Opponent's opinion of the doctoral thesis of Pavel Eichler

Doctoral thesis title: Mathematical modeling of fluid flow using lattice Boltzmann method

The main topic of the doctoral thesis is analysis and practical use of the Lattice Boltzmann Method in modelling of incompressible fluid flow.

The text of the thesis is 136 pages long; it is written in English and divided into 9 chapters including an introduction and a conclusion. There are also three appendices on 12 more pages. The bibliography contains 200 entries including 15 papers authored or co-authored by the author of the thesis.
The chapter 2 presents the used mathematical model of incompressible viscous fluid flow including viscosity models and basics of the kinetic and boundary layer theory. The Lattice Boltzmann Method (LBM) is presented in the chapter 3. The next chapter includes the analysis of the LBM and its relation to the incompressible viscous fluid flow model. The chapter 5 shows performance of various boundary conditions in LBM on two benchmark problems. The chapters 6 to 8 are devoted to numerical experiments, especially comparison of LBM models of several fluid flow problems with Finite Difference and Finite Volume models and experimental data. The chapter 6 deals with turbulent boundary layer flow, the chapter 7 deals with turbulent fluid flow in the combustion chamber of the fluidized bed boiler, and the chapter 8 deals with a non-Newtonian fluid flux in a phantom of an aortic vessel.

The text of the thesis is written in solid English. I found only a few typos or grammatical errors. Nevertheless, the text is not very readable and at first I had difficulties to understand the author's formulations well. I had enough time to get used to the author's style, as the scope of the work is enormous. The amount of text and topics included in the thesis obviously shows that the author has approached the topic from many perspectives and is capable of a really broad scientific scope. Parts of the thesis appear to have been taken from several of the author's publications, leading to some notation not being followed quite consistently throughout the thesis, which slightly reduces the readability of the text. However, the notation is logical in all cases and the text is quite comprehensible with careful reading.

The topic of the thesis is highly topical, the origins of LBM are several decades old but there is still a lot of open problems in its analysis and also many open questions in its practical usability. The Lattice Boltzmann Method is rather computationally consuming. That is why it can be successfully applied for real-world problem solutions only when a big computational power is available. Only its implementation on current graphical processor units allows the real practical testing of LBM performance which is the main topic of the doctoral thesis. The thesis includes also some theoretical results, especially the analysis of cumulant LBM in the chapter 4 and proposition of so-called interpolated outflow boundary condition with decomposition in the chapter 5. The topicality of the author's results is confirmed also by the list of papers related to this thesis that were authored or co-authored by the author of the doctoral thesis.

The declared goals of the doctoral thesis included theoretical objectives like a formulation of the mathematical model of the incompressible single-phase isothermal viscous fluid flow, its LBM representation, and a new boundary condition proposal, implementation objectives such as implementation of a parallel variant of the LBM with a grid refinement technique and implementation of a viscosity model, and numerical testing objectives based on validation of the implemented method with other codes or experimental data.

The declared objectives cover a wide range of issues, which makes the work very extensive and difficult to review. The author tries to explain all the related details. Especially in the introductory chapters he introduces a number of terms from which he immediately distances himself “for the sake of simplicity”. However, **the author fulfils all his pre-declared goals**.

The wide range of goals requires the wide range of used methods. In the theoretical part standard mathematical derivation and a mathematical proof conducting are being used, in the numerical experiments many mathematical and statistical methods for data analysis and comparison are being applied. All methods are chosen appropriately and used properly.

There are several contributions of the doctoral thesis to the scientific knowledge. I see the greatest contribution in formulation of the novel boundary condition and in practical confirmation of usability of LBM for solution of various types of problems involving incompressible isothermal fluid flow. The novelty and originality of the presented results is also confirmed by the fact that the author has published a substantial part of his doctoral thesis in peer-reviewed journals.

I have several questions to the author that could be answered during the thesis defense:

1) Comment please the term “equation of state” that you use in several forms and contexts. First time as “the equation of state for ideal gases” (2.10), then as “the equation of state for the incompressible Newtonian fluid in the isothermal domain” (2.15), and finally in the dimensionless form (3.8). Especially explain the meaning of the term “incompressibility” to explain which terms are constant and which terms vary in the equations of state.
2) Comment please, how the viscosity is controlled in your code especially in the case of non-Newtonian models.
3) At the very end of the section 7.1.1 you declare that the maximal velocity comes from prescribed parabolic profile at the inflow. Is the parabolic profile realistic for such high Reynolds numbers?
4) At the end of the page 124 you hypothesize that the possible source of disagreements between LBM and MRI results is the accuracy of the MRI measurement. I do not reject your hypothesis at all but I hesitate that it can fully explain the differences. On the figures 8.8 and followings some model predictions differ even more than 2*sd from the measurements. Do you mean that the supposed sd is undervalued?

The presented thesis includes original results of author’s research. In my opinion Pavel Eichler showed his ability of independent scientific work by the submitted doctoral thesis and by the published journal papers during his studies. I recommend the submitted doctoral thesis to be accepted for its defense before the Ph.D. committee and I recommend the author for the award of the degree of Ph.D.

in Liberec on April 16, 2023 Jan Šembera