Finite element method based computational time reversal in elastodynamics

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Motivation

The aim of this thesis is to develop a methodology to correctly reconstruct the source in the time domain using the computational time reversal method. This capability would allow identification of existing or forming flaws in the material by analyzing the reconstructed signals. Localization of sources using refocusing of energy is a well-established method, but reconstruction of these sources is still an unsolved problem. A dedicated explicit solver based on the finite element method is developed to solve the elastic wave propagation in solid material. The developed methodology is numerically tested and also verified using experimental data. It is proved that the reconstruction is feasible and correct with a tuned model. Thanks to the presented research, the practical deployment of the computational time reversal method is more promising and it can be included in the concept of the so-called digital twin. On the other hand, the method is very sensitive to the input parameters and very computationally demanding, which are aspects that open up possibilities for further research.

Aim of the Thesis

1. *Development of a numerical linear elastodynamic solver suitable for time reversal application*
2. *Development of a suitable methodology for FEM based TR (correct reconstruction of the source signal)*
3. *Sensitivity analysis of computational TR process*
4. *Reconstruction of a real source using experimental data*

State of the art

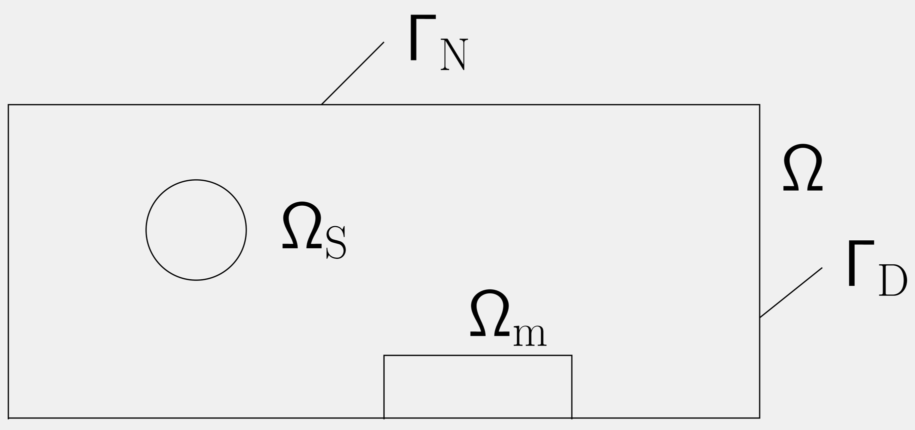


Figure 1. Scheme of TR problem.

Frontal problem

$$\nabla \cdot \sigma = \rho \ddot{\mathbf{u}}, \quad \sigma = \mathbf{C} : \varepsilon, \quad \varepsilon = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathsf{T}}), \quad 0 \le t \le T, \quad \mathbf{x} \in \Omega,$$

with boundary conditions

$$\mathbb{B}\mathbf{u} = \mathbf{0} \text{ on } \partial\Omega,$$

and with initial conditions

$$\mathbf{u}(\mathbf{x}, t = 0) = \mathbf{u}^0(\mathbf{x}), \quad \dot{\mathbf{u}}(\mathbf{x}, t = 0) = \mathbf{v}^0(\mathbf{x}), \quad \mathbf{x} \in \Omega,$$

and with measured output

$$\dot{\mathbf{u}}_{\mathrm{m}}(\mathbf{x}, t), \quad \mathbf{x} \in \Omega_{\mathrm{m}}.$$

Reverse problem

$$\nabla \cdot \bar{\sigma} = \rho \ddot{\mathbf{w}}, \quad \bar{\sigma} = \mathbf{C} : \bar{\varepsilon}, \quad \bar{\varepsilon} = \frac{1}{2}(\nabla \mathbf{w} + (\nabla \mathbf{w})^{\mathsf{T}}), \quad 0 \le \tau \le T, \quad \mathbf{x} \in \Omega,$$

