Acta Polytechnica CTU Proceedings **40**:1–7, 2023
https://doi.org/10.14311/APP.2023.40.0001

THE EXPERIMENTAL EVALUATION OF TENSILE FORCES IN STAYS OF A CABLE-STAYED FOOTBRIDGE

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Abstract. The topic of this paper is an experimental analysis of the dynamic behaviour of stays of an existing footbridge focusing on determination of the tensile forces in stays. The examined structure was the footbridge across the Vltava river in Lužec nad Vltavou. It is a cable-stayed footbridge, the horizontal load-bearing structure consists of UHPC components, which are prestressed by a pair of external cables. The horizontal load-bearing structure is suspended on a 39.85 m high A-shaped steel pylon by means of a system of 17 pairs of stays. As a part of the work the tensile forces in stays were determined using the vibration frequency method. The method is based on the knowledge of natural frequencies of the stays, which were investigated by the experimental analysis.

Keywords: Vibration, footbridge, cable-stays, vibration frequency method, tensile force.

1. Introduction

The reason of choosing the footbridge in Lužec nad Vltavou town as examined structure was the frequent appearance of a problem with oscillation of its longest stays caused by wind. The dynamic experiment focused on the oscillation of stays was carried out in April 2021. Natural frequencies of each stay were evaluated using Fast Fourier Transform (FFT).

The aim was to determine the tensile forces in stays using two simple models within the vibration frequency method (a string model and a simply supported beam), compare the obtained results and specify, which results are more accurate. Since the stays are structural elements with a relatively long free vibrating length and with a relatively low bending stiffness, the using of the vibration frequency method was suitable here [1].

There was also an experimental modal analysis and dynamic load test carried out on the structure in October 2021 focusing on the dynamic behaviour of the horizontal load-bearing structure. Also, a theoretical analysis of the dynamic behaviour of the structure was performed. The results of the experimental and theoretical dynamic analysis are described in [2].

2. Description of the footbridge

The footbridge is located in Lužec nad Vltavou town nothwards from Prague. It crosses Vltava river and it creates a pedestrian and cycling connection between Lužec nad Vltavou town and Bukol town. It was put into operation in 2020.

It is a cable-stayed footbridge with two asymmetric spans, which are 99.18 m and 31.90 m long. The horizontal load-bearing structure is supported by pair of longitudinally sliding bearings on the O1 abutment and it is frame-connected with the O3 abutment. The height of the frame-connection varies from 500 mm to 1000 mm and its shape is parabolic. The width of the horizontal load-bearing structure is 4.50 m and its total length is 131.58 m. The width of free space for walking and cycling is 3.00 m. The longitudinal slope of the footbridge deck varies along the length from +8.00 % to −8.00 %.

The horizontal load-bearing structure consists of UHPC components. The components are made of concrete C 110/130. The length of typical component is 3.998 m and the length of atypical component is 1.998 m. The footbridge deck consists of 31 typical and 2 atypical components. The minimum width of the girders, which are a part of the component, is 400 mm and it varies along the height up to 500 mm in the direction of longitudinal axis of the component. There are linear rises between the girders and the components desk, which thickness is 60 mm. The width of cross girders, which are in ¼ and ¾ of the length of the component (in ½ of the length of the atypical component), varies symmetrically along the height from 200 mm to 400 mm and there are linear rises between the cross girders and the components desk. The longitudinal slope and transversal slope of the upper surface of the components is ±0.00 % and the longitudinal slope of the footbridge is created by the suitable composition of the components. The height of the cross girder above the abutment O1 varies from 750 mm to 500 mm.

The UHPC components are prestressed by a pair of external cables. The diameter of the cables is 15.7 mm and the cables consist of 19 strands. The type of strands is St 1640/1800 MPa by VSL.

The horizontal load-bearing structure is suspended on a 39.85 m high A-shaped steel pylon by means of a system of 17 pairs of stays, 5 pair of stays are reverse (see Figure 1). The stays are anchored into the massive

Figure 1. The footbridge in Lužec nad Vltavou town.

head of the pylon and into certain components of the horizontal load-bearing structure or into the anchor block on the O3 abutment. The type of stays is "Full Locked Coil Strands" by Redaelli. The diameter of stays varies from 32 mm to 64 mm.

