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EXTRACTING GENERAL KNOWLEDGE OF MODEL PARAMETERS FOR CLAYS OUT OF NUMEROUS LABORATORY TESTS

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Abstract. This paper is concerned with the process of determining the material parameters of advanced constitutive models for clays. Provided results are applicable already in the early stages of geotechnical survey when no detailed data from laboratory tests of soils expected at construction site are known. A worldwide database of laboratory tests obtained during four year operation of web calibration application ExCalibre was used for this purpose. This application allows automatic calibration of three advanced constitutive soil models based on detailed data from triaxial and edometric tests. The paper describes the process of determining the confidence intervals of each material parameter of modified Cam-clay and hypoplastic clay models for low (CL) and high (CH) plasticity clay according to unified soil classification system (USCS) and tabulates their typical ranges. The purpose of presented work was to contribute to the use of advanced constitutive models among practical engineers.

Keywords: Hypoplastic clay, modified Cam-clay, calibration, material parameters.

1. Introduction

In engineering practice, it can be observed that majority of geotechnical engineers limit themselves to the use of basic non-linear soil models of the Mohr-Coulomb or Drucker-Prager type when describing the interaction of the structure and the subsoil and mostly avoid critical state models when solving standard tasks. The general lack of knowledge of these advanced material models among practical engineers may be one of the reasons. Also standard civil engineering studies and the most commercial geotechnical programs primarily promote only basic models.

The second reason is that it is significantly more difficult to determine the input parameters of advanced non-linear models in comparison to the basic linear models. While the inputs for the use of basic non-linear models are shear strength parameters that can be determined directly from the results of standard laboratory or field tests, and their ranges are even tabulated for each soil type, critical state models require prescription of parameters describing the behavior of the given material not only under isotropic compression but also parameters characterizing the current state of the soil. A practical engineer often encounters a situation where, although he knows that there exist models which are capable to describe the behavior of modeled soil significantly more accurately than linear constitutive models, he is no longer able to determine the necessary parameters that govern the model.

2. Critical state models

Both, the hypoplastic clay [1] and modified Cam-clay [1, 2], constitutive models are based on the theory of critical state. The main features of these models are similar but unlike the hypoplastic model the modified Cam-clay model distinguishes the elastic and plastic part of strain components and therefore allows the straightforward visualization of permanently strained plastic zones in FEM calculations. On the other hand, the model of hypoplastic clay can correctly represent the associated gradual reduction in stiffness upon activation of plastic behavior rather than a sudden drop as predicted by the modified Cam-clay model.

2.1. Hypoplastic clay

The hypoplastic model for clay takes into account the following important properties of soils:

- The stiffness depends on the current value of the mean effective stress σ_m^{eff} and the direction of loading.
- The strength depends on the value of the mean effective stress σ_m^{eff} and current density characterized by the void ratio e.

The model distinguishes between material parameters that are constant for all possible states of a particular soil and state variables that evolve during straining. In this sense the hypoplastic model is fundamentally different from the Mohr-Coulomb model which employs distinct values of its material parameters for the same soil in different density states. In

Parameter	Name	Unit
Material parameters		
N	Position of the normal consolidation line	[-]
λ^*	Slope of the normal consolidation line	[-]
κ^*	Slope of the swelling line	[-]
φ_c	Friction angle in the critical state	[°]
ν	Parameter controlling the value of the shear modulus	[-]
State parameter		
e	Void ratio	[-]

TABLE 1. Material parameters of hypoplastic clay model.

Parameter	Name	Unit
Material parameters		
e_0	Position of the normal consolidation line	[-]
λ	Slope of the normal consolidation line	[-]
κ	Slope of the swelling line	[-]
M_{cs}	Slope of the critical state line	[-]
ν	Poisson's ratio	[-]
State parameter		
p_c	The preconsolidation pressure	[Pa]

TABLE 2. Material parameters of modified Cam-clay model.

