THESIS REVIEWER'S REPORT

I. IDENTIFICATION DATA

Thesis title:	**Calculating Lyapunov Exponents fromTime Series**
Author's name:	**Lejla Dobrić**
Type of thesis :	bachelor
Faculty/Institute:	Faculty of Electrical Engineering (FEE)
Department:	Department of Cybernetics
Thesis reviewer:	Shekhovtsov Oleksandr Mgr.,Ph.D.
Reviewer's department:	Department of Cybernetics

II. EVALUATION OF INDIVIDUAL CRITERIA

Assignment — **ordinarily challenging**

How demanding was the assigned project?

The assignment requires to implement and compare several methods on a small synthetic data. Implementing Rosenstein's or Wolf's methods appear to have complexity of an ordinary lab assignment.

Fulfilment of assignment — **fulfilled with minor objections**

How well does the thesis fulfil the assigned task? Have the primary goals been achieved? Which assigned tasks have been incompletely covered, and which parts of the thesis are overextended? Justify your answer.

The assignment is open regarding how many methods should be compared, but from the list of the recommended literature only two methods were described and implemented. Other methods may be more challenging.

Methodology — **correct**

Comment on the correctness of the approach and/or the solution methods.

It is left out when the limit defining the Lyapunov spectrum exists and is independent of the starting point.
An infinitesimal ball in the limit of infinite time is a double limit which is not formalized in the definition (1).
Estimates from randomized experiments (e.g. noisy data) should be reported with a standard deviation with respect to the noise.

Technical level — **D - satisfactory.**

Is the thesis technically sound? How well did the student employ expertise in the field of his/her field of study? Does the student explain clearly what he/she has done?

The thesis gives a basic introduction to chaotic dynamical systems and Lyapunov exponents. The presentation of the estimation methods, especially Wolf's method, is very unclear (e.g. not defined what is a feducial point, an evolved and the replacement elements, how the length L(t_0) depending on t and t_0 should progress with time, etc.). The time delay method is mentioned in both algorithms but not explained (in the experiments I believe the whole phase space is observed, making it irrelevant). The experiments are described rather well except of unclear notation (e.g. what is s in t(s)?).

Formal and language level, scope of thesis — **C - good.**

Are formalisms and notations used properly? Is the thesis organized in a logical way? Is the thesis sufficiently extensive? Is the thesis well-presented? Is the language clear and understandable? Is the English satisfactory?

The language level and the presentation with illustrations are fine. The structure has some issues.
Lyapunov exponents are not formally defined, only as a consequence of the spectrum definition given afterwards.
The synchronization is not formally defined and it is not clear why it is relevant for the thesis at all. In 3.3.1 variable X denotes both the time series and the reconstructed trajectory (from X) in the phase space. It appears that X is used in the Rosenstein's algorithm in 3.3 before it is defined in 3.3.1.
Runge-Kutta methods are formally introduced using lots of undefined notation – it would be better to avoid the formal definition altogether. Definitions of N,M,n,m are scattered throughout the paper or undefined. In (8) d_j(i) is not defined. (10) does not define a single value which should be the largest Lyapunov exponent, but a vector of values y(i).

Selection of sources, citation correctness	**B - very good.**
Does the thesis make adequate reference to earlier work on the topic? Was the selection of sources adequate? Is the student's original work clearly distinguished from earlier work in the field? Do the bibliographic citations meet the standards?	
I am not expert in this area, but the introduction and overview reads well and some further methods beyond implemented are cited. However, [13] is cited incorrectly, basically its title contradicts to what the thesis claims on p.11.	

Additional commentary and evaluation (optional)
Comment on the overall quality of the thesis, its novelty and its impact on the field, its strengths and weaknesses, the utility of the solution that is presented, the theoretical/formal level, the student's skillfulness, etc.
I am missing an introduction regarding practical applications: in which practical scenarios it is needed to estimate Lyapunov exponents from time series? I do not see that it is needed for synchronization because the presence or absence of synchronization can be measured more directly. Such discussion can perhaps lead to a selection and development of specialized methods and increase the utility and impact.

III. OVERALL EVALUATION, QUESTIONS FOR THE PRESENTATION AND DEFENSE OF THE THESIS, SUGGESTED GRADE

1. In Table 3.1, a negative spectrum in 3 dimensions is called a "fixed point" but in 1 dimension a "periodic orbit". Is that right?

2. Does a positive Lyapunov exponent imply that the ellipsoid in Fig.3.2 will ever grow and the trajectory will be unbounded in the phase space? If yes, then why the Lorenz attractor is chaotic?

The grade that I award for the thesis is **C - good.**

Date: **24.8.2022** Signature: