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ESTIMATION OF MACHINE TOOL ACCURACY BASED ON MONTE CARLO SIMULATION USING MOVEMENT AXES GEOMETRIC ERRORS MODEL

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Abstract

Accuracy is one of the most important feature of a machine tool. Being able to estimate the overall accuracy of a machine tool represented by the total volumetric error can be a huge advantage in the design process. This paper provides overview of methods used for geometric error transfer modeling and volumetric error modeling. An error transfer rigid body model is derived and after that, a random linear guideway straightness error profile generator is proposed. This allows to simulate a movement axis with random straightness errors of rails and computed geometric errors of an assembled axis. Volumetric error of a 3-axis machine tool is then modeled using its kinematic scheme and homogenous transformation matrices with 21 kinematic errors. Using simulated axes as input to the volumetric model, an entire machine tool can be simulated. From that, the total volumetric error can be obtained and a Monte Carlo simulation carried out. This allows to analyze statistics of the total volumetric error and with slight input changes (rail straightness tolerance), a sensitivity analysis can be performed to determine the most influential errors in the machine tool.

Keywords:
Volumetric accuracy, Monte Carlo simulation, Error budget

1 INTRODUCTION

Machine tool accuracy is a one of the most important feature of machine tools. Current industry demands increasingly higher accuracy, and machine tool producers need to deliver this. The accuracy of machine tool has multiple contributing factors [Schwenke et al. 2008]:

- Geometric accuracy
- Thermal expansion errors
- Static and dynamic compliance
- Control errors

This paper focuses on geometric accuracy of machine tools, thus regarding accuracy under no load or quasistatic machining conditions.

Achieving higher accuracy through tight tolerances on structural parts and through very strict assembly requirements is very expensive and uneconomical. This is the main motivation for implementation of the error budgeting method, where the aim is to control the total error of a system at the design stage by analyzing and balancing error contributions from individual subsystems. [Donaldson 1980]

Error budgeting of first level studies the error field effect of basic elements, e.g. a guideway. This means optimizing individual rail straightness errors to achieve higher overall axis accuracy and service life of bearings. Error budgeting of second level then studies an error field of multiaxis systems consisting of basic elements, optimizing individual axis accuracy or machine structure to achieve a better overall volumetric accuracy [Treib 1987].

Some research papers presented the effect of individual straightness errors of rails on resulting straightness of axis [Majda 2012], [Rahmani 2015], [Ni 2019] [Tong 2020] and thus contributed to first level error budgeting knowledge. Some other authors also studied and used the modeling of volumetric errors [Treib 1987], [Kiridena 1993], [Dang 2015] using homogenous transformation matrices [Stejskal 1996] for purposes of second level error budgeting optimization. The lesser common way of modeling and computing volumetric errors is screw theory [Liu 2018], [Zhong 2019].

In this paper, the first and second level error budgeting are combined to achieve an increase in overall volumetric accuracy through optimizing straightness tolerances of linear rails, and thanks to recent research [Zhong et al. 2019], consequently the straightness tolerances of linear rail mounting surfaces. Zhong's experiments have shown

that the linear rail, when mounted properly, will almost perfectly adopt the straightness profile of its mounting surface. A force-deflection geometric error model of a 4-carriage moving table is used together with HTM to model the resultant volumetric error.

To be able to predict the volumetric error and to optimize the tolerances, a huge set of straightness error profiles is needed. No measurement can supply this amount of data, so a random straightness error profile generator is derived and used. Some research has been done on the character of error profiles [Ekinci 2007], [Qi 2016], [Fan 2018] and it is being generalized as a sine curve. This however is not very accurate and thus an error profile character is described.

Another factor to consider is the probability distribution of straightness given a tolerance. [Treib 1987] uses a normal distribution for individual straightness errors, [Shen 1993] uses a uniform distribution. [Elmaraghy 2018] is however a much more recent source and recommends using a Rice distribution, or a noncentral chi distribution. However, thanks to a set of own measured data of C-frame machine linear rail straightness errors, a beta distribution is proposed.

