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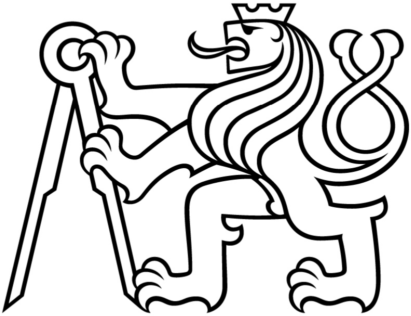

# DOCTORAL
# THESIS
# STATEMENT

CZECH TECHNICAL UNIVERSITY IN PRAGUE

FACULTY OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICS, BIOMECHANICS AND MECHATRONICS

DOCTORAL THESIS STATEMENT

# APPROXIMATE METHODS FOR CALCULATING NOTCH TIP STRAINS AND STRESSES UNDER MULTIAXIAL CYCLIC LOADING

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Doctoral Study Programme: Mechanical Engineering

Study Field: Mechanics of Rigid and Deformable Bodies and Environment

Supervisors:

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A doctoral thesis statement for fulfilment of the requirements for the degree of Doctor, shortly "Ph.D."

Prague March 2022

# Title: Approximate Methods for Calculating Notch Tip Strains and Stresses under Multiaxial Cyclic Loading

This doctoral thesis is an outcome of a full-time form of doctoral study at the Department of Mechanics, Biomechanics and Mechatronics of Faculty of Mechanical Engineering of Czech Technical University in Prague.

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doc. Ing. Martin Fusek, Ph.D. - FME TUO, Ostrava,
doc. Ing. Pavel Hutař, Ph.D. - IPM CAS, Brno.

The doctoral statement was sent on: ..............................

The thesis defense is planned on ............................... at ……… o’clock.

in Boardroom 17 (ground floor) Faculty of Mechanical Engineering of Czech Technical University in Prague, Technická 4, Prague 6,

in the presence of a Doctoral committee for examining doctoral theses in the study field Mechanics of Rigid and Deformable Bodies and Environment.

It is possible to familiarize with the thesis at the department of Science and Research of the Faculty of Mechanical Engineering CTU in Prague, Technická 4, Prague 6.

…………………

prof. Ing. Michael Valášek, DrSc.

Head of Doctoral Study Field Mechanics of Rigid and Deformable Bodies and Environment

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# Contents

# 1. STATE OF THE ART

Most initially isotropic engineering materials exhibit elastic-plastic behavior. To evaluate the damage imposed on components made from such materials, one has to obtain strains and stresses of the components in question. Achieving this is possible with the help of experimental measurements, finite element analyses (FEA), or approximate calculation methods. Experimental approaches, although being most precise, are usually expensive and, for some applications, are quite difficult from a realization point of view. With FEA, financial costs of obtaining stresses and strains might be reduced, but calculations of large assemblies especially with non-elastic material behavior could take up to several days. If several of such calculations need to be performed, weeks could be consumed before the final result is achieved.

In order to speed up the process of obtaining stresses and strains, an approximate calculation might be carried out. The results would not be as precise as in the case of experimental measurements or finite element analyses, but they would be obtained faster. That is the motivation behind the approximate methods for stress and strain calculations.

Many methods have already been suggested for the estimation of elastic–plastic stresses and strains. The first group of approximate methods contains those intended for monotonic loading only [1–5]. The methods do not take cyclic hardening or cyclic softening into account and they do not describe the movement of the yield surface. Therefore, this group of methods is not suitable for cyclic loading.

The second group of methods [6–15] deals with cycling loading and incorporates plasticity models to describe cyclic hardening or softening and yield surface movement. Unlike finite-element analyses (FEA), approximate methods do not deal with elastic–plastic stiffness matrices to obtain a solution. Instead, they use an elastic solution that they convert into an elastic–plastic solution by using a relation either between pseudo material and real material or between linear–elastic and elastic–plastic strain energies.

