Review of thesis "Phenomenological studies of QCD at high energies" by Marek Matas.

The dissertation thesis by Marek Matas is studying the Balitsky-Kovchegov evolution equation within the dipole model in several approximations. The main motivation of the work is to study the onset of gluon saturation effects and to help understanding the puzzle of rising gluon distributions in protons. The main part of the thesis is organized as follows. First, a solution of the evolution equation for a limit of infinite target is shown, then more realistic configurations with impact parameter dependence are discussed along with an appropriate treatment of non-perturbative region giving rise to so called Coulomb tails. Collinear logarithms are resumed along with incorporation of running coupling into the kernel of the evolution equation and new form of initial conditions is suggested. These new initial conditions are adopted to the case of proton-ion collisions which allows not only to fit DIS data but also to make prediction for proton-ion data for the Large Hardon Collider. The thesis targets important open questions in the field and pushes our knowledge towards answering them.

The thesis reads very well. It provides a useful basic introduction to the problematics and a well written description of the work the author did. The presented work uses standard methods and problem formulations. Formally, there is only very small number of typos or imprecisions (e.g. on page 30, proton is not depicted as a gray blob in the center of figure 3.12).

The thesis is equipped with reprint of six papers out of which three were published in journals of high impact parameter. Further the thesis lists four proceedings that are connected with presentation of author's results at international conferences.

I general, there is no doubt about the quality of the presented work. Nevertheless, I'd like to ask several questions and I'd like to kindly ask the author of the thesis to prepare answers for a discussion during the defense of the thesis:

a) Page 12. Can you just briefly outline a derivation of the formation of the color dipole, please? The color dipole is a central object you use. Why to work in the position space representation instead of momentum space representation? What is the advantage? To my knowledge, formulation of problems in position space is often more complicated than in the momentum space (see e.g. QFT propagators).

b) Table 3.1. What is the method you use to perform the fits with a numerical solution of evolution equations, please?

c) Table 3.1. A value of constant C is said to be obtained from the fit to the data. I guess this value also reflects a freedom in the choice of the renormalization / factorization scale. Do you agree? In general, do you know how correlated are your parameters in the fits? If they are correlated then e.g. an uncertainty on C would also translate into the uncertainty on other parameters.

d) Table 3.1. Question related to point c). You provide estimates of your parameters to three or more significant digits, but you don't provide an estimate of the precision. What is their precision, please? Fits may provide you with some error estimates and you may also have some other sources of uncertainties.

e) Eqn. (3.10) has the logaritmic term compared to (3.9). The amplitude will steeply fall for some large r values if there is this log term (compared to (3.9) which will remain flat at unity). Do you agree? Does a presence of this logarithmic term and the steep falling at large r have some physical interpretation?

f) Related to e). You spend a significant effort on getting under the control the Coulomb tails. The log term introduced in (3.10), I mean $\ln(1/rL + e)$, could also help to get that under the control. Do you agree? Did you try? In any case, why you don't use initial conditions with this log term in section 3.6 and onwards?

g) Page 23. Why constituent quark masses are used instead of current quark masses? Is that connected with the Vector Meson Dominance model? The use of the constituent masses suggests that you already work with dressed quarks which seems counterintuitive.

h) Page 23, How does N look for r>10 GeV^-1 in Fig. 3.4? Is it constant at unity for larger r? I guess it will start falling down if you use (3.10). How did you arrive at the interval "from 0.02 to 4 GeV^-1" quoted in the text?

i) Sec 3.5. Do you or other authors consider color coherence effects? These effects were shown to shape the parton showers and may be of a relevance here as well. An example: if the r of the dipole is significantly smaller then the size (defined by the wavefuntion) of the object inside the proton, then that object should not be able to recognize the structure of the dipole and the dipole radiates coherently (that is being the gamma*).

j) page 39: you say that "B_G defines the size of the proton". B_G of 3.2 GeV^-2 implies 0.36 fm. How to explain such small proton size?

k) Sec. 4.7: Unfortunately, there is no comparison of the calculation with existing data where some interesting effects are seen. Is it possible to say how the calculation compares e.g. with Fig.6 bottom of arxiv:1901.10440?

Irrespective of these questions I should stress that the thesis is of a high quality and that I recommend this thesis as a valid dissertation thesis.

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