RNDr. Daniel Reitzner, PhD.

Research Center for Quantum Information Institute of Physics, Slovak Academy of Sciences Dúbravská cesta 9, 84511 Bratislava, Slovak Republic

# Dissertation thesis report 

Thesis title: State Transfer in Imperfect Networks Přenos stavu na sítích s poruchou<br>Author: Ing. Antonín Hoskovec<br>Supervisor:<br>Institution:<br>Year:<br>Opponent: RNDr. Daniel Reitzner, PhD.

In his dissertation thesis titled State transfer in imperfect networks Ing. Hoskovec puts forward his results on the topic that are not only excerpts from his two published works $[7,11]$ and one book chapter [25], but he includes results from the ongoing research as well. Overall, the student has 4 WOK entries (all prior to 2017) related to this area with all together 10 citations.

This work is a resubmission of a thesis I reported on before. Previous work had many insufficiencies mainly in the style of writing and contained one major flawed conclusion for which I suggested it should have been revised. Otherwise I found the results of good quality, and interesting. In the current work, the student has improved on all the important criticisms from my previous report and reforged the thesis to a very well readable form, while keeping good quality of obtained results.

Present work consists of 6 chapters out of which the first and the last one are Opening Remarks and Concluding Remarks respectively. The structure of the work is logical and builds up knowledge starting from basics in Sec. 2, Introduction. Sections 3 (Dynamical Decoupling) and 4 (QST and Orthogonal Polynomials) present important methods in the study of state transfer, while some interesting unpublished observations are provided. In Sec. 5, Perturbed Linear Chains, presented methods are used to obtain major results of the student - the results show recovery of state transfer for perturbed NN and NNN chains. While the overall quality has improved significantly, the style of presentation could have been improved even further.

In particular, the presentation of methods and results could have been more rigorous with a more detailed explanations of conditions and settings. To just name a few:

- On p. 19, Eq. (2.40) it is assumed without stating that the permutations $P^{(j)}$ act on disjoint sets of sites. If this would not hold, then vectors in (2.42) would not be orthogonal and, in turn, this would not allow expression (2.43).
- On p. 35 an algorithmic approach to finding $\vec{e}$ is presented, yet it would be useful to see expectations one has on the results - while it is desired that $D$ is as small as possible, I
would welcome discussion on how this translates to particular choice of $\vec{e}$. As a result, it is unclear why it is reasonable to take $\gamma_{0}=\min \left\{e_{k}\right\}$.

These points are not essential, but their presence would improve readability of the work.
In addition, the student seems to have a habit of drawing conclusions without considering all the possibilities. This can lead to false reasoning, such as was present in the previous version of the work. In current work I have found one such place, although with benign consequences. On p. 47 there is a conclusion that if we want to combine chains, these have to be the same $\left(H_{1}=H_{2}\right)$. But this is not completely true, as one can consider to combine three same chains $\mathcal{C}$. Combining first two, one gets a chain of double length $\mathcal{C}-\mathcal{C}$ with possibility of state transfer, but one can join the third chain as well to obtain state transfer even though at this point the two chains, the doubled one $\mathcal{C}-\mathcal{C}$ and the third one $\mathcal{C}$, are not the same. The resulting Hamiltonian would be persymmetric as well.

Previous criticisms are, concerning the quality of the work, only minor ones, and besides these shortcomings there is only a small number of mistakes and typos. As mentioned, presented work contains relevant results of good quality showing that the student is capable of delivering interesting results using lengthy and complicated calculations. Though the student seems to have a habit of hastily drawing conclusions without stopping to consider all the relevant cases, he is able to employ a multitude of different approaches and methods to obtain desired results.

Questions to the student. Before drawing the final conclusion, to better assess the insight of the student into the topic, I would like him to answer the following questions:

1. What would you say is the motivation behind Cartesian product of graphs. Is it not that the state transfer on combined graphs can be seen just as particular state transfers on separate graphs?
2. In Sec. 4.2.1 I find the explanation rather vague. Trying to find a rectangular lattice allowing the state transfer means that one identifies the Hamiltonian parameters $I_{j, k}, J_{j, k}$ and, possibly $B_{j, k}$, yet the section ends with just stating that a state transfer is possible. Can you show, at least schematically, how one can arrive at the desired Hamiltonian parameters?
3. On p. 67 it is stated that $\sigma_{z}$ pulses can be eliminated by considering solutions to the homogeneous equation. Could you show how this works?
4. In your work you have employed minimization of parameter $D$. Would it make sense to optimize parameter $m$, the number of dynamical decoupling pulses necessary to get the ideal evolution, instead?
5. It is mentioned that the more $g_{j}=\mathbb{1}$ a scheme has the easier it is to be implemented. Is this really so? In Eq. (3.2) it is defined that $g_{k}=p_{k} p_{k-1} \ldots p_{0}$ where $p_{j}$ are the particular pulses. Then if $g_{0} \neq \mathbb{1}$, all subsequent $g_{j}$ 's will be also different from identity. Can you comment on this point?

Conclusion. In the light of high quality of the presented work and its significant improvement over the previous version, I recommend accepting this dissertation thesis for the public defense. On the usual grade scale A-F, I would suggest grade B.