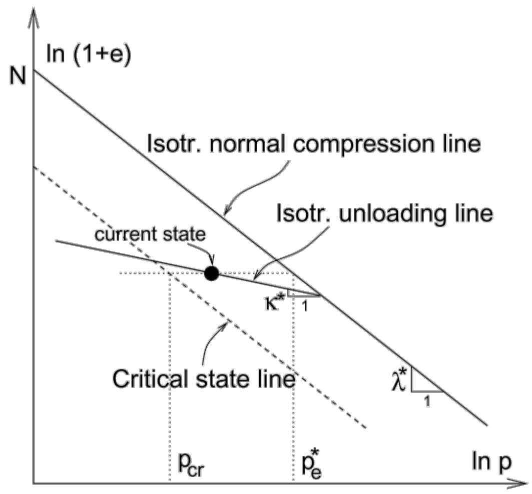


FIGURE 1. Normal consolidation line and swelling line assumed in hypoplastic model for clay [1].

particular, the hypoplastic model for clay requires five material parameters and one state variable summarized in Table 1.

2.1.1. Parameters N and λ^*

Parameters N and λ^* define the position and the slope of the isotropic normal compression line (NCL) in the $\ln(\sigma_m^{\text{eff}}) \times \ln(1+e)$ space as illustrated in Figure 1. The normal compression line defines a theoretical response of the soil sample during isotropic compression test. Since the model assumes incompressible grains the resulting volumetric strain depends on the decrease of volume of pores only and thus on the void ratio.

2.1.2. Parameter κ^*

Parameter κ^* defines the slope of the swelling line in the $\ln(\sigma_m^{\text{eff}}) \times \ln(1+e)$ space as illustrated in Figure 1. Unlike the NCL which is fixed by parameter N the position of the swelling line depends on the previous compression stress. In addition, unlike simple elasto-plastic models, the slope of the unloading-reloading line is non-linear in hypoplasticity, so that the parameter κ^* should thus be calibrated by means of simulation of a laboratory test, rather than by directly evaluating its slope.

2.1.3. Parameter φ_c

Parameter φ_c is the critical angle of internal friction. The angle of repose can not be measured for fine-grained soils and the critical friction angle has to be determined from triaxial shear test. The critical friction angle correspond to the maximal mobilized friction angle for normally consolidated samples where no softening occurs during the triaxial shear test. Or the critical friction angle corresponds to the mobilized friction angle after the peak for overconsolidated samples.

2.1.4. Parameter ν

Parameter ν controls the value of the shear modulus. The bulk and shear modulus are linked through the parameter ν in the isotropic normally consolidated state. During shear test the shear modulus evolves non-linearly and its magnitude is being controlled by the parameter ν and by the distance from the failure state.

2.2. Modified Cam-clay

The modified Cam-clay takes into account the following important properties of soils:

- The stiffness depends on the current value of the mean effective stress σ_m^{eff} and the direction of loading.
- The strength depends on the value of the mean effective stress σ_m^{eff} a current density characterized by the void ratio e or by the preconsolidation pressure p_c.

The model introduces five material parameters that are constant for all possible states of a particular soil and one state variable that evolves during straining. All parameters are summarized in Table 2.

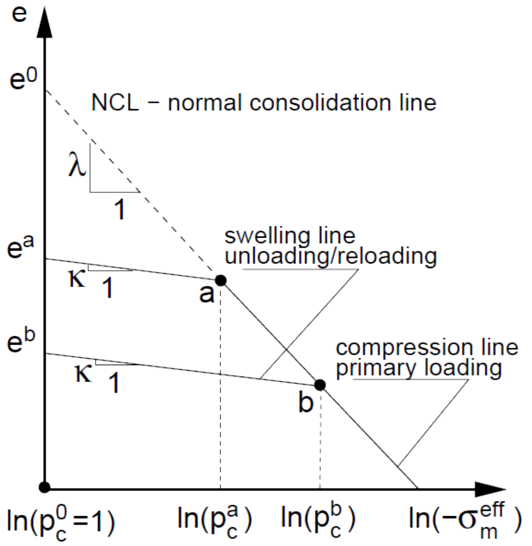


FIGURE 2. Normal consolidation line and swelling line assumed in modified Cam-clay model [2].

