Review on Two-particle quantum walk on percolated graph by Bc. Magdalena Parýzková

Master's Degree Project

This paper extends quantum walk model on the percolated graph with one-particle [27] to the two particle. Both distinguishable and non-distinguishable cases are considered. The underlying graphs treated in this paper are the cycle and the finite path. The total Hilbert space is given by the tensor product of the vector spaces labeled by the half edges of the original graph $G = (V, E)$. The graph connectivity is randomly given at each time step: each edge e is independently closed with probability $p(e)$ while open with probability $1 - p(e)$. The unitary time evolution operator with the configuration of the edges $\sigma \in \{0, 1\}^E$, which appears with probability p_σ, is given by $U_\sigma^{\otimes 2}$ with $U_\sigma = S_\sigma C$ is defined as follows: the shift operator S_σ is isomorphic to the direct sum of the two-dimensional unitary matrices determined by the configuration σ: $S_\sigma \cong \oplus_{e\in E} X^{\sigma(e)}$, where X is the pauli matrix, while the coin operator C is isomorphic to the direct sum of the two-dimensional unitary matrices independent of the configuration: $C \cong \oplus_{u\in V} C_u$, where C_u is a $d(u)$-dimensional unitary matrix labeled by half edges of the vertex u; in this paper's case, the 2-dimensional Hadamard matrix. The average probability with respect to all the possible configurations until the time step t can be obtained from the density matrix ρ_t satisfying $\rho_{t+1} = \Phi(\rho_t)$, where Φ is the super operator defined by $\Phi(\rho) = \sum_\sigma p_\sigma (U_\sigma \otimes U_\sigma)\rho(U_\sigma \otimes U_\sigma)^*$. This paper's interest is the asymptotics of the time evolution in the sufficiently large steps t.

The time evolution driven by Φ is attracted to the stable orbit which is included in the attractor space $\oplus_{|\lambda|=1} \ker(\Phi - \lambda I)$ in the long time limit. The useful way to build the attractor space $\ker(\Phi - \lambda I)$ is applying the two conditions for $X \in \ker(\Phi - \lambda I)$ named the coin condition and the shift condition; that is, $(C' \otimes C')X(C' \otimes C')^* = \lambda X$ and $(S_\sigma \otimes S_\sigma)X(S_\sigma \otimes S_\sigma) = X$ for any configurations $\sigma \in \{0, 1\}^E$. A sufficient condition for $X \in \ker(\Phi - \lambda I)$ is to find the common eigenstates ϕ_α's satisfying $U_\sigma \phi_\alpha = \alpha\phi_\alpha$ for any $\sigma \in \{0, 1\}^E$; $\rho = \phi_\alpha \phi_\beta^*$ with $\lambda = \alpha\beta^*$. The common eigenvector satisfies $C'\phi_\alpha = \alpha\phi_\alpha$ and $\phi_\alpha = S_\sigma\phi_\alpha$ for any $\sigma \in \{0, 1\}^E$. Such an eigenattractor is called the p-attractor.

The author determined the attractors both p-attroactor and non-p-attractor of two-particle quantum walk on percolated path graph and cycle graph. In particular, for one particle case, there is only the trivial attractor as the non-p-attractor. On the other hand, the author showed that for the two-particle cases, there are a lot of non-p-attractors. The attractors on the indistinguishable particles in the symmetrical (bosons) and antisymmetrical (fermions) subspaces are also considered and determined concretely.

This paper is organized well and the computational results are correct and also impressive. I think this manuscript has enough quality to be the master thesis. The grade is B (very good). There are several new interesting results but it seems that they are buried because it is a little bit hard to find them immediately. I think if possible, the presentation in this paper of the authors results should be improved for example, using tables.

The followings are my suggestions :

1. The attractor space no longer depends on the open probability on edges $p(e)$ $(e \in E)$, but I think the following is worth to state somewhere in Section 2: a dependency of the convergence speed to the stable orbit of this dynamical system on this open probability.

2. p.30: It seems to be kind to the readers if the derivations of (2.50) and (2.51) were provided because these equations are starting points for all the cases.

3. p.44: There are at least three parts such that "then" should be "than".

4. p.45 (4.5): $\bigotimes$ should be $\bigoplus$ or $I \otimes H$

5. p.46 (4.8) and (4.9): there are exceptional "="'s.

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