with boundary conditions

$$\mathbb{B}\mathbf{w} = \mathbf{0} \text{ on } \partial\Omega,$$

and with initial conditions

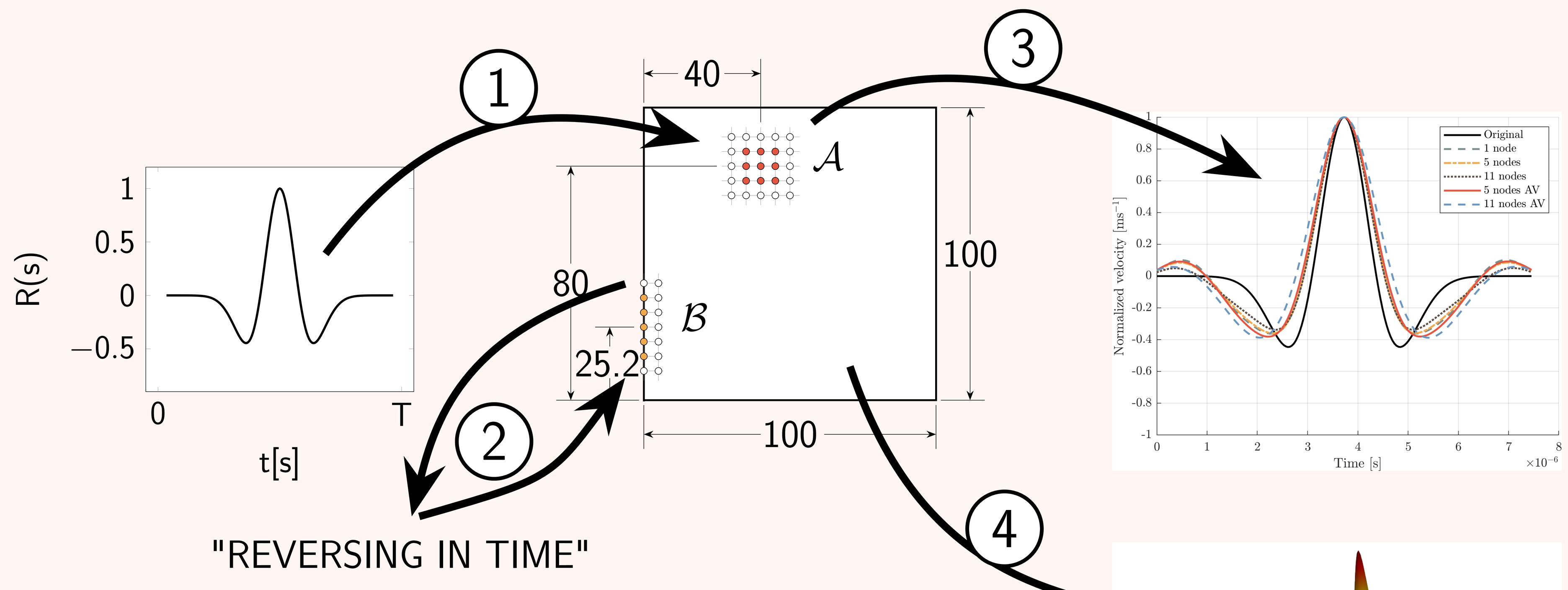
$$\mathbf{w}(\mathbf{x}, 0) = \mathbf{0}, \quad \dot{\mathbf{w}}(\mathbf{x}, 0) = \mathbf{0}, \quad \mathbf{x} \in \Omega,$$

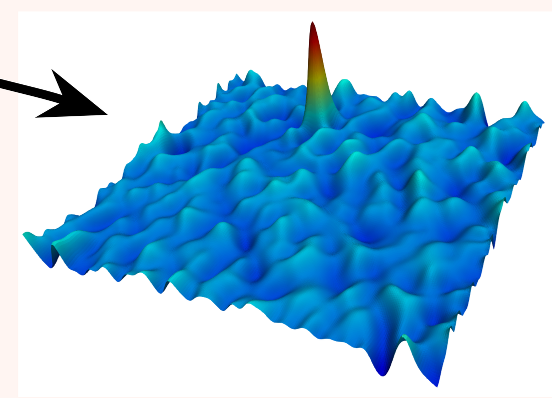
and with kinematic conditions on Ω_{m} for TR identification as follows

$$\mathbf{w}(\mathbf{x}, \tau) = \mathbf{u}_{\mathrm{m}}(\mathbf{x}, t), \quad \dot{\mathbf{w}}(\mathbf{x}, \tau) = -\dot{\mathbf{u}}_{\mathrm{m}}(\mathbf{x}, t) \quad \mathbf{x} \in \Omega_{\mathrm{m}}$$