The vertical load-bearing structure is made of pair of pillar struts. The pillar struts are rectangular-shaped and the heel dimensions are 1000×600 mm, the head dimensions are 600×600 mm. The material of the pillar struts is steel S355J2+N and sheet metal thickness is 20 mm. The axial distance of the pillar struts in the heel is 7060 mm, the struts converge into the massive head. There are 12 stays anchoring levels towards Lužec nad Vltavou and 5 stays anchoring levels towards Bukol. The pylon is filled with C 30/37 – XC2 – SCC – D_{max} 16 concrete up to the level of the footbridge deck.

3. The experimental analysis

As was said above, one of the aims of the experimental analysis was to find out the current natural frequencies of vibration of the stays of the footbridge, to be able to determine the tensile forces. The experiment was carried out in April 2021.

3.1. Measurement line

The vibration of the stays was measured by piezoelectric acceleration transducers Brüel&Kjær Type 8344. The working range of these sensors is from 0.2 Hz to 3 kHz, their sensitivity is very high such as approximately 2500 mV/g and their natural frequency is 10 kHz.

During the measurement, the sensors were magnetically attached directly to each stay and via cables connected to the 8-channel data acquisition station SIRIUS Type 6ACC-2ACC+ made by DEWESoft. All channels have their own unique 2×24-bit A/D converter (so called DualCore) that is able to measure with dynamic range up to 160 dB.

3.2. Measurement system

At first, the system of marking the stays was introduced. The level with longest stays, which are the closest to Lužec nad Vltavou, was signed as "Y01", where "Y" means "L" for the left side of the footbridge deck and "R" for the right side (see Figure 2). Then the sign of the pairs of the stays gradually grows from "Y01" to "Y17", where "Y17" signs the pair of stays closest to Bukol.

Twenty-six records of vibration of stays in time were acquired. The minimum number of records of vibration of each stay was two. The acceleration transducers were placed in the plane of the stays perpendicularly to the longitudinal axis of the stay (see Figure 3). During some records, the transducers were placed also perpendicularly to the plane of the stays. However, these records were then not used for evaluation of natural frequencies of the stays, because the properties of the anchoring of stays (the boundary conditions) in the perpendicular direction are not clear enough. The wind velocity varied between 0 km/h and 20 km/h and was measured by anemometer.

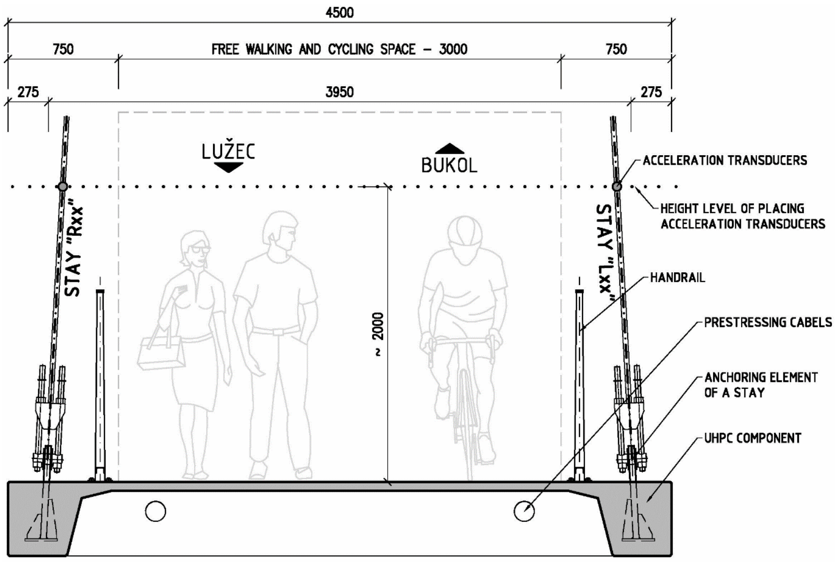

Figure 2. Marking the stays – cross section.