Only a few methods are based on the equality of linear–elastic and elastic–plastic strain energies. The more popular approach is in creating a so-called pseudo material that allows to relate a theoretical purely elastic solution with the elastic-plastic one. In Figure 1, a pseudo stress-strain curve and a real stress-strain curve are shown. In this example, the real plastic strain and the pseudo plastic strain are related. When a plasticity model is applied to the history of pseudo stress, the real plastic strain is obtained, as it is equal to the pseudo plastic strain. The real stress is then calculated by applying the plasticity model again, but this time, to the real plastic strain.

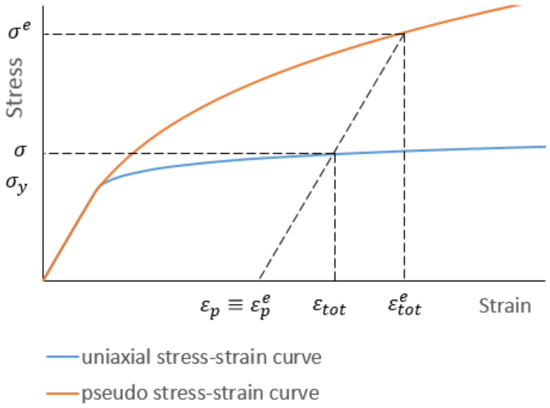


**Figure 1. Illustration of the pseudo stress-real plastic strain principle.**

One of the drawbacks of the current state of the approximate methods, which becomes obvious when a researcher is trying to recreate the methods, is the lack of detailed descriptions of how the plasticity part and the notch correction part are combined. This was also noticed by other authors in [12]. That might be the probable reason for why the methods have not gained wide practical use, despite the fact that some of them were invented several decades ago. There are many works on plasticity models by themselves, but since the approximate methods work on different principles, a lot of guesswork remains when a notch correction and a plasticity model must be combined.

The precision of previously proposed methods in predicting an accumulation of plastic strain under constant cyclic loading, commonly called ratcheting, remains unverified, as experimental loading paths with mean stress have not been measured and used for the validation of the methods estimates. Such loading paths cause, depending on the material, a noticeable ratcheting response and provide the possibility to evaluate the precision of the ratcheting estimates.

Another drawback of the current development of the approximate methods is a very limited amount of experimental data of the notch strain responses. Beside steels, only titanium alloy and GH4169 superalloy were used for the validations so far. These types of experiments might be not so popular due to their specific utilization in notch tip strain testing and a complicated way of measurement.

## 2. AIMS OF THE THESIS

**Based on the observations discussed above, the main target of the thesis is set as the development of a novel pseudo-curve-based approximate method for calculating notch tip elastic-plastic stresses and strains under multiaxial cyclic loading condition.**

The main target consists of the following sub-tasks:

1. Develop a methodology on how to combine a notch correction and a plasticity model, which will be the main parts of the novel approximate method for notch tip stresses and strains estimation. Provide a detailed description and an implementation code.
2. Propose a new and original approximate method for calculating elastic-plastic stresses and strains at the notch tip under multiaxial cyclic loading that provides results of better or of similar precision compared to the other existing methods.
3. Obtain new and original experimental data of notch tip strains to validate the method predictions on specimens manufactured from a different than steel material. The experimental program must include a loading path with a mean stress.

## 3. USED METHODS/ TOOLS

The plasticity model of Abdel-Karim-Ohno (AKO) [16] was chosen for the plasticity part. In addition to requiring only a few material parameters, the model also represents other plasticity models as its special cases, depending on its settings. The nonlinear kinematic rule for the AKO model has the following form:

$$\boldsymbol{da}^{(i)} = \frac{2}{3} C_i d\boldsymbol{\varepsilon}_p - \mu_i \gamma_i \boldsymbol{a}^{(i)} dp - \gamma_i H(f_i) \langle d\lambda_i \rangle \boldsymbol{a}^{(i)}, \tag{1}$$

where

$$f_i = \frac{3}{2} \boldsymbol{a}^{(i)} : \boldsymbol{a}^{(i)} - \left(\frac{C_i}{\gamma_i}\right)^2, \tag{2}$$

$$d\lambda_i = \boldsymbol{d\varepsilon}_p : \frac{\boldsymbol{a}^{(i)}}{C_i/(\gamma_i)} - \mu_i dp. \tag{3}$$

