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REPORT OF A THESIS ADVISOR

Master's thesis title: "Generalized uncertainty relation and its use in cosmology"
Candidate: Jan Masák
Institution: Department of Physics, Czech Technical University in Prague, Faculty of Nuclear Science and Physical Engineering, Břehová 7, 115 19 Czech Republic
Assessment: A (výborně)

It is a pleasure to report on Master's Thesis of Jan Masák. Mr. Masák worked under my supervision throughout the period September, 2022 – August, 2023. During that time he focused his main attention on the Generalized Uncertainty Principle (GUP) and its applications in cosmology and astrophysics.

The Generalized Uncertainty Principle tries to coherently incorporate a fundamental minimal length scale into Heisenberg uncertainty relations. The existence of a minimal length is a great prediction from different approaches related to Quantum Gravity such as the black hole physics and the String theory. GUP was originally introduced by D. Amati, G. Veneziano and D. Gross in 1980's, even though the incentives can be traced back to works of J.A. Wheeler and H.S. Snyder from 1940's. Since then, GUP has gained momentum with numerous applications ranging from quantum mechanics and particle physics to condensed matter theory.

In his work was Mr. Masák inspired mainly by works of A.N. Tawfik and A.M. Diab [1] (conceptual background for GUP), G. Amelino-Camelia *et al.* [2] (GUP and gravity), R.J. Adler *et al.* [3] (GUP and black hole physics) and K. Sundermeyer [4] (quantization of constrained systems). He approached the subject of his Thesis with a wide knowledge of the prerequisite material.

The Thesis itself is divided into 5 chapters. After the motivational Chapter 1 where various pros and cons of GUP are discussed, Mr. Masák goes on and puts forward a number of motivations for quadratic GUP, which are needed in Chapter 3. In Chapter 3, the deformation of geodesic motion caused by quadratic GUP is explored. This serves as a convenient conceptual starting

point for the discussion of the deformed geodesic motion of test particles for both Schwarzschild and Kerr black-hole solutions in a pseudo-Newtonian limit. The latter is discussed in Chapter 4, which also forms a core part of the Thesis with a number of original solutions. The concept of Hawking radiation and ensuing modification of the radiation formula in the quadratic GUP framework are briefly analyzed in Chapter 5. It is expected that issues from Chapters 4 and 5 will be further explored in more detail during Mr. Masák next-year research project. Last but not least, the Thesis is concluded with Conclusion where key points of the Thesis are re-emphasized.

All in all, Bachelor Thesis of Mr. Masák has in my opinion a high quality. It offers an interesting and in many respects original selection and discussion of topics that are indispensable for understanding GUP and its implications in cosmology. Key aspects of the problems are worked out logically and clearly. GUP is conceptually and numerically demanding endeavor within the steadily growing field of Quantum Gravity and I am sure that expertise gained by Mr. Masák will be beneficial to him in the years to come.

[**1**] A.N. Tawfik and A.M. Diab, *Generalized Uncertainty Principle: Approaches and Applications*, International Journal of Modern Physics D **23**, 1430025 (2014).

[**2**] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, and L. Smolin, *The principle of relative locality*, Physical Review D **84**, 084010 (2011).

[**3**] R.J. Adler, P. Chen, and D.I. Santiago, *The Generalized Uncertainty Principle and Black Hole Remnants*, General Relativity and Gravitation **33**, 2101 (2001).

[**4**] K. Sundermeyer, *Constrained Dynamics*, volume 169 of Lecture Notes in Physics, (Springer- Verlag, Berlin, 1982).

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