Sign of the stay	f_1	f_2	f_3	f_4	f_5 [Hz]	f_6	f_7	f_8
L01	1.09	2.17	3.24	4.32	5.40	6.47	7.56	8.62
R01	1.12	2.22	3.32	4.43	5.53	6.63	7.75	8.95
L02	1.14	2.25	3.37	4.49	5.58	6.73	7.85	8.95
R02	1.13	2.22	3.33	4.43	5.53	6.64	7.76	8.88
L03	1.34	2.66	4.00	5.34	6.62	7.98	9.31	10.63
R03	1.32	2.63	3.94	5.25	6.53	7.86	9.18	10.45
L04	1.41	2.81	4.21	5.61	7.00	8.41	9.80	11.18
R04	1.44	2.84	4.27	5.69	7.12	8.53	9.94	11.35
L12	2.37	4.74	7.12	9.43	11.49	13.16	15.19	17.56
R12	2.41	4.85	7.25	9.61	11.69	13.34	15.44	17.88
L15	1.99	3.97	5.96	7.96	9.93	11.83	13.59	15.13
R15	2.04	4.09	6.13	8.16	10.18	12.15	13.87	15.47
L16	1.83	3.65	5.46	7.29	9.12	10.94	12.67	14.33
R16	1.85	3.68	5.52	7.35	9.20	11.03	12.79	14.46
L17	2.97	5.92	8.86	11.75	14.62	17.24	19.40	21.67
R17	2.95	5.89	8.83	11.75	14.59	17.22	19.27	21.47

Table 1. The comparison of the values of the tensile forces of the selected stays obtained from the calculation according to the various models.

Figure 3. Acceleration transducers Brüel&Kjær Type 8344 attached directly to a stay by neodym magnet, the silver tape is used as a security tool only.

3.3. Evaluation of natural frequencies of the stays

At first, the measured data from all channels were transformed using FFT from the time domain to the frequency domain (see Figure 4 and Figure 5). The peaks in a grid of integer multiples were considered as natural frequencies of the investigated stays because this phenomenon is typical for a string behaviour (see Figure 4 and Figure 5). Some of the peaks were relevant to natural frequencies of the footbridge deck (marked by grey colour in Figure 4 and Figure 5), as was proved within another experiment carried out on this structure [2], they were ignored for this case.

There is a summary of evaluated first eight natural frequencies of selected stays in following Table 1.