In Equations (1)–(3), $\boldsymbol{a}^{(i)}$ is the $i^{th}$ part of the total backstress $\boldsymbol{a}$; $C_i$ and $\gamma_i$ are material parameters; symbol $\langle x \rangle$ represents Macaulay brackets ($\langle x \rangle = (x+|x|)/2$) and $H(f_i)$ is the Heaviside step function. $\mu_i$ is the ratcheting parameter. The same value of the ratcheting parameter is usually set for all backstress parts, and its value varies between 0 and 1. When it is set to 0 for all *i*, the model corresponds to the multilinear model of Ohno and Wang type I [17]. When $\mu_i$ is set to 1 for all *i*, the model corresponds to Chaboche's kinematic hardening model of plasticity [18].

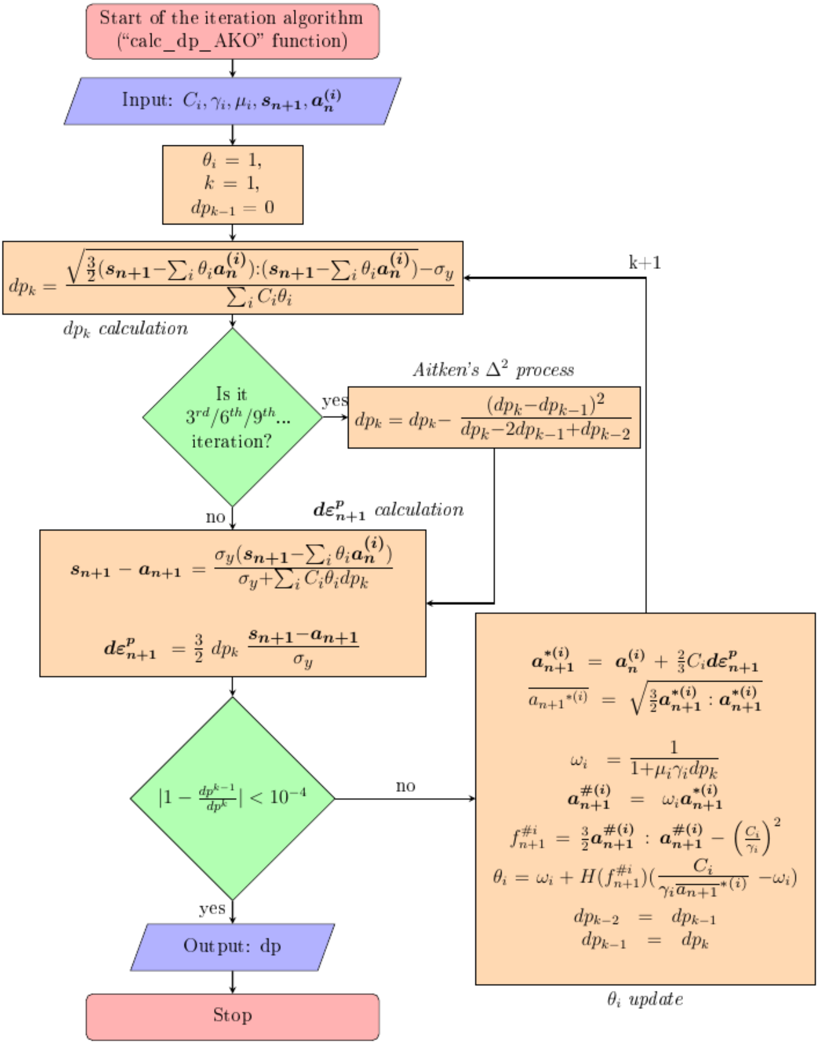


**Figure 2: Iteration algorithm for calculating accumulated plastic strain increment *dp*.**

A substitution algorithm that allows to solve the non-linear scalar equation for accumulated plastic strain is an important part of an approximate method for cyclic loading. In this work, the algorithm presented in [19] for the combined AF and OW model is used. The algorithm is depicted in Figure 2.

