CHARLES UNIVERSITY
Faculty of Mathematics and Physics
Department of Numerical Mathematics
Sokolovská 83, 186 75 Prague, Czech Republic
Tel.: (+420)221 913 364, e-mail: knm@karlin.mff.cuni.cz

Fakulta jaderná a fyzikálně inženýrská
prof. Ing. Igor Jex, DrSc.
děkan

October 19, 2020

Review of the PhD thesis of Matěj Klíma

The thesis deals with the development of the Arbitrary Lagrangian-Eulerian (ALE) methods which exhibit a very important tool of the simulation of various evolution and dynamics problems. The thesis focuses on simulation of multi-materials, particularly the mixture of fluids, solids and voids (vacuum). Namely the numerical simulation of arising void zones is problematic by the Eulerian technique hence the ALE approach is much more promising. The core of the thesis is the Interface-Aware Sub-Scale Dynamics (IASSD) multi-material model which allows to capture the interface among different materials within mesh cells.

The thesis has two parts. In the first one, the very well balanced overview of the thesis is presented. Sections 2-3 describe the used methods and techniques. The governing equations are discretized by finite volume method and the time-integration is carried out by second order predictor-corrector method. Some quantities are taken as the node values and the others as the cell averages. Section 4 deals with the IASSD model consisting of bulk stage, sub-scale dynamics scale and the treatment of the void. Section 5 contains a detailed explanation of the mesh smoothing and remapping which exhibit the fundamental technique for the practical usability of the method. Several interesting numerical examples are given in Section 6, they clearly demonstrate the robustness of the proposed method.

The second part of the thesis is the collection of 5 papers published recently in recognized international journals. Three of them belong among the first quartile of the journal ranking, the two other are in the second quartile. Obviously, the development of the whole machinery is not the work for alone researcher but for a (medium size) research team. Therefore, it is natural that these papers have several co-authors. However, the contribution of Matěj Klíma is essential. as stated in the declaration of co-authors. Moreover, the work of Matěj Klíma is the continuation of the previous research, namely of his supervisor. Nevertheless, the progress during the PhD study of Matěj Klíma is significant.

The subject of the thesis is topical and challenging. The development of the robust and accurate IASSD model is a complex and complicated problem since many aspects and practical restrictions have to be taken into account. The most critical seems me the mesh modification and the re-computation of the solution to the new mesh. The goals of the thesis were achieved without any doubts. The work of Matěj Klíma has contributed to the development of the advanced techniques in computer simulation of complex evolution problems.

On the other hand there are several items which are not treated in the thesis but could serve fora example as subjects of further research. The thesis does not contain a numerical analysis of the proposed technique. It is clear to me that this is a very complicated task beyond the frame of PhD thesis. Although numerical experiments give a reasonable justification of the validity of the developed method, theoretical results would increase the worth of the developed method. Furthermore, there is no study of the efficiency and accuracy of this technique, e.g., a comparison with the other approaches. So it is not clear how expensive is the performing of the computations. Another interesting problem is the study of a possible violation of the accuracy during the mesh modification and the recomputation of the solution.

Additionally, I have the following points to the discussion:

- The second-order predictor-corrector method is used for the time discretization. In Section 3.3, there is explained the limitation of the size of the time steps in order to guarantee the stability of the method. Is there any benefit of the use of the corrector step in comparison to a purely explicit scheme?

- The mesh smoothing exhibits a step where the perturbed quadrilateral grid is shifted back to the uniform mesh. Does it mean that the number of degree of freedom during the simulation is kept constant? Is it possible to adapt the number of degree of freedom during the simulation process?

- The benefit of ALE technique is clear for many situation. However, some of the examples given in Section 6 could be solved by "traditional" techniques, i.e., the Eulerian description on fixed meshes. Do you have a quantitative and/or qualitative comparison of these technique for an example?

- The used discretization is explicit with respect to time. Do you have idea how complicated would be a possible extension to implicit time discretization which is demanding for stiff problems?

I can conclude that the thesis of Matěj Klíma is excellent and I can warmly recommend to award him the title PhD.

Prof. RNDr. Vít Dolejší, Ph.D., DSc.
Charles University
Faculty of Mathematics and Physics
Sokolovská 83, 186 75 Prague 8, Czech Republic
tel.: (+420) 221 913 373
e-mail: dolejsi@karlin.mff.cuni.cz