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STRUCTURAL RELIABILITY OF EXISTING RC BEAMS STRENGTHENED WITH UHPFRC TENSILE LAYERS

Lenganji Simwanda*, Nico De Koker, Celeste Viljoen, Adewumi John Babafemi

Stellenbosch University, Department of Civil Engineering, Private Bag X1, Matieland 7602, South Africa

* corresponding author: 24429902@sun.ac.za

Abstract. A methodology for reliability analysis of reinforced concrete (RC) beams strengthened with ultra high-performance fibre reinforced concrete (UHPFRC) tensile layers is presented. The proposed methodology includes stochastic stress-block analysis of a section, assuming a perfect bond between the RC beam and the UHPFRC layer. Annual reliability analysis of the RC beam before and after the strengthening operation is conducted. Deterioration induced by chloride corrosion is incorporated into the analysis via a chloride induced corrosion model based on Fick's law of diffusion and described stochastically to account for the epistemic uncertainty in the time to corrosion initiation and rate of corrosion. A plot for determining the required thickness of the UHPFRC tensile layer to upgrade to the required reliability level is also given, considering the time from construction to when the strengthening operation is conducted. The proposed approach is easy to apply for routine practice.

Keywords: Corrosion damage, rehabilitation of existing structures, reliability analysis, UHPFRC tensile layers.

1. Introduction

Addition of ultra high-performance fibre-reinforced concrete (UHPFRC) layers on the tensile side is a newly developed technique for strengthening reinforced concrete (RC) constructions, such as floor slabs, beams and bridge decks [1]. Other conceptual ideas include additional layers on the compression side or even including on the sides (referred to as jackets) to improve the bending resistance of RC beams.

Efforts have been made in the recent past to investigate the efficacy of the UHPFRC strengthening techniques for retrofitting and rehabilitation of existing RC structures. For example, Farhat et.al [2] observed an increase in the load carrying capacity and reduction in autogenous shrinkage strains of damaged RC beams retrofitted with UHPFRC and proposed a load-carrying prediction model based on fracture mechanics.

Habel et.al [3] investigated, through a parametric study, the flexural response of composite UHPFRC-RC elements via the traditional analytical cross-sectional approach and observed that addition of steel reinforcement in the UHPFRC layer is the most effective approach to increase the moment capacity. A most recent work by Lampropoulos et.al [4] assessed, experimentally and numerically, the efficacy of UHPFRC layers and jackets for the strengthening of existing RC beams, and observed that the three side UHPFRC jacket approach was superior in terms of performance.

With efforts to apply these techniques to routine practice, analytical and numerical models [1–4] have been proposed for the estimation of bending moment capacity of RC beams strengthened with UHPFRC layers. Despite several uncertainties present in the formulations of the present analytical approaches, the application of structural reliability method has received very little attention in this research area.

The objectives of this paper are two-fold. The first being to assess the viability of the stress-block section analysis in the estimation of nominal bending moment capacity of RC beams strengthened on the tensile side with UHPFRC layers, and the second being to present and apply a statistical methodology for conducting reliability analysis of existing RC beams strengthened with UHPFRC tensile layers.

2. Stochastic modelling of deterioration of RC beams prior to strengthening

2.1. Corrosion modelling

A significant factor to the structural deterioration of RC beams exposed to saline conditions is the corrosion of steel reinforcements. Therefore, their remaining service life when strengthened with UHPFRC is considered in this study in the context of chloride induced corrosion. The effect of corrosion-induced deterioration on the cross-section area of steel rebars is discussed while ignoring its effects on other geometrical and material properties of steel and concrete.

