchers and manufacturers almost exclusively claim the sensor noise "in zero field", and rarely show noise figures at frequencies below 1 Hz [6], [7] because of its dependence on the electronics, setup and location.

An alternative to magnetic shields for estimating the sensor noise is measuring its output in a quiet ambient (Earth's) field [8]. The method is cheap and benefits from the real-world operation of the sensor, i.e., it is not artificially exposed to zero magnetic field. If ambient noise is lower than the predicted sensor noise, direct measurements can be performed, however, the ambient field cancellation method with two sensors, as described below, is utilized mostly.

A. Out of the shield – difference measurement

The most common method to reject ambient magnetic field and its noise is subtracting the outputs of two identical magnetic sensors; although more advanced methods might yield better results [9]. If we assume that the two sensors exhibit sensor noise $n_1(t)$ and $n_2(t)$, then it follows for the two noisy observations $y_1(t)$ and $y_2(t)$ of the ambient field $a(t)$:

$$y_1(t) = n_1(t) + a(t) \quad \text{and} \quad y_2(t) = n_2(t) + a(t) \tag{1}$$

If observations are subtracted, the common (correlated) noisy ambient field $a(t)$ is removed. The basic condition is that $a(t)$ is the same at the two sensors. This can be met where the noise gradient is negligible (i.e. sufficiently far away from anthropogenic noise). Natural ambient field fluctuations (diurnal changes of the Earth's magnetic field, magnetic storms / field oscillations induced by Sun activity, thunderstorm discharges etc. - [8]) can be regarded as homogeneous on a local scale and thus $a(t)$ for two aligned and calibrated sensors will be the same.

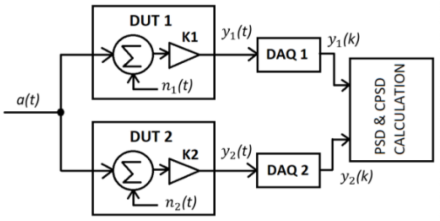

Fig. 1. Difference measurement method

If the condition that $n_1(t)$ and $n_2(t)$ are uncorrelated is met, and both signals are of the same magnitude and distribution, we can then write for Amplitude Spectral Density (ASD=√PSD, T/√Hz) of the individual sensor noise:

$$ASD\big(n_{1,2}(t)\big) = \frac{1}{\sqrt{2}} \cdot ASD\big(y_1(t) - y_2(t)\big) \tag{2}$$

This method is convenient, however suffers from drawbacks. First, the two sensors have to be of similar noise performance, otherwise the estimation yields more towards the noise of the inferior sensor. Also, the noise floor is limited by coherent sampling, alignment and perfect calibration of the two sensors, i.e. $k_1a(t) \neq k_2a(t)$. Further, any gradient or de-correlation (i.e. due to the presence of ferrous objects) in the ambient noise is not suppressed, requiring lowering the sensor-to-sensor distance [9] and/or using a low-gradient environment.

B. *Proposed method – cross-spectrum subtraction*

To overcome the first two limitations of the difference method, we propose to use a modified cross-spectrum method, as described i.e. in [10]-[12]. The cross-spectrum method was used to suppress the noise of the preamplifiers for magnetoresistive sensors [13] - here the uncorrelated noise was suppressed and noise of the magnetoresistors, far less than the noise floor of the preamplifiers, was revealed. In our case, we will however assume that the ambient magnetic noise is the only correlated component when logging the ambient field with two magnetic sensors. By calculating the cross-spectra and using the spectral subtraction method [14], however, we do not reconstruct the signal our case and stay in the frequency domain.

II. CROSS-SPECTRAL NOISE ESTIMATION IN AMBIENT MAGNETIC FIELD

With the “modified cross-spectrum method”, we can calculate individual sensor noise while overcoming the drawbacks of the direct-subtraction method. Specifically, we can estimate the noise when using two sensors with different noise characteristics, which will be illustrated below even in the case of more than one order of magnitude difference (fluxgate vs SQUID sensors). Also, this method allows us to estimate the noise of a single SQUID sensor, as the ULF noise can differ significantly from sensor to sensor. In our setup [18] we have only one SQUID axis running; therefore the difference method cannot be used.

A. *Method description*

Firstly we have to obtain power spectrum densities (PSD) for measured signals and cross power spectrum densities (CPSD) for DUTs. As the PSD/CPSD is a Fourier transform of auto/cross correlation $R(k)$ of sampled signals [15], we can write for the first observation $y_1(k)$ - using the notation of equation (1) and skipping some mathematical operations after substituting $y_1(k) = n_1(k) + a(k)$:

$$R_{y_1}(k) = \frac{1}{N}\sum_{n=1}^{N} y_1(n)y_1(n-k) = \cdots =$$
$$= R_a(k) + R_{an_1}(k) + R_{n_{1a}}(k) + R_{n_1}(k) \qquad (3)$$