Thanks to the robust random error profile generator, the entire error budgeting model can be used for Monte Carlo simulation and thus overall volumetric accuracy can be analyzed and optimized, based on tolerances of individual structural parts. This similar goal has been described [Wu 2020], but Wu et al. took the liberty of generalizing all machines' rail straightness curve as a sine curve at the limit of tolerance, and also did a very basic calculation of axis geometric errors given the rail's straightness. This paper is thus more thorough in modeling and the stochastic approach results in more results and information. The simulation approach is presented only. The accuracy model of a three-axis milling machine is presented in the section 2. The model is used in section 3 for analyze of key design parameters influencing the machine tool final accuracy. Discussion of results is presented in section 4.

2 BUILDING THE SIMULATION MODEL

2.1 Random straightness error profile generator

The following machine tool accuracy model uses as an input the movable axis rail straightness error profile for calculation of the TCP errors. Thus, the first proposed model of this paper is the random straightness error profile generator – a function that generates data for further use in the model, substituting physical measurements of straightness error. To build such generator, samples of measured data need to be examined (see Fig. 1 for reference) and the character of straightness error profiles described. These error profiles vastly differ according to machine type and size – the generator built in this paper focuses on mid-size C-frame vertical milling machines and would need adjustment for other types of machine tools.
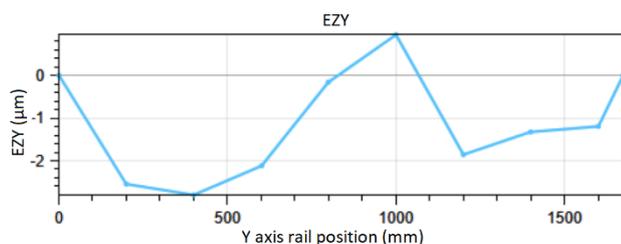
Fig. 1: An example of a linear rail straightness profile.

The character and properties of the linear rail straightness profile can be described as such:

- Starts and ends with a zero error (adjusted straightness)
- The profile is scalable in both directions
- An error change frequency can be observed
- The rate of change has its bounds

Beta distribution of straightness value given a tolerance is used to generate the value for the profile. Thanks to a set of own measured data of C-frame machine linear rail straightness errors, a beta distribution with coefficients $\alpha=6$ and $\beta=2$ and limits between 0 and tolerance value is proposed. This distribution and its coefficients were verified using a chi-squared test. Inputs to the function are as follows:

- Length of rail
- Straightness tolerance
- Error period
- Error multiplier
- Beta distribution parameters
- Output data resolution (simulation step)

See Fig. 2 for explanation of terms.
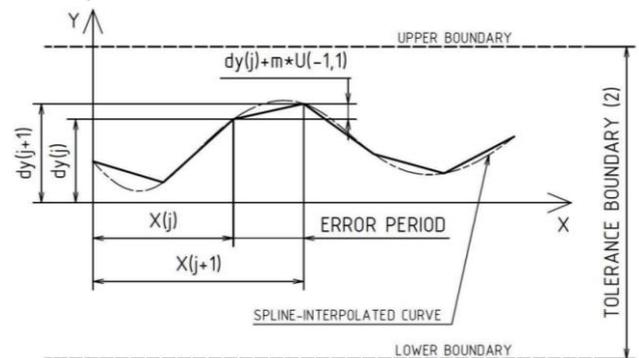
Fig. 2: Explanation of some inputs of the function.

