ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE
FAKULTA JADERNÁ A FYZIKÁLNĚ INŽENÝRSKÁ
KATEDRA MATEMATIKY

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Supervisor's report of doctoral thesis

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Thesis: Numerical and machine learning methods for medical image processing

The submitted doctoral thesis deals with numerical and machine learning methods for processing of data in magnetic resonance imaging (MRI). The thesis is divided into several main parts:

1. signed-distance function based registration of images with varying image intensity (chapter 2),
2. mechanical and imaging models in image registration (chapter 3),
3. estimation of left ventricular torsion (chapter 4),
4. image enhancement by solving inverse diffusion equation (chapter 5),
5. estimation of T_1 relaxation time from cardiac MRI data (chapter 6).

In chapter 2, the author presents a new method for registration of images with varying intensity. Necessity to register such images appears in processing of MOLLI (Modified Look-Locker Inversion recovery) sequences for evaluation of ECV (extracelular volume) in myocardium. The difficulty comes from the fact that we need to register two images with different image intensity. Common image registration methods cannot be used since they rely on constant image intensity of related pixels. The method proposed by the author is not based on registering pixels with the same intensity but rather on registration of shapes. The object of interest, which is the myocardium in this case, is segmented in the first step. The segmentation contours are then used as an initial shape for computation of the signed-distance function (SDF). This function allows extrapolation of the shape even to regions further from the segmentation contours. The registration is then performed on SDF instead of the image intensity function. The new method is compared with a state-of-the-art method for image registration - the mutual information maximization. The comparison shows that in this specific problem, the new method performs better. The new method described in this chapter is a subject of paper [2].

In chapter 3, the author presents another registration method. In this case, MRI data with tagging are subject of the registration. Registration of such data allows better analysis of the motion of myocardium – we talk about motion extraction. The main difficulty here comes from varying quality of MRI data. The ratio of noise to signal can be very high in MRI which may significantly affect the final registration. To make the method more robust, the author proposes a new method using mechanics-based regularization to express the elastic properties of myocardium and an imaging model which generates synthetic tagged images. The new method has been tested only on synthetic data so far. Its application on real data will be subject of future research. The method is a subject of paper [1].

In chapter 4, estimation of left ventricular torsion is studied. The candidate again deals with the motion extraction, specifically with detection of twisting motion during ventricular contraction. Common methods for the motion estimation exhibit large variance of obtained results. The new method proposed by the author is based again on mechanical regularization used in the previous chapter. In this case, 3D MRI data are first segmented and the segmented region is covered by 3D mesh which is used for the numerical approximation of the elastic motion of the myocardium. The new method was tested on data measured on patients tetralogy of Fallot disease. The method gives better results, however it also suffers from high variance of measured twist. The method is a subject of paper [3]

In chapter 5, the author describes a new method for image enhancement by solving the inverse diffusion equation. Image enhancement is one of the key operations in image processing. In most cases the aim is to perform image deblurring or denoising. Often one needs to perform both operations at the same time. Image blurring can be simulated by solving the heat equation on the image intensity function. Therefore the solution of the inverse heat equation can be a good way for image deblurring. Unfortunately, the inverse heat equation is a typical example of an ill-posed problem. In this chapter, the author proposes a new method for solving the inverse heat equation based on the solution of the optimization problem with constraints given just by the heat equation. In other words, we aim to find the initial condition for the heat equation such that the final solution fits the given blurred images best. This optimization problem can be solved by the gradient descent method. Common evaluation of the gradient is computationally too expensive. Alternative approach is to derive a so-called adjoint equation which leads to a much more efficient algorithm for the computation of the gradient. Numerical experiments show that this method is numerically stable and it gives very good results. Unfortunately, it turned out that a very similar method was published in [5]. In this paper, the same problem is solved by a deconvolution method which leads to an identical algorithm. The method proposed by the candidate, however, could be used even for the solution of inverse problems with other PDEs than just the heat equation. This will be a subject of future research.

Chapter 6 deals with estimation of T_1 relaxation time from cardiac MRI data. T_1 time is a parameter which reflects important properties of the cardiac tissue. Common methods for the estimation of T_1 time are based on various simplifications. The author describes a new method which should provide more accurate results. It is based on the solution of the inverse problem, similar to the previous chapter. The relaxation process during MRI acquisition could be simulated by the Bloch equations. The main parameters for these equations are T_1 and T_2 relaxation times. The proposed method is based on use of a so-called Bloch simulator which does not solve only the Bloch equations but it also takes into account number of artifacts caused by imperfections of the MRI scanner. Next, we search for input parameters for the Bloch simulator which best fit the data measured by MRI MOLLI sequence. This can be formulated as an optimization problem which is solved by the gradient descent method. To get a good initial guess, a multilayer neural network is trained to approximate the inverse problem. The computational tests performed on synthetic (in-vitro) and real (in-vivo) data show that the new method gives better results. It also shows that the two stage method profits from both stages. If we do not have an initial guess made by the neural network the optimization method would not converge on one hand and the second stage really improves the initial guess made by the neural network on the other hand. The new method is a subject of paper [4].

The work on the thesis was conducted as part of the OP RDE project no. CZ.02.1.01-0.0-0.0-16-019-0000765: *Research Center for Informatics* of Ministry of Education, Youth and Sports of the Czech Republic, AZV project No. NV19-08-00071: *Analysis of flow character and prediction of evolution in endovascular treated arteries by magnetic resonance imaging coupled with mathematical modeling* of the Ministry of Health of the Czech Republic. During her study, the candidate visited research center Inria Saclay in Paris, France (short stay for three weeks) and University of Texas, Southwestern Medical Center in Dallas, USA (two short stays for two and four weeks). She has also actively attended eight international conferences and she is co-author of two impacted scientific papers listed on Web of Science, one scientific paper listed in Scopus, one book chapter and one paper in conference proceedings listed in Web of Science. The candidate also assisted in the education of students by guiding exercises for four different lectures in calculus and programming.

The candidate proved her ability to deal with complex mathematical and interdisciplinary problems. She was able to profit from her detailed knowledge of data acquisition in magnetic resonance, numerical mathematics, optimization methods but also her ability to design neural networks and programming in various languages like C++, Python or Julia. Based on these skills she was able to design new methods in the form of workflows and software pipelines for processing of data from the magnetic resonance imaging. She was also able to design new algorithms for image registration and the solution of inverse problems.

The thesis fulfills all requirements for Ph.D. theses. Hence, I recommend the candidate to the committee for the doctoral theses to award her the title of Doctor of Philosophy.

Prague, December 14, 2022

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