For simplicity, a model that assumes uniform corrosion and proposed in [5, 6] is considered here, but more detailed corrosion models may be applied in appropriate cases following the methodology presented in this paper. In [5, 6], a model is presented such that the evolution in time of the diameter ϕ of the corroded rebars is computed via Equation (1)

$$\phi(t) = \begin{cases} \phi_{\text{initial}} & t \leqslant t_0 \\ \phi_{\text{initial}} - 0.0232 i_{\text{CORR}}(t - t_0) & t > t_0 \end{cases} \tag{1}$$

where ϕ_{initial} is the initial bar diameter, t_0 is the time (in years) to corrosion initiation, and i_{CORR} is the corrosion ratio (in $\mu A/cm^2$) computed empirically via Equation (2)

$$i_{\text{CORR}} = \frac{37.8(1 - w/c)^{-1.64}}{c'} \tag{2}$$

where w/c and c' is the water to cement ratio of concrete and concrete cover to reinforcement, respectively. With this model, the reduction of the cross-section area of the steel reinforcement $A_s(t)$ as a function of time can be computed from the time varying diameter $\phi(t)$.

2.2. Corrosion stochastic simulation

It is worth noting that Equations (1) and (2) contain parameters associated with given levels of uncertainty, that needs to be quantified and accounted for in the reliability analysis of RC beams exposed to marine environments. Specifically, the espistemic uncertainty in the time to corrosion initiation t_0 and corrosion ratio i_{CORR} needs statistical description.

Following a previous work [7], the epistemic uncertainty in t_0 and i_{CORR} were reported to be well described by Weibull and Uniform probability distributions, respectively. For the problem considered in this work, t_0 was estimated to have Weibull parameters ($\alpha = 11.3$, $\beta_w = 4.81$), whereas i_{CORR} was taken to be uniformly distributed between the closed interval $i_{\text{CORR}} \in [2, 3]$. Figure 1 shows the mean time series and its corresponding 10% confidence interval of the stochastic Process describing A_s and estimated via Monte Carlo sampling with 10^6 samples.

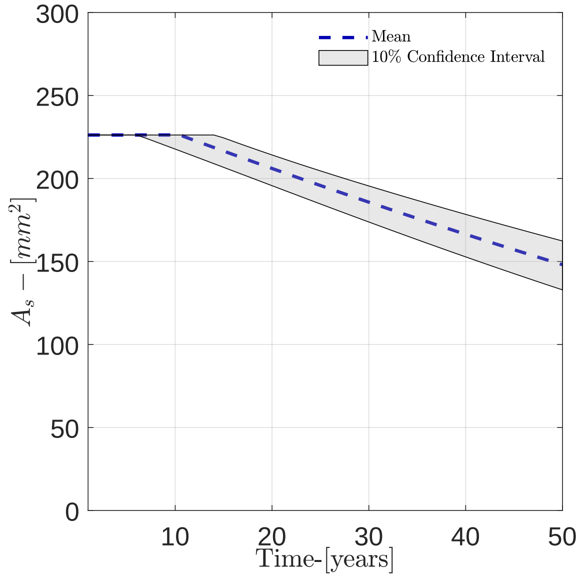

Figure 1. Uncertainty quantification of deterioration of steel cross-sectional area in the RC beam over its lifetime.

3. Analysis of RC beams strengthened with UHPFRC

3.1. Moment capacity: Proposed approach

The moment capacity of non-strengthened RC beams can readily be computed by analytical models reported in literature [8–10]. The Eurocode model [9] of singly reinforced concrete beams is used in this work.

Building on the assumptions of the Eurocode model [9], a procedure for estimating the moment capacity of RC beams strengthened with tensile UHPFRC layers is proposed. The approach is based on the stress-block analysis of a balanced section is, assuming a perfect bond between the RC beam and the UHPFRC layer and that the section cracks in the RC zone of the strengthened beam transferring the section forces to the longitudinal reinforcements and the fibres in the UHPFRC layers[11].

The analysis of the cracked section assumes strain compatibility across the cross-section (see Figure 2), maximum strain in concrete, and yielding of steel fibres in the UHPFRC layer is reached first. With this assumption the characteristic moment capacity of the RC beam strengthened with layers may be estimated via the following equations.