This generator works by first randomly generating a number between -1 and 1 from a uniform distribution, and then for every other point after the error period it adds another random number from the uniform distribution, but this time multiplied by the error multiplier. If at any point the result is outside of the boundary (from -1 to 1), it sets the point to be equal to the boundary. After generating these initial points, a cubic curve is fitted through the points and adjusted, so it begins and ends at zero. Then, the entire curve is multiplied by a random number from the tolerance beta distribution and output points with desired resolution generated.
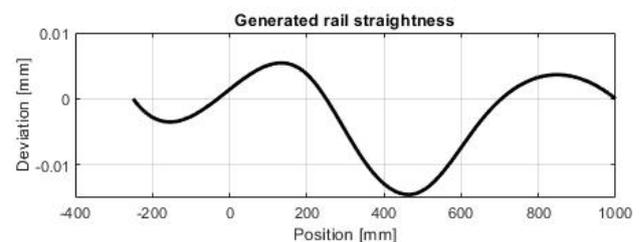
Fig. 3: An example of a generated straightness error profile.

In Fig. 3, an example of a randomly generated straightness error profile is shown. This was generated using a 1250 mm long rail with a straightness tolerance of 0,03 mm. Error period and error multiplier were set to 150 mm and 0,65 respectively. These parameters are kept for further use in the model.

2.2 Deflection model of a 4-carriage table

A 4-carriage table is example of a basic structural group of the machine tool. Two pairs of carriages follow two rails of the linear guideways. The model enables to predict final movement errors in the center of the table. With a given deviation from the perfect straightness at each carriage, the total deflection of the center point of the table can be determined, with a set of boundary conditions. These are:

- Rigid structural parts
- Linear deformation behavior of carriages
- Small deformations and angle errors
- Stiffness center of carriages as a point of substitution

For the purpose of this model, linear rail carriages are substituted with lateral and normal springs, each with its respective stiffness k_{zi} and k_{yi} (Fig. 4).
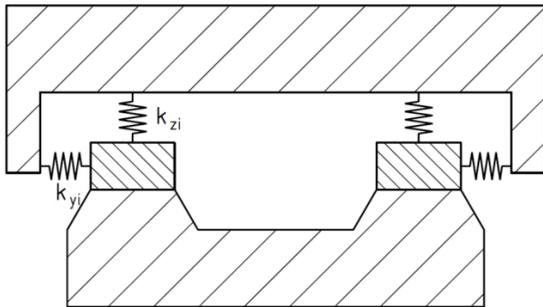
Fig. 4. Carriage to spring substitution.

Indexing of rails and carriages and table dimensions are determined by Fig. 5 and straightness deviation nomenclature can be seen in Fig. 6.
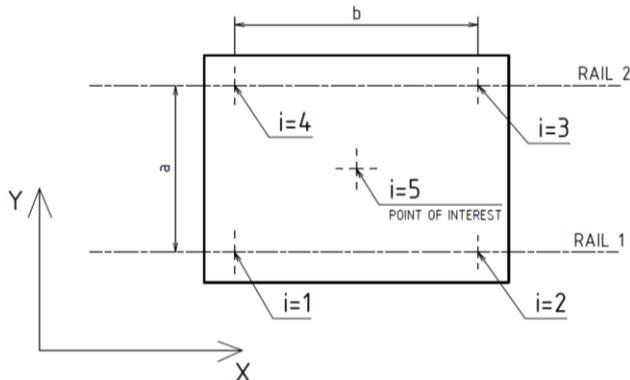
Fig. 5: Indexing of rails and carriages, table dimensions.
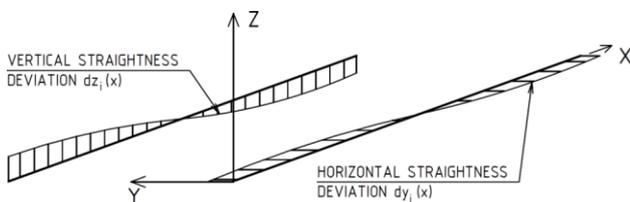
Fig. 6: Straightness deviation nomenclature on example of X linear axis.

A rigid mechanical model was derived, using deviations of each rail at the carriage and carriage stiffness as input and outputting resultant deviation of the carriages, deviation of the center point and angle errors. First, internal forces of substitute springs F_{zi} and F_{yi} are calculated using matrix equations (3) and (4).

