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THE SUPERVISOR'S REPORT
ON THE PHD THESIS BY
ING IVETA SEMORÁDOVÁ

"Nonstandard perturbation methods and non-Hermitian models in Quantum Mechanics"

(the planned date of defense: October 11th, 2021)

The thesis summarizes the results of research performed by Ing. Semorádová (with coauthors) which were published as a series of papers in several top-level international refereed scientific Journals. Very recently, the project climaxed by the completion of results which are also already submitted for publication.

In the role of supervisor, I cannot resist mentioning, in introduction, that in spite of a highly topical nature of the research devoted to the non-Hermitian models in Quantum Mechanics, the original formulation of the subject (putting emphasis upon the role of perturbation methods) remained unchanged during the whole term of the study. This reflects, in some sense, one of the most characteristic personal features of Ing. Semorádová, viz., her ability of a long-time concentration and of a deep analysis of the subjects in question.

Not too surprisingly, the concentration of IS on the project led not only to a highly non-standard, over-the-average publication activity and success (see the Appendix below) but also (and, perhaps, first of all) to multiple parallel and equally over-the-average communication activities of IS documented via her homepage and manifested, first of all, by a truly impressive intensity of her participation in various international conferences and schools where she presented her results and received the necessary feedback and encouragement.

The IS' selection of the form of the thesis reflects her above-mentioned communication strategy: The main information is given in Appendices, containing the copies of the five most important papers forming the essence of the message. The readability of this message is facilitated by the newly written and comparatively compact introductory text.

The latter, explanatory text (i.e., in a narrower sense of the word, the thesis proper) is separated into chapters and sections. Their ordering offers a picture of the innovative theory which is given a natural "evolution of the idea" structure. We are shown that and how the presented amendments of the underlying standard formalism (viz., of Quantum Mechanics) can be branched into three sub-units, viz., into Quantum Mechanics of conventional textbooks (perceived as formulated in the so called Schödinger representation *alias* picture), a more general Quantum Mechanics of unitarily evolving systems (formulated in two non-equivalent Hilbert spaces, better known under the nickname of quasi-Hermitian Quantum Mechanics) and, thirdly, the recently most quickly growing framework and formalism of Quantum Mechanics of non-unitary systems (in which people found, i.a., an opportunity of making ample use of the new mathematics developed during the preceding studies of quasi-Hermitian quantum Hamiltonians).

The text of the thesis deserves a separate appreciation as offering a self-contained presentation of quantum theory in the form which might serve not only as a useful introduction in the subject but also as a well organized review of the role and consequences of a formal non-Hermiticity of operators (and, mainly, of the operators representing observables) and of their "small" modifications in quantum mechanics. The main emphasis is put upon the role played by the specific perturbation-approximation techniques.

In this language the innovations achieved by IS are threefold. Firstly, in a way dating back to the Trefethen and Embree's monograph it is emphasized that one of the most adequate formal bridges between the conventional Quantum Mechanics (CQM) and the quasi-Hermitian Quantum Mechanics (QQM) may be seen in the use of the concept of pseudo-spectrum. Indeed, she showed that and how this concept enables us to find and formulate an abstract background of the weak- and strong-coupling expansions, with a particular emphasis upon the applications known as large-N expansions.

Secondly, IS addressed also a parallel challenging problem of our under-

standing (and of some non-standard perturbation-formalism description) of certain effective non-linear models of quantum phenomena (incidentally, let me add that this branch of the IS's PhD-related research has been inspired by the collaboration of our team in Řež with another supervisor's team in Durban (DUT, under the guidance of K. Zloschchastiev) where the the role of non-linearities is currently being studied, albeit not necessarily just by the non-Hermitian or perturbation-expansion techniques).

In parallel, thirdly, the communicative capabilities of IS contributed significantly to a build-up of productive scientific collaborations with the research teams in USA (T. Curthright, branched-Hamiltonian quantum models), India (B. Bagchi, local potential models) and in Algeria (M. Maamache and his colleagues). In all of these contexts, the key contribution by IS is to be seen in the expertise she acquired in perturbation techniques which she was able to implement and put in the broader methodical context of quantum field theory.

This being said, it is necessary to add that from my current point of view of the IS' supervisor, one of my most successful managerial deals, still connected with the above-mentioned "covering mathematics" of pseudo-spectra, was the collaboration contact established (or rather re-established, during one of many international conferences) with my previous PhD student Petr Siegl (currently a professor of mathematical physics in Belfast). Indeed, shortly before the Coronavirus crisis I already proposed that IS should prepare her dissertation. Nevertheless, PS was against. He proposed that (for some rather technical and purely administrative reasons) we should postpone the IS' defense. In facct, he proposed that she should use the incidental, not quite expected opportunity of his invitation of IS to join, for a year or so, his newly established research team in Belfast.

On these grounds, IS spent an Erasmus-supported full year of her study in Belfast. Summarizing: not only that she proved well integrated in the PS' team. What was even more important was that she got excited by their non-Hermitian and perturbation-related projects. This enabled her to add, to her current PhD thesis, another "third", viz., the material devoted to the so called domain truncation method.

Many experts on the strategy of science use to say that in contrast to the start of PhD study, the student should become, at the end of the PhD

study, a better expert in the field than his supervisor. And I feel afraid that, mainly due to the PS's scientific-leadership contribution, it is true also in the IS's case.

In summary, my overall evaluation of the IS's work and thesis is very positive. The author's range of interests is broad, her knowledge of the field is deep, and her collaboration potential is strong. By my opinion, the thesis clearly demonstrates that IS will be, undoubtedly, capable of an active and independent research work in the future.

I am sure that also on the legal level, all of the requirements needed for the award of the PhD degree to Ing. Semorádová are satisfied.

Řež, September 8th, 2021

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Appendix

A. Publications in Journals (incorporated in the PhD thesis)

Miloslav Znojil and Iveta Semorádová
Log-anharmonic oscillator and its large-N solution
Mod. Phys. Lett. A 33 (2018) 1850223
arXiv:1809.07590

Miloslav Znojil and Iveta Semorádová
Quantum square well with logarithmic central spike
Mod. Phys. Lett. A 33 (2018) 1850009
arXiv:1712.03672

Bijan Bagchi, Syed M. Kamil, Tarun R. Tummuru, Iveta Semorádová,á Miloslav Znojil
Branched Hamiltonians for a class of Velocity Dependent Potentials
Journal of Physics: Conference Series 839 (2017) 01201
arXiv: 1701.02280

Miloslav Znojil, Iveta Semorádová, Frantisek Ruzicka, Hafida Moulla, Ilhem Leghrib
The problem of coexistence of several non-Hermitian observables in PT-symmetric quantum mechanics
Phys. Rev. A 95, 042122 (2017)
arXiv: 1610.09396v2

B. The most recent preprint incorporated in the PhD thesis

Iveta Semorádová and Petr Siegl
Diverging eigenvalues in domain truncations of Schroedinger operators with complex potentials
arXiv:2107.10557

C. Further publications of IS, not incorporated in the PhD thesis

Iveta Semorádová
Crypto-Hermitian approach to the Klein-Gordon equation
Acta Polytechnica 57(6):462-466, 2017
arXiv: 1801.09602

Francisco M. Fernández, Javier Garcia, Iveta Semorádová, Miloslav Znojil

Ad hoc physical Hilbert spaces in Quantum Mechanics
Int. J. Theor. Phys. 54 (2015) 4187-4203
arXiv: 1405.7284

D. Theses (available online)

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Quantum Mechanics of Klein-Gordon equation
Master Thesis, supervisor Miloslav Znojil

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Two-dimensional Lie groups and their Poisson brackets
Bachelor Thesis (in czech), supervisor Ladislav Hlavatý