



Supervisor's report of doctoral thesis

Candidate: Ing. Pavel Eichler

Thesis: Mathematical modeling of fluid flow using lattice Boltzmann method

The submitted thesis deals with the mathematical modeling of incompressible fluid flow under isothermal conditions using the lattice Boltzmann method (LBM). The main goal of the thesis is to investigate the applicability of LBM to simulate Newtonian and also non-Newtonian fluid flow in 3D geometries. Concerning the method, the objective is, if possible, to propose improvements for the treatment of boundary conditions, to analyze and implement the grid refinement technique, and to perform an asymptotic analysis of the cumulant variant of LBM.

The thesis is divided into nine chapters and besides the introduction and conclusion, these chapters form the following three topical parts: I) theoretical background and mathematical model (Chapter 2), II) description of the numerical model, its asymptotic analysis, and investigation of boundary conditions (Chapters 3–5), and III) applicability of LBM to problems with experimental or reference data (Chapters 6–8).

In Part I, the candidate outlines the theoretical background of mathematical modeling of fluid flow both inside and outside the boundary layer from the macroscopic point of view. He also briefly discusses initial and boundary conditions and present analytical solutions for flow between two parallel plates and in a rectangular duct. Then, he summarizes modeling concepts for the approximation of non-Newtonian fluids and emphasizes non-Newtonian models suitable for hemodynamics. Finally, a brief overview of the kinetic theory of fluids, which is essential for the understanding of LBM, is presented.

In Part II, LBM is presented in a concise but comprehensible way with a special attention given to the description of the cumulant collision operator together with various types of boundary conditions used in LBM. Inspired by the boundary conditions reported for the D3Q19 model in literature, the candidate proposes new, moment-based boundary conditions for the D3Q27 LBM model and presents the main idea behind their derivations in Chapter 3. Further in Chapter 5, these boundary conditions are confronted with commonly used LBM boundary conditions and their superior accuracy is demonstrated. This work resulted in paper [49] which is currently under consideration for publishing in *Computers and Mathematics with Applications* (IF 3.218). At the end of Chapter 3, the candidate also describes the grid refinement technique and LBM implementation. Chapter 4 is devoted to the backward analysis of LBM with the cumulant collision operator using the asymptotic expansion approach showing that the numerical method indeed solves the Navier-Stokes equations.

In Part III, the aforementioned numerical scheme is used to simulate several laboratory-scale problems within three different research projects in collaboration with (a) the Institute of Thermomechanics of the Czech Academy of Sciences (Chapter 6), (b) the Faculty of Mechanical Engineering CTU in Prague (Chapter 7), and (c) the Institute for Clinical and Experimental Medicine in Prague (Chapter 8). All experimental work carried out at (a), (b), and (c) involved

fluid flow under isothermal conditions and, therefore, these problems were suitable for modeling using LBM. In Chapter 6, LBM, as a DNS solver, was found to be able to sufficiently well simulate the flow in the boundary layer above a rough surface and the results are summarized in [48]. Additionally, the applicability of the mesh refinement technique is demonstrated. In Chapter 7, LBM was used to simulate a part of a combustion chamber of the fluidized bed boiler. The results, published in [9], were compared with the results obtained using the finite volume method implemented in Ansys Fluent. In Chapter 8, Newtonian and non-Newtonian variants of LBM were compared to experimental measurements using the phase-contrast magnetic resonance imaging and the findings are summarized in a submitted paper [50].

All chapters that contain original results developed by the candidate are conveniently concluded with a chapter summary, where his main contributions are emphasized and references to related published or submitted papers are given.

During his doctoral studies, the candidate participated in eight research projects and collaborated in international research teams, mainly at the following research institutions: AGH University of Science and Technology, Kraków, Poland; University of Texas Southwestern Medical Center, Dallas, USA; and CESEP, Colorado School of Mines, Golden, USA (part of the work on the thesis was conducted as part of the Czech-U.S. scientific collaboration within the framework of the Inter-Excellence project No. LTAUSA19021 of the Ministry of Education, Youth and Sports of the Czech Republic between CTU in Prague and CESEP, CSM Golden). The candidate visited CESEP two times (1 week in January 2020 and 1 week in February 2022). Other (longer) stays at CESEP planned for 2020-2022 could not take place due to COVID19 pandemic restrictions. Furthermore, the candidate visited Kanazawa Institute of Technology in Kanazawa, Japan (1 week in July 2018) and attended two summer schools abroad (PUMPS+AI Summer School in June 2019, Barcelona, Spain and 5th Spring School Lattice Boltzmann Methods with OpenLB Software Lab. in June 2022, Kraków, Poland).

He has an excellent publication record. He has co-authored ten research papers in impacted journals (8 published, 2 under review) of which he is the lead author of four papers (2 published, 2 under review). Additionally, he has also contributed to a book chapter. Based on Web of Science, 9 publications are listed that have been already cited 7 times (without self-citations). One research paper is currently in preparation.

He attended several international conferences and contributed in conference proceedings. In September 2022, the candidate was invited to give a lecture at the Institute of Mathematics of the Czech Academy of Sciences.

The candidate also assists with the educational process at FNSPE. He has been guiding exercises in Calculus 1, 2 (7 semesters in total) and Mathematics 1, 2 (5 semesters in total). He also teaches a course about the lattice Boltzmann method (since 2020). In total, he has supervised 1 BSc and 1 MSc students and co-advised 3 BSc and 1 MSc students at FNSPE. He has been involved in several promotion activities of FNSPE. In 2018, he won the Award of Nadace Josefa, Marie a Zdeňky Hlávkových for excellent master's thesis. In 2022, he won the Award of 5th Spring School Lattice Boltzmann Methods with OpenLB Software Lab for best scientific poster.

During his work, the candidate proved his ability to master difficult problems of interdisciplinary character. The submitted thesis is of high quality and represents major scientific contributions which is endorsed by a number of papers in Q1/Q2 journals. The thesis fulfills all requirements for Ph.D. thesis. Hence, I recommend the candidate to the committee for the doctoral thesis to bestow him the title of Doctor of Philosophy.

Praha, January 20, 2023

doc. Ing. Radek Fučík, Ph.D.