Because $R_{\mathrm{an1}} = R_{\mathrm{n1a}}$, we can write:

$$R_{y_1}(k) = R_a(k) + 2R_{an_1}(k) + R_{n_1}(k) \qquad (4)$$

The same applies for observation y_2(k):

$$R_{y_2}(k) = R_2(k) = R_a(k) + 2R_{an_2}(k) + R_{n_1}(k) \qquad (5)$$

Cross-correlation between the two observations is then:

$$R_{y_1y_2}(k) = \frac{1}{N}\sum_{n=1}^{N} y_1(n)y_2(n-k) = \ldots =$$
$$= R_a(k) + 2R_{an_2}(k) + R_{n_1n_2}(k) \qquad (6)$$

When subtracting the cross-correlation (4) from $R_{y1}(k)$:

$$R_{y_1}(k) - R_{y_1y_2}(k) = R_a(k) + 2R_{an_1}(k) + ..$$

$$..+R_{n_1}(k) - \left(R_a(k) + 2R_{an_2}(k) + R_{n_1n_2}(k)\right) \qquad (7)$$

If we can assume that $2R_{an_1}(k) = 0$ and $2R_{an_2}(k) = 0$ due to no correlation between DUT intrinsic noise and external noise, and that also $R_{n_1n_2}(k) = 0$ due to no correlation between the noise of both DUTs, it is left that:

$$R_{y_1}(k) - R_{y_1y_2}(k) = R_{n_1}(k) \qquad (8)$$

which proves our method to be correct. Conversion between correlations $R(k)$ and power spectral density $S(f)$ can be expressed using Fourier transform F:

$$S(f) = F\{R(k)\} = \sum_{k=1}^{N} R(k)e^{-i2\pi fk} \qquad (9)$$

The Welch’s periodogram [15] estimates the averaged $S(f)$ with number of averages m, defined by window length and overlap. The number of averages influences the variance of the spectrum – $PSD = S(f)$ variance is approximately inversely proportional to m. During the estimation of $a(t)$ noise by calculating the CPSD, we need to suppress the non-correlated part of the y_1 and y_2 observations. The suppression further depends on the number of averages available, and is inversely proportional to √m [10]. The minimum number of averages for Gaussian signals was shown to depend on the inverse coherence function γ^2 [16]: $m_{\mathrm{min}} = 1/\gamma^2$.

B. *Verification with synthetic data*

For the initial testing of the proposed method, we generated synthetic data in MATLAB (white noise with additional pink noise). Using three arrays $n_1(k)$, $n_2(k)$, $a(k)$ of uncorrelated noise we obtained two “composite noise” observations $y_1(k)$, $y_2(k)$ - see equation (1). We simulated a frequent scenario with one “good” low-noise sensor and one “poor” sensor with higher noise. In this case the subtraction method cannot be used as it automatically leads to the noise of “poor” sensor. Results can be seen in Fig. 2.

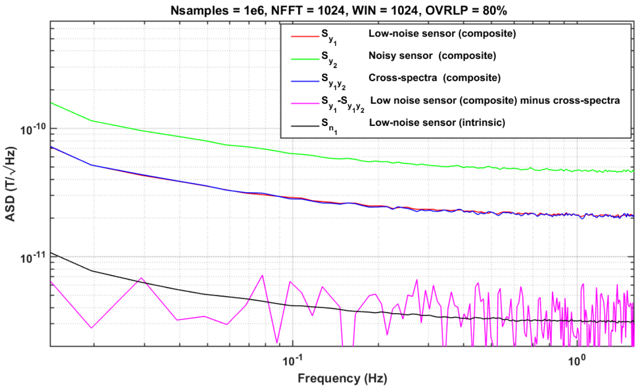

Fig. 2. Verification with simulated data – 1×10^6 samples, m ~4900

C. *Effects of imperfections – alignment, sample rate, gain*

For practical use of the method, multiple imperfect scenarios have been tested to verify its usability. Since we are doing computations in the frequency domain instead of the time domain, and the process can be assumed ergodic, it gives us inherent independence to time alignment (lead/lag has no effect on the CPSD subtraction method). Errors in sampling rate leads to worsening of results as shown in Fig.3 for real data. Error in gains (>10%) caused by imperfect calibration also lead to worse results, however, we have verified that even a 1% gain error is acceptable.

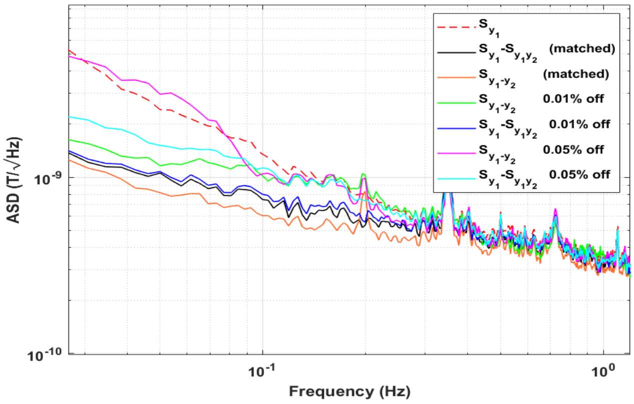

Fig. 3. Samplerate error simulation - effect on CPSD subtraction

D. *AMR magnetometer noise estimation*

The real-world measurement has been done using two identical AMR magnetometers [17] mounted on a wooden desk with 18 cm spacing, shown in Fig. 4. Both magnetometers streamed data over a serial link with the same sampling rate, and the data were recorded on a PC. Even at such a small distance, crosstalk between compensation windings of AMR sensors is negligible, as the compensation windings are on the AMR chip, and thus yield negligible magnetic moments. In Fig. 4 we see that the calculated noise of AMR#1 (black trace) matches the direct measurement of S_{n1} in a 6-layer shield (magenta) closely.