Results

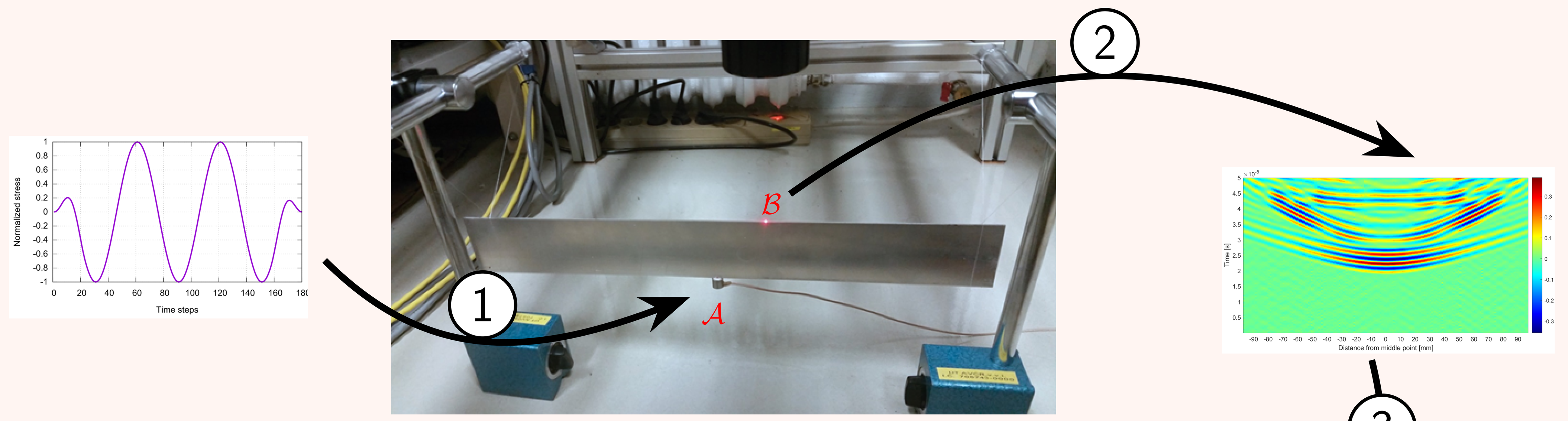
Source location and reconstruction - numerical simulation





1. Loading numerical model with signal
2. Loading with time-reversed output signal
3. Reconstruction of original signal
4. Reconstruction of velocity field (refocusing)

Source reconstruction - TR with experimental data



1. Loading real model with signal
2. Export of output signal
3. Loading numerical model with time-reversed output signal
4. Reconstruction of original signal

Conclusions

- This research was conducted to improve the quality of the reconstruction of the original source using the computational time reversal method.
- A procedure for correctly prescribing the loading signal was developed and verified on experimental data, and several numerical tests were performed as a benchmark.
- It can be concluded that with a sufficient amount of information loaded (temporal and spatial) it is possible to reconstruct the original source in the time domain correctly.
- The use of the computational time reversal method in practice is feasable if the parameters of the computational model are sufficiently calibrated.

Author's publications

[1] Mračko, M., Kober, J., Kolman, R., Převorovský, Z., Tkachuk, A., Plešek J. *Finite Element Method Based Computational Time Reversal in Elastodynamics*, Mathematics and Computers in Simulation, 2021.

[2] Mračko, M., Adámek, V., Berezovski, A., Kober, J., Kolman, R. *Finite Element Method Based Computational Time Reversal in Elastodynamics*, Applied Acoustics, 2021.

[3] Adámek, V., Berezovski, A., Mračko, M., Kolman, R. *A Two-Layer Elastic Strip under Transverse Impact Loading*, Mathematics and Computers in Simulation, 2021.