The stress σ_f and strain ε_f in the extreme steel fibres of the UHPFRC layer may be computed via Equations (3) and (4)

$$\sigma_f = 2\tau_f \frac{l_f}{d_f} \tag{3}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f} \tag{4}$$

so that the position of the the neutral axis of a balanced section can be obtained via

$$a = \frac{f_{yk}A_s + f_{tk}bt_\Gamma}{f_{ck}b} \tag{5}$$

$$c = a/0.80. \tag{6}$$

The position where the force in the steel rebars and the force in the steel fibres effectively act (denoted as ζ) is

$$\zeta = \frac{f_{yk}A_s(h - c' - c) + f_{tk}bt_\Gamma(h - c - bt_\Gamma)}{f_{yk}A_s + f_{tk}bt_\Gamma} \tag{7}$$

and finally, the characteristic moment capacity M_c of the beam strengthened with a tensile UHPFRC layer is

$$M_c = \zeta(f_{yk}A_s + f_{tk}bt_\Gamma) \tag{8}$$

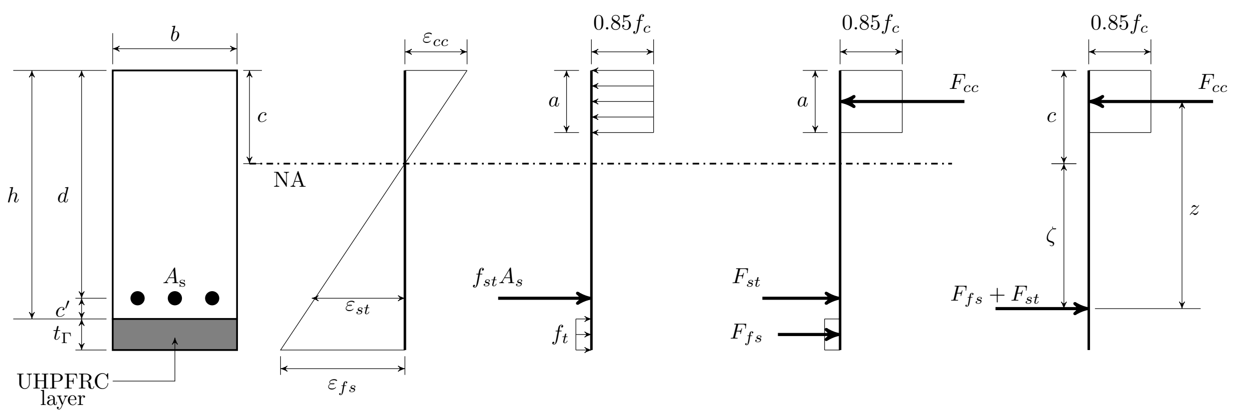

Figure 2. Stress block geometries for the for the proposed approach considered with tensile capacity of UHPFRC layer due to steel fibres considered

3.2. Comparison of proposed approach to test results

12 RC test beams strengthened with tensile UHPFRC layers from previous experimental protocols [3, 12, 13] are used to validate the proposed stress-block approach. Figure 3 shows a comparison in moment capacity prediction between the proposed analytical procedure and what was experimentally reported. A significant correlation between the proposed approach and test results is observed.

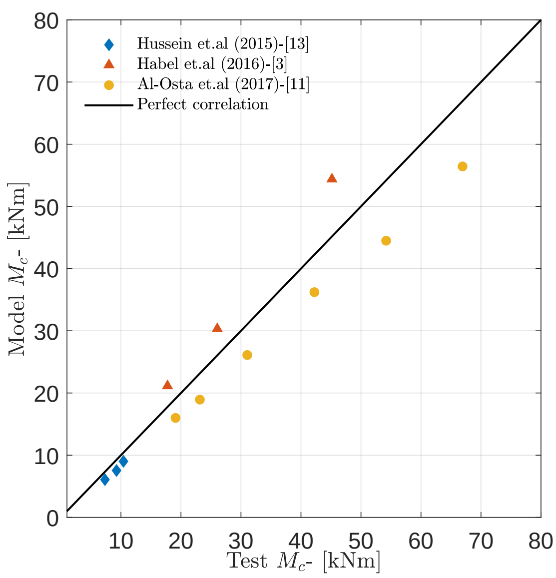

Figure 3. Comparison of a proposed analytical procedure with experimentally predicted moment capacity.

