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COMPARISON OF FAILURE CRITERIA FOR WOOD IN TENSILE-SHEAR STRESS STATE

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Abstract. An orthotropic failure criterion enhancing the Lourenco's criterion by a shear strength multiplier and a maximum shear strength upper bound has been recently proposed and validated for timber under tensile and shear loading by the authors. The paper discusses its applicability for predicting strength in comparison with Tsai-Hill criterion, Hankinson's and Hyperbolic formula applying the two above mentioned enhancements of the Lourenco's criterion. Experimental data available in the literature for off-axis tensile and shear test of Sitka spruce (*Picea sitchensis Carr.*), Katsura (*Cercidiphyllurn japonicurn Sieb. and Zucc.*), Douglas fir (*Pseudotsuga menziesii*), Douglas fir laminated veneer and Cupiúba (*Goupia glabra*) are used for the purpose of this study.

Keywords: Orthotropic failure criterion, timber, off-axis tensile test, shear strength upper bound.

1. Introduction

A large number of phenomenological strength criteria, that apply to the failure phenomenon in general instead of the failure mechanism itself, has been proposed for composite materials in recent decades. Similarly to composites, wood is typical for two-dimensional character and organized non-isotropic structure and as such can be categorised as orthotropic or transversely isotropic. It follows that strength theories developed for artificial composites can be used for prediction of wood strength [1, 2]. Unfortunately, for some of these theories, it may be difficult to obtain specific coefficients experimentally, let us mention the issue of the interaction term [3–5]. All strength parameters may not be viewed in a deterministic way. Stochastic analysis is an alternative approach that is on the rise. It takes into consideration, among others, correlation of strength properties that we can treat as independent random variables [2].

The off-axis test has become a widely used experiment for validation of failure criteria of non-isotropic materials. At a relatively low cost and operational requirements, it provides the strength of tensile or compressive specimens under uniaxial loading at various load-grain angles. Specimens with the angle of 0°and 90°yield tensile strength parallel and perpendicular to the grain, respectively. The off-axis tensile test has been involved in numerous studies, especially in those related to the complex problem of determining shear strength of wood [6–10] or the applicability of failure criteria for a specific wood species [11, 12].

In the present study, we compared four existing failure criteria: (i) Lourenco's criterion modified for wood by the authors [13, 14], (ii) Tsai-Hill criterion [15], (iii) Hankinson's formula [16] and (iv) Hyperbolic formula [17]. Furthermore, we applied some of the modifications of the Lourenco's criterion to the other criteria. We made the comparison against the data available in the literature for Sitka spruce, Katsura [7], Douglas fir [18], Douglas fir laminated veneer [19] and Cupiúba [20].

2. Failure criteria

In this section, we give a brief overview of four failure criteria that are involved in this study. The formulas are expressed in terms of either a 2D stress state or the off-axis test using the following relations:

$$\sigma_x = \sigma_0 m^2 \tag{1}$$

$$\sigma_y = \sigma_0 n^2 \tag{2}$$

$$\tau_{xy} = -\sigma_0 nm \tag{3}$$

2.1. Lourenco's criterion modified for timber

Failure criterion that has been recently proposed by the authors [13, 14] is defined by means of Rankine failure surface modified for orthotropy [21]. It is further enhanced by (i) parameter p_s that multiplies shear strength f_s in the failure surface formula, (ii) shear strength as an upper bound of the shear stress and (iii) crack type criterion. The first two enhancements enable a calibration of the failure surface through off-axis test results. Crack type criterion defining whether a crack is initiated either along or across the grain is out of the scope of this work. Failure surface and failure criterion are expressed in a 2D stress space by Equation (4) and (5), respectively.

$$\tau_{xy,fs} = \begin{cases} \frac{p_s f_s}{f_{tx} f_{ty}} \sqrt{f_{tx} f_{ty} (\sigma_x - f_{tx})(\sigma_y - f_{ty})} \\ \quad \text{for } \tau_{xy} < f_s \\ f_s \\ \quad \text{for } \tau_{xy} \geq f_s \end{cases} \tag{4}$$

$$\tau_{xy} = \tau_{xy,fs} \tag{5}$$

2.2. Tsai-Hill criterion

The most frequently used failure theory for wood was proposed by Norris [15]. It is also referred to as the Tsai-Hill criterion. It is an extension of the von Mises-Hencky distortion energy hypothesis. In a 2D stress state, it has the form of Equation (6).