To verify the proposed method, fatigue experiments were carried out on two types of notched samples (Figure 3 and Figure 4). The specimens were manufactured from the aluminum alloy 2124-T851.

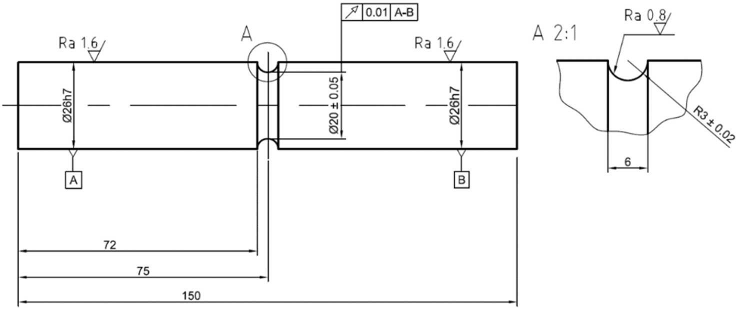


**Figure 3: Specimen with U-notch.**

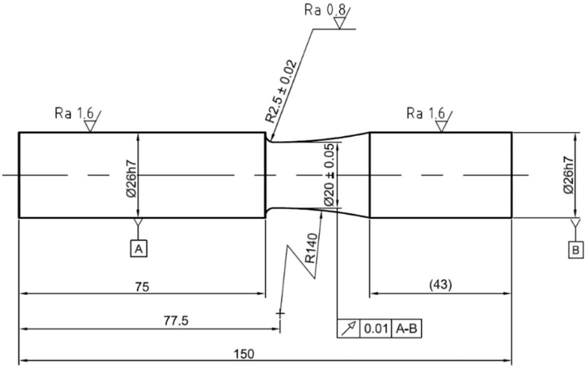


**Figure 4: Specimen with fillet.**

Experiments were carried out under force and moment control. The testing machine used for the experiments was INOVA FU 250 (distributed by Inova Praha s.r.o.), multiaxial tension–compression and torsion load frame with hydraulic actuator for dynamic loading. Seven different loading paths were chosen from the literature to test the specimens (Figure 5). Two stress ratios of nominal axial stress to nominal shear stress were tested. The notch strains were measured using the digital image correlation method.

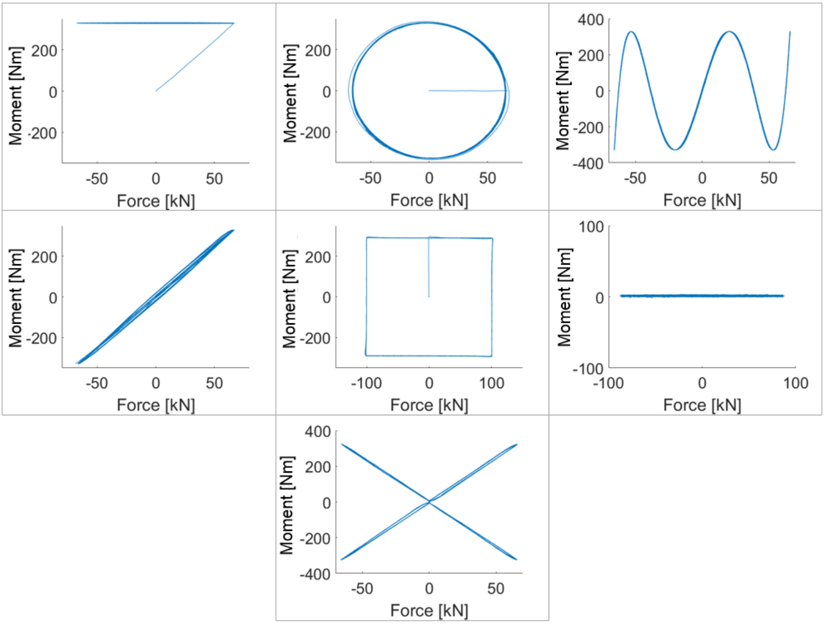


**Figure 5: Loading paths: (top left) ``7"; (top middle) Circle; (top right) NV; (middle left) proportional; (middle) Square; (middle right) uniaxial(bottom) Circle; (bottom) X.**