4. Reliability assessment of RC beams strengthened with UHPFRC

4.1. Basic concepts

The current set of recommendations for the assessment of existing concrete structures given by the ISO 13822:2010 [14], Probabilistic Model Code by the Joint Committee on Structural Safety (JCSS PMC 2010 [15]), and the ISO 2394:2010 [16] are strongly based on reliability concepts. The *fib* bulletin 80 [17] gives a codified reliability-based framework that is consistent with ISO 2394:1998 [18] for the upgrading of existing concrete structures.

Generally, reliability targets of $\beta_{up,50} = 3.3$ and $\beta_{0,50} = 2.3$ are given in *fib* bulletin 80 [17] for upgrading and assessing existing structures related to the ultimate limit states for reliability class 2 and 50 years reference period, respectively. For a reference period of 1 year, these values correspond to reliability indices of about $\beta_{up,1} = 4.3$ and $\beta_{0,1} = 3.5$, respectively.

Considering the ultimate limit state of flexure, for a RC beam strengthened with a UHPFRC tensile layer and originally designed to take an annual peak moment M_{app}, the performance function g required for reliability analysis can therefore be cast as

$$g = \theta M_c - K_E M_{app} \tag{9}$$

where θ and K_E are the uncertainties in moment capacity and applied moment respectively (see Table 1). Statistical properties of θ were assessed by comparing the present stress block approach with the test results in Figure 3.

Using similar reliability formulations as stated in [27], annual failure probability $p_{f,1}$ and reliability index β_1 can readily be computed by First Order Reliability Method (FORM) and Monte Carlo Simulation (MCS).

Parameter	Statistical model	Char	Mean	CoV	Ref.
b, mm	Gaussian	150	152.50	0.01	[19]
h , mm	Gaussian	250	258.20	0.02	[20]
$^{1)}A_s$,	Gaussian	A_s	$1.021A_s$	0.0125	[21]
E_{fs}, GPa	Lognormal	200	202	0.01	[21]
f_y, MPa	Lognormal	500	573.15	0.083	[20]
f_c, MPa	Lognormal	39.50	52.25	0.17	[22]
f_t, MPa	Lognormal	12	19.66	0.3	[22]
ρ_c, kg/m^3	Gaussian	2268	2315	0.012	[21]
$^{2)}Q_k$, kN	Gumbel		$0.682Q_k$	0.25	[23]
c', mm	Gamma	25	$26.20'$	0.03	[23]
l, m	Constant		2.2		[23]
$^{3)}t_\Gamma$, mm	Constant		t_Γ		[23]
τ_f, mm	Lognormal	4.15	4.72	0.06	[24, 25]
l_f/d_f, m	Constant		6.5		
θ, [-]	Lognormal		1.08	0.16	
K_E, [-]	Lognormal		1.01	0.1	[26]

NOTES:
1) The values of A_s are updated using the stochastic model in Figure 1.
2) Q_k is adjusted for each beam so that the strengthened beam is critical when analysed. and designed via the present stress block approach.
3) t_Γ is varied to determine its variation with β_1 (Section 4.3).

Table 1. Statistics of stochastic parameters in the load, geometric, and resistance models

4.2. Reference beam and data collection

The RC beam strengthened with a tensile UHPFRC layer to be considered in this work is based on a previous study [4]. The cross sectional dimensions of the beam were 150 mm by 250 mm, with a span of 2.2m. The RC beam was strengthened with 50mm thick layer of UHPFRC on the tensile side. The beam was reinforced with two 500MPa high strength steel rebars with a diameter of 12mm, at a cover of 25mm. The 28 days characteristic compressive strength of the beam and of the UHPFRC layer were 39.5 MPa and 164 MPa, respectively. The average tensile strength of UHPFRC was reported as 12 MPa.

To account for the uncertainty in each of the parameters of the beam describing the performance function, a stochastic description of each parameters in terms of probability distribution, mean value and coefficients of variation were defined based on previous literature (Table 1).