$$\left(\frac{\sigma_x}{f_{tx}}\right)^2 - \frac{\sigma_x \sigma_y}{f_{tx} f_{ty}} + \left(\frac{\sigma_y}{f_{ty}}\right)^2 + \left(\frac{\tau_{xy}}{f_s}\right)^2 = 1 \tag{6}$$

2.3. Hankinson's formula

This formula [16] has been used mostly for predicting the ultimate compressive strength of wood. Some researchers have applied it also for predicting tensile strength [22]. The formula is as follows:

$$\sigma_0 = \frac{f_{tx} f_{ty}}{f_{tx} n^h + f_{ty} m^h}. \tag{7}$$

2.4. Hyperbolic formula

The hyperbolic formula has been developed to provide a better fit to Douglas Fir off-axis test results compared to other commonly used failure theories [17]. The definition is following:

$$\sigma_0 = \frac{2 f_{tx} f_{ty}}{exp^{0.01\theta}(f_{tx} + f_{ty}) + exp^{-0.01\theta}(f_{ty} - f_{tx})}. \tag{8}$$

3. Results and discussion

Using the Equations 1 - 3, the failure criteria are expressed in terms of the off-axis tensile test, i.e. $\sigma_0(\theta)$, and compared with the experimental data from the literature for off-axis tensile and shear tests. For the purpose of calibration, two enhancements of the Lourenco's failure criterion proposed by the authors (Section 2.1) are applied to the other failure conditions, if possible. These are the use of:

(1.) a shear strength multiplier p_s in the failure surface formula ($p_s f_s$ instead of f_s),

(2.) shear strength as an upper bound of shear stress in a 2D stress space.

3.1. Sitka spruce and Katsura

Yoshihara and Ohta conducted off-axis tensile and shear test for Sitka spruce (*Picea sitchensis Carr.*) and Katsura (*Cercidiphyllurn japonicurn Sieb. and Zucc.*) [7]. Specimens were conditioned at 65% relative humidity. For the first experiment type, dog-bone specimens with the outer dimensions of 140×10×8 mm were cut, five pieces for each grain angle out of 0°, 5°, 10°, 15°, 20°, 25°, 30°, 45°, 60°, 75° and 90°. Shear strength was obtained from both off-axis tensile test (by transformation of the axial strength to the stress components of orthotropic symmetry) and torsion test. For the latter, ten dog-bone specimens with the outer dimensions of 180×20×5 mm were prepared. In this experimental campaign, shear strength from torsion test coincided well with that from the off-axis test for 15°– 30°.

Off-axis tensile test and shear test results for Sitka spruce together with the uncalibrated failure curves ($p_s = 1.0$, $h = 1.0$) are shown in the upper part of Figure 1 in terms of off-axis tensile test variables. We can see that out of the uncalibrated curves, the Tsai-Hill curve represents a good estimate. Calibrating the modified Lourenco's, Tsai-Hill and Hankinson's curves by the parameters of $p_s = 2.0$, $p_s = 1.2$ and $h = 1.8$, respectively, and activating the shear strength upper bound, we obtain the best fit of the average experimental data, see the lower part of Figure 1. On the contrary, the Hyperbolic curve does not reproduce the data well.

Experimental data for Katsura are plotted together with uncalibrated and calibrated failure curves in the upper and lower part of Figure 2, respectively. Similarly to the results for Sitka spruce, Tsai-Hill curve is the best estimate out the uncalibrated curves. Nevertheless, by the calibration of the modified Lourenco's, Tsai-Hill and Hankinson's curves by the parameters of $p_s = 2.3$, $p_s = 1.3$ and $h = 2.2$, respectively, and by application of the shear strength limit to the shear stress, we get a good prediction of the data. The Hyperbolic curve strongly underestimates the data.

3.2. Douglas Fir

Woodward and Minor measured tensile strength parallel and perpendicular to fibers and shear strength for Douglas Fir (*Pseudotsuga menziesii*) [18] according to the specimen configuration and testing procedure specified by ASTM D-143 [23]. They used eight specimens for each type of experiment. For determination of the ultimate strength at grain angles of 15°, 30°, 45° and 60°, they used eight rectangular specimens for each grain angle. The specimens were 38 mm wide, 10 mm thick and they were of different length, excluding the gripping area: 480 mm, 330 mm, 250 mm and 200 mm for the grain angles of 15°, 30°, 45° and 60°, respectively. The specimens were dried to the moisture content of $12 \pm 1\%$.