2.2.1. PARAMETERS e_0 AND λ

Parameters e_0 and λ define the position and the slope of the isotropic normal compression line (NCL) in the $\ln(\sigma_m^{\text{eff}}) \times e$ space as illustrated in Figure 2. The normal compression line defines a theoretical response of the soil sample during isotropic compression test. Since the model assumes incompressible grains the resulting volumetric strain depends on the decrease of volume of pores only and thus on the void ratio.

2.2.2. PARAMETER κ

Parameter κ defines the slope of the unloading/reloading line in the $\ln(\sigma_m^{\text{eff}}) \times e$ space as illustrated in Figure 2. Unlike the NCL which is fixed vertically by parameter e_0 the position of the swelling (unloading/reloading) line depends on the maximal previous compression stress denoted as the preconsolidation pressure p_c.

2.2.3. PARAMETER M_{cs}

Parameter M_{cs} is the slope of the critical state line plotted in the $\sigma_m^{\text{eff}} \times J$ space, see Figure 3. The value is computed from the critical state angle φ_c by:

$$M_{cs} = \frac{2\sqrt{3}\sin\varphi_c}{3 - \sin\varphi_c}\,. \tag{1}$$

2.2.4. PARAMETER ν

Parameter ν is the Poisson ratio known from linear elasticity. It controls the ratio of radial and axial elastic strains under uniaxial loading and consequently the ratio between the stress dependent bulk and shear modulus.

2.2.5. PRECONSOLIDATION PRESSURE p_c

The model has one state variable – the preconsolidation pressure p_c. For an isotropic compression test the preconsolidation pressure p_c corresponds to the maximal mean stress σ_m^{eff} to which the soil has been subjected. The kinematics of the model is evident from Figure 2. Soil sample with an initial state a (it has void ratio e^a and preconsolidation pressure p_c^a) is loaded isotropically up to the compressive mean stress $\sigma_m^{\text{eff}} = p_c^a$. The deformation during this loading is nonlinear but elastic, i.e. if the soil is unloaded from this point it returns along the κ-line back to the state with $e = e^a$ and no permanent volumetric strain occurs. If the soil is loaded beyond the point p_c^a it moves along the λ-line. During this loading both the elastic and the plastic volumetric strain develop. When the soil is unloaded from the point $\sigma_m^{\text{eff}} = p_c^b$ it moves along the new κ-line defined by the new value of preconsolidation pressure $\sigma_m^{\text{eff}} = p_c^b$. As evident from the initial and final (in unloaded state) values of the void ratio $e^a > e^b$ the permanent volumetric strain developed during this loading sequence [1].

3. AUTOMATED CALIBRATION OF CONSTITUTIVE MODELS

Even though the development of more advanced constitutive soil model theories which take into account bounding surface plasticity, whether those based on the theory of plasticity or hypoplasticity [3, 4], or barodesy [5] continues steadily forward, the development of automated calibration procedures stays behind. The calibration procedures are regarded as the inverse analysis, since the parameters of a constitutive model are not known in advance but the reaction of the system is known. The goal of the calibration is to minimize the objective error function $E(U)$ which is defined as a function of a difference between the observation or experiment U_{exp} and simulation U_{num}. The most common is the least square method. Nevertheless, more sophisticated dimensionless formulas can be used [6].

3.1. EXCALIBRE

As mentioned in the introduction, the Faculty of Civil Engineering of CTU in cooperation with the Faculty of Science of Charles University developed a ExCalibre web application [7] which performs automatic deterministic calibration of the hypoplastic model for sands [8], hypoplastic model for clays [9] and modified Cam-clay model [10]. From 2018, the application [1] is available free of charge for registered users who agree to provide the uploaded data for research and development purposes. A typical workflow of model calibration in ExCalibre is the following:

(1.) The user downloads a laboratory protocol template which is prepared for the user in the form of an MS Excel file. The template is different for clays and sands.