The RC beam strengthened with the UHPFRC tensile layer is analysed and designed with characteristic imposed load Q_k adjusted for the beam to be critical. Specifically, the characteristic imposed load needed to obtain the strengthened beam via the present stress block approach is determined, after which the corresponding mean imposed load is obtained using the assumed coefficient of variation. In addition, the dead load is considered as the self-weight of the beam with its uncertainty accounted for via uncertainties in the geometric properties of the strengthen beam and density of concrete.

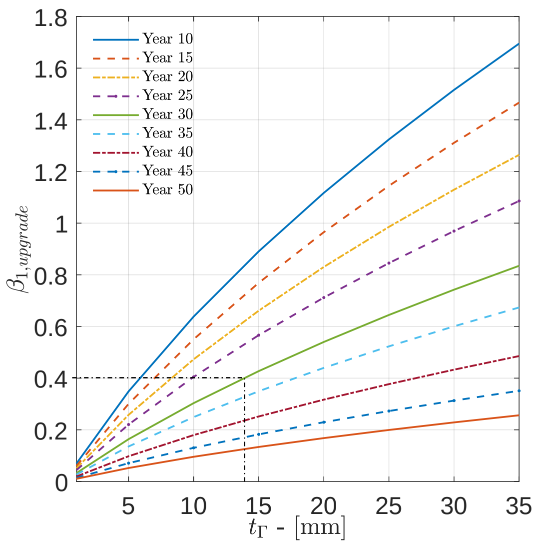

Figure 4. Required upgrading reliability increase $\beta_{1,upgrade}$ as a function of the thickness of the UHPFRC layer t_Γ and the year when the strengthening operation is performed.

4.3. Data analysis

To apply the proposed methodology for assessing and upgrading the reliability of RC beams strengthened with UHPFRC layers, lets consider a RC beam under the effect of corrosion whose annual reliability index at year n has been assessed and say deteriorated to $\beta_{up,1} = 4.3$. Techniques presented in [28] may be applied for this purpose. The evolution of the reliability

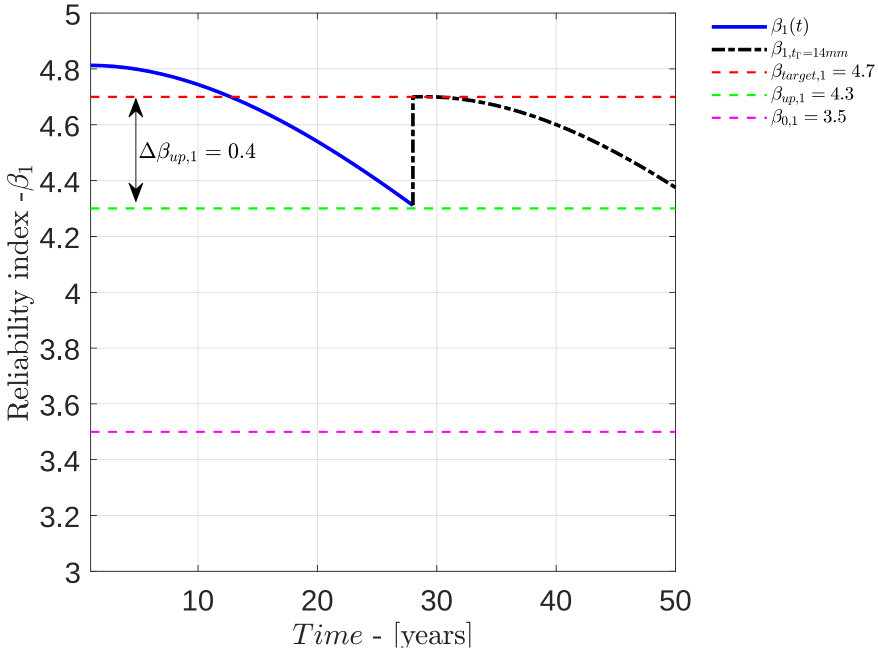

Figure 5. Evolution of the reliability index of the RC beam prior to strengthening strengthening (blue solid line) and after addition of 14mm thick UHPFRC tensile layer at 28 years old (black dotten line).

index of the RC beam prior to strengthening up to the time β_1 reaches a value of $\beta_{up,1} = 4.3$ is given by the blue curve in Figure 4.