Then, individual carriage resultant deflection z_i and y_i are calculated using equation (1). Center point deflection is then just a simple average of individual corner deflections.

$$z_i = d_{zi} + \frac{F_{zi}}{k_{zi}}; \quad y_i = d_{yi} + \frac{F_{yi}}{k_{yi}} \tag{1}$$

With this computed, angle errors can also be calculated (2).

$$roll = \frac{z_4 - z_1}{a}; pitch = \frac{z_2 - z_1}{b}; yaw = \frac{y_1 - y_2}{b} \tag{2}$$

This model can be fed either measured data, or data generated with the random straightness error profile generator from chapter 2.1, resulting in a randomly generated machine axis.

2.3 Calculating the total volumetric error of a 3 axis C frame machine tool

This section describes enhancing of the previously presented model to the whole machine tool structure. With either computed or measured error components of all linear axes, the total volumetric error can be calculated using homogenous transformation matrices (HTM) and the kinematic model of the machine tool. The simplified kinematic scheme of the structure and its dimensions are in Fig. 7. HTM is a widely used method for computing the volumetric error amongst most authors.
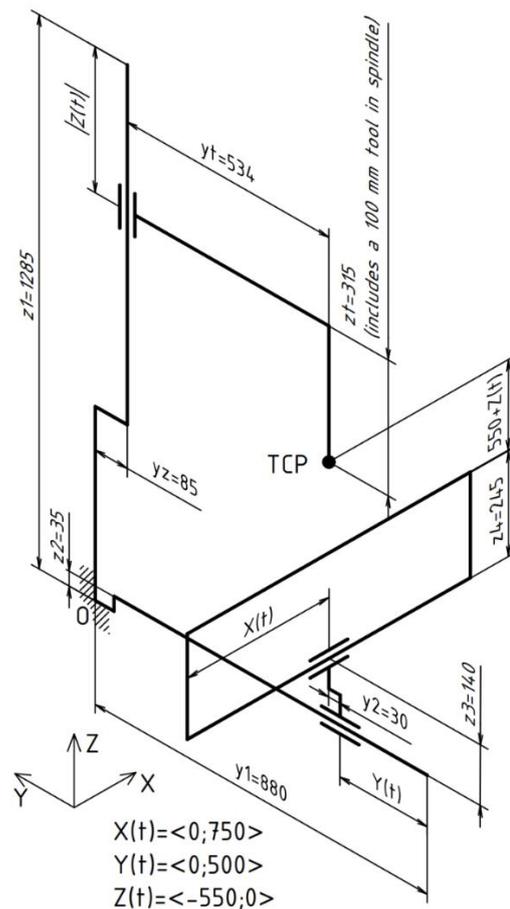
Fig. 7: Simplified mechanics of a midsize C frame machine tool.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ \frac{1}{k_{z1}} & \frac{-1}{k_{z2}} & \frac{1}{k_{z3}} & \frac{-1}{k_{z4}} \end{bmatrix} \begin{bmatrix} F_{z1} \\ F_{z2} \\ F_{z3} \\ F_{z4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -d_{z1}(x) + d_{z2}(x) - d_{z3}(x) + d_{z4}(x) \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ \frac{1}{k_{y1}} & 0 & 0 & \frac{-1}{k_{y4}} \\ 0 & \frac{1}{k_{y2}} & \frac{-1}{k_{z3}} & 0 \end{bmatrix} \begin{bmatrix} F_{y1} \\ F_{y2} \\ F_{y3} \\ F_{y4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -d_{y1}(x) + d_{y4}(x) \\ -d_{y2}(x) + d_{y3}(x) \end{bmatrix} \tag{4}$$

The entire structure of the machine tool is modeled in two directions towards the TCP – from the origin O through the workpiece and through the tool. Translational volumetric errors are calculated as a difference between an ideal structure transformation and a transformation with individual axis error components introduced in their respective points. (5) is an example of a transformation matrix with all error components of an X axis in accordance with ISO 230-1.

$$\boldsymbol{T}_{EX} = \begin{bmatrix} 1 & -ECX & EBX & EXX \\ ECX & 1 & -EAX & EYX \\ -EBX & EAX & 1 & EZX \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The transformation itself is trivial, the resultant volumetric error calculation is, as mentioned, a difference between the ideal and error-filled transformation. To get the relative translational volumetric error with respect to the workpiece origin, the equation (6) is used, where the initial volumetric error in the workpiece coordinate system is subtracted.

$$(r^*_{0TTCP} - r_{0TTCP}) + (r^*_{0WTCP} - r_{0wTCP}) - E^0_{volT} = \begin{bmatrix} E_X \\ E_Y \\ E_Z \\ 1 \end{bmatrix} \tag{6}$$

Rotational volumetric errors are a simple sum of individual rotational errors.

The highest Euclidean distance of the volumetric error (7) in the machine's working area can serve to represent its total accuracy, when comparing machine tools.

$$\|E_{volT}\| = \sqrt{E_X^2 + E_Y^2 + E_Z^2} \tag{7}$$

The values of individual error components can be either obtained by measurement or by previous models. Randomly generating squareness errors and using the random straightness error profile generator and deflection model, an entire machine tool error description can be obtained, with random errors.

Volumetric errors can be visualized with a 3D map, where the total Euclidean distance deforms a section plane of the machine's working area. Pictured in Fig. 8 is an example of a volumetric accuracy 3D map of a machine tool with errors generated by previous models, with straightness tolerances on all axes od 0,03 mm, positioning accuracy of 0,01 mm and maximum squareness error of 40 μm /m.
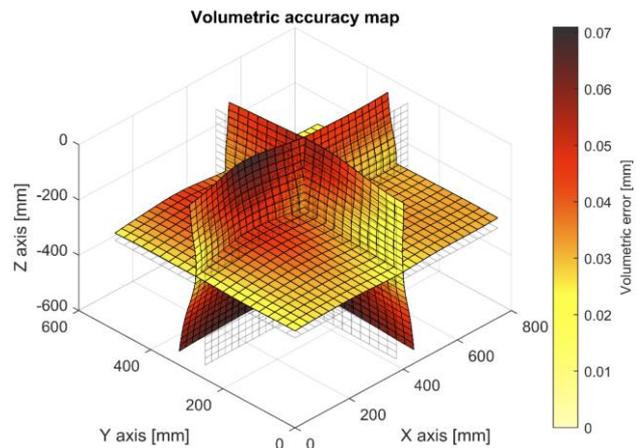
Fig. 8: An example of a volumetric accuracy 3D map.

This field of volumetric errors can be very useful for selecting the optimal area of machining. Calculating the maximal local volumetric error for all possible machining areas and selecting the one with the smallest total error is a simple operation, but it can improve the machining accuracy by up to 80%.

3 MONTE CARLO SIMULATION AND ANALYSIS OF VOLUMETRIC ERROR

3.1 Overall volumetric accuracy and sensitivity analysis

With the entire simulation model built, a Monte Carlo simulation can be carried out, stochastic estimating the total volumetric accuracy of the machine tool given its individual rail straightness tolerances.
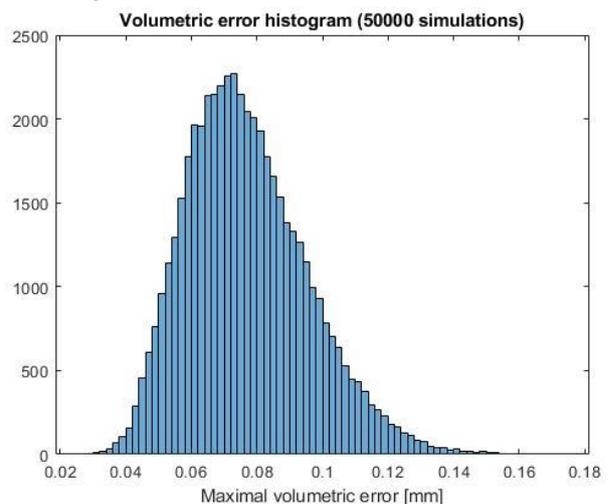
Fig. 9: Maximal volumetric error histogram.