(2.) The user fills in the results of laboratory tests. A minimum of one consolidated isotropically undrained (CIUP) triaxial test with pore pressure measurement for clay soils or one consolidated isotropically drained (CID) triaxial test for sandy

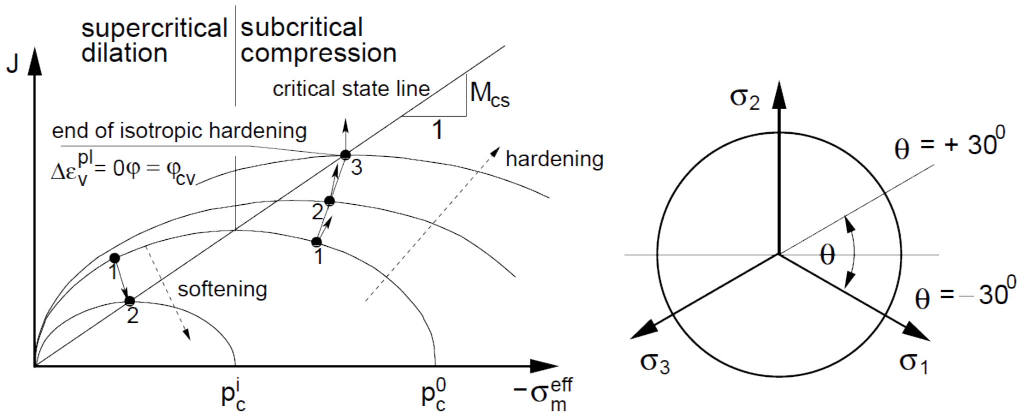


Figure 3. Yield surface of the modified Cam-clay model [2].

soils and one standard oedometric (OED) test must be completed for successful calibration. In addition to those already mentioned, it is possible to fill in the classification according to USCS and index characteristics of the soil. These parameters are currently not used in calibration but they are stored for research purposes.

(3.) The user uploads the laboratory protocol back to the web application. As a result, the application proposes optimal model parameters and graphs comparing the course of measured and simulated laboratory tests. These graphs serve to visually check the calibration of the model.

(4.) It is possible to manually modify individual model parameters to adjust the calibration or to examine the influence of individual parameters on model predictions after the calibration.

Large pool of available data collected worldwide propose an effective basis for statistical processing, determination of typical ranges of individual parameters and search for correlations between individual parameters presented in this paper.

4. Data processing

All data work was concentrated in universal scripts written in the Python programming language due to the repeatability of laboratory test data analysis. It is a higher-level programming language with dynamic support for data types and support for programming paradigms. The programming language [11] is freely available.

The format conversion of laboratory protocols from Microsoft Excel Spreadsheet XLSX ExCalibre template to Comma-separated values CSV was the first step of data processing. It was a necessary step due to the complexity of the XLSX file structure. XLSX file is a zip package that contains XML files representing individual sheets of the spreadsheet which are always supported by RELS files containing specifications for linking individual XML files together to form a complete file. Compared to complex XLSX file, CSV is a simple file format that is intended for exchanging tabular data between different systems. The data of the CSV file are stored in rows just like in the default table while the individual items in the row are further separated by a predetermined character. The individual sheets of the input XLSX laboratory protocol form separate CSV files which are, for organizational reasons, stored in a folder with a name corresponding to the original name of the XLSX file so that one folder represents one complete laboratory protocol.

Written script cycles through the folders representing individual laboratory protocols with CSV files replacing Microsoft Excel Spreadsheet and extracts characteristic information about the laboratory protocol using predefined classes. The extracted data is further inserted into a dictionary-type data structure. A dictionary is a data type sometimes called an associative array. It consists of key-value pairs. Each key-value pair maps a key to an associated value. In the algorithm described , the key corresponds to the name of a folder representing one laboratory protocol. Key values are assigned using classes so that one line of the dictionary represents the data of one laboratory protocol. The first value of the dictionary corresponds to the relative path to the file, assigned attributes are: USCS classification of soil, basic index characteristics such as specific gravity, Atterberg's limits or angle of repose. It is followed by representation of particle size distribution and loading and quantity of performed oedometric and triaxial tests.

The ExCalibre application contains a public laboratory test database so that these protocols can be used for testing the functionality of the calibration application by a new user. Knowing that, filtering of duplicate laboratory protocols was another step of data processing. A simple condition consisting of

- USCS classification of soil,
- the largest sieve and the corresponding passing in the grain size analysis,
- the initial void ratio of the oedometric test of the naturally consolidated soil sample,
- the initial void ratio of the oedometric test of the reconstituted soil sample

	Hypoplastic clay					Modified Cam-clay				
	λ^*	κ^*	N	ν	φ_c	λ	κ	e_0	ν	M_{cs}
Count	7	7	7	7	7	7	7	7	7	7
Mean value	0.119	0.010	1.396	0.276	25.3	0.297	0.016	2.784	0.307	0.997
Standard deviation	0.082	0.003	0.659	0.062	2.0	0.365	0.012	2.521	0.065	0.088
Minimum value	0.063	0.006	0.872	0.170	22.8	0.104	0.010	1.237	0.220	0.888
25 % quantile	0.068	0.009	0.960	0.245	23.9	0.107	0.010	1.383	0.250	0.935
50 % quantile	0.094	0.010	1.173	0.280	25.6	0.155	0.010	1.752	0.340	1.008
75 % quantile	0.123	0.010	1.521	0.320	26.2	0.249	0.018	2.734	0.355	1.032
Maximum value	0.296	0.014	2.763	0.350	28.8	1.107	0.039	8.263	0.380	1.147

TABLE 3. Basic statistical description of calibrated model parameters for CH soil.

was used for the determination and removal of duplicate laboratory protocols. The results of this class-driven filtering revealed that only 58 of the total 1 924 laboratory protocols are non-duplicated. The remaining protocols are duplicates of another 161 protocols in various frequencies. In total, the database would therefore contains 219 unique laboratory protocols. Considering USCS composition of non-duplicate protocols only low (CL) and high (CH) plasticity clay soils were selected for further processing. All of the laboratory protocols of soils classified as CH or CL have been uploaded to the ExCalibre web application and material parameters for hypoplastic model for clay soils and modified Cam-clay were calibrated and added to the dictionary-like dataframe collecting protocol information. Thus the already existing dataframe was extended by 5 columns with material parameters of hypoplastic clay model (see Table 1) and another 5 columns for modified Cam-clay model material parameters (see Table 2).

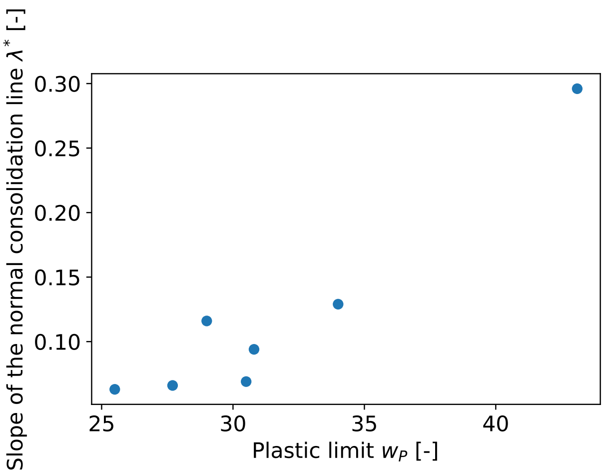


FIGURE 4. Correlation between plastic limit and slope of the NCL assumed in hypoplastic model for clay.