Uncalibrated and calibrated failure curves are shown in the upper and lower part of Figure 3, respectively, together with the experimental results for Douglas fir. Both the modified Lourenco's and Hyperbolic criterion fit the average off-axis tensile test data well without any calibration. On the other hand, we can reproduce the data even better applying $p_s = 0.8$

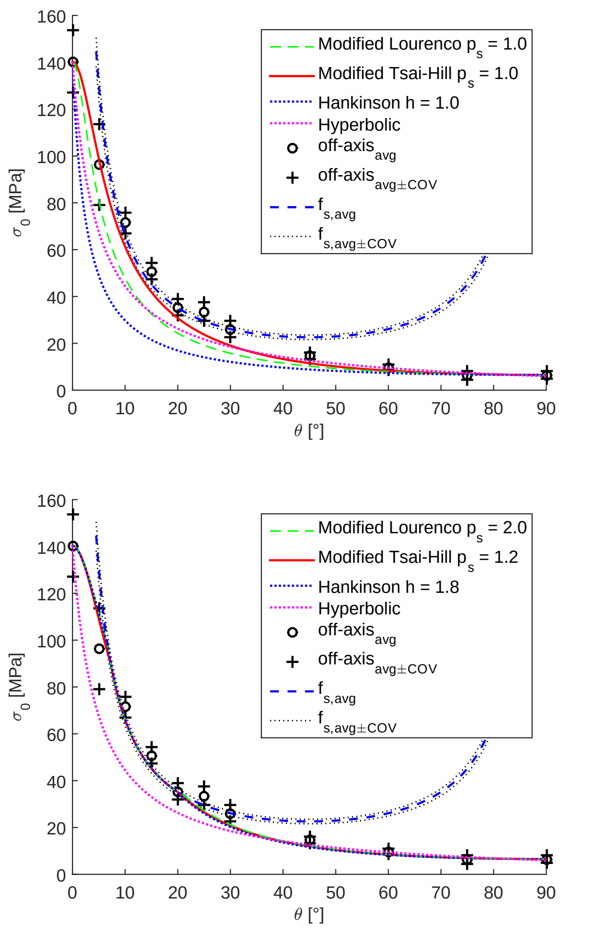

Figure 1. The experimental data of Sitka spruce [7] with uncalibrated (top) and calibrated (bottom) failure conditions.

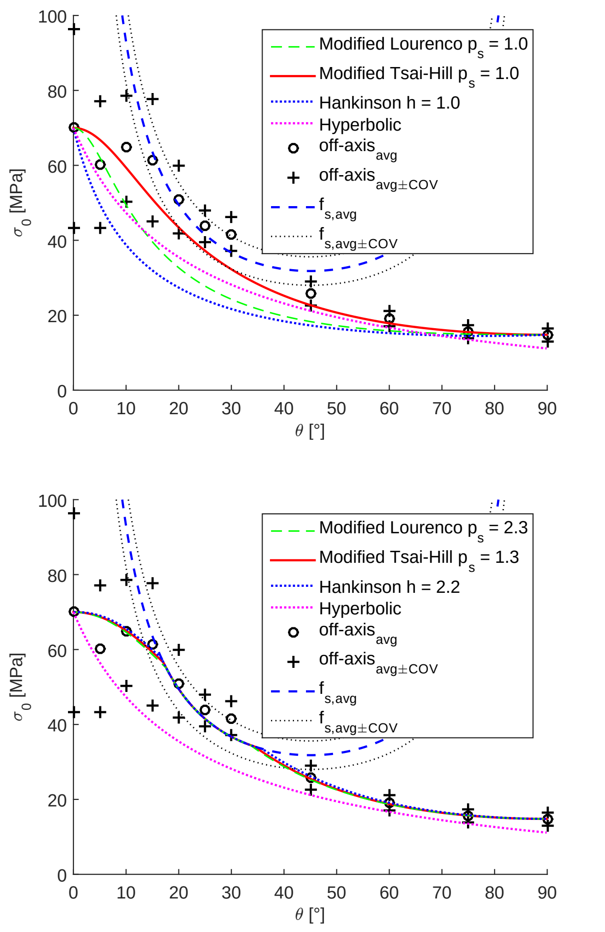

Figure 2. The experimental data of Katsura [7] with uncalibrated (top) and calibrated (bottom) failure conditions.

and $h = 1.5$ to the Tsai-Hill and Hankinson's formulas. Let us note that for this wood species, the shear strength upper bound is not activated. In this study, we disregard size effect following conclusions of a related experimental work [24].