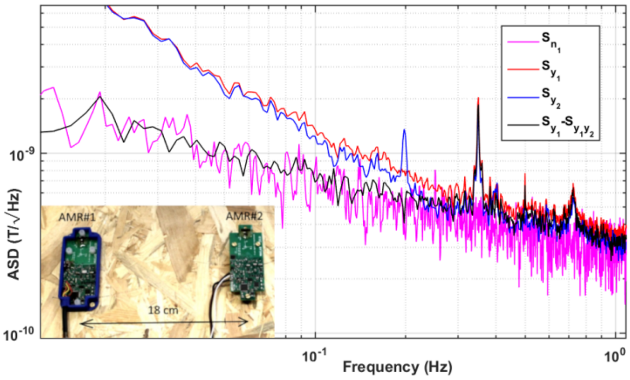

Fig. 4. AMR – noise measurement results and photo of the setup

III. FIELD ESTIMATION OF SQUID NOISE

At SANSA Space Science, we are operating an unshielded HTS SQUID for geomagnetic observations [18]. A single Z-axis sensor type M2700 (StarCryo, USA), is connected to flux-feedback-loop electronics type SEL-1 (Magnicon, Germany) - see Fig. 5. The analog output is digitized with two 24-bit cards, NI-9252 and AD24-ETH. For this study we used the latter, as it offers lower noise at the expense of bandwidth. The ULF noise of the SQUID could not be estimated yet as it is masked by ambient noise and we are lacking a second sensor performing equally well.

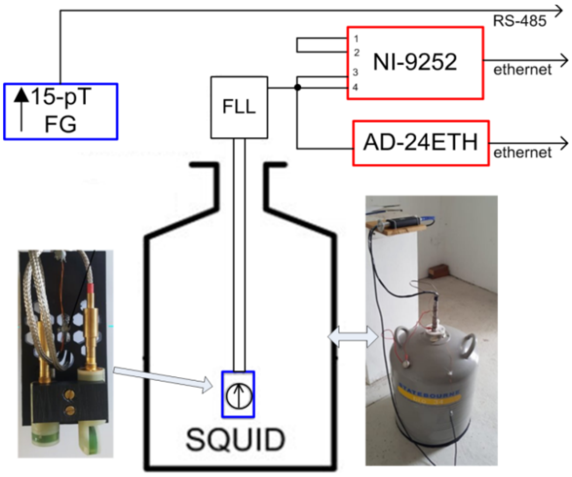

Fig. 5. HTS SQUID and fluxgate sensor setup

The AMR fluxgate and SQUID do not share a common ADC (the fluxgate output is digital). The only possible correlation would be from a noisy power supply, which is 12V DC buffered by lead-acid batteries. Fig. 6 shows a short record of sampled data and resulting spectra. Correlation between the fluxgate and SQUID data is evident from the time record; we also see that the "composite" fluxgate noise is also almost order of magnitude higher than of the SQUID.

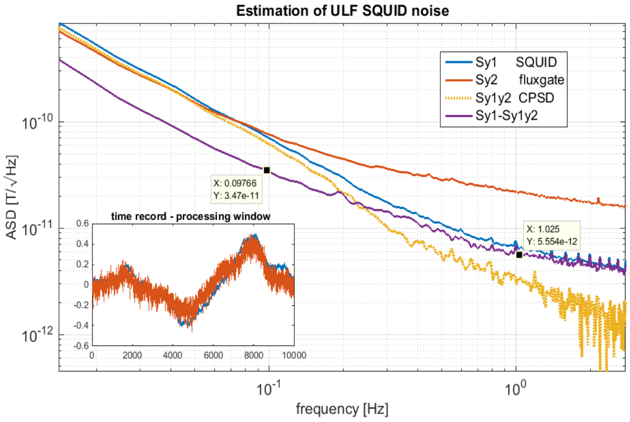

Fig. 6. SQUID and fluxgate data - time domain and noise estimation

The level of correlated ambient field noise of about 3 pT/√Hz at 1 Hz is reasonable since we observed similar values before with a 1-pT fluxgate magnetometer [19]. We can see that above approximately 200 mHz, the SQUID noise is dominating, with about 5 pT/√Hz at 1 Hz. Below 100 mHz the coherent ambient field noise dominates. The SQUID noise at 100 mHz (upper limit estimation) was established as about 30 pT/√Hz. A future measurement with the 1-pT fluxgate [19] as a third instrument will bring us more confidence in the potential of the presented method and reliability of the established SQUID noise.

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