	Hypoplastic clay				Mod. Cam-clay		
	λ^*	N	ν	φ_c	λ	e_0	M_{cs}
w_L	0.92	0.95		0.41	0.91	0.94	0.40
w_P	0.95	0.92	0.52	0.44	0.89	0.88	0.44

TABLE 4. Correlations between Atterberg's limits of high-plasticity clay soil and model's parameters.

5. RESULTS

The database contained only 7 laboratory protocols of soil classified as high-plasticity clay after removing all duplicates. All of them were successfully calibrated for both of constitutive models – hypoplastic clay and modified Cam-clay. The summarization of basic statistical description of calibrated model parameters is summed up in Table 3. Strong correlations between Atterberg's limits (liquid limit plastic limit w_L and plastic limit w_P) and slope of the NCL line of both calibrated models have been observed. Example of correlation between plastic limit and slope of the NCL assumed in hypoplastic model for clay can be seen in Figure 4 and the same trend applies to the position of the NCL. All considerable correlations are summarized in Table 4 but limited number of protocols has to be considered.

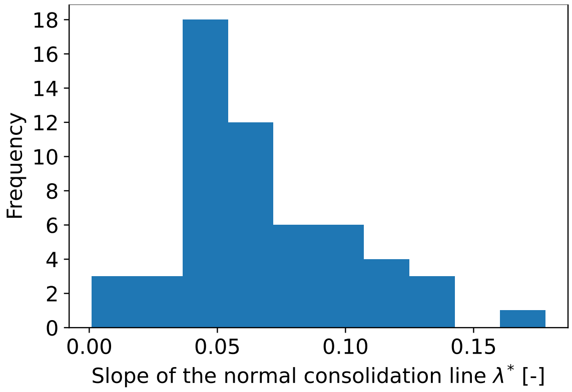
Overall 72 laboratory protocols of soil classified as low-plasticity clay contained the database after removing all duplicates of which it was possible to calibrate 56 for hypoplastic clay and 61 for modified Cam-clay model. The reason why not all of the protocols were successfully calibrated will be further examined. Basic statistical description of calibrated model parameters is also summarized and can be examined in Table 5. Examples of parameter's distribution are shown in histogram in Figure 5. In comparison to high-plasticity clay, only moderate correlations between Atterberg's limits and model parameters have been observed for low-plasticity clay. In particular, higher soil moisture at Atterberg's limits indicates higher parameter N with correlation coefficients about 0.41. The similar trend also applies to modified Cam-clay's parameter e_0 with correlation coefficients about 0.46 for the liquid limit and the 0.41 for plastic limit. Correlation 0.40 has been found between moisture at liquid limit and the slope of the NCL line λ. Another correlations seem insignificant for practical applications.

6. CONCLUSIONS

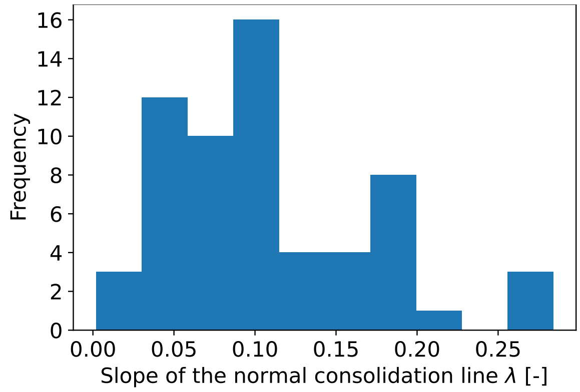
This paper briefly explained each individual input parameters of both hypoplastic and modified Cam-