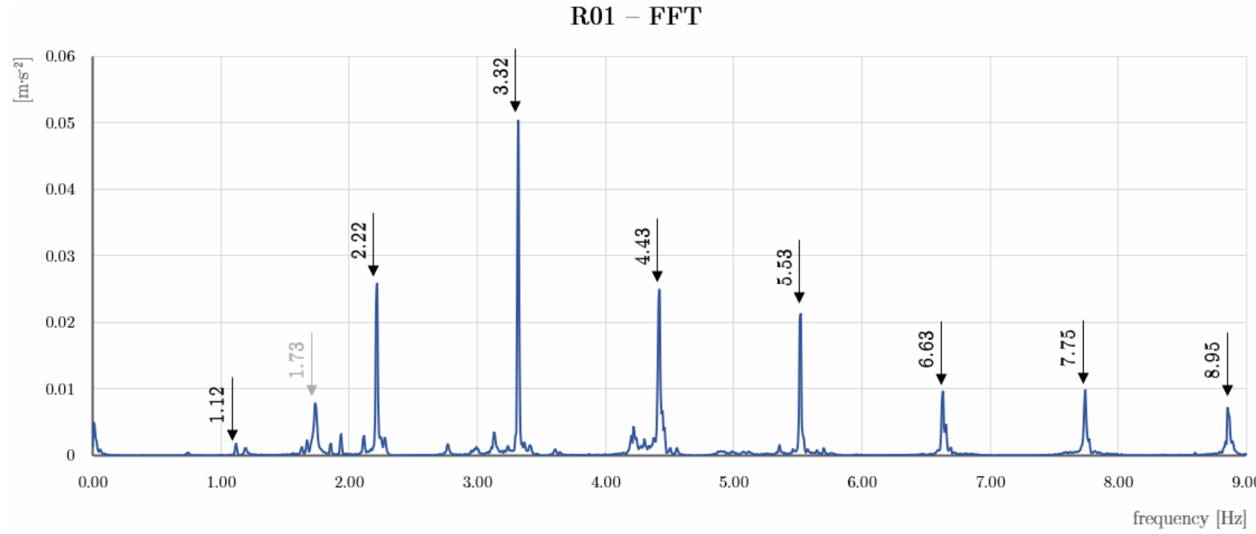

Figure 4. Natural frequencies evaluated using FFT – the stay no. R01.

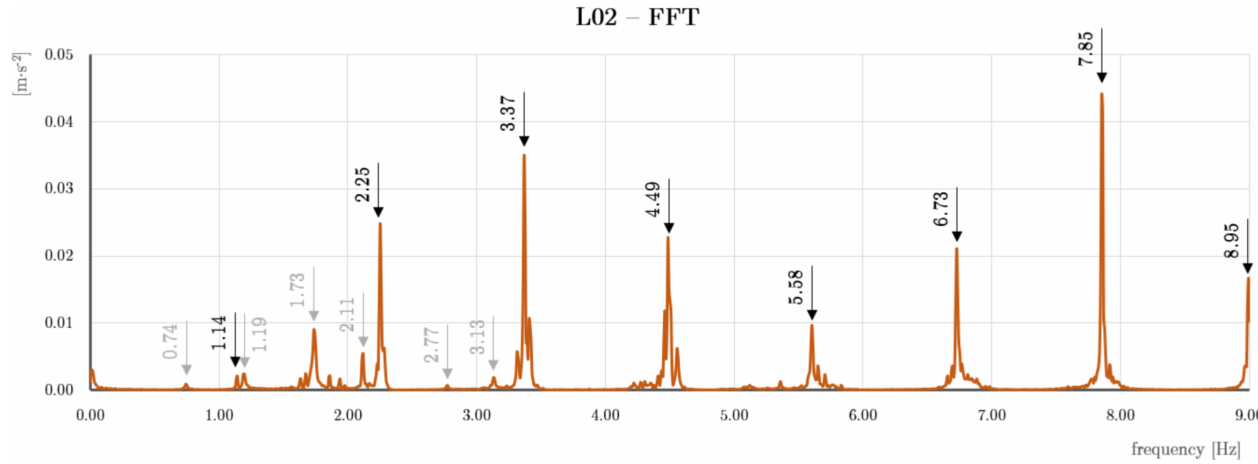

Figure 5. Natural frequencies evaluated using FFT – the stay no. L02.

4. The determination of tensile forces in stays

There are more approaches how to determine the tensile forces in stays within the vibration frequency method. A rod can be modelled as a string, a simply supported beam, an elastically fixed beam or as a fixed beam [2, 3]. In this case two simplest theoretical models of the stays (the string model and the simply supported beam) were used for evaluation of the results. The first reason for choosing these two simplest models was because of the length of the investigated stays, that was large enough. The second reason were the boundary conditions, which correspond with the simple support.

4.1. String model

According to this theory, the stay is modelled as a string with no bending stiffness, perfect flexibility, constant mass per metre along the entire length of the string and very low damping. The tensile force $N^S_{(j)}$ in the string can be calculated as [4]:

$$N^S_{(j)} = \nu \left(\frac{2 \cdot f_{(j)} \cdot L}{j} \right)^2 , \qquad (1)$$

where

μ mass per metre, the data were taken from the manufacturer of the stays,

j the serial number of the natural frequency of the stay,

$f_{(j)}$ the j-th natural frequency of the stay,

L the length of the stay according to the realization documentation [5].