Now, regarding the requirement of upgrading this RC beam back to $\beta_{target,1} = 4.7$ by strengthening with UHPFRC tensile layers, the thickness of the UHPFRC layer t_Γ that guarantees a reliability upgrade ($\beta_{1,upgrade} = \beta_{target,1} - \beta_{up,1} = 4.7 - 4.3 = 0.4$) needs to be determined. By computing the increase in reliability index for increasing t_Γ of the RC beam considered to be upgraded at year n, Figure 4 was obtained and may be used for estimating t_Γ given the required increment in the reliability ($\Delta\beta$). Assuming continuation of the corrosion process, the reduction of the reliability index after the strengthening operation can be computed to estimate the remaining service life of the beam (shown by a dotted black curve in Figure 5).

To decipher the relative importance of the stochastic basic variables on the computed reliability indices, a FORM-based sensitivity analysis is conducted (see Figure 6). The live load Q has the highest influence on the computed β values followed by the model uncertainties in the resistance and load models, respectively. As expected, the variables with the lowest variability (Table 1) have insignificant influence of the computed β values. Most importantly, the α value of model error θ serves as an indication of the required partial resistance factors for the stress-block formulation presented in this work. Detailed investigation on the statistical characteristics of θ based on a larger database remains an avenue for future studies

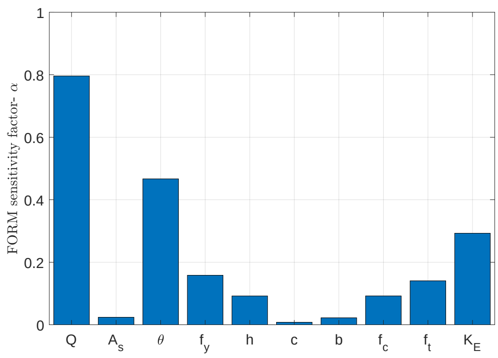

Figure 6. First order reliability method (FORM) sensitivity factors of the stochastic basic variables. Deterministic basic variables have FORM α values of zero.

5. Conclusions

A methodology for assessing and upgrading the reliability of RC beams strengthened with UHPFRC tensile layers has been presented. The applied methodology includes stochastic stress-block analysis of a section, assuming a perfect bond between the RC beam and the UHPFRC layer, and then conducting life-time reliability of the RC beam before and after the strengthening operation. Deterioration induced by corrosion can be incorporated into the analysis via stochastic corrosion models. A plot for determining the required thickness of the UHPFRC tensile layer to upgrade to the required reliability level is also given, considering the time from construction when the strengthening operation is conducted.

The present work has only considered the time-dependent deterioration model in terms of the corrosion of steel reinforcing area. Consideration of time-dependent behaviour of concrete, UHPFRC and steel strength, other geometric properties of concrete and the UHPFRC layer, and the time-dependent correlations in reliability analyses should be considered in future studies, with more detailed models and availability of information on the time-dependent correlation of deteriorating RC beams strengthened with UHPFRC layers.

6. List of symbols

a, depth of a rectangular stress block

A_s, area of steel rebar in tension

b, width of the beam

c, depth to the neutral axis

d, depth from extreme compressive fiber to centroid of rebar steel

E_{fs}, modulus of elasticity of fibers

f_c, compression strength of UHPC

f_t, the tensile stress of UHPC

f_y, yield strength of steel rebar

f_{st}, stress in steel rebar

ρ_c, density of concrete

h, height of the beam

c' cover to steel rebar

l_f, length of fibers

d_f, diameter of the fibers

ρ_s, rebar percentage

σ_{fy}, fiber yielding stress

τ_f, fiber-concrete bond strength

β_1, stress block parameter

ρ_c, density of UHPFRC

ρ_{st}, density of steel

V_f, percentage fibre content

t_Γ, thickness of the UHPFRC layer.

Acknowledgements

Reliability analysis computations were performed using the UQLab reliability analysis library [29].

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