A single output variable was selected – the highest total volumetric error in the working area of the machine tool – and with 50 000 simulation samples the resultant probability distribution was computed (Fig. 9). Straightness tolerances were set at 0,03 mm on all axis in both horizontal and vertical directions. Maximal squareness error was set to 40 µm/m and a positioning error was also generated, with a maximal error of 0,01 mm. With a 3σ rule (95%), it can be estimated, that a machine of described mechanics and tolerances should be working with a maximal volumetric error of 0,11 mm at most.

Thanks to this simulation model, input variables can be tweaked and the effect on the resultant volumetric error observed. This has been done for the straightness tolerances of rails one at a time. The tolerances used are {0,005 mm; 0,015 mm; 0,03 mm; 0,045 mm; 0,06 mm}. Each change was simulated with 50 000 samples and the 90th percentile of the maximal volumetric error was recorded. In total, 30 different Monte Carlo simulations were performed. In these simulations, the squareness error influence was suppressed.

Fig. 10: Volumetric error relation to vertical straightness tolerance of Y axis rails.

When only adjusting a single straightness tolerance, the relationship between the tolerance and the resulting volumetric accuracy appears to be quadratic (Fig. 10). To understandably compare individual influences, multiple linear regression was performed and coefficients of significance for individual tolerances were obtained (Tab. 1, VtX = vertical straightness tolerance on X axis rails, etc.).

Tab. 1: MLR coefficients of significance

VtX	HtX	VtY	HtY	VtZ	HtZ
1,13	0,16	1,02	0,07	0,64	0,20

Comparing the individual coefficients, it can be observed that the most weight lies upon the vertical straightness tolerance of X axis rails. Generally, it seems that the horizontal straightness tolerances are much less significant to overall accuracy.

3.2 Optimal machining area analysis

Other use case for this type of simulation is finding the optimal area for machining in a machine tool. As described, the optimal area for machining can be found. The origin of this area can be saved as a point in space and used in a 3D histogram to show which point or area is most likely to result in the most accurate volumetric machining.

For this simulation, the initial error tolerances were used together with a machining area of 200x100x50 mm. Using 50 000 samples again, Fig. 11 shows the resultant 3D histogram of optimal machining areas – the point with the most occurrences is most likely to be evaluated as the origin of an optimal area.

Another visualization method is viewing the average local volumetric error for all possible origins of machining areas (see Fig. 12).

In both cases, it appears that it is generally better to place workpieces near the table and thus achieving the lowest local volumetric error.
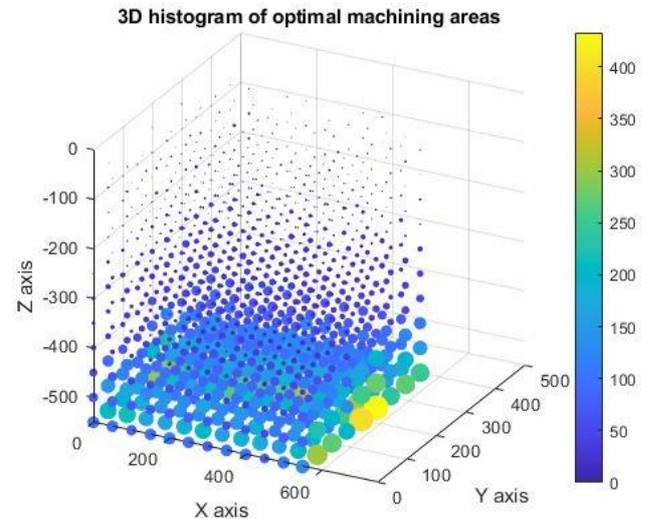
Fig. 11: 3D histogram of optimal machining area origin occurrences.
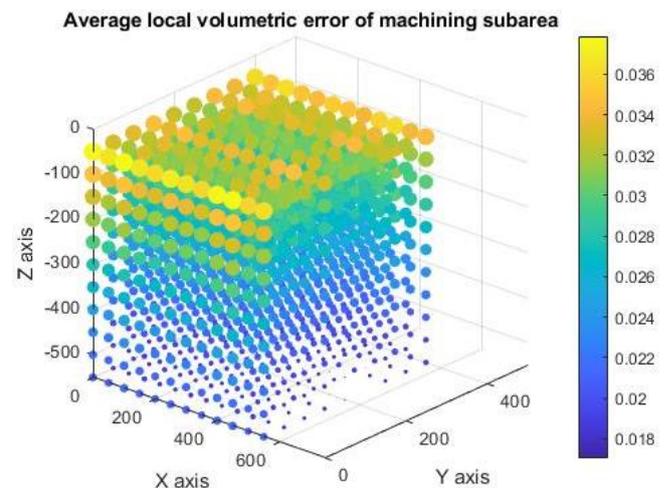
Fig. 12: Average local volumetric error of machining subareas.