	Hypoplastic clay					Modified Cam-clay				
	λ^*	κ^*	N	ν	φ_c	λ	κ	e_0	ν	M_{cs}
Count	56	56	56	56	56	61	61	61	61	61
Mean value	0.067	0.008	0.948	0.217	29.1	0.107	0.010	1.388	0.270	1.120
Standard deviation	0.035	0.005	0.283	0.109	6.6	0.062	0.006	0.597	0.112	0.320
Minimum value	0.001	0.000	0.507	0.010	2.9	0.002	0.000	0.112	0.010	0.101
25 % quantile	0.042	0.005	0.725	0.155	25.0	0.059	0.008	0.949	0.200	0.984
50 % quantile	0.064	0.007	0.899	0.230	30.0	0.096	0.010	1.246	0.260	1.199
75 % quantile	0.090	0.010	1.170	0.290	33.3	0.149	0.011	1.797	0.390	1.274
Maximum value	0.178	0.025	1.667	0.400	43.6	0.284	0.036	3.075	0.400	1.790

TABLE 5. Basic statistical description of calibrated model parameters for CL soil.



(A). Hypoplastic clay model.



(B). Modified Cam-clay model.

FIGURE 5. Histograms of slopes of the NCL for hypoplastic clay and modified Cam-clay model.

clay constitutive model and it summarized the issue of automatic parameter calibration. ExCalibre web calibration application, a practical tool for calibrating these material parameters based on the results of standard laboratory tests, was introduced in Section 2 and Section 3. Then a method of how could existing data be used to obtain general knowledge of the properties of these individual parameters was presented in Section 4 and finally, the paper summarized extracted basic statistical data of model parameters with respect to USCS classification in Section 5.

The main aim of this research is to help with development of a robust automatic calibration procedure which will be applicable at any stage of geotechnical survey to promote the use of advanced models in geotechnical practice. The determination of the confidence intervals along with correlations between material parameters and index soil properties was only the first step of data analysis and other more complex studies will follow.

ACKNOWLEDGEMENTS

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REFERENCES

[1] ExCalibre – SoilModels Automatic Calibration. [2022-07-26]. `https://soilmodels.com/excalibre-en/`

[2] M. Šejnoha, J. Pruška, T. Janda, M. Brouček. *Metoda konečných prvků v geomechanice: Teoretické základy a Inženýrské aplikace.* České vysoké učení technické, 2015. ISBN 978–80–01–05743–8.

[3] J. Jerman, D. Mašín. Hypoplastic and viscohypoplastic models for soft clays with strength anisotropy. *International Journal for Numerical and Analytical Methods in Geomechanics* **44**(10):1396–1416, 2020. `https://doi.org/10.1002/nag.3068`

[4] D. Mašín. Clay hypoplasticity model including stiffness anisotropy. *Géotechnique* **64**(3):232–238, 2014. `https://doi.org/10.1680/geot.13.P.065`

[5] D. Kolymbas. Introduction to barodesy. *Géotechnique* **65**(1):52–65, 2015. `https://doi.org/10.1680/geot.14.P.151`

[6] T. Kadlíček. *Parameters identification of advanced constitutive models of soils.* Ph.D. thesis, Czech Technical University in Prague, 2019.

[7] T. Kadlíček, T. Janda, M. Šejnoha, et al. Automated calibration of advanced soil constitutive models. Part II: hypoplastic clay and modified Cam-Clay. *Acta Geotechnica* **17**:3439–3462, 2022. `https://doi.org/10.1007/s11440-021-01435-y`

[8] P.-A. Von Wolffersdorff. A hypoplastic relation for granular materials with a predefined limit state surface. *Mechanics of Cohesive-frictional Materials: An International Journal on Experiments, Modelling and Computation of Materials and Structures* **1**(3):251–271, 1996. `https://doi.org/10.1002/(SICI)1099-1484(199607)1:3<251::AID-CFM13>3.0.CO;2-3`

[9] D. Mašín. Clay hypoplasticity with explicitly defined asymptotic states. *Acta Geotechnica* **8**(5):481–496, 2013. https://doi.org/10.1007/s11440-012-0199-y

[10] K. Roscoe, J. Burland. On the generalized stress-strain behaviour of wet clay. In *Engineering plasticity*, pp. 235–609. Cambridge University Press, 1968.

[11] Python. [2022-07-26]. https://www.python.org