3.3. Douglas fir laminated veneer

Clouston et al. performed off-axis tensile test and shear test for Douglas fir laminated veneer [19]. Off-axis specimens of equal size (610×63×35 mm) were cut with grain angles of 0°, 15°, 30°, 45°, 60° and 90°, sixteen to eighteen pieces for each angle. Shear strength was evaluated on nineteen standard ASTM shear block specimens using shear adjustment factor [25]. Average moisture content of the specimens was 7.9%. Looking at uncalibrated failure curves and experimental data in the upper part of Figure 4, we can see that the modified Lourenco's curve represents the best estimate. We can get similar results if we calibrate the Tsai-Hill's and Hankinson's criteria by $p_s = 0.65$ and $h = 1.8$, respectively, see the lower part of Figure 4. We can notice that the shear strength upper bound is not activated.

3.4. Cupiúba

Todeschini conducted off-axis tensile and shear test for Cupiúba (*Goupia glabra*) [20] in accordance with the Brasilian norm NBR 7190 [26]. For the purpose of the off-axis test, dog-bone specimens with the outer dimensions of 280×20×20 mm were cut, twelve pieces for each grain angle out of 0°, 15°, 30°, 45°, 60°, 75° and 90°. Shear strength was measured using shear block specimens. The moisture content of the specimens varied from 12% to 14%.

Plotting the uncalibrated curves together with the experimental results of Cupiúba (the upper part of Figure 5), we can see that the Hyperbolic curve yields the closest estimate of the average off-axis data. The lower part of Figure 5 shows that we can reproduce well the data utilizing $p_s = 0.6$, $p_s = 0.4$ and $h = 1.7$ for the modified Lourenco's, Tsai-Hill and the Hankinson's criteria, respectively. Similarly to the case of Douglas fir and Douglas fir laminated veneer, the shear strength upper bound is not activated.

4. Conclusion

The paper compares four orthotropic failure criteria against off-axis and shear experimental results avail-

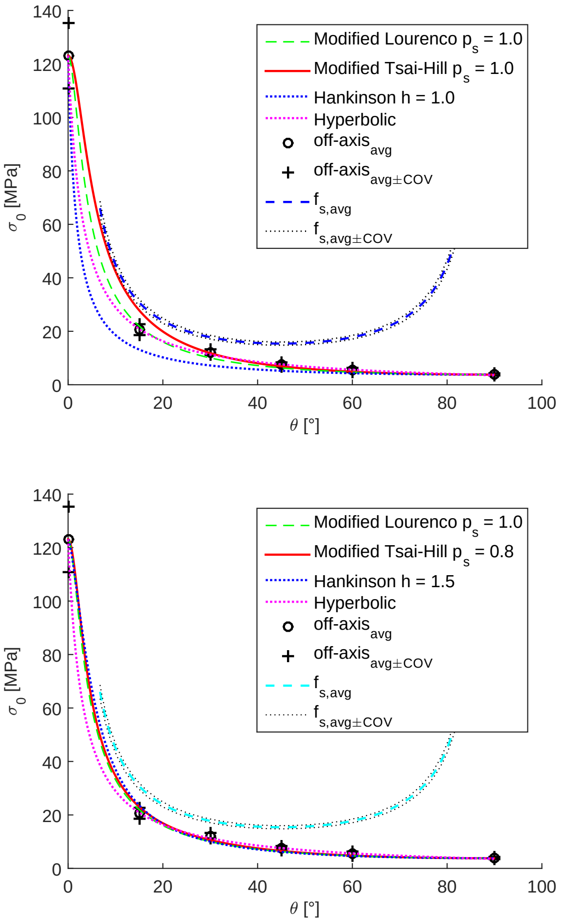

FIGURE 3. The experimental data of Douglas fir [18] with uncalibrated (top) and calibrated (bottom) failure conditions.

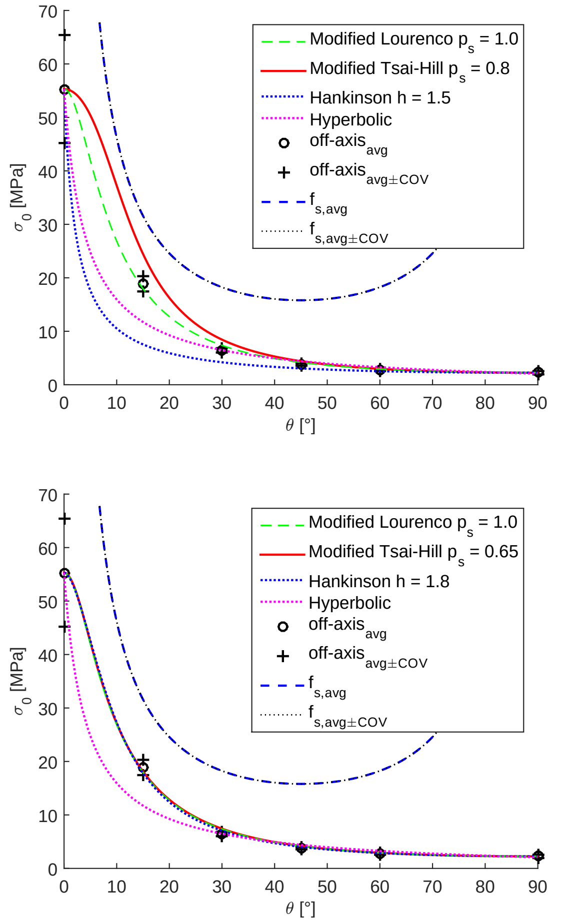

FIGURE 4. The experimental data of Douglas fir laminated veneer [19] with uncalibrated (top) and calibrated (bottom) failure conditions.

able in the literature for the wood species of Sitka spruce, Katsura, Douglas fir, Douglas fir laminated veneer and Cupiúba. The enhancements of the modified Lourenco's criterion, which has been recently proposed and validated by the authors, are applied to the Tsai-Hill, Hankinson's and Hyperbolic formula, to a certain extent. Specifically, (i) the shear strength multiplier and (ii) the shear strength upper bound are used. Following conclusions can be drawn:

(1.) The shear strength multiplier enables calibration of a failure formula that contains shear strength. In this way, both, the modified Lourenco's and Tsai-Hill criteria provide a better estimation of the average off-axis data for Sitka spruce, Katsura, Douglas fir, Douglas fir laminated veneer and Cupiúba.

(2.) The shear strength upper bound, that can be activated as a maximum limit of the shear stress component, provide a better fit for average off-axis data of Sitka spruce and Katsura. For these wood species, average shear strength measured by torsion test coincides well with the average off-axis strength.

(3.) Calibration of the Hankinson's formula by the parameter h yields good estimates for all wood species involved in this study, similarly as the modified Lourenco's and Tsai-Hill criteria do. In contrast, the Hyperbolic formula predicts well only Douglas fir data as it does not contain any calibration or shear strength parameter.

LIST OF SYMBOLS

θ load-grain angle [°]

f_{tx} tensile strength parallel with grain [MPa]

f_{ty} tensile strength perpendicular to grain [MPa]

f_s shear strength [MPa]

h parameter of the Hankinson's criterion [–]

m $\cos\theta$ [–]

n $\sin\theta$ [–]

p_s shear strength multiplier [–]

σ_0 ultimate strength at a load-grain angle θ [MPa]

σ_x normal stress parallel with grain [MPa]

σ_y normal stress perpendicular to grain [MPa]

τ_{xy} shear stress [MPa]

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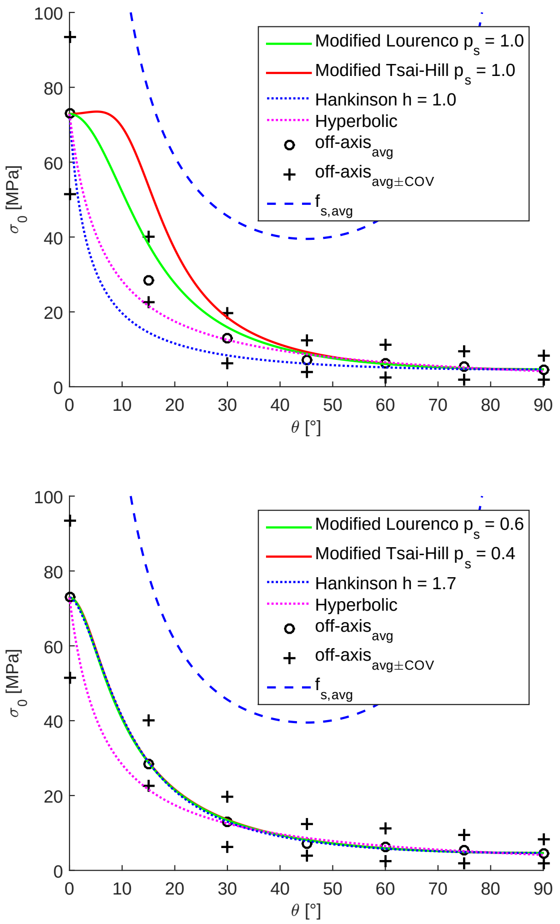

Figure 5. The experimental data of Cupiúba [20] with uncalibrated (top) and calibrated (bottom) failure conditions.

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