The point with the most occurrences as the optimal origin of machining area and concurrently the point of origin with the lowest average local volumetric error was X550 Y250 Z-550, which is the point on the right end of X axis, middle of Y axis and bottom of Z axis.

4 DISCUSSION

A Monte Carlo simulation model of total volumetric accuracy was derived. The total model contains 3 partial models – random straightness error profile generator, a rigid deflection model of a 4-carriage table and an HTM kinematic model for volumetric error calculation. Results of sensitivity analysis in Tab. 1 show a very dominant influence of vertical straightness errors on the overall accuracy for the modelled machine and boundary conditions. This remains to be confirmed by measurement, but if confirmed, it would mean that horizontal straightness tolerances on structural parts of midsize C-frame machine

tools could be looser by an order of a magnitude without the machine showing a significant decrease of accuracy and thus would allow for cheaper manufacture. Because of the quadratic relation from Fig. 10, a multiple quadratic regression could be carried out for the simulated data and a one-equation relationship between individual rail straightness tolerances and resulting volumetric accuracy obtained.

During the volumetric error modelling stage, a calculation of optimal machining area was derived. This allows to find an optimal subspace in the machine with the lowest maximal local volumetric error and thus achieve the highest possible accuracy while machining. This is not only usable while using measured data of a real machine tool, but can also be analysed with a Monte Carlo simulation. The result is being able to define a point of origin of the highest accuracy for a given machining area of a workpiece. This was done with two different approaches – a 3D histogram of points being determined as the origin of an optimal machining area and a map of the machine's workspace with displayed average local volumetric error for each possible workpiece origin. In both cases, the same origin point arose as the either most occurrent or most accurate – X550 Y250 Z-550 – or the point on the right side of X axis, in the middle of Y axis and at the bottom of Z axis. Generally speaking, it is best to place workpieces as near the table as possible to achieve highest machining accuracy.

5 SUMMARY

This paper proposed a new method for Monte Carlo simulation of a machine tool's accuracy. This method will benefit new approaches to error budgeting in machine tools thanks to volumetric error estimation and tolerance sensitivity analysis. The stochastic approach allows to predict machine tool volumetric accuracy based on tolerances of structural parts and thus influence the machine's accuracy in the design phase. An example of input tolerances and parameters was given, resulting in an estimation of a basic midsize C-frame 3 axis milling machine's overall volumetric accuracy. The sensitivity analysis has shown that vertical straightness tolerances on rail mounting surfaces are of greater significance than horizontal straightness tolerances. Other type of accuracy analysis was finding the optimal workpiece origin with the smallest local volumetric error. Two approaches were used with the same general result – it is best to place workpieces near the table, in case of the modelled machine. The further research would benefit from verification of the force deflection model or its further deriving with compliant structural parts in mind.

6 ACKNOWLEDGMENTS

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