FAKULTA APLIKOVANÝCH VĚD
ZÁPADOČESKÉ UNIVERZITY
V PLZNI

In Plzeň, June 30, 2020

Peer review on Ph.D. Thesis

Author: Ing. Josef Navrátil
Title: Reaction Diffusion Systems and Pattern Formation in Biology
Supervisor: prof. RNDr. Milan Kučera, DrSc.
Consultant specialist: PD Dr. Martin Väth

The aim of this dissertation thesis is to study systems of reaction-diffusion equations originating from morphogenesis and pattern formation in biology. This is very classical, difficult and still intensively studied topic originating from the seminal work by A. Turing [Philos. Trans. R. Soc. Lond. Ser. B, 237 (1952), pp. 37-72] and later developed, e.g., by Gierer and Meinhardt [Kybernetik 12, (1972), pp. 30-39] (to name a few). In general, main concern of the critics of these models are (possibly) unrealistic hypothesis of high ratio between diffusion coefficients of the two substances needed for diffusion driven instability and pattern formation. It has been numerically observed in Vejchodský et al. [Phys. Rev. E 96, 022212 (2017)], that pattern formation occurs at more realistic ratios (closer to 1) of these diffusion coefficients for systems with unilateral terms. The author of this dissertation thesis focuses on systems with unilateral terms and systematically develops adequate theory, thus significantly contributing to better understanding of these type of systems.

The dissertation thesis is based on two published papers (with co-authors) and one submitted preprint (written by the author alone). The structure of the thesis is the classical one, it is not a collection of papers. It consists of seven chapters including introduction and appendix. The results from the papers are organized in chapters mostly not by the corresponding paper, but by the type of results, e.g. Chapter 2 contains various abstract formulations of the of reaction-diffusion systems taken from both papers and preprints and Chapter 3 contains various abstract methodology (variational methods, topological methods, implicit function theorem) taken from both papers and preprint. Such organization would not be a bad idea, in general. But, unfortunately, Theorems and Lemmas in the text do not contain reference to respective papers. Thus it is very difficult to guess what are original results presented only in the thesis and what are author's results already published in journals and what are general results. After a request, I received a separate list of Theorems and Lemmas from the author, which match them to the published papers and preprint. With this help, I got to the conclusion that big portion of the results is published and already gained some attention by other researchers (3 citations without self-citations).

Here, I address requested specific questions:

Topicality of the dissertation thesis. From what was said above, it follows that covered topics are up-to date and very interesting not only for mathematicians, but also for biologists studying emergence of patterns and morphogenesis.

Methodology. Wide range of advanced methods of nonlinear functional analysis are used in the dissertation thesis. In particular, bifurcation theory is heavily used. Before application to the problems

Katedra matematiky
Západočeská univerzita v Plzni
Univerzitní 8, 301 00 Plzeň
IČ: 49777513
DIČ: CZ 49777513
Email: pgirg@kma.zcu.cz
Telefon: +420 377 632 644

with unilateral terms, these methods had to be sometimes significantly modified: global bifurcation of Rabinowitz (modification for positively homogeneous operators), Crandall-Rabinowitz Theorem (modification for non-smooth perturbations of smooth mappings), Krasnosel'ski Potential Bifurcation Theorem (applied to skew symmetric problem). Numerical experiments for investigation of reaction-diffusion systems are also used in the dissertation thesis.

Meeting the objectives. The objectives of the dissertations were met and also exceeded. Systematic theory for treatment of diffusion driven instability in reaction-diffusion systems with unilateral terms was developed.

Results and scientific achievements. Dissertation thesis contains new results of nontrivial character. Known methods were not directly applicable and their (sometimes very nontrivial) modifications were developed. Recently published results already gained attention by other researchers (3 citations without self-citations).

The author of the dissertation thesis demonstrated deep knowledge from nonlinear functional analysis. Especially, he showed that he can further develop existing theory. He also demonstrated knowledge from numerical mathematics including implementation skills.

Taking into account all what was said above, **I recommend the dissertation thesis for defense.**

Questions

1. Is it possible to theoretically study the influence of the unilateral terms and show that for "stronger" unilateral terms the ratio of diffusion coefficient can be closer to one as it was numerically observed in Vejchodský et al. [Phys. Rev. E 96, 022212 (2017)] ?
2. Proofs of Propositions 9 and 10. How to understand the sentence: "The following proposition has been proved by numerical computation, therefore we are not giving the proof here. "

Typos and minor corrections

1. The classical model ascribed to "Schnackenberg" is frequently mentioned in the dissertation thesis without referencing to the original paper in the list of literature. Moreover, the name "Schnackenberg" is misspelled throughout the thesis, it should be "Schnakenberg".
2. Proof of Lemma 2 on page 16. The way the inequality (2.18) is written with quantifiers, it cannot be true (unless C depends on φ). There are terms involving φ missing in inequality (2.19). This is just a negligence, since similar inequalities appears on page 17, where they are written correctly. Moreover, the proof is standard and the reader could be referred to Kučera [Czechoslovak Math. Journal, 47(3), (1997) pp. 469-486]. Indeed, the author omits the most interesting part of the proof and refers to this paper anyway.
3. The English needs some improvements in several places.

.....

doc. Ing. Petr Girg, Ph.D.

Katedra matematiky
Západočeská univerzita v Plzni
Univerzitní 8, 301 00 Plzeň

IČ: 49777513
DIČ: CZ 49777513

Email: pgirg@kma.zcu.cz
Telefon: +420 377 632 644