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DIPLOMA THESIS ASSIGNMENT FORM

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Study branch/spec.: Advanced Masters in Structural Analysis of Monuments and Historical Constructions

II. DIPLOMA THESIS DATA

Diploma Thesis (DT) title: Numerická a experimentální analýza kostelní věže sv. Jakuba v Kutné Hoře

Diploma Thesis title in English: Numerical and experimental analysis of the Bell Tower of St. Jacob's Church in Kutna Hora Town

Instructions for writing the thesis:
To explore of the Bell Tower of St. Jacob's Church in Kutna Hora focusing on the historical background, past restorations and current state of its structural system. The study will continue to take photographs and take part in a dynamic experiment. After that a simplified computational finite element model of the tructure. It will be fitted on the known experimental results of the first three frequencies of natural vibration of the bell tower. Finally, the experimentally estimated characteristics of the bell tower will be compared with the corresponding theoretical ones.

List of recommended literature:

Name of Diploma Thesis Supervisor:

DT assignment date: 5.4.2022

DT submission date in IS KOS: 6.7.2022
see the schedule of the current acad. year

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III. ASSIGNMENT RECEIPT

I declare that I am obliged to write the Diploma Thesis on my own, without anyone's assistance, except for provided consultations. The list of references, other sources and consultants' names must be stated in the Diploma Thesis and in referencing I must abide by the CTU methodological manual "How to Write University Final Theses" and the CTU methodological instruction "On the Observation of Ethical Principles in the Preparation of University Final Theses".

01 04 2022
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DECLARATION

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Title of the Msc Dissertation:	Numerical and Experimental Analysis of the Bell Tower of St. Jacob's Church in Kutna Hora Town
Supervisor(s):	Petr Fajman
Year:	2021/2022

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

I hereby declare that the MSc Consortium responsible for the Advanced Masters in Structural Analysis of Monuments and Historical Constructions is allowed to store and make available electronically the present MSc Dissertation.

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Date:	06-07-2022
Signature:	

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Abstract

Dynamic analysis techniques are a way of identifying the global behaviour of a structure. It is beneficial in heritage engineering works because the testing causes no damage to the building and can indicate a lot about its current properties, as well as potential concerns for damage. Dynamic testing results can also be used to calibrate FEM models to be used as an engineering tool in design and analysis.

The objectives of this paper are to use dynamic testing and analysis to determine the safety of the bell tower in the St. James Church in Kutná Hora, Czech Republic. All numerical results are compared with experimental values to verify acceptability.

First, dynamic testing is completed to get experimental results of the fundamental frequencies of the tower. These are then used to develop a numerical model. The model is then used to simulate the ringing of the bell to check for signs of resonance, such as large displacements. Conclusions are made about the safety of continued bell use and the viability of the results.

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Abstrakt

Numerická a experimentální analýza kostelní věže sv. Jakuba v Kutné Hoře

Dynamická analýza nám pomůže identifikovat parametry, které jsou potřeba pro zjištění globálního chování konstrukce. To je zvláště cenné při výpočtu historických konstrukcí, kde není možné odebrat vzorky pro destruktivní testy. Pomocí výsledků z dynamických experimentů můžeme kalibrovat FEM model a ten pak použít pro další účely.

Cíle disertační práce jsou využít dynamické testování k zjištění stavu zvonové věže v kostelu sv. Jiří v Kutné Hoře. Všechny výsledky získané numerickým modelem jsou porovnány s výsledky získanými experimentálním měřením.

Setování okrajových podmínek a materiálových vlastností numerického modelu bylo provedeno z vlastních frekvencí, získaných při měření. Po spočtení sil od zvonění se provedl výpočet vynuceného kmitání. Získané výsledky posunů byly v souladu s experimentem. Zároven lze říci, že zvonění nezpůsobuje žádné resonance a konstrukce je zezpečná.

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1 Introduction

Kostel svaty Jakub, or in English, the Church of Saint James, is located in Kutná Hora, Czech Republic and has been a local landmark for many centuries. While its significance as a church is less than that of the Sedlec Church and ossuary, and the St. Barbara Cathedral nearby, its architecture dominates the Kutná Hora skyline, as clearly seen in Figure 1. With one tall imposing tower and a highly decorated interior, it is one of the many historic structures that helped designate Kutná Hora as a UNESCO World Heritage Site. It calls out its presence by ringing its bell thrice daily, at 12:00, 15:00, and 17:00.

Figure 1: Kutná Hora skyline featuring the St. James Church Tower, which looms over the surrounding buildings. (Photo by author)

Kutná Hora is located about 70 kilometres east of Prague, in the Czech Republic. The town has played a significant part in the history of the country as a centre for mining silver and minting coinage, and more recently as an exciting tourist destination. It is home to many museums, historic churches, mines, and green spaces, making it a good place to visit for all types of tourists. The church itself is located near the centre of the historic core of the city and is a short walk from the St. Barbara Church (Chrám svaté Barbory), as seen in Figure 2.

Figure 2: Location of the Church of St. James within the town of Kutná Hora. The historic centre surrounds St. James Church, and the St. Barbara Church, also indicated, is only about five minutes away on foot. (Captured from Google Maps)

The Church of St. James has a three-nave floorplan with two towers positioned opposite the altar. One tower is significantly taller than the other, so tall in fact, it defines the architecture of the church. This tall tower was also chosen to house the bells in one of its upper floors. Originally, both towers were meant to be built to the same height, but poor ground conditions prevented this endeavour. While the tall tower is the defining feature of the church and the local skyline, the interior of the building also holds many historic decorative elements that have amassed over the centuries.

In 2017, a new bell was installed in the tall northern tower to replace the bells lost 75 years prior under unknown circumstances. Before the installation, restoration works were completed on the tower that included a preliminary analysis of the effect of the bell on the tower. Tall structures like the St. James bell tower are subject to dynamic effects. These properties can be exaggerated by objects like bells, and this can lead to damage and must therefore be considered before and after installation of bells. In this case, the pre-installation analysis was completed by Doc. Ing. Petr Fajman, CSc. and Prof. Ing. Michal Polák, CSc. and it concluded that should the bell remain within the recommended parameters; the effects of the ringing should not cause any level of significant damage. The recommended parameters include not only the size and shape of the bell, but how it rings, including things like angles of rotation and beats per minute. [1]

The purpose of this paper is to measure the current conditions of the bell and confirm this hypothesis. This will be done by completing dynamic testing and structural modelling. These two methods in combination will present a clear picture of the current state of the tower and its safety with respect to the bell vibrations.

1.1 History

Construction of the church began in 1330, marking it as one of the oldest surviving churches in Kutná Hora. Funding for the construction came from wealthy locals involved in the highly lucrative silver mining business that put Kutná Hora on the map. The church was built near the Italian Court, which served as both the mint and housing for royalty and the rich. As this church was supposed to be a show of the wealth in the area, it was designed with a sense of lavishness that can be seen throughout both the interior, and exterior of the church. The most notable symbol of this, of course, being the bell tower itself, which stands at 86 m, much taller than its surroundings, as can be seen in Figure 3. Originally, the second tower was supposed to be as imposing as the first, however due to poor soil conditions and mining in the area, the second tower was completed to the same height as the church because the ground could not support more height. [1]

Figure 3: The St. James Church tower in Kutná Hora stands much higher than all the surrounding buildings, making it a notable landmark within the town. [1]

After construction, the church was dedicated to the Virgin Mary, although thanks to the tower, it was mostly called the 'Tall' church. Unfortunately, shortly after completion, the church was severely damaged in the Hussite Wars and repairs were not started until after the Wars, about forty years later [2]. In 1424, the Utraquist priests took over management of the church from the Catholics until the end of the Thirty Years' War in 1648. It was in this time that the church became dedicated to St. James the Apostle. [1]

While small changes have been made to the church over time, like additions of Renaissance and Baroque elements, significant preservation efforts were not made until recently. The tower fell into such disrepair that there was concern for public safety, and so in 2016, reconstruction efforts were started to stabilize and strengthen the tower. These works were completed in 2017 and commemorated with the placement of a new bell and bell frame within the tower, the raising of which can be seen in Figure 4. [3]

Figure 4: The raising of the bell in 2017 was completed using traditional lifting methods and was carefully supervised by the bell maker and project engineer. [4]

1.2 Geometric Survey

St. James Church is a traditional three nave church with large columns between the naves. The altar is found at the east end of the church, while the two towers and the choir are opposite, at the west end. Figure 5 illustrates the floor plan in a simplified schematic. The interior of the church is highly ornamented with statutes and other elements dating back to construction. The structure itself is Gothic, with large, tall columns and quadripartite vaulting. There are also elements of Renaissance and Baroque architecture within. These decorative and structural elements can be seen in Figure 6 and Figure 7.

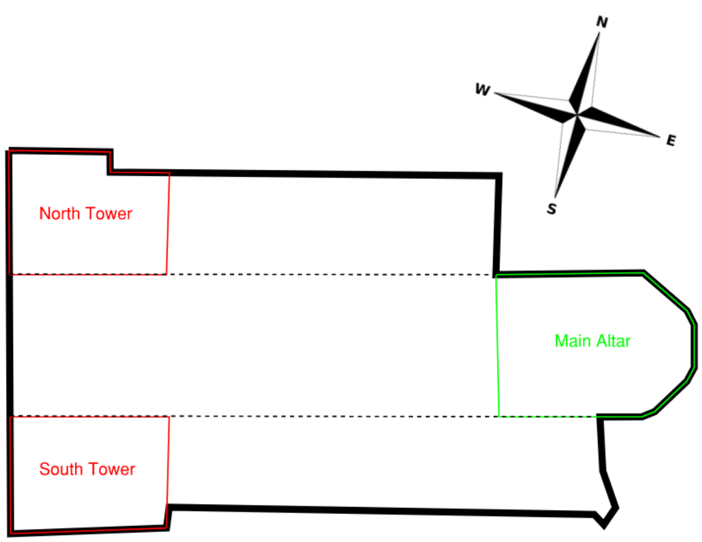


Figure 5: Floor plan of the church with the north and south towers indicated in red, and the main altar indicated in green. Dashed lines through the church floor plan indicate the separation of the naves.

Figure 6: Facing east to the altar. The ornamentation in the altar and surroundings can be seen, as well as the tall gothic vaulting. (Image captured from Google Street View)

Figure 7: Facing west towards the towers. While the towers cannot be seen, the large columns, as well as the choir and supporting arch are visible. Additionally, there are more Baroque and Renaissance decorative elements to be seen. (Image captured from Google Street View)

In the roofing above the vaults, there are two small masonry walls above the column lines. Above them, a large roofing truss system was built to make the peaked roof. An extensive walkway system was also built for ease of access to the different parts of the upper structure. All of these elements are visible in Figure 8.

Figure 8: Upper roof structure of the church. The small masonry wall is found on the left side of the image. The large truss is part of the roof system and contains old and new elements. It also supports the walkway, as seen on the right side of the image. (Photo by author)

For the purposes of this study, a further in-depth inspection was conducted for the towers. Structural elements, openings, materials, and conditions were all reviewed for their potential impact on the structural model. Inside the northern tall tower, the first set of stairs are stone, up to the level of the church roof, with one exit on route, into an intermediate floor, currently used as a storage space. After that, a timber stair system has been designed to allow passage to the top of the tower. In fact, most of the interior structure on the tower is timber, leading to flexible floors and minimal rigidity impact on the structure by the flooring. The newer Baroque style roof on the tower is also made of a complex timber frame system, which is pictured in Figure 9. Ties of varying age are located within the tower connecting the external walls together to create a system that moves together, instead of as separate walls. The older ties are historically significant as they are Renaissance era wrought iron ties, which have now been reinforced with modern steel rods [5].

Figure 9: Looking up at the Baroque timber roof truss. The complexity of the truss, with all its diagonal members is a sight to behold. Many of the members have been replaced or reinforced with new timber. (Photo by author)

The walls themselves are a mixed-stone three-leaf masonry system, about a metre thick. In some areas the walls appear to have cut blocks on both sides, whereas in others, the interior stone layout is more sporadic. What should be noted is that the entire tower seems to have well-maintained mortar. There are no gaps or areas of missing mortar on the tower that would indicate weakness, damage, or deterioration. In the most recent renovations, it was understood that a re-pointing and capping technique was used, which would explain the excellent state of the mortar. This means that compatible mortar was applied to the exterior jointing and the top of the walls, covering any defects or loss of material that may have happened with time. The capping is illustrated in Figure 10. In addition, some stitching work was completed in the upper levels, which means that extra material was added into the mortar joints to connect the masonry pieces and create a more stable and cohesive system [5]. However, neither of these techniques ensure a lack of voids in the interior leaf of the wall, such that the overall wall strength may be less than a solid, consolidated wall. Also, there is a higher quality of masonry found towards the top of the tower as opposed to the bottom of the tower. This can be clearly seen by looking at the stones themselves and their mortar joints. Higher quality masonry has larger, more regular stones with even, thin joints. While there are elements of the higher quality masonry in the lower part of the tower, it is not consistent enough to be considered the same as the top of the tower.

Figure 10: Concrete overlay ‘capping’ on top of the masonry walls to connect the wall leaves together. Metal and timber components are also connected through the capping process. (Photo by author)

In the upper level of the northern tower is the bell, situated in an independent frame that is shown in Figure 11. The frame sits within the existing truss of the tower and takes up the entire floor. The current bell was crafted by Petr Rudolf Manoušek, a famed bell-maker. Casting occurred in the Netherlands before transportation to St. James, where it was blessed before installation. It was lifted into place using historical man-powered winching methods to preserve authenticity. [4] The bell now runs on an electronic motor that causes it to ring on a schedule or can be triggered reomtely. There is also an electronic bell speaker in the tower to signal different times and reduce the usage of the new bell.

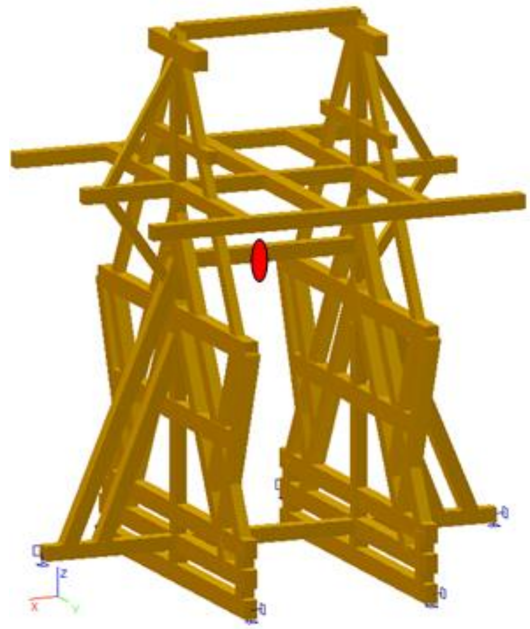


Figure 11: Bell frame geometry where the bell hangs from the red dot.

The shorter tower was created with the intention of being as tall as its pair, but due to instability in the soil structure, it was never fully finished and is only slightly taller than the church itself, with the roof level being very near that of the rest of the structure. Access to it is only allowed through other areas of the church, such as the roof level or main floor, but there are no stairs that ascend the tower like in its taller pair.

As was discovered during construction, the soil conditions under the church are highly variable. Under the entire city, there has been extensive mining works, leaving much of the ground weakened and dangerous for large constructions. Additionally, the church is located near an elevation drop off to the south, which may also contribute to the weakening of the southern foundations. Therefore, the church is assumed to be on a severe gradient of soil strength decreasing towards the south.

1.3 Objectives

The purpose of this paper is to follow up on the work of Fajman (2016) [6] and confirm that the new bell does not cause damage to the tower. The first step of this is completing new dynamic testing of the tower. This will provide natural frequency values that can be compared to pre-installation values and be used as a calibration reference for the structural model. The second part of the analysis will be done by creating a structural model in Dlubal RFEM 5 software. This model will be subjected to the forced vibration of the bell and then the results will be checked for resonance, the main type of damage a bell can cause. In the end, it will be determined if the continued use of the bell will be safe for the church and the community.

2 Literature Review

2.1 Dynamic Analysis

One of the most basic and yet helpful analyses that can be done to a tower is the analysis of its fundamental frequencies. Due to the scientific concept that all matter is in constant motion, it can also be deduced that all structures are constantly in motion. They move at their own rate, and this is called a fundamental frequency. These frequencies are a description of the vibrations of a structure and can be determined from solving the basic equation of motion, Equation 1, under conditions that are unique to the structure. The basic analysis considers a system with of only one degree of freedom, that is to say, an object with only one way it can move. In this case, the equation can be solved analytically. However, in a structure, there can be nearly infinite degrees of freedom, typically called N number of degrees of freedom, or it can be called a multiple degree of freedom (MDOF) system. When this occurs the equation of motion becomes a series of N equations to be solved using numerical methods.

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

Equation 1: General equation of motion

In the equation of motion, p(t) is the excitation function, otherwise known as the input function. When the excitation function is equal to zero, the system is in free vibration. When the damping matrix, c, is also neglected, the equations can be solved for the fundamental frequencies of a structure. These frequencies show the natural motions of the structure and can be very important for understanding induced vibrations on a structure.

When p(t) is not zero, the system is under forced vibrations. Most structures are designed to withstand vibrations in both the design and serviceability levels, however there is an exception. When the induced vibrations approach the fundamental frequency of a structure, something occurs called resonance. In physics, wave theory teaches that when two different waves meet, like the induced force and the natural vibrations, the amplitudes of the waves will be added together. This is called superposition. Therefore, when the frequencies of the structure and the induced force are different, the amplitudes cancel because the peaks and zeroes of the waves do not align, causing decreased and more irregular amplitudes, as seen in Figure 12. On the other hand, when the frequencies match, the amplitudes grow significantly, such as in Figure 13, and can start to cause unprecedented damage and movement in structures.

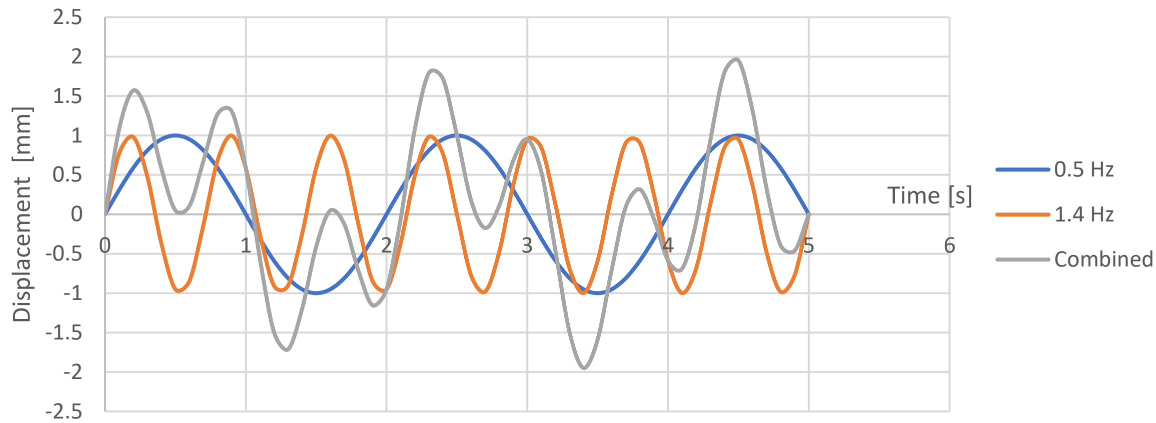


Figure 12: Superposition of two waves with dissimilar frequencies. Note that the maximum amplitude of the combined wave is always less than 2 mm and it not regularly reaching a constant peak. This is ideal.

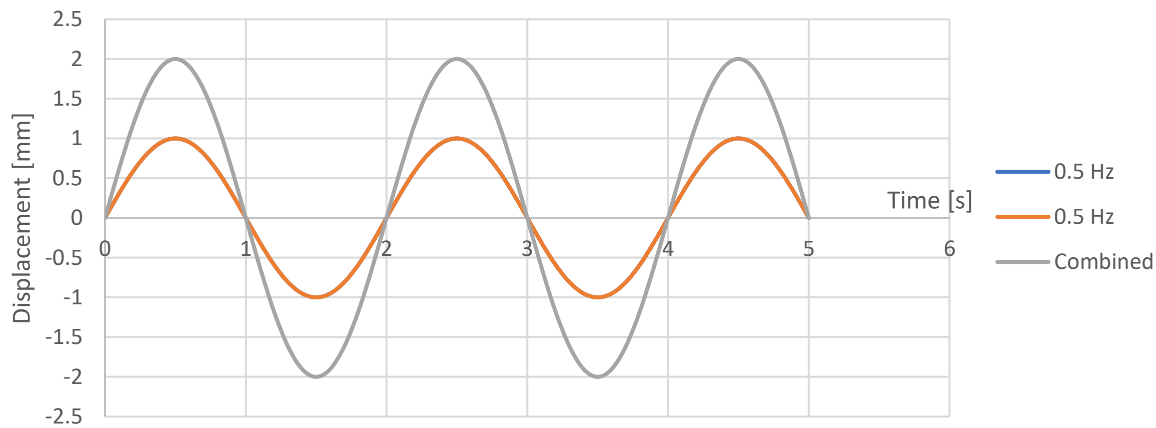


Figure 13: Superposition of two waves with matching frequencies. A full maximum of 2 mm is regularly reached. This is resonance and the constant high peaks can lead to damage.

The motion of a structure depends on its mass, stiffness, and the vibrational damping the building. Such that, the fundamental frequencies are also dependent on this. More specifically, frequency can be defined in proportion to stiffness and mass through the relationship in Equation 2.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Equation 2: Relationship between frequency, mass, and stiffness

Mass and stiffness are two of the more easily determinable properties as they rely directly on the materials within the structure, however, in the heritage field these values can only be found through testing and are often inconsistent within the structure, so modal analysis is typically used in conjunction with material testing to achieve an understanding of the global stiffness of the structure. In contrast, the mass is often more easily estimated as determining the specific weight, or mass per volume, is much simpler and can often be generalized by type of material. On the other hand, stiffness depends on materials and construction methods, and can vary dramatically with damage, deterioration, time, and many other factors.

Modal analysis has its limitations, and while it can provide some fundamental data about a structure, it does not consider forced vibrations or damping effects. To account for these, a time history analysis is typically used. This will apply an excitation function and monitor the structure over time, measuring outputs due to the vibration. Time history analyses can be done with linear and non-linear systems, and can show damage, displacements, and even resonance.

One of the important differences between a modal analysis and a time history analysis is the inclusion of damping. Damping is the inherent capability of a structure to take a vibrational energy input and convert it into another form of energy to reduce the force of the vibrations over time. There are many different approaches to damping in the field of civil engineering, but the most used is classical damping. Classical damping allows for the assumption that the natural fundamental frequencies are the same as the damped fundamental frequencies. This is a good simplification for analysis. However, this type of damping can only be used when the damping is expected to be uniform throughout the system, including through the foundations. Fortunately, classical damping is assumed for St. James tower.

A common form of classical damping is the Rayleigh's Method. This method uses a linear combination of the mass and stiffness matrices in an MDOF system to create the damping matrix, as show in Equation 3.

$$\xi = \alpha \boldsymbol{M} + \beta \boldsymbol{K}$$

Equation 3: Linear damping relationship for Rayleigh's Method

To do this, it is important to determine the coefficients for the linear combination, α and β. It also requires assuming an original damping level, ξ. Since damping is not accounted for in modal analysis, the coefficients can be found using two of the fundamental frequencies. Typically, the first, and one of the significant later frequencies are used for a more stable result. The coefficients are calculated using the angular frequencies, which can be found simply with $\omega=2\pi f$, and these can be directly used in analysis software. Equation 4 and Equation 5 show how to find these values.

$$\alpha = \xi \frac{2\omega_i \omega_j}{\omega_i + \omega_j}$$

Equation 4: α - Rayleigh's coefficient

$$\beta = \xi \frac{2}{\omega_i + \omega_j}$$

Equation 5: β - Rayleigh's coefficient

2.2 Dynamic Testing

There are two types of dynamic testing widely used: deterministic and stochastic. Deterministic dynamic testing requires knowledge of the input excitation, which is applicable in laboratory situations, but for projects like the St. James Church tower, the stochastic method is preferred. Stochastic input means that the dynamic measuring is determined by the ambient vibrations, like wind, human activity, and other day-to-day influences.

To complete a dynamic test on a structure, accelerometers are connected at strategic points on the building. These are, in turn, connected to a Data Acquisition System (DAQ) that reads the information from the accelerometers and saves it to be interpreted later. The DAQ can also provide on-site data processing. Given the nature of technology and the randomness of the input parameters, it is prudent to correctly process the data to get workable results. Signal processing involves applying different processes to remove and clean up superfluous data that could interfere with understanding of the true output. This can be done using different techniques like filtering, which eliminates frequencies which are outside of an expected range, and decimation, which decreases the sampling rate of a test. Another process that the DAQ will often do is take the time-based input and turn it into a frequency-based reading. This is done using a process called a Fourier transform. This transformation is critical to the testing process as the desired results are often in the frequency domain, such as the fundamental frequencies of the structure, but measurements occur in the time domain. Once in the frequency domain, peaks at certain frequencies form. These peaks represent the fundamental frequencies and are selected using a method called Peak Picking, as seen in Figure 14. The process can be done by hand or by the DAQ software, but it is important to note that there is an element of subjectivity to the process and an experienced user is required.

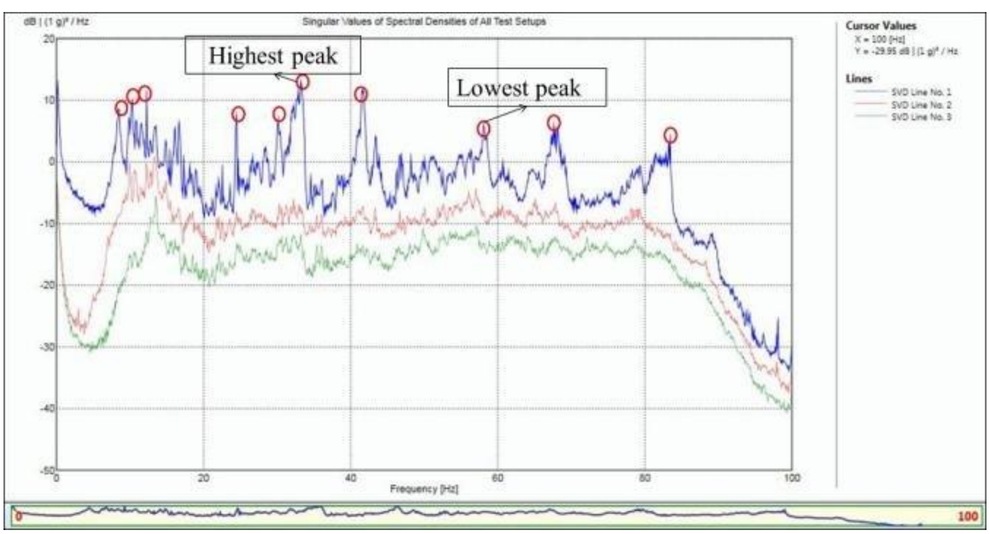


Figure 14: Peak Picking method as seen in the Artemis software. [7]

Running the experimental analysis on a regular basis can be beneficial for monitoring the state of the building. As damage occurs in a building the fundamental frequencies decrease due to a loss of stiffness. By tracking the changes in frequencies, in conjunction with the changes in environmental conditions, it can display trends of material deterioration before visible damages appear, allowing for earlier interventions and more cost-effective solutions. Monitoring can also help measure the quality of interventions and repair strategies too.

2.3 FEM Modelling

Finite Element Method modelling, or FEM modelling, is a computer modelling method that breaks complex structures down into smaller parts and then uses numerical methods, which are simple iterative equations repeated until they converge on a value, to compute various engineering parameters, such as stresses or displacements. This methodology of solving is ideal for computers because it is a lot of little steps that are simple and straightforward, which in computers is fine because they can store the data easily, whereas a human may prefer a more complex method with fewer steps, as this aligns better with human brain capacity.

In FEM software, models are constructed of element types, like beams, planes, and solids. Each different element type has different assumptions that are used in solving, which in turn modify the results depending on the complexity of the overall model. A 3D element will solve differently than a 2D element of the same thickness, although the results are likely to be similar. For example, there are two similar beam theories, Bernoulli and Timoshenko. While both will solve for the desired parameters of a beam, Timoshenko takes more of the shear behaviour into account. Therefore, under minimal shear effects, the theories will solve similarly, but in locations of high shear forces, Timoshenko will present a more accurate result. Therefore, it is important to consider what kind of element is used and where.

For the purposes of this project Dlubal RFEM 5 with the RF-Dynam Pro module, pictured in Figure 15 was used for FEM modelling. The add-on module was used for dynamic analysis of both natural and forced vibrations.

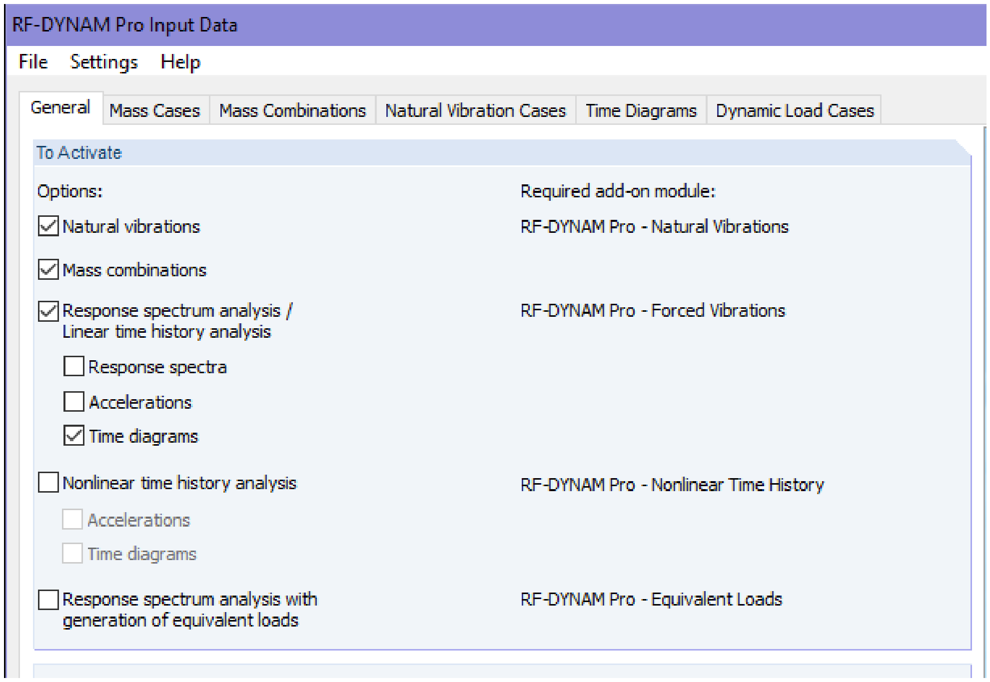


Figure 15: Dlubal RFEM 5 RF-Dynam Pro Module home screen where the different types of analysis can be selected for use. In this project, only the natural vibrations and linear time history analysis were used.

In Dlubal modelling, the mathematical selections that affect the assumptions of FEM are contained within the program and designed to be adaptable to models, allowing for a more user-friendly and less theory-based design approach. To that end, Dlubal creates the small elements in the form of a mesh, where shape and size of the mesh are automatically generated, using square based shapes in 2D and 3D first, then resulting to triangular based shapes where needed to complete an encompassing mesh over even more complex shapes. Each of these mesh shapes create nodes at the corners that represent the degrees of freedom, and these are then set for displacement and rotation for all points, except at specific boundary conditions. The theories used in the modelling are then applied across the mesh. In Dlubal, these theories are many, as they account for all the different element types, however more information is always readily available through the online manual. [7]

2.4 Bell Dynamics

The way in which bells swing is classified into three main systems: English, Spanish, and Central European, as seen in Figure 16. Both the English and Spanish systems allow the bell to swing in a full circle while the Central European system limits this to typically less that 80° from the vertical. The difference in the way the bells swing has a significant impact on the forces the bell exerts on the structure. [8] The bell in St. James Church swings in a Central European system.

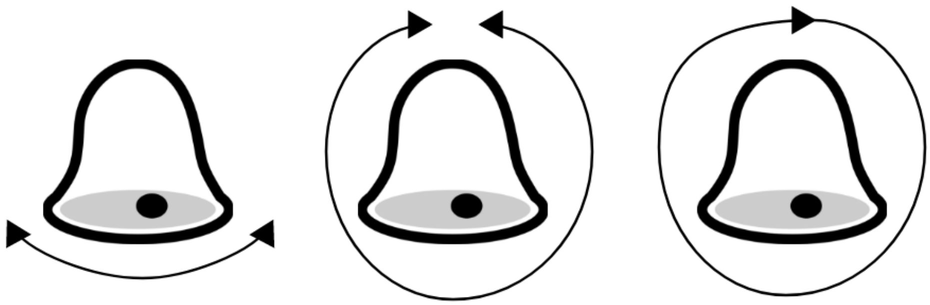


Figure 16: The Central European system (left) swings back and forth to prescribed angle that is typically less than 80° form vertical. The English system (middle) swings all the way to the top before returning the way it came to complete a full circle in the other direction. The Spanish system (right) swings in a continuous circle in one direction.

Knowing the swinging system, the applicable standard that provides the force of a swinging bell is DIN 4178. The forces are calculated independently for horizontal and vertical components, and both components rely on the mass and dimensions of the bell, the maximum rotation of the bell and the current rotation of the bell. Rotation is measured from the centre of mass of the bell, not the clapper, with 0° at the vertical and angle increasing positively or negatively depending on the direction. The current angle of the bell at any point is then a function of time, with respect to the period of the swing, allowing the forces to be mapped as a time history function. The equations of the forces, both horizontal and vertical, are calculated by Equation 6 and Equation 7.

$$H = \frac{m * g}{1 + k^2} * \left(2 * \frac{cos\varphi_0}{cos\varphi} - 3\right) * cos\varphi * sin\varphi$$

Equation 6: Horizontal applied force of a bell

$$V = \frac{m * g}{1 + k^2} * (k^2 + 3 * cos^2\varphi - 2 * cos\varphi * sin\varphi_0)$$

Equation 7: Vertical applied force of a bell

In these equations $k = \frac{i_s}{r}$, which is the radius of gyration divided by the imbalance of the system. The imbalances are due to the individual rotating masses of the clapper versus the main body of the bell, illustrated in Figure 17. Within a bell there are two points of rotation; one at the top of the shell, and one at the connection of the clapper to the shell. The clapper is hinged at its connection, allowing it to rotate freely. Therefore, as a force is applied to the whole bell, due to the difference in mass between the shell and the clapper, they will swing at different speeds. With less mass to propel, the smaller clapper will swing more quickly, striking the bell on the high side of the swing, which causes the bell to give its distinct tolling sound.

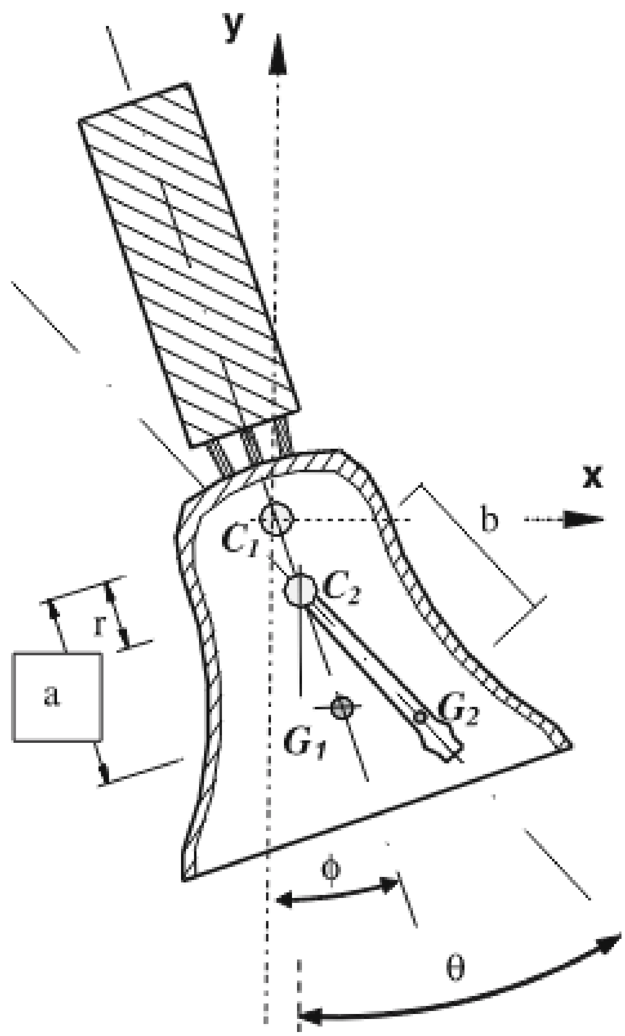


Figure 17: Simplified bell schematic. C represents the centre of rotation and G represents the centre of mass. 1 is for the shell of the bell while 2 is the clapper system. The misalignment of a (distance between C1 and G1) and b (the distance between C2 and G2 causes the imbalance in the systems that causes the swinging and forces in the bell. [8]

3 Dynamic Testing

To fully understand the effect of the new bell on the tower, it was important to be able to develop a computer model of the tower that behaves in a similar dynamic manner to reality. The best way to do this is to calibrate the model based on the fundamental frequencies of the existing structure, and therefore a dynamic test of the tower was completed. A model for the tower had been previously constructed for Dlubal for Fajman (2016) [6] but it was not calibrated for the dynamic properties.

3.1 Testing Procedure

Testing took place in the top floor chamber above the bell. The floor of the testing room vibrates noticeably under the movement of people on the floor. Given that the floor is a stone overlay on a timber joist system at a very tall height, this can be understood, but is not ideal for visitor comfort. It should be noted that when the bell rings, the floor does not vibrate as much as it does under the direct movement of people. Even within the Baroque roof, where the more flexible timer may be more subject to displacements, the vibrations from the bell are not really felt. The bell tolling is more significant, however, than the sound used to mimic the bell, which is in use for the regular tolling at the hour and each quarter.

The testing took place in three phases.

I. Accelerometers in initial positions
    a. With bell chiming
    b. With no induced vibrations
    c. With vibrations induced by walking
II. Accelerometers in varied positions
    a. With vibrations induced by walking
III. Accelerometers placed on the bell frame
    a. With bell chiming

Accelerometers were set up at varying locations on the walls. Some were placed on the tops of the walls, like in Figure 18, while others were placed in the windows and on the floor of the top level. The supports were positioned to ensure that the accelerometers were facing the correct orthogonal direction, either x, y, or z.

Figure 18: Accelerometers placed on a magnetic stand placed in a corner of the tower. Each of the accelerometers are set to measure a single direction (x, y, or z,). Several of the magnets with accelerometers were placed in many locations around the top of the tower. (Photo by author)

The first phase of testing collected a baseline set of data for each of the different types of vibrations the tower could experience. The second phase then repeated the induced vibrations test with the accelerometers in new locations. Induced vibrations are an important testing stratagem because having a random but applied force creates a more pronounced excitation that is easier for the DAQ to read and analyse over the minimal vibrations occurring without the induced vibrations. The second phase was done to corroborate the results of the first phase. The results should be the same no matter the location of the accelerometers in the tower and the repetition of the tests confirm this. Finally, the measuring of the accelerations by the bell as it rings, gives both the frequency of the bell and local displacements that can be found din the bell frame. This test was also video recorded. Through this it was confirmed that the period of the bell was 2.5 seconds, and therefore rings at a frequency of 0.4 Hz.

3.2 Results Processing

Results processing was completed by the testing team, using the Peak Picking method. The first three fundamental frequencies were found to be 0.98 Hz, 1.13 Hz, and 2.88 Hz for the x-direction, y-direction, and torsional direction, respectively. Further analysis, completed by using frequency analysis software, also determined the mode shapes for each frequency by creating a simplified tower model with very basic properties. These can be seen in Figure 19, Figure 20, and Figure 21. Additionally, testing revealed a displacement of 3 mm at the bell frame, and less than 1 mm within the tower itself.

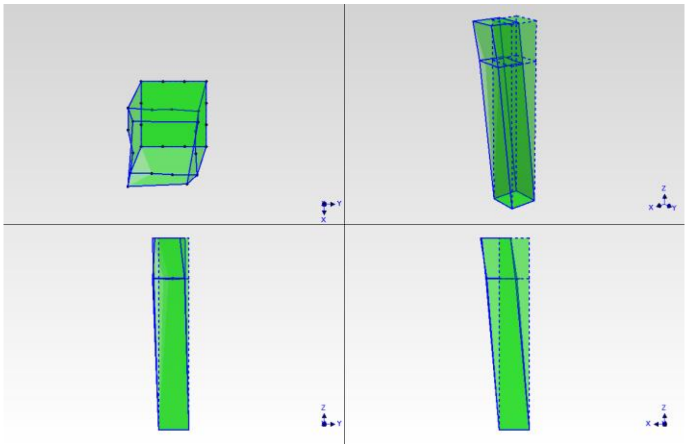


Figure 19: Mode shape in x-direction @ 0.98 Hz. Components can be seen in the x and y directions, but the movement is dominantly in the x-direction.

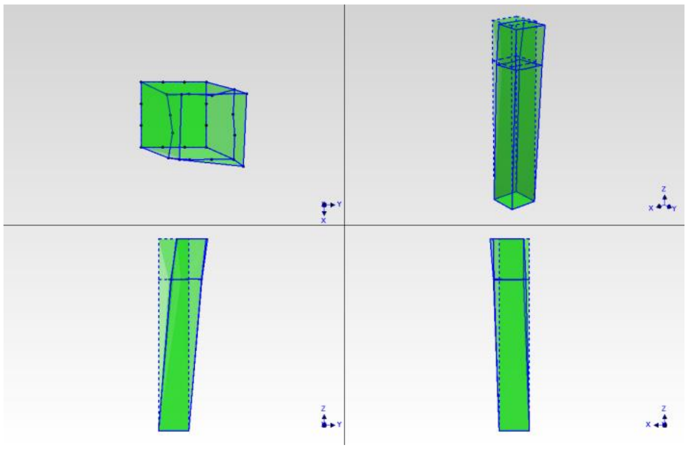


Figure 20: Mode shape in y-direction @ 1.13 Hz. Components are in both the x and y directions, but predominantly in the y direction. The top also moves more than the rest of the structure.

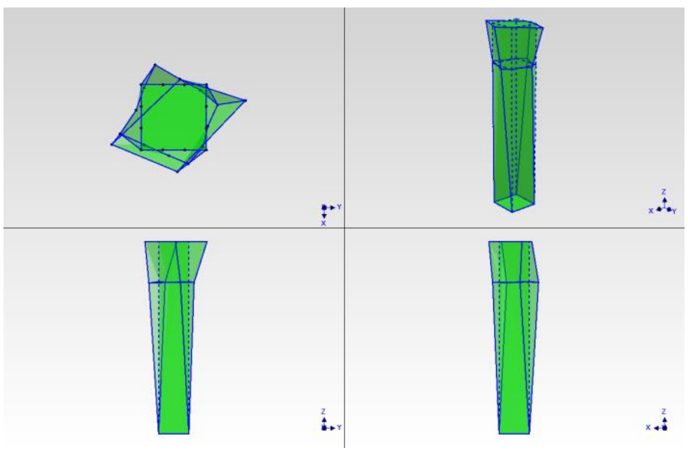


Figure 21: Mode shape in torsional direction @ 2.88 Hz. Top moves separately than the rest of the structure.

4 Structural Modelling

To check for resonance a structural model was developed in Dlubal RFEM 5. The Dlubal model is different from the model generated by the testing software as now the boundary conditions, various materials, and complex geometry can be taken into account.

4.1 Model Geometry

The St. James tower has been modelled in the past for the purposes of structural assessment of the restoration work for the report by Fajman in 2016 [6], before the installation of the current bell. 3D models were developed for both the tall tower and the short tower, including interior vaulting, trusses, and the bell frame in the taller tower. The surrounding elements from the church, up to the first row of columns, were also included around each tower. The structural ties were omitted as the model connections between walls account for the box behaviour of the system that is provided in the real structure. The final result can be seen in Figure 22. Originally, the two towers were modelled separately, but for the purposes of this analysis, they were combined to allow for a more accurate calibration, up to and including the surrounds to the first columns.

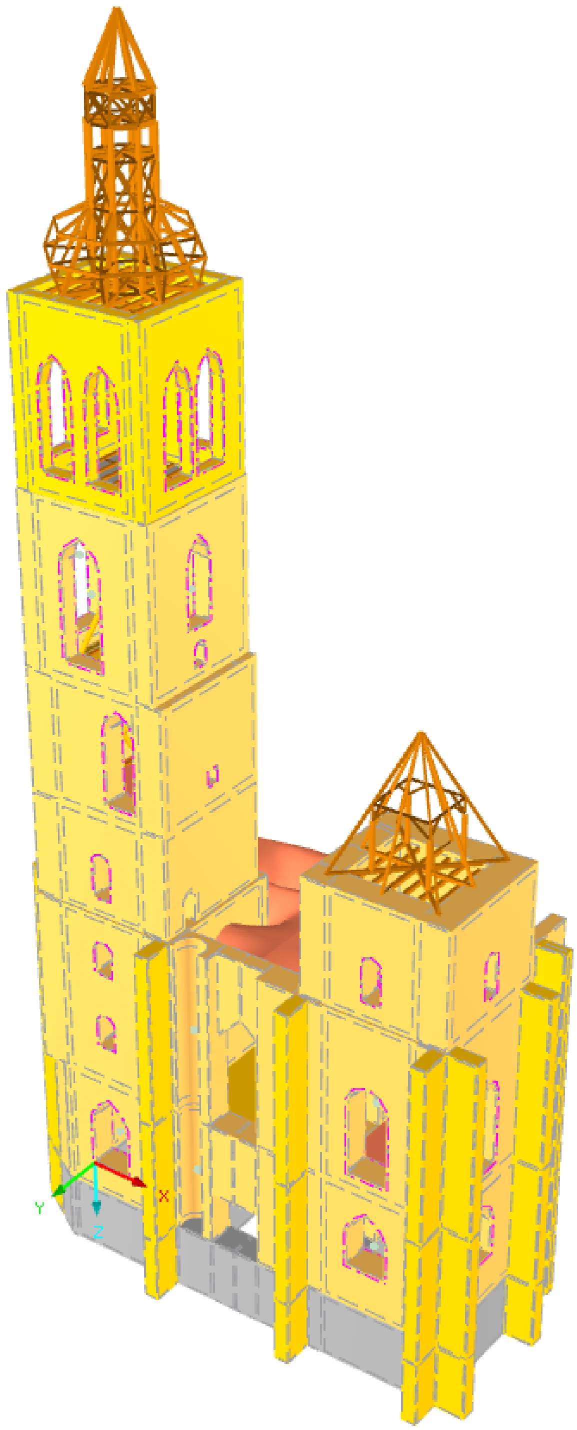


Figure 22: Dlubal 3D model of both towers, exterior viewpoint. The towers are made of a combination of 3D elements and 2D elements with varying materials throughout the structure. Geometry is slightly simplified for modelling purposes, including omission of the timber roof.

The walls and timber structures of the towers were modelled as 3D elements, while the vaulting was modelled as a surface element to represent the much thinner conditions since the vaults were made layers of brick and plaster, as opposed to the three leaf walls found in the towers. The choir floor and arch above the nave entrance were also modelled as surface elements due to the minimal impact they were expected to have on the structural frequencies. The choir arch and floor can be seen in the dark yellow in Figure 23, below the red vaults, connecting the two towers.

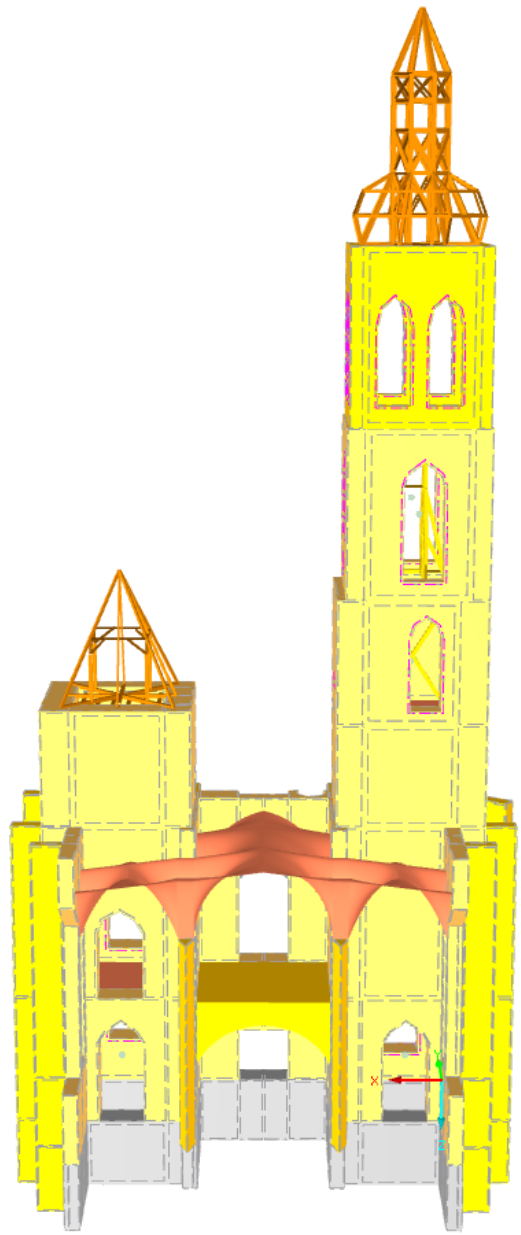


Figure 23: Dlubal 3D model of both towers, interior viewpoint with simplified choir and supporting arch modelled as high quality masonry made of 2D elements connecting the two towers.

4.2 Materials and Boundary Conditions

There are five main materials within the model and reference may be made to Figure 22 and Figure 23 to see the distribution within the structure:

- Brick masonry: A low strength masonry used to create the ceiling vaults. Modelled in red.
- Lower quality sandstone: A medium strength sandstone used in most of the walls. Lower quality due to less consistent arrangement of stonework with smaller stones and larger mortar joints. Modelled in light yellow.
- Higher quality sandstone: A medium strength sandstone used in the top of the tower, buttresses, columns, and the choir. Higher quality due to larger stones with more regular mortar joints. Modelled in dark yellow.
- Timber: Class C20/21 used in trusses and the bell frame. Modelled in brown.
- Concrete: Class C12/15 used in foundations. Modelled in grey.

Boundary conditions were considered on the foundations and at the connections to the rest of the church. They were all modelled as spring supports, meaning that there was some movement expected and the connections were not perfectly rigid. Final values for the supports’ spring constants were calibrated to the model, as testing these parameters was not included in the scope of the work, however several assumptions were made. On the foundations, the horizontal confining pressure was modelled as surface springs and given the lowest overall values, while the ground under the two columns received the highest values. The vertical foundation supports were also modelled as surface springs however, the stiffnesses are varied across the ground to mimic the loss of structural stiffness towards the south that prevented the increased height of the second tower. The last set of modelled supports are two sets of line spring supports modelling the interaction with the rest of the church. The upper line supports represented the interactions due to the roofing and vaults, while the lower indicated the interaction with the walls. Given this, a stiffer lower support value was considered over the upper supports, creating more of a cantilever effect that increases torsional stiffness while keeping the horizontal stiffness relatively even. In the next section, Table 2 shows locations of all the different supports and their corresponding calibrated stiffness.

It is important to note that the properties used to represent the materials and boundary conditions are not their true values. When working with historical buildings, the material properties are difficult to determine because they can vary wildly within the structure itself, as it lacks the uniformity required in modern construction. So, while the values are not unreasonable for what they represent, they are also potentially inaccurate. For the purposes of understanding the global dynamic behaviour, however, a model calibrated with the right assumptions, as explained above, is a good enough representation without completing the extensive testing scheme that would be required to get true values.

4.3 Calibration

Calibration was completed by comparing the results of the natural vibration testing done on-site with the natural vibration analysis results from Dlubal. In the tower, the first three frequencies were measured, but due to the unpredictability of the software, the first five frequencies were considered in calibration. Using the visible mode shapes, the three relevant modes, x-, y- and torsion, could then be selected.

Running natural vibration cases in the Dlubal RFEM 5 is a simple process. In the Dynam-Pro module, simply select the number of desired eigenvalues to be output. The simplicity of this analysis is what makes it ideal for calibrating the model. For the purposes of calibration, five eigenvalues were found, along with their mode shapes to ensure that the frequency and shape aligned with the experimental results.

The key parameters used in calibration were the elastic modulus of the material and the stiffness coefficient of the supports. These were varied until the fundamental frequencies of the tower matched those exhibited by the real structure, as determined by the testing. It quickly became apparent that the values that affected the frequencies most significantly were the line supports representing the contribution of the rest of the church, highlighted in green in Figure 24. This was particularly challenging as it was also the component with the least readily available knowledge.

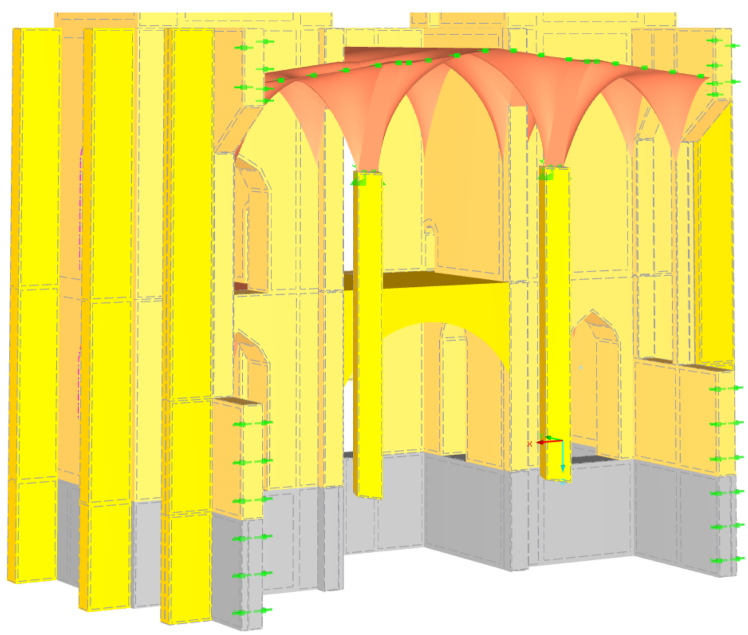


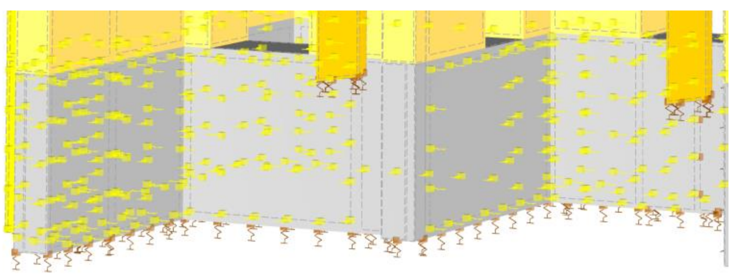
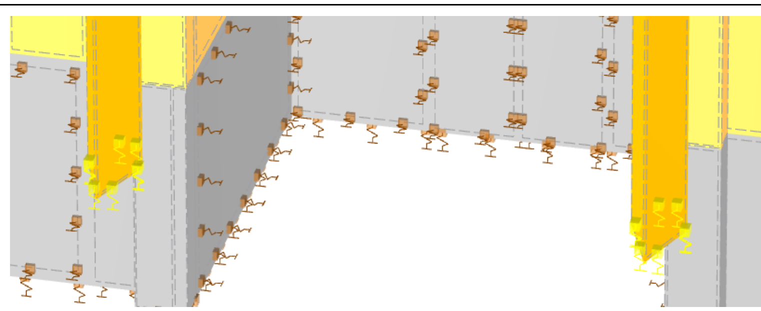
Figure 24: Line supports, in green, representing the effects of the rest of the church. The supports in the walls vary from the supports in the vaults to mirror the realism of the situation as the walls will be stiffer than the vaults due to geometry and materials.

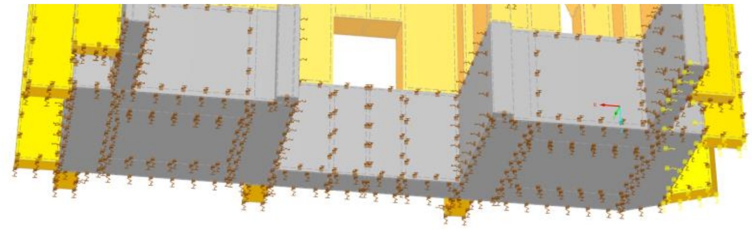
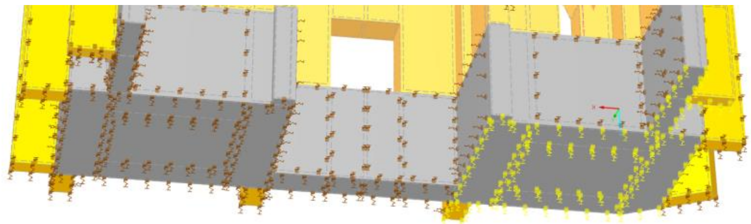
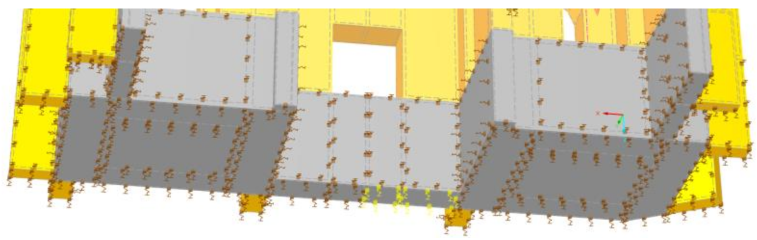
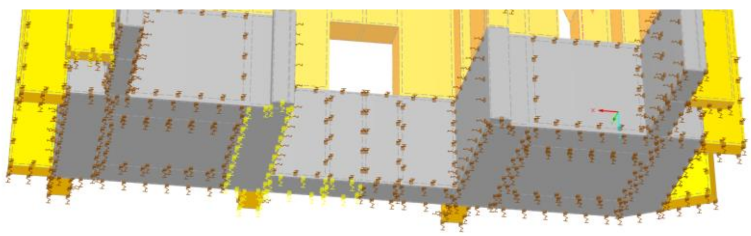
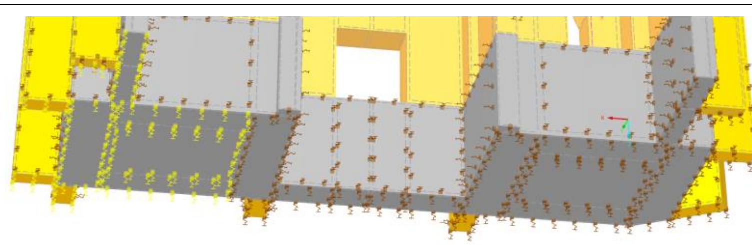
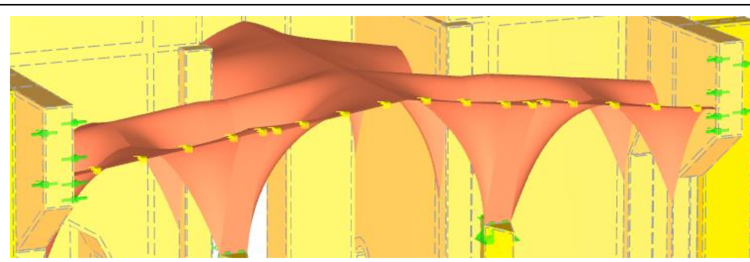
Another challenge was that the fundamental frequencies did not increase and decrease proportionally to the testing frequencies, with the torsional frequency regularly presenting a larger discrepancy than the other two. To correct that, measures were taken to change the torsional stiffness, such as creating a dramatic stiffness variation between the effect of the church roof and the effect of the church walls on the model. After significant efforts, the final model was considered to be calibrated with the values found in Table 1, with a comparison of the testing frequencies to the final model values in Table 2.

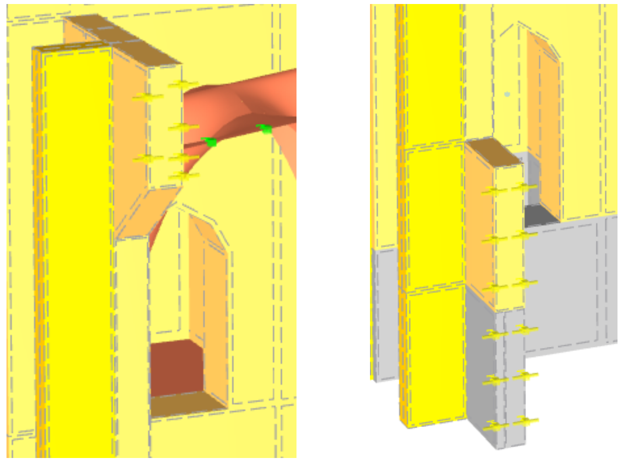
Table 1: Final values for the modulus of elasticity for the materials used in the church

Material	Modulus of Elasticity [GPa]
Brick Masonry	3.50
Sandstone Masonry	5.50
Higher Quality Sandstone Masonry	6.00

Table 2: Final spring constants for boundary conditions affecting the church. Indicated supports are in yellow.

Support	Spring Constant	Location
Confining Pressure Supports	10 MN/m^3	
Column Supports	1000 MN/m^3	

Northern Buttress Supports	150 MN/m^3	
North Tower Supports	150 MN/m^3	
Northern Entryway Supports	130 MN/m^3	
Southern Entryway Supports	110 MN/m^3	
Southern Tower Supports	80 MN/m^3	
Roof Interaction Supports	120 MN/m^2	

Wall Interaction Supports	170 MN/m^2	

Furthermore, a final comparison between the testing and model frequencies and mode shapes can be seen in Table 3, Table 4, and Table 5. These confirm that both the frequencies are similar and have matching mode shapes. This is important because computer models can sometimes have local modes that do not reflect the behaviour of the tower, so it is critical to ensure that the shapes and the frequencies match, which they do for this tower.

Table 3: Comparison of the first fundamental frequency of the St. James Church tower between the testing results and the calibrated model

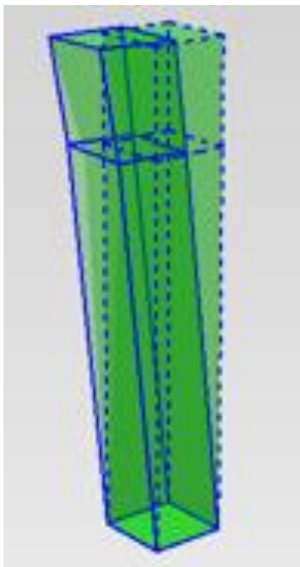
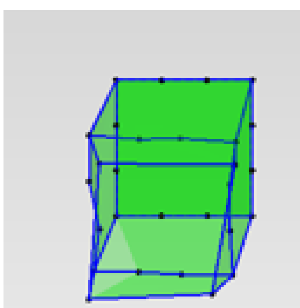
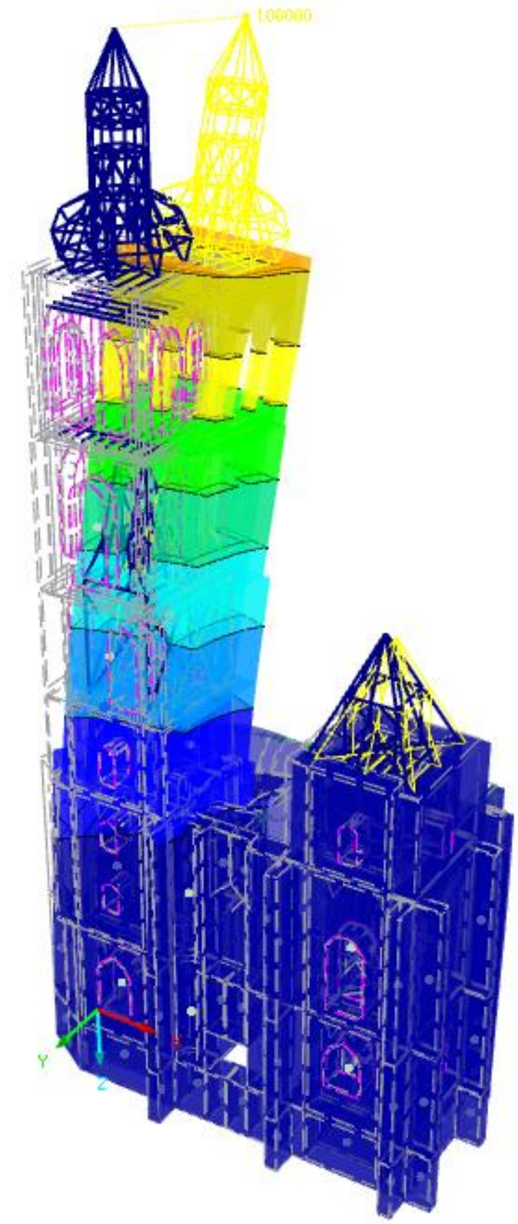
Test	Model
 	
0.977 Hz	0.989 Hz

Table 4: Comparison of the second fundamental frequency of the St. James Church tower between the testing results and the calibrated model

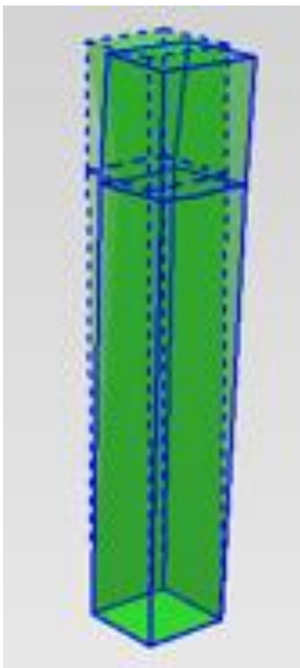
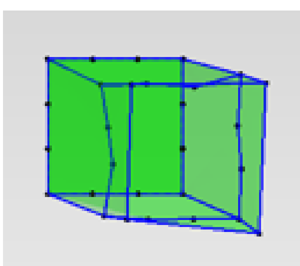
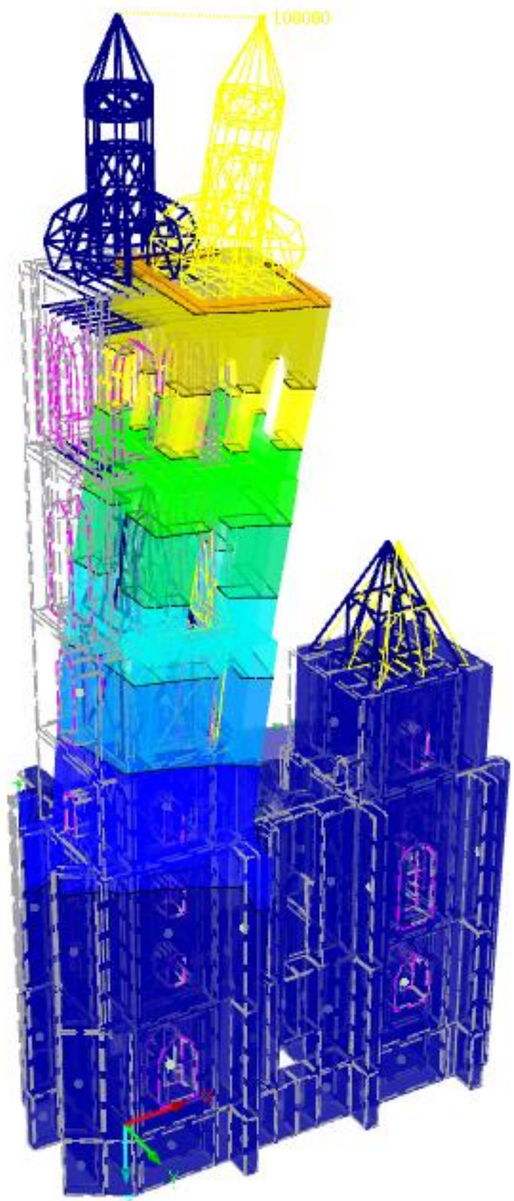
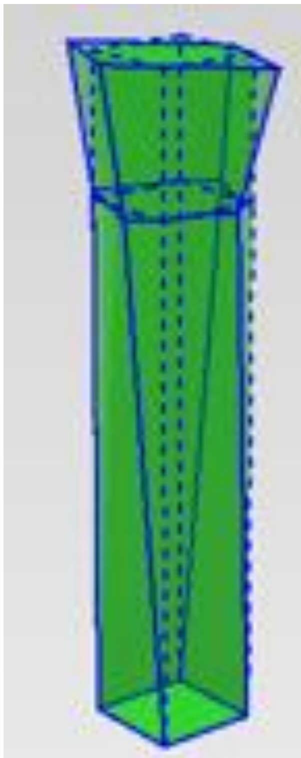
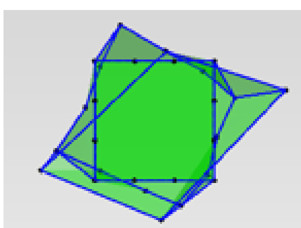
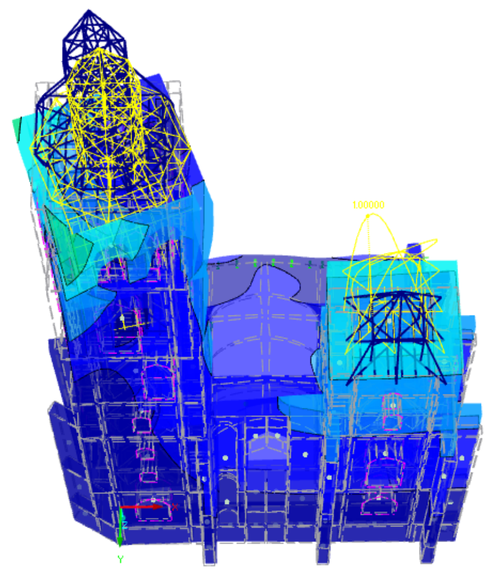
Test	Model
 	
1.129 Hz	1.075 Hz

Table 5: Comparison of the third fundamental frequency of the St. James Church tower between the testing results and the calibrated model

Test	Model
 	
2.88 Hz	2.831 Hz

5 Analysis

There were two points of consideration once the model had been calibrated. The first was determining the changes in the frequencies pre- and post-installation of the bell in the tower. The second was checking if the bell was resonating with the tower and therefore could be causing significant damage.

5.1 Comparison to Previously Tested Frequencies

From Fajman (2016) [6], the tested frequencies of the tower before the installation of the bell can be seen in Table 6 next to the corresponding frequency from the most recent tests.

Table 6: Comparison of pre-installation tower experimental frequencies to post-installation experimental frequencies

	2016	**2022**
1st Frequency	0.97 Hz	0.977 Hz
2nd Frequency	1.125 Hz	1.129 Hz
3rd Frequency	2.91 Hz	2.88 Hz

From the table, it can be seen that very little changes have happened to the frequencies over the last six years, despite the installation and continued use of the bell on a regular basis. In the cases of the first two frequencies, the small increases may be attributed to ongoing restoration works, or even small data collection differences. Although the third frequency weakened, indicating some damage may have occurred, the difference is very small and may also be attributed to variation in data collection. Overall, the frequencies appear to be very consistent over the time period.

5.2 Verification of the Safety of Continued Bell Usage

Checking the bell safety required running a linear time history analysis. Given the small, predicted displacements from the dynamic testing of 3 mm in the bell frame and less than a millimeter in the masonry, a non-linear time history analysis was deemed superfluous. There were several steps of assumptions that were required before the analysis could be input into the Dlubal model. In order to apply the forced vibration, the magnitude of the horizontal and vertical forces with respect to time had to be calculated using the equations from DIN 4178 (see Equation 6 and Equation 7). Such that, the assumptions in Table 7 were made.

Table 7: Fixed variables used in the calculation of the horizontal and vertical bell forces

Variable	Symbol	Value
Radius of gyration	i_s	0.8 m
Eccentricity	r	0.93 m
I_s/r	k	0.861538
Mass of the bell	m	2400 kg
Maximum angle of the bell	Φ_o	70°

With these variables defined, the force caused by the bell acting on the structure can be calculated for any angle, however, to use these equations as a forced vibration application, it must be done with respect to time, not angle. Looking more closely at Equation 6 and Equation 7, all the variables are fixed, except for the current rotation of the bell, φ, because it changes as the bell swings. A full period of the bell means that it swings out to its maximum, 70°, back to centre, out to -70° in the other direction, and back to 0°. From the dynamic testing, it is known that this process takes 2.5 s, or 280° in 2.5 s., which gives a rate of 112°/s. Therefore, for every force per degree, it now occurs at a time related to the angle. What this means is that the forces also experience a repeating period of 2.5 s and vary with respect to the angle. This can be seen in Figure 25 below, where both the horizontal and vertical forces return to their starting force every 2.5 s.

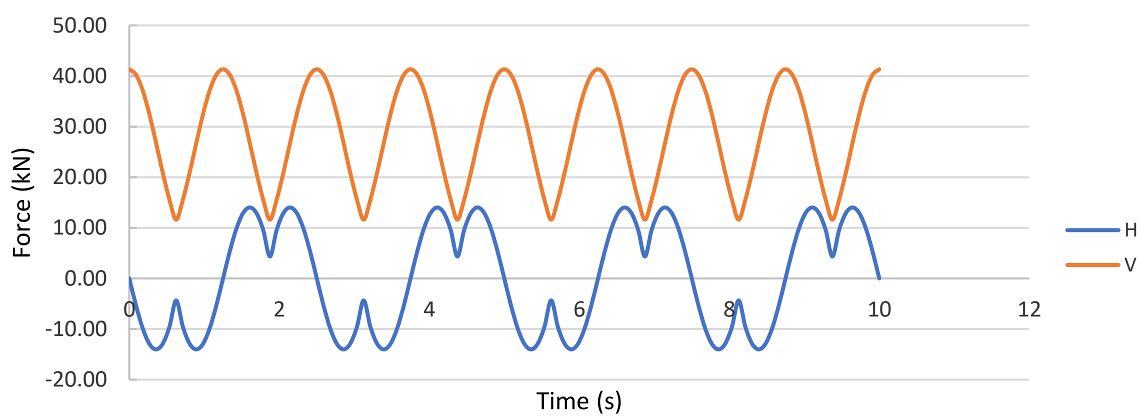


Figure 25: Force of bell v. time. The vertical force is show in orange and is always positive as there is always a vertical component acting downwards. The blue force is acting in the horizontal direction and crosses the x-axis because the bell causes forces in both directions.

This is also a good check of the equations' accuracy. The horizontal force should change in each direction, creating positive and negative values, while the vertical force should only cause downward force.

To apply this in Dlubal, two forces must first be applied at the location of the acting forces. For this, the force of both horizontal and vertical components was calculated at 50°, which came to -12.88 kN in the horizontal direction and 20.84 kN in the vertical direction. These forces were applied at the connection point between the centre of the bell and the frame. The vertical force was applied as a downward force, while the direction of the horizontal force was inconsequential provided it remained perpendicular to the frame in the correct direction due to the alternating nature of the force. These forces can be seen at their application point in Figure 26.

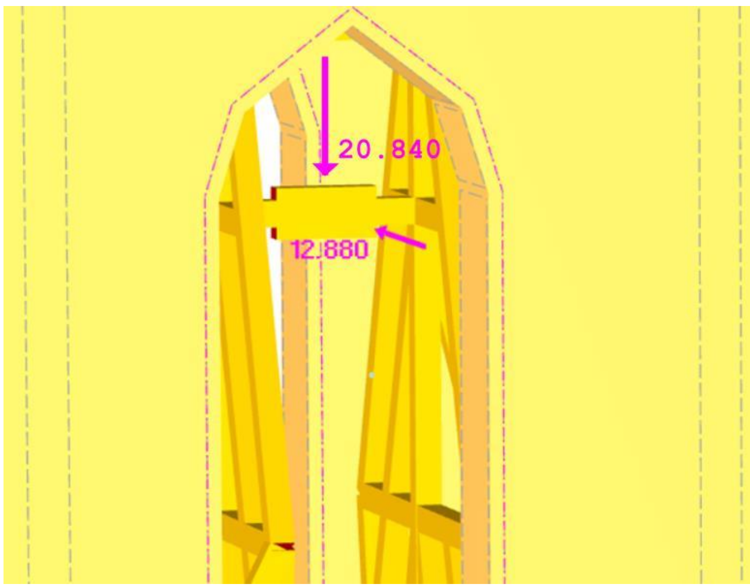


Figure 26: Forces acting from the bell at 50°. These forces are used as base forces for the transient time functions used to complete the dynamic analysis in Dlubal.

In Dlubal, to apply these forces over time, each of these loads are multiplied by a factor with respect to time, as shown in Figure 27 and Figure 28. Therefore, for the graphical values from above, they were all divided by the corresponding horizontal or vertical load at 50°. That way, when the data was inserted as a transient function into Dlubal's time history module, the graphs would look similar, but when the applied force was multiplied, the true force for that time step would be applied.

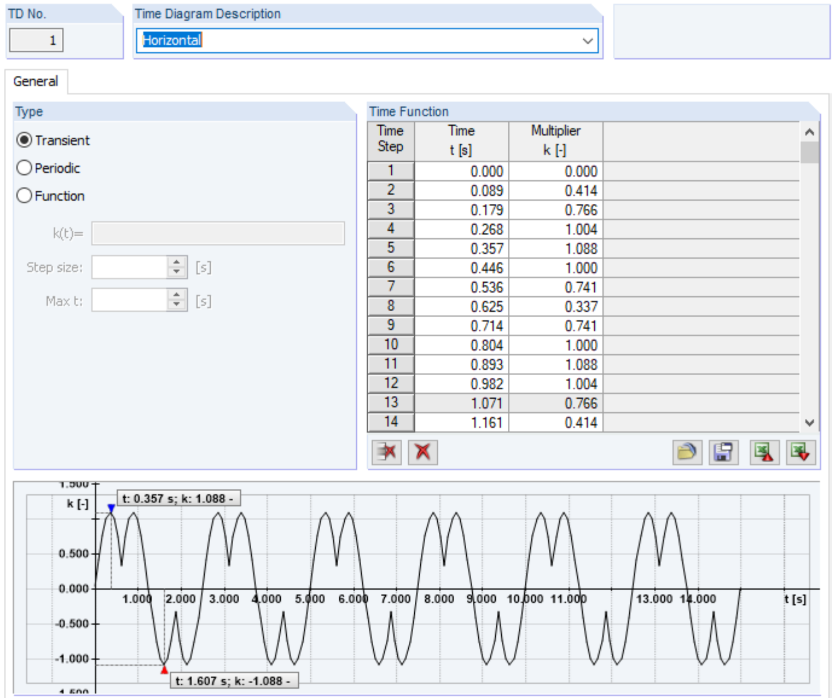


Figure 27: Horizontal force factor over time as a transient function in Dlubal. Time-dependent force as a multiplier of the calculated force applied at 50° in the horizontal direction.

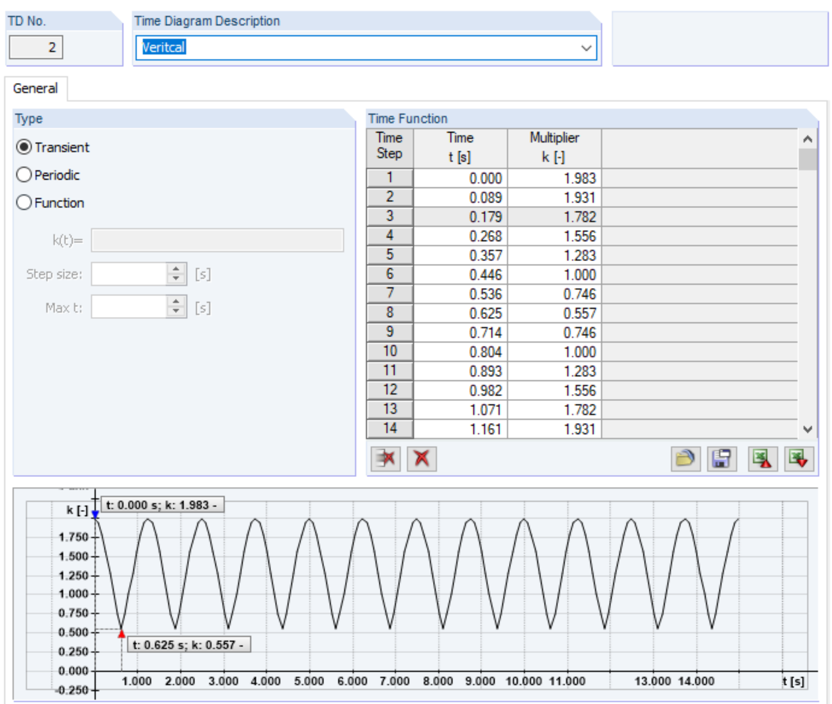


Figure 28: Vertical force factor over time as a transient function in Dlubal. Time-dependent force as a multiplier of the calculated force applied at 50° in the vertical direction.

While the bell in Kutná Hora rings thrice daily for up to five minutes, for the purposes of discovering resonance while reducing computational time, a segment of 15 seconds at full swing was applied. Additionally, for mathematical computation purposes it was important to have a minuscule time step for the numerical calculations in the program, in this case 0.01 s, however not all data points were required for analysis after the results were done, so only results at every 0.1 s were saved, as shown in Figure 29.

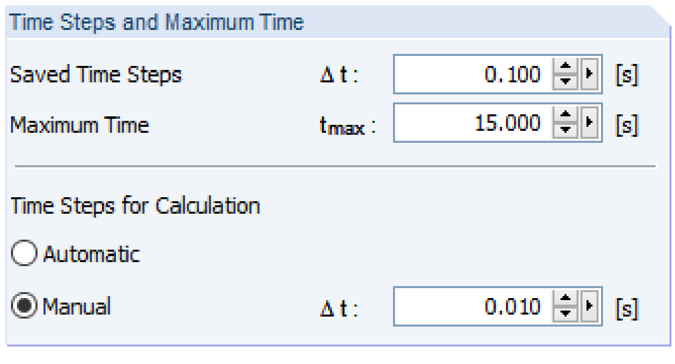


Figure 29: Time step settings. To run an accurate analysis, a small time step of 0.01 s was chosen for the 15 s analysis. However, to save all those time steps consumes a lot of computer data and is not relevant for an analysis that focuses on the overall displacements, so only the data for each 0.1 s was saved.

Furthermore, before the time history analysis could be run, the Rayleigh's damping coefficients had to be calculated as described in Section 2.1. Using modes that are not next to each other often gives a more stable result, so for this reason Mode 1 and Mode 3 were used, being 0.989 Hz and 2.831 Hz, or 6.21 rad/s and 17.79 rads/s, respectively. Hence, the coefficients were determined to be $\alpha = 0.460$ and $\beta = 0.004$, from Equation 4 and Equation 5,which means that the damping relied more heavily on the mass of the tower than the stiffness, and understandable outcome given the weight of the walls of the system.

With all the settings defined, the time history analysis could be run, and the results analysed. By analysing results in the dynamic envelope, the maximum values at any point in time can be seen in Figure 30. The maximum overall displacement in the tower under the forced vibration of the bell was 2.8 mm. This is very closely in line with the results of 3 mm from the testing, at a 0.2 mm difference. Also, these larger displacements are in the timber trussing of both the bell frame and the Baroque roof. The masonry tower does not displace more than a millimetre at any point, which at a height of 86 m provides a relative drift of little consequence to the structure.

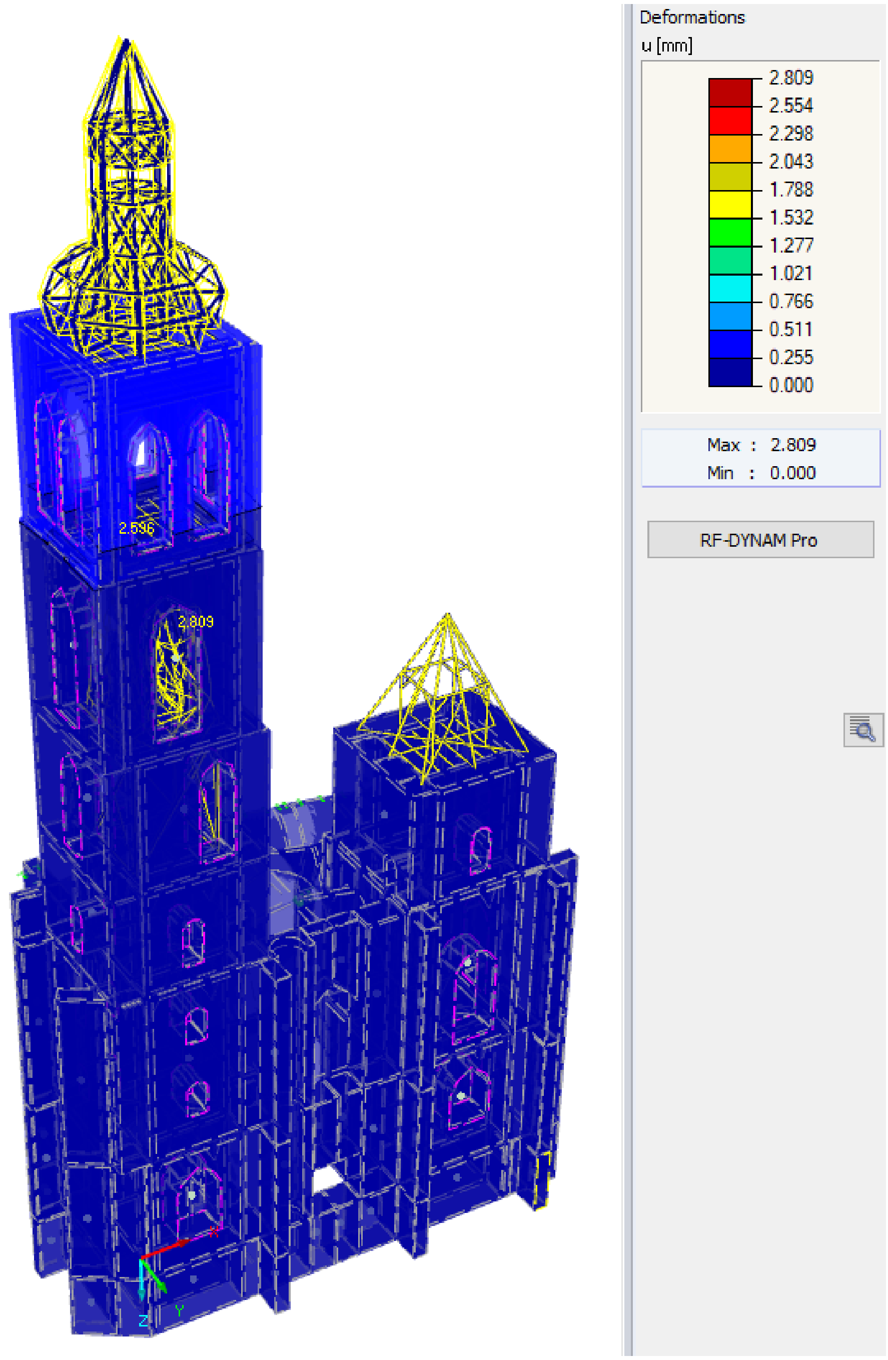


Figure 30: Maximum displacements in tower due to the bell ringing. The masonry does not move any significant amount at any time. The timber has higher displacements, but still nothing large enough to cause damage.

It is important when working with a computer model to ensure that the structure is behaving the way the inputs are telling it to. One way to check this is by analysing the displacements at the location in which the forces are acting, as seen in Figure 31. In the figures below, the displacements in the y and z direction mirror the amount of force applied, as shown in Figure 32 and Figure 33, indicating that the beam supporting the bell is moving in conjunction with the forces. The effect this has on the x-direction is also cyclical, shown in Figure 34, which means that the loading is causing displacements in all directions.

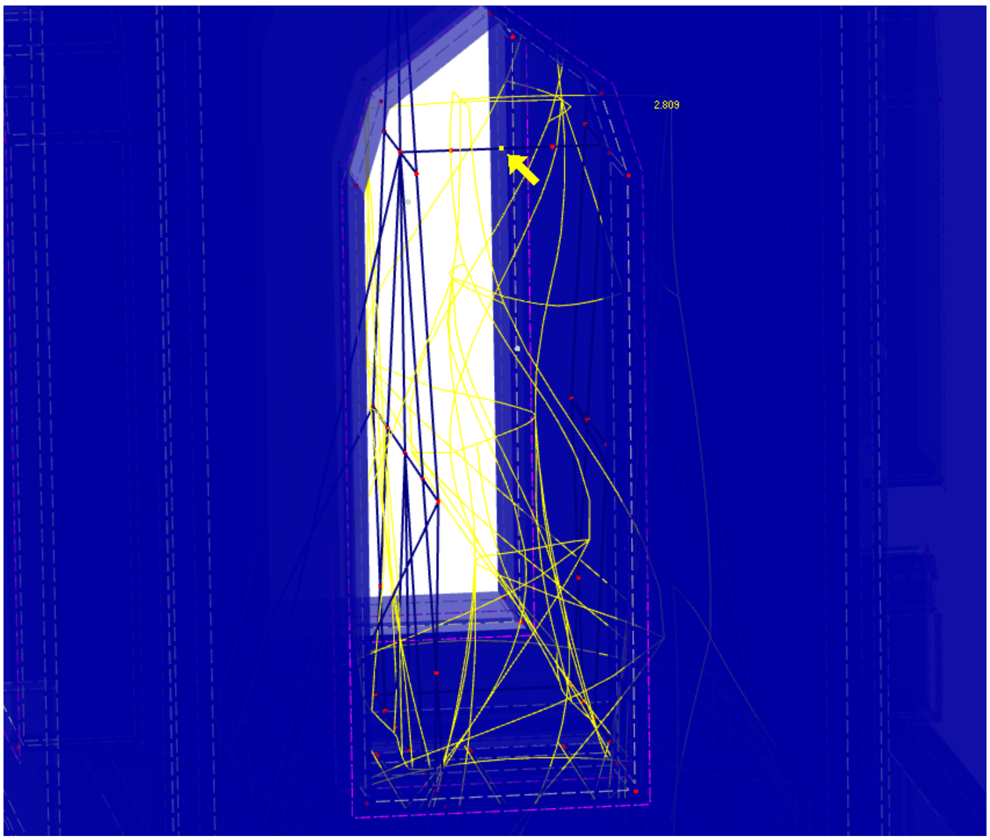


Figure 31: Location of the force loading point in the results display of the Dlubal wireframe model. Deformations in the bell frame are displayed in yellow (exaggerated scale), while the original location of the frame is shown in dark blue. The frame warps and moves away from the centre, towards the rest of the church.

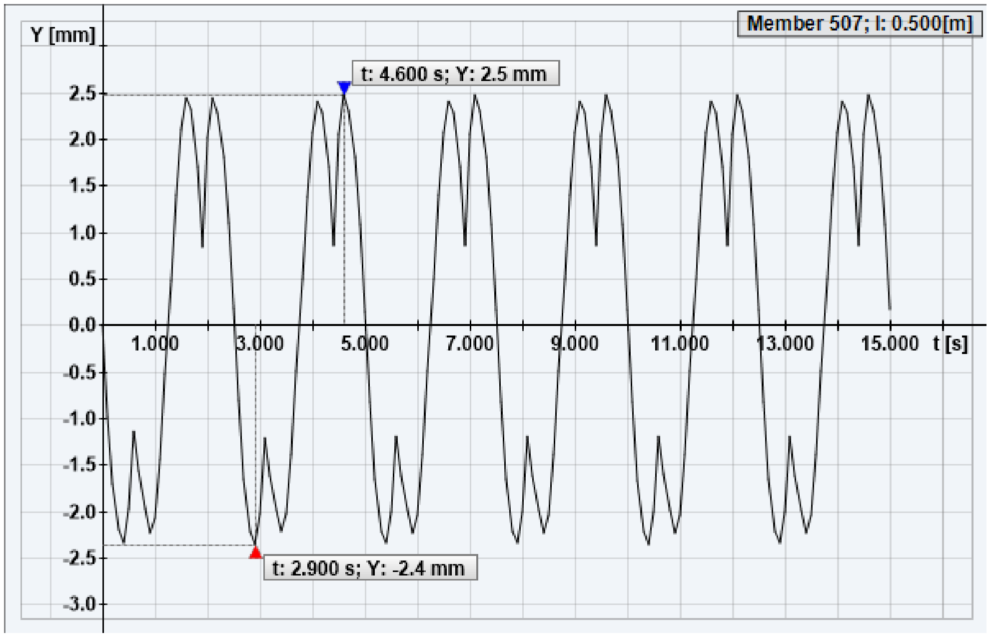


Figure 32: Movement in the bell frame at the connection to the bell in the direction of the bell swing. The displacement mirrors the amount of applied force in the horizontal direction which means that the force was applied correctly.

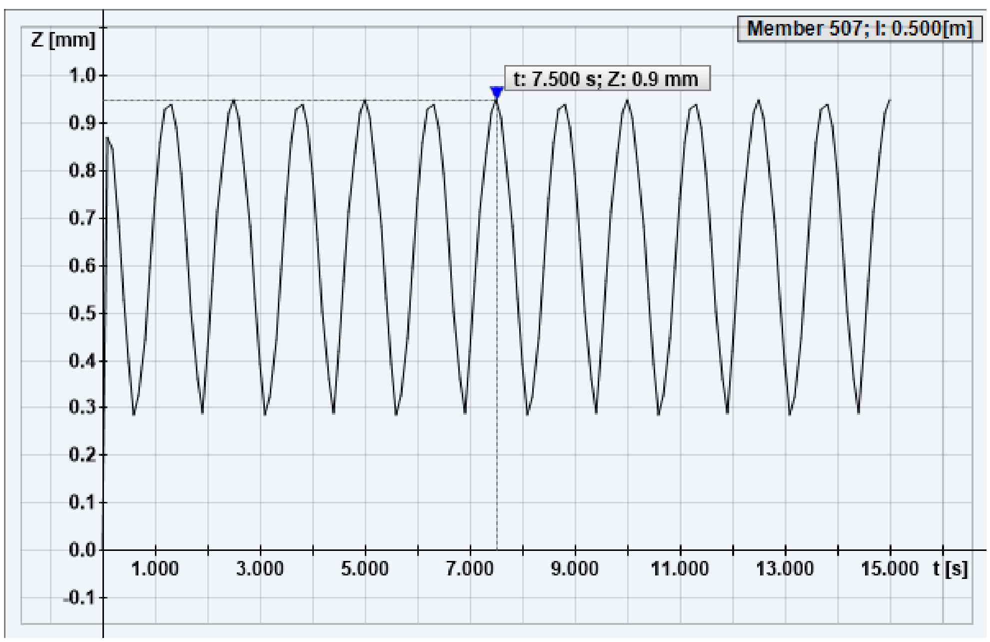


Figure 33: Movement in the bell frame at the connection to the bell in the vertical direction. The displacement mirrors the amount of applied force in the vertical direction which means that the force was applied correctly.

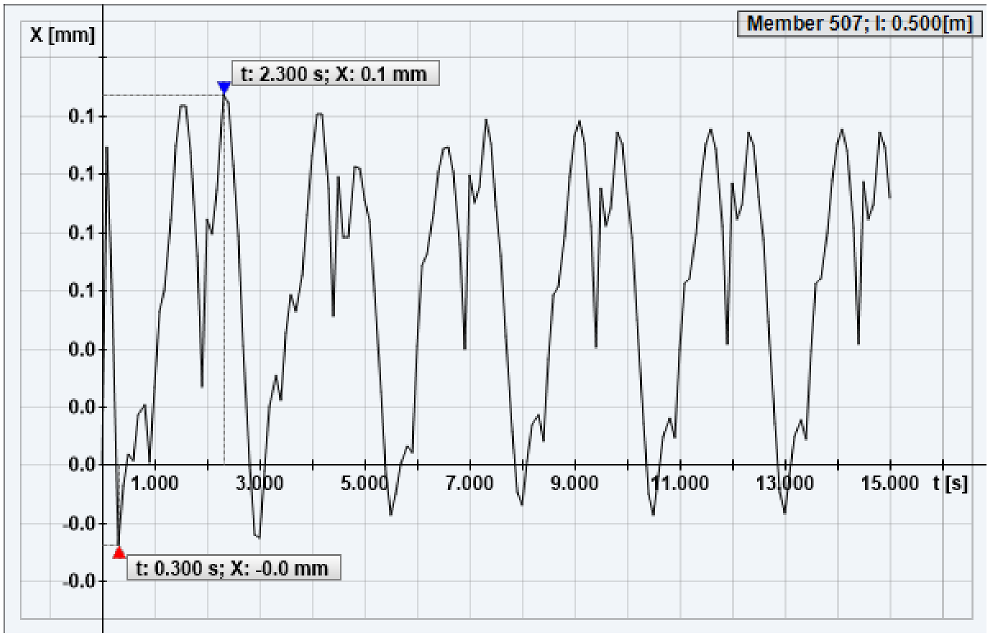


Figure 34: Movement in the x-direction (perpendicular to the bell). Although no load was applied in this direction, movement is expected as the entire structure vibrates. Notably, the displacement is much less than in the other directions.

Given the small displacements, it is highly unlikely that any resonance is occurring, however this can also be checked by looking at the displacement of a FE mesh point. Due to the small global displacements, a mesh point in the masonry that has a relatively high displacement was selected. Additionally, the point was chosen in a location where an accelerometer could be found so the results could also be compared to the readings from the experimental results. This location is indicated in Figure 35.

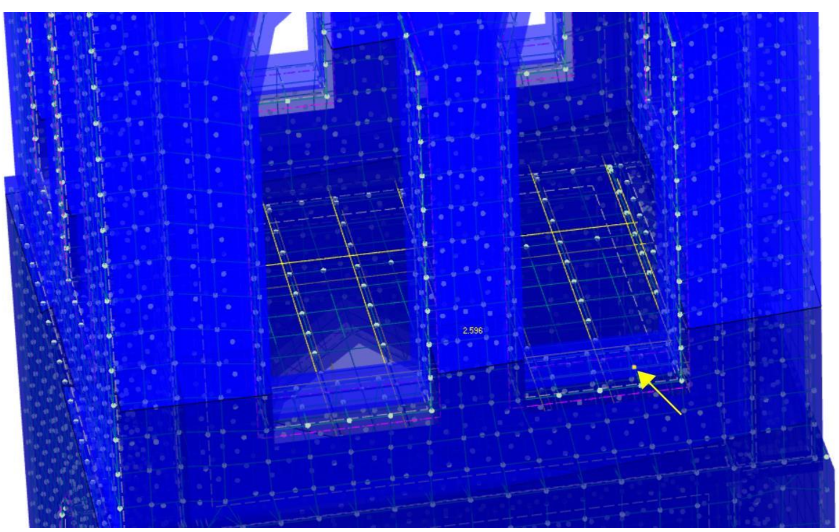


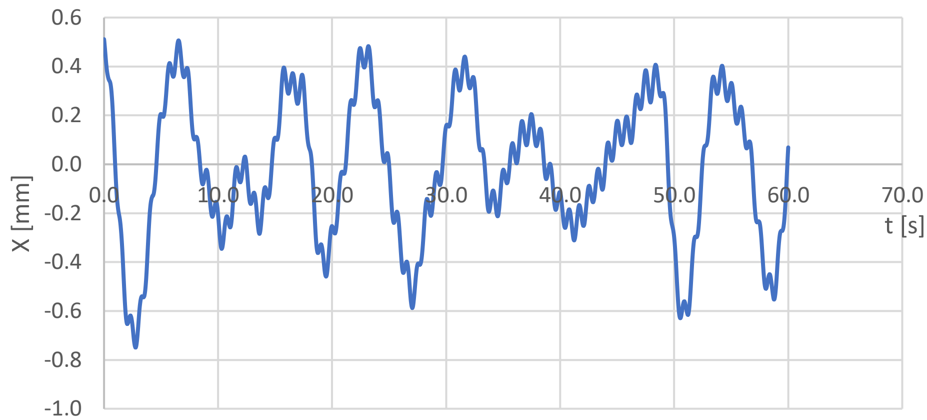
Figure 35: Location of FEM node point in similar location to actual accelerometer for comparison of displacements.

Displacements in the model appeared to mirror the applied forces in the bell to a certain extent, despite the distance from the bell, but the results of the displacements from testing were not as similar.

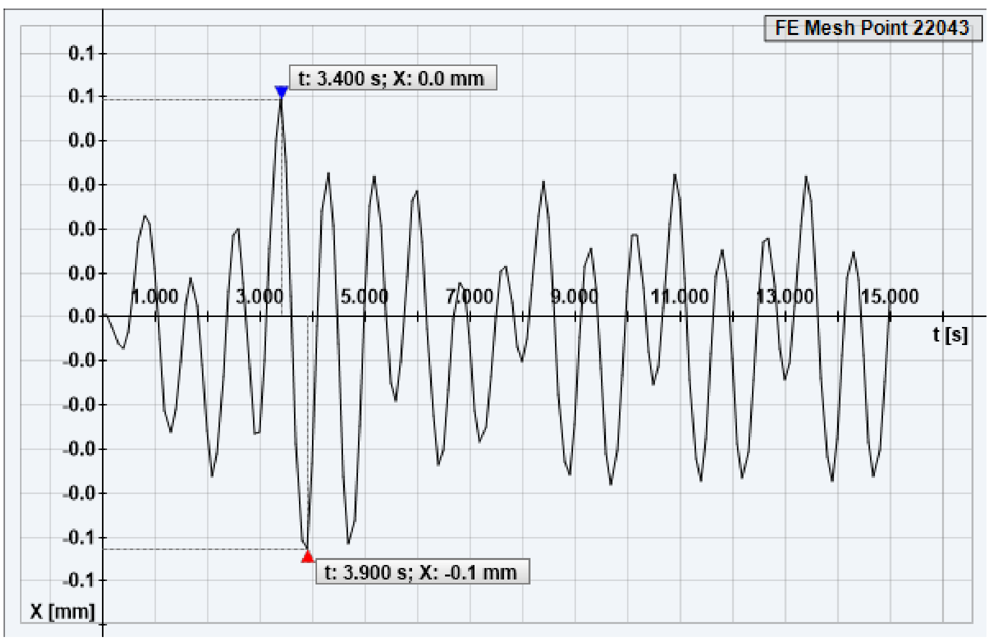
Table 8, Table 9, and Table 10 compare 60 seconds of displacements from the dynamic testing to the 15 seconds of analysis from the model. The first comparison of note is that the true displacements are larger than the ones predicted by the model. While the magnitude of the displacements does not change, the exact values are different. This is understandable, as simulated structures rarely mimic reality exactly.

Additionally, it can be seen that the displacements from the Dlubal model follow the cyclical pattern of the applied force. This is much less obvious in the real displacements. However, the structure does still move under a similar pattern, indicating that it does feel the forces from the bell and the displacements are not occurring due to another source. It is important to note that while the differences are noticeable on a time scale, the overall displacements of the tower were also very similar, meaning that on a small scale these discrepancies seem like a negative comparison, but on the global scale of the structure, there is little need for concern. So, while the graphs in Table 8, Table 9, and Table 10 are different, they are not so different that the data they present is invalid. This confirms the model did work in the way it was supposed to.

Table 8: Comparison of displacements at an accelerometer in the x-direction

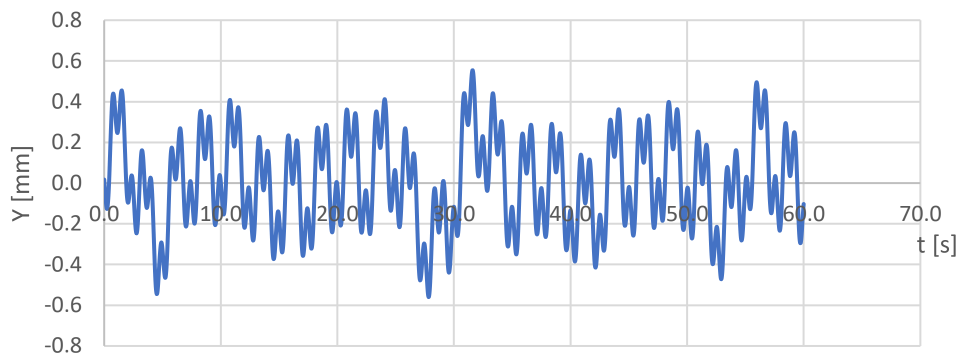


X – Direction: Testing

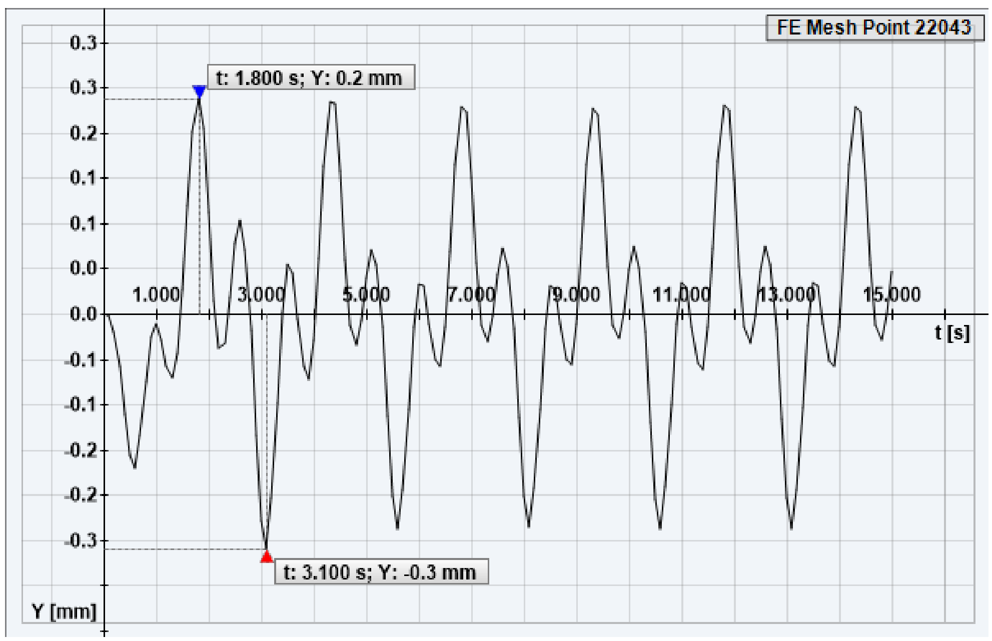


X – Direction: Model

Table 9: Comparison of displacements at an accelerometer in the y-direction

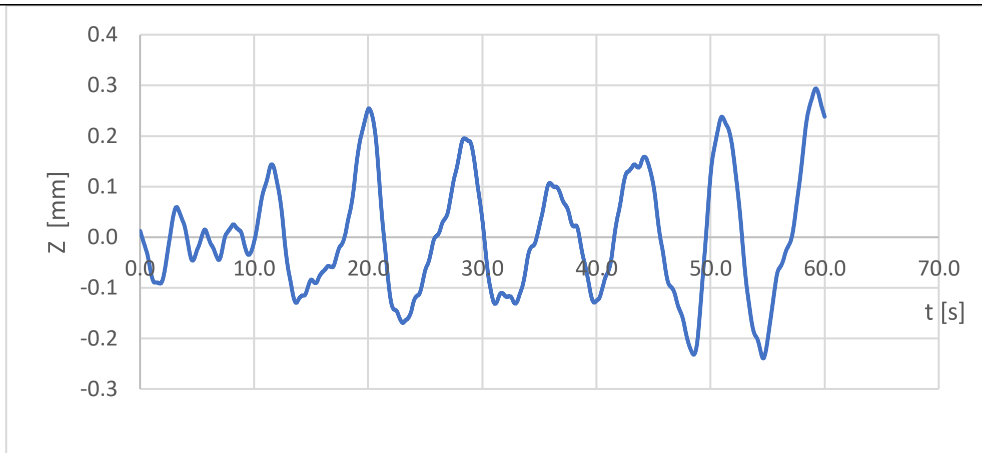


Y – Direction: Testing

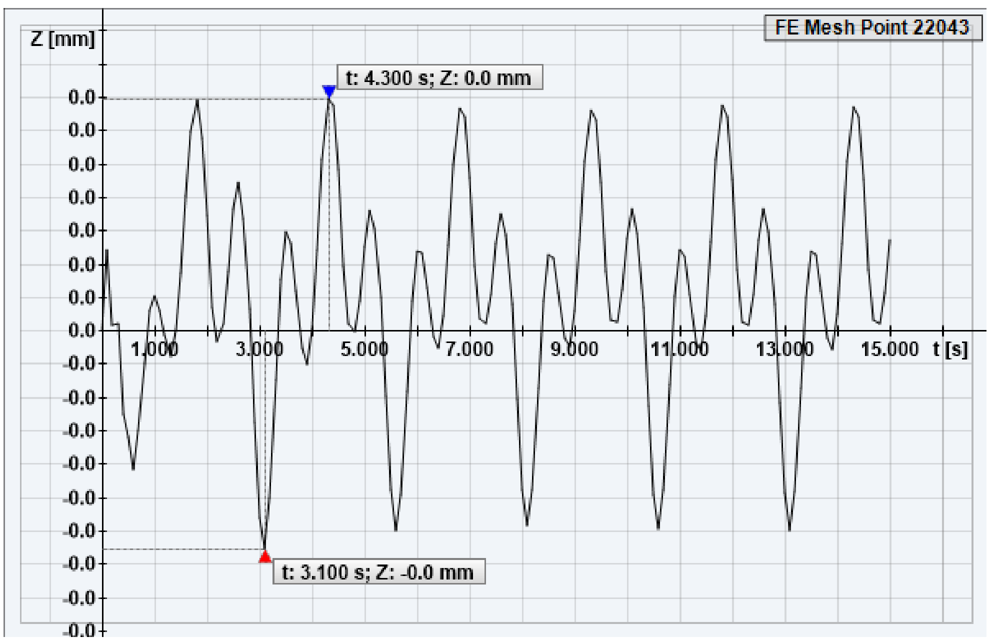


Y – Direction: Model

Table 10: Comparison of displacements at an accelerometer in the z-direction



Z – Direction: Testing



Z – Direction: Model

Through the results, it can be confirmed that the swinging of the bell does not cause harm to the tower. The continued use of the bell has done very little in changing the stiffness of the structure, indicating that it has not caused harm since its installation. Furthermore, modelling of the tower and the bell have presented no signs of resonance, such that it is likely not occurring. To that end, no changes are recommended at this time, although further monitoring is recommended to ensure no unforeseen circumstances occur in the future.

Although a comparison of the dynamic testing results does not provide an exact match to the results from the model, they are considered similar enough for the model to be considered a good representation of the structure. There are many reasons a model may not perfectly match the true state. In this case, the materials in the model were homogenised across the different groups of materials. In the tower walls, they consist of sandstone and mortar, in varying combinations but in the model the walls are one shape, a simplification that can mildly affect results. Also, no other loading was considered in conjunction with the bell, such as wind or live loads. The bell may cause the most significant loading, but the factors from the other loads may impact some of the displacements and keep the model from matching perfectly. Overall, the model is good for the situation in which it was used.

6 Conclusion

6.1 Results

Dynamic testing revealed fundamental frequencies of the St. James tower to be 0.98 Hz in the x-direction, 1.13 Hz in the y-direction, and 2.88 Hz in the torsional direction. Displacements in the bell frame were measured at 3 mm with tower displacements found to be less than 1 mm. A Dlubal RFEM 5 model was then calibrated using these frequencies and mode shapes. The final frequencies of the model were 0.989 Hz, 1.075 Hz, and 2.831 Hz. From there, a time history analysis of the tower while the bell is ringing revealed no signs of extreme displacements or resonance. The model was compared to the displacements from testing as well, and were found to be similar, confirming the model as an accurate representation. Additionally, since the bell was installed, negligible changes in the frequencies have occurred. Therefore, it is determined that the continued use of the bell in St. James tower is safe.

6.2 Recommendations

Although no structural interventions are recommended at this time, continued monitoring is encouraged. Regular checks of the fundamental frequencies can indicate a trend in loss of stiffness, likely allowing for earlier interventions, which in turn leads to lower costs overall, and may help to mitigate future damage. Also, since the displacements due to the force of the bell are largely carried in the timber frame, it is important to inspect it regularly for any fatigue or damage.

6.3 Future Works

Further studies could be completed on the St. James tower that would improve the knowledge of the structure of the church. Materials and foundation testing could be done to quantify the material properties and boundary conditions that were assumed and changed in calibration. The dimensions of the bell were assumed, but the time history analysis could be re-run with accurate bell measurements to confirm the results. It was also determined that the impact of the rest of the church had significant effects on the structural frequencies, therefore it would be good to re-analyse the model with a full model of the church, as opposed to just the towers. Finally, it may be beneficial to run the dynamic tests in conjunction with wind or snow loadings and check for peak stresses. A combination of loads may cause problems.

7 Bibliography

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