For the notch correction part, the pseudo stress-plastic strain approach was chosen. The pseudo curve was established by combining elastic stress with plastic strain. The plastic strain values were the same as the plastic strain values of the real cyclic stress–strain (CSS) curve. The key part in calculating the real response from the pseudo variables was the equivalence of the plastic strain tensor of the pseudo curve and of the real stress–strain curve. This was ensured by the way the pseudo curve was established.

The following steps summarize the approximate method:

1. The pseudo material curve is established.
2. The pseudo stress history is obtained either by elastic FEA or using stress concentration factors.
3. The plasticity model is applied to the pseudo stress history. In this step, the plasticity parameters obtained for the pseudo material are used. The plastic strain tensor and the accumulated strain are calculated.

4. The plasticity model is applied to the obtained plastic strain tensor and to the accumulated strain. In this step, the plasticity parameters for the real material are used. Real stress and real backstress are calculated.

## 4. RESULTS

Relative errors of differences between estimated and measured strain ranges were calculated and are presented for the first 100 cycles (42 and 50 in the case of path NV for U-notched samples) in Table 1 and Table 2. The relative error of the estimates was calculated according to Equation (4):

$$RE = \frac{Calculated\,strain\,range - measured\,strain\,range}{measured\,strain\,range} \tag{4}$$

In Table 1 and Table 2, the green filling of the cells means that the absolute value of the relative error is within the 0-10% interval. The yellow filling corresponds to 10-20% of the relative errors, and the orange color means a higher relative error.

Positive values of relative errors mean that the estimated strain range is greater than the measured strain range. Such a result is considered conservative. The relative error values for the axial and shear strain ranges, as well as combined values, are presented. The combined values are calculated as the square root of the corresponding axial and shear components.

**Table 1: Relative errors in percents between measured and calculated strain ranges for 2124-T851 U-notched specimens.**

| path | $\sigma_{nom}$ [MPa]/ $\tau_{nom}$ [MPa] | number of cycles | axial | shear | combined |
|---|---|---|---|---|---|
| Circle | 1 | 100 | -6 | -12.2 | 13.6 |
| Circle | 1.73 | 100 | -7.6 | -4.7 | 8.9 |
| NV | 1 | 50 | -6.8 | -13.9 | 15.5 |
| NV | 1.73 | 42 | -1.2 | -5.4 | 5.5 |
| Proportional | 1 | 100 | -6.7 | -21.9 | 22.9 |
| Square | 1 | 100 | -2.8 | -12.6 | 12.9 |
| Uniaxial | 274.5/- | 100 | -13.5 | - | 13.5 |
| Uniaxial | 319.8/ - | 100 | -11.8 | - | 11.8 |
| X | 1 | 100 | -0.2 | -4 | 4.0 |
| X | 1.73 | 100 | 3.1 | -2.2 | 3.8 |

The precision of the estimates for the axial and shear components is different. In the case of the axial component, 94% of the studied cases lie within 20% of the relative error in absolute values. The shear component for the same range shows a slightly lower value of 87.5%.

**Table 2: Relative errors in percents between measured and calculated strain ranges for 2124-T851 single fillet specimens.**

| path | $\sigma_{nom}$ [MPa]/ $\tau_{nom}$ [MPa] | number of cycles | axial | shear | combined |
|---|---|---|---|---|---|
| 7 |  | 100 | -2.8 | 17.7 | 17.9 |
| Circle | 1 | 100 | 9 | -4.8 | 10.2 |
| Circle | 1.73 | 100 | -