If the stay was a perfect string, the calculated tensile force would be the same for all used natural frequencies. Since that is in fact not possible, the calculation of the standard deviation was performed to be able to compare the calculation according to the various theories. There are the calculated tensile forces, their average value and the standard deviation in Table 2.

Sign of the stay	Mass μ [kg/m]	Length L [m]	Serial number of the natural frequency j 1	2	3	4	5	6	7	8	Avg. force [kN]	Std. dev. [kN]
			Tensile forces [Hz]									
L01	5.7	96.599	252.8	249.4	248.8	247.9	247.8	247.6	248.5	247.0	248.70	1.69
R01	5.7	96.599	267.2	261.8	261.1	260.4	259.9	259.8	261.0	266.0	262.15	2.66
L02	5.7	88.816	234.3	227.9	226.9	226.4	224.1	226.2	226.4	225.1	227.16	2.91
R02	5.7	88.816	228.3	222.3	221.2	220.3	219.7	220.5	220.8	221.6	221.83	2.55
L03	5.7	81.111	267.7	266.1	267.2	267.3	263.1	265.6	265.6	264.8	265.94	1.43
R03	5.7	81.111	263.1	259.2	258.6	258.4	256.0	257.7	257.7	255.8	258.32	2.13
L04	5.7	73.510	244.9	242.3	242.1	241.9	241.5	242.1	241.5	240.4	242.09	1.22
R04	5.7	73.510	255.5	248.4	249.6	249.3	249.8	249.0	248.4	248.0	249.76	2.24
L12	5.7	28.413	103.1	103.5	103.7	102.2	97.2	88.5	86.7	88.7	96.70	7.06
R12	5.7	28.413	107.2	108.1	107.6	106.3	100.6	91.0	89.6	91.9	100.28	7.66
L15	7.2	36.640	153.1	152.3	152.6	153.0	152.5	150.4	145.7	138.3	149.74	4.90
R15	7.2	36.640	161.4	161.4	161.4	160.9	160.3	158.5	151.8	144.6	157.54	5.77
L16	20.4	42.519	491.3	490.0	488.7	490.0	490.8	490.4	482.9	473.3	487.18	5.80
R16	20.4	42.519	502.2	498.1	498.5	497.4	499.4	498.5	492.1	482.0	496.04	5.93
L17	23.2	48.709	1935.6	1925.8	1918.2	1899.9	1881.2	1816.7	1691.1	1615.8	1835.54	112.21
R17	23.2	48.709	1909.6	1906.3	1905.3	1899.9	1873.4	1812.5	1667.7	1585.8	1820.05	117.31

TABLE 2. The tensile forces calculated according to the string model.

Sign of the stay	Mass E [GPa]	Length $I\times10^{-8}$ [m^4]	Serial number of the natural frequency j 1	2	3	4	5	6	7	8	Avg. force [kN]	Std. dev. [kN]
			Tensile forces [Hz]									
L01	164.5	3.69	252.8	249.4	248.7	247.7	247.6	247.4	248.1	246.6	248.54	1.78
R01	164.5	3.69	267.2	261.8	261.0	260.3	259.7	259.6	260.7	265.6	261.99	2.66
L02	164.5	3.69	234.3	227.9	226.8	226.3	223.9	225.9	226.0	224.6	226.97	3.00
R02	164.5	3.69	228.3	222.2	221.1	220.1	219.5	220.2	220.5	221.1	221.64	2.63
L03	164.5	3.69	267.7	266.1	267.1	267.2	262.9	265.3	265.1	264.3	265.71	1.54
R03	164.5	3.69	263.1	259.2	258.5	258.3	255.8	257.4	257.3	255.3	258.09	2.27
L04	164.5	3.69	244.9	242.3	242.0	241.7	241.2	241.7	240.9	239.7	241.80	1.40
R04	164.5	3.69	255.5	248.4	249.5	249.1	249.6	248.6	247.9	247.3	249.48	2.38
L12	164.5	3.69	103.0	103.2	103.0	101.0	95.3	85.9	83.0	83.9	94.81	8.53
R12	164.5	3.69	107.1	107.8	106.9	105.1	98.7	88.3	85.9	87.2	98.39	9.12
L15	164.7	5.91	153.0	152.1	152.0	151.8	150.7	147.8	142.2	133.7	147.92	6.30
R15	164.7	5.91	161.4	161.1	160.8	159.8	158.5	155.9	148.3	140.0	155.71	7.19
L16	165.2	47.85	490.9	488.3	484.8	483.1	480.0	474.9	461.8	445.7	476.18	14.30
R16	165.2	47.85	501.7	496.4	494.7	490.5	488.7	483.0	471.0	454.4	485.03	14.52
L17	164.9	61.90	1935.2	1924.1	1914.4	1893.1	1870.5	1801.4	1670.3	1588.6	1824.71	120.95
R17	164.9	61.90	1909.2	1904.6	1901.4	1893.1	1862.8	1797.2	1646.9	1558.6	1809.22	125.90

TABLE 3. The tensile forces calculated according to the simply supported beam theoretical model.

4.2. SIMPLY SUPPORTED BEAM

In contrast to the string model, in this theoretical model the stay is modelled as a simply supported beam with bending stiffness. The same for both theories is constant mass per metre along the entire length of the stay. The tensile force $N^B_{(j)}$ can be calculated as [6]:

$$N^B_{(j)} = \nu \left(\frac{2 \cdot f_{(j)} \cdot L}{j} \right)^2 - \left(\frac{j \cdot \pi}{L} \right)^2 EI, \qquad (2)$$

where

μ mass per metre, the data were taken from the manufacturer of the stays,

j the serial number of the natural frequency of the stay,

$f_{(j)}$ the j-th natural frequency of the stay,

L the length of the stay according to the realization documentation [5],

E Young's modulus, the data were taken from the manufacturer of the stays,

I moment of inertia.

The moment of inertia was calculated for alternative circular cross section, where the area of the alternative cross section was the same as the area of the real cross section, which states the manufacturer of the stays. Since the values of mass per metre μ and length L are the same as in Table 2, they are not shown again in Table 3.

Sign of the stay	**Mass μ [kg/m]**	**Length L [m]**	**N [kN]**	**EI [kNm^2]**
L01	5.7	96.599	250.1	−50.5
R01	5.7	96.599	262.0	4.0
L02	5.7	88.816	229.2	−63.8
R02	5.7	88.816	223.2	−41.9
L03	5.7	81.111	266.9	−25.3
R03	5.7	81.111	260.2	−48.0
L04	5.7	73.510	243.2	−23.4
R04	5.7	73.510	251.3	−32.5
L12	5.7	28.413	104.6	−25.2
R12	5.7	28.413	108.8	−27.2
L15	7.2	36.640	155.1	−28.4
R15	7.2	36.640	164.0	−34.3
L16	20.4	42.519	493.0	−41.8
R16	20.4	42.519	502.2	−44.0
L17	23.2	48.709	1967.3	−1241.9
R17	23.2	48.709	1955.2	−1273.9

TABLE 4. Results.

4.3. The identification of the cable bending stiffness and the cable tensile force

Since the value of the bending stiffness EI of the stays was not exactly known, it was possible to perform the identification of the bending stiffness EI and the tensile force N using the Equation (2) transformed into (3) [7].

$$N + \left(\frac{j \cdot \pi}{L}\right)^2 \cdot EI = \mu \left(\frac{2 \cdot f_{(j)} \cdot L}{j}\right)^2. \tag{3}$$

The eight lowest evaluated natural frequencies were used for calculating the bending stiffness and the tensile force in the stays. Based on the Equation (3) [7], it was possible to create eight equations for two unknowns.

$$\begin{pmatrix} 1 & \left(\frac{1\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{j\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{n\cdot\pi}{L}\right)^2 \end{pmatrix} \cdot \begin{pmatrix} N \\ EI \end{pmatrix} = \begin{pmatrix} \mu\left(\frac{2\cdot f_{(1)}\cdot L}{1}\right)^2 \\ \vdots \\ \mu\left(\frac{2\cdot f_{(j)}\cdot L}{j}\right)^2 \\ \vdots \\ \mu\left(\frac{2\cdot f_{(n)}\cdot L}{n}\right)^2 \end{pmatrix}. \tag{4}$$

This equation system can be solved using Gauss Markov theorem [7].

$$\begin{pmatrix} 1 & \left(\frac{1\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{j\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{n\cdot\pi}{L}\right)^2 \end{pmatrix}^T \cdot \begin{pmatrix} 1 & \left(\frac{1\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{j\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{n\cdot\pi}{L}\right)^2 \end{pmatrix} \cdot \begin{pmatrix} N \\ EI \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & \left(\frac{1\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{j\cdot\pi}{L}\right)^2 \\ \vdots & \vdots \\ 1 & \left(\frac{n\cdot\pi}{L}\right)^2 \end{pmatrix}^T \cdot \begin{pmatrix} \mu\left(\frac{2\cdot f_{(1)}\cdot L}{1}\right)^2 \\ \vdots \\ \mu\left(\frac{2\cdot f_{(j)}\cdot L}{j}\right)^2 \\ \vdots \\ \mu\left(\frac{2\cdot f_{(n)}\cdot L}{n}\right)^2 \end{pmatrix}.$$

Results of the identification are shown in following Table 4. It is clear from the negative values of the bending stiffness of the stays that this method of determining the bending stiffness and the tensile forces is not suitable for this case.

5. Conclusions

Since the experiment was focused on the vibration of stays, twenty-six records of vibration in time were acquired. Natural frequencies of each stay were evaluated using FFT.

As was said above, the aim was to determine the tensile forces in stays using various theoretical models within the vibration frequency method and compare the obtained results. In this case the simplest models were used, such as the string model and the simply supported beam. Also, the identification of the bending stiffness and the tensile forces in stays was performed. As is shown in Table 4, the results of the identification were unsatisfactory because of the negative values of

Sign of the stay	String model			Simply supported beam		
	Avg.	Std. dev.	Coeff. of var.	Avg.	Std. dev.	Coeff. of var.
L01	248.7	1.69	0.68 %	248.54	1.78	0.72 %
R01	262.1	2.66	1.01 %	261.99	2.66	1.01 %
L02	227.2	2.91	1.28 %	226.97	3.00	1.32 %
R02	221.8	2.55	1.15 %	221.64	2.63	1.19 %
L03	265.9	1.43	0.54 %	265.71	1.54	0.58 %
R03	258.3	2.13	0.82 %	258.09	2.27	0.88 %
L04	242.1	1.22	0.50 %	241.80	1.40	0.58 %
R04	249.8	2.24	0.90 %	249.48	2.38	0.95 %
L12	96.7	7.06	7.30 %	94.81	8.53	8.99 %
R12	100.3	7.66	7.64 %	98.39	9.12	9.27 %
L15	149.7	4.90	3.27 %	147.92	6.30	4.26 %
R15	157.5	5.77	3.66 %	155.71	7.19	4.62 %
L16	487.2	5.80	1.19 %	476.18	14.30	3.00 %
R16	496.0	5.93	1.20 %	485.03	14.52	2.99 %
L17	1835.5	112.21	6.11 %	1824.71	120.95	6.63 %
R17	1820.1	117.31	6.45 %	1809.22	125.90	6.96 %

Table 5. The comparison of the values of the tensile forces of the selected stays obtained from the calculation according to the various models.

the bending stiffness of the stays, it implies that they are not usable in this case.

It was expected that the results obtained from the calculation according to the simply supported beam are going to be more accurate. To be able to compare the results, the values of standard deviations of the tensile forces were calculated. Since the values of standard deviation and coefficient of variation are lower for the values of the tensile forces calculated according to the string model, the author considers them more accurate. There is a comparison of the values of the tensile forces of the selected stays obtained from the calculation according to the string model and the simply supported beam in following Table 5.

Acknowledgements

The results presented in this paper are outputs of the research project No. SGS22/089/OHK1/2T/11 supported by the Czech Technical University in Prague.

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