May 26th, 2023

To: TU Prague

Re: Evaluation of a thesis by Dominika Burešová

To whom it may concern,

Quantum logics (alias orthomodular partially ordered sets) are sometimes assumed to be the algebraic equivalent of ‘event structures’ of quantum experiments---an empirical logic of sorts. This bachelor thesis addresses some natural open questions of the theory of such quantum logics and provides their complete or partial answers. As it is quite often the case, the theoretical findings may in turn shed light on the problems of 'practical' physics. Obviously, the theory of quantum logic lives its own life in the formal realm of mathematical ordered structures.

In her thesis, the author presents main results of her research papers---one published alone and two with a co-author---and a paper submitted for publication. In what follows I shall briefly review these efforts.

After introducing the subject, Dominika Burešová asked when finite sets in a lattice quantum logic generate finite or infinite 'substructures'. The situation is much more complex than the situation in Boolean algebras where 'finite generates finite'. It is known, for instance, that three atoms aka projection operators aka nonzero vectors in the associated projection logic of three-dimensional vector spaces often generate infinite sets of vectors and the associated projection operators. Dominika Burešová shows by a rather involved construction that a similar phenomenon may occur for set-representable lattice quantum logics, too. She also finds a link between the 'finite-generates-finite' property and the extension of states.

In the second and the third chapters, the author studies the quantum logics that allow for an introduction of a (generalized) symmetric difference (a kind of a XOR operation). This part of the quantum logic theory (the quantum logics) initiated by the Prague quantum logic group, and recently pursued by other authors, belongs to the line of 'almost Boolean' quantum logics, meaning that these quantum logics have intrinsic similarities with Boolean algebras rather than with projection quantum logics.

Further, an example of a quantum logics is constructed whose state space is ‘small’ and whose 'degree of non-compatibility' can be arbitrarily high; this is again a non-Boolean phenomenon. In the third chapter, the Abbott algebras can be enriched with other operations to obtain structures which are categorically isomorphic to the class of quantum logics. This result positions these enriched Abbott algebras closer to quantum theories. Properties of these
Abbott algebras related to compatibility, state properties, and so on are investigated in the thesis of Dominika Burešová, too.

In the fourth chapter, the author asks whether each set-representable quantum logic has an isomorphic copy of a quantum logic that is in a sense analogous to the topological Hausdorff space (i.e., any pair of distinct points of the underlying set can be distinguished by a set belonging to the quantum logic). She proves that such a copy can indeed be constructed by analyzing certain equivalence. Moreover, she shows how such a copy can be alternatively obtained by using the Stone technique borrowed from the theory of Boolean algebras.

The results of the thesis present a substantial contribution to the theory of quantum logic. They by far exceed the requirements standardly asked for bachelor thesis.

The methodology is correct and scientifically sound.

Therefore, I rate the thesis of Dominika Burešová very highly. More concretely, my assessment of this thesis is the best available score “A” (excellent).

Karl Svozil