HZDR
HELMHOLTZ ZENTRUM
DRESDEN ROSSENDORF

Helmholtz-Zentrum Dresden-Rossendorf e. V.

Czech Technical University in Prague
Faculty of Nuclear Sciences
and Physical Engineering
Prof. Ing. Igor Jex, Dr. Sc.
Dean

**Department of Theoretical Physics**
Dr. Uwe Günther
Senior Research Scientist

Phone +49 351 260-2415

u.guenther@hzdr.de

Our reference:
19.09.2021

**Referee's report on the PhD thesis**

by Ing. Iveta Semoradova

"Nonstandard perturbation methods and non-Hermitian models
in Quantum Mechanics"

submitted for defense to the Faculty of Nuclear Sciences and
Physical Engineering of the Czech Technical University in Prague

During the 120 years of exploration and description of quantum phenomena the corresponding theory for their mathematical formalization has undergone many directions of development and generalization to include more complicated and more sophisticated system classes. One of the more recent directions of research concerns the spectral properties of operators non-self-adjoint in Hilbert spaces, so called non-Hermitian operators. Such operators naturally emerge, e.g., in the description of open and effective quantum systems with decaying and amplifying components, corresponding interaction channels and generalized effective observables. In contrast to the relatively well developed theory of self-adjoint operators in Hilbert spaces with their mostly purely real spectra, the spectra of non-self-adjoint operators may remain purely real or they may form certain patterns deep in the complex spectral plane. Small perturbations of non-Hermitian operators may lead to very strong and very wild deviations in their spectra. Techniques well tested for Hermitian operators may fail, turn out useless and become no longer applicable for non-Hermitian operators. More adequate theoretical and mathematical tools have to be invented, developed and explored for the study of such non-Hermitian operators. Many research groups world wide are currently very actively working on these topics of utmost importance not only for the internal progress of mathematical sciences but even more for the analysis and design of large classes of novel applications. Noteworthy examples of applications range from novel lasing systems, optical meta-materials, quantum wires and circuits of nano-meter sizes over possible applications in quantum computing up to unexpected new phase transition scenarios in condensed matter systems, elementary particle physics and quantum gravity.

Iveta Semoradova's PhD thesis fits exactly into these quickly evolving leading edge research activities and contributes remarkable novel mathematical tools

**Helmholtz-Zentrum Dresden-Rossendorf e. V.**

Bautzner Landstraße 400
D-01328 Dresden
www.hzdr.de

and techniques to characterize the spectral properties of certain classes of non-self-adjoint operators. From the chronology and contents of Iveta Semoradova's publications one sees an impressive learning-by-doing process showing a strongly progressing evolution from investigating certain nontrivial and exotic model examples at the initial stage of her studies towards the development of highly generalized operator-theoretic methods and techniques at the current final stage. The PhD thesis provides a summary of the obtained results within a general unified framework which meets highest standards of advanced operator theory.

Main topic of the thesis is the description and characterization of the spectral behavior of unbounded non-self-adjoint differential operators of Schrödinger type defined over general open domains in real Euclidean spaces of arbitrary finite dimension and Dirichlet boundary conditions imposed at the domain boundaries. To tackle the spectral problems Iveta Semoradova uses sophisticated combinations of various operator-theoretic techniques. Basic tools are rescalings of coordinates and coupling constants such that the original operators may be reduced to rescaled and shifted versions of well known exactly solvable model components supplemented by suitably adapted and well controlled perturbative correction terms. As exactly solvable model components figure operators with linear and quadratic potentials yielding eigenfunctions in terms of Airy functions and Hermite functions of quantum oscillator type, respectively (chapter 1). The perturbative corrections are estimated with the help of pseudo-spectral techniques in terms of resolvent bounds as well as resolvent differences (chapter 3). The techniques are explicitly demonstrated for systems with enhanced dissipation as well as for two types of PT-symmetric systems which show PT phase transition behavior, i.e. parameter dependent passages from sectors with purely real eigenvalus to sectors with pairwise complex conjugate eigenvalues. Spectral components of non-self-adjoint operators over unbounded domains are investigated as spectral limits of sequences of operators with truncated domains (chapter 4). As result, well-controlled spectral approximations for these Schrödinger type operators are obtained, including approximations for eigenvalues diverging in the strong coupling limit (chapter 3) as well as for eigenvalues induced by the domain truncation regularization which diverge when the truncation regularization is removed in a well controlled way (chapter 4). Details and subtleties are illustrated for Schrödinger operators with odd and even imaginary potentials, i.e. PT-symmetric and anti-PT-symmetric ones, as well as for radially symmetric potentials on annuli (chapter 4). Topically slightly aside from these main research subjects is the investigation of quasi-self-adjoint quantum observables, the derivation of conditions for the mutual compatibility of several such quasi-self-adjoint observables and the existence of corresponding positive definite metric operators in auxiliary Hilbert spaces (chapter 2).

The PhD thesis is clearly written in a good scientific style. It can be stated that in developing general pseudo-spectral approximation techniques for the relevant spectral branches of non-self-adjoint operators and their application to a large variety of various concrete PT-symmetric and non-PT-symmetric model operators the goal of the PhD thesis has been achieved in all relevant points. The results have been published in internationally recognized high quality journals like Phys. Rev. A, Mod. Phys. Lett. A, J. Phys. Conf. Ser.. The general operator-theoretic results have been finalized most recently, are published up to now only as e-print but have been presented at several international conferences and discussed with colleagues from the world wide math-physics com-

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

munity. It can be anticipated that the results will serve as valuable technical tools for future research in this community and that they have the potential to become very well cited.

In summary, I can recommend Iveta Semoradova's PhD thesis for defense without the least hesitation.

Dr. Uwe Günther (PhD)