Referee's report on doctoral thesis

Title: *Nonstandard perturbation methods and non-Hermitian models in Quantum Mechanics*

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The doctoral thesis is based on five scientific articles. Four papers are published in international physically oriented journals. The mathematically most interesting and valuable as regards its content and mathematical rigour is the fifth paper, which is just an arXiv preprint so far. All the papers are with co-authors; the four published are with the PhD supervisor, three of which with a non-standard (non-alphabetic) order of authors.[1]

The doctoral thesis consists of an introduction, four chapters summarising the content of the aforementioned papers and the papers themselves added as an appendix. The introduction appears to me unsatisfactory for it neither represents a global overview of the subject the thesis was supposed to cover nor links the following chapters with the attached papers. Indeed, there are just three pages of a rather disordered text, in which the author quickly passes to the very concrete models covered by the thesis. The remaining chapters are equally an unclear mismatch of general facts and own results. The text is in many parts logically inconsistent and often difficult to read, even for a reader familiar with the topic. My general impression is that the thesis was not written carefully, in any case it is difficult to get oriented therein.

As the title of the thesis suggests, the subject of the papers are typically non-self-adjoint operators in quantum mechanics and their spectral analysis by means of perturbative methods, allegedly not orthodox. However, it is not clear in which aspects the used perturbative methods are unorthodox.[2] At any rate, the obtained results, especially those in the preprint paper, are scientifically relevant, up-to-date and interesting for the scientific community.

Despite the formal complaints above, I recommend the thesis to be defended and the candidate to be awarded the doctorate title of PhD.

Further questions for the defence and minor criticisms about the text can be found on the following pages.

[1] An explanation why the author of the doctoral thesis is located in non-priority places in some of the papers should be given during the defence.

[2] Maybe the author can explain the heterodox nature of the used methods during the defence.

Further questions for the defence

3. Explain the following claim from page 41:

   > *This phenomenom of diverging eigenvalues is common on certain types of open domains with corners (such as bounded interval)...*

   What does "domains with corners" mean? An open interval is a *smooth* domain according to the standard definition of smoothness for Euclidean domains.

4. Explain Hypothesis 3.1.1 from page 29. What is the significance of the power $3/2$ at the potential Q? Is this power optimal for this realisation of the operator T?

Minor criticisms

1. Page 4, Abstrakt: Schrödingerovských → schrödingerovských.

2. Page 9, 2nd paragraph: The first sentence of the paragraph

   > *Self-adjoint operators are similar to quasi-self-adjoint operators for which [potential] Q may be complex.*

   is unclear. The author probably wishes to say:

   > A Schrödinger operator (1) with complex Q is said to be *quasi-self-adjoint* if it similar to a self-adjoint operator.

   The exact significance of the similarity transform should be explained.

3. Page 9, 3rd paragraph: $C \to \mathbb{C}$.

4. Page 11–12: The first sentence of the first paragraph of Section 1.1 is obscure. In reality the alternative definition (1.3) says that the spectrum of the perturbed operator $H + E$ with $\|E\| < \varepsilon$ is precisely described by the ε-pseudospectrum. The author probably wishes to say that, in the special case of self-adjoint H, the ε-pseudospectrum (and thus the spectra of the perturbed operator $H + E$) coincides with the ε-tubular neighbourhood of the spectrum H (because of the formula (1.2)). The main message should be that, in the general case, the pseudospectrum can be much larger.

5. Page 13–14, opening of Section 1.2: Why does the author restrict to self-adjoint A_g here? More general claims are used later.

6. Page 15, above (1.25): It is not clear what Example 1.1 is.
7. Page 17, above (1.40): and $\to$ an?
8. Page 19: In formula (1.53) there is comma instead of full stop. Similar typographical discrepancies can be encountered in the whole text of the doctoral thesis, before as well as after (1.53).
9. Page 23: The claims in the paragraph above Section 2.1 should be proved or a source reference should be given.
10. Page 23: The first two sentences of Section 2.1 are unclear.
11. Page 23, above (2.7): It is not clear what "form a complete biorthonormal system" means precisely. Just property (2.7)?
12. Page 23, below (2.7): The term "diagonalizable operator" should be explained.
13. Page 23, below (2.9): positive and properly bounded $\to$ *uniformly* positive and bounded?
14. Page 27, below (3.2): In the definition of the operator $\mathcal{T}$ there is a complex conjugation missing. The second paragraph of Chapter 3 is mathematically incomprehensible.
15. Page 33, Corollary 3.3.2: The term "spectral pollution" should be explained here. (The definition is given on page 40 only.)

Given in Rome, 15 September 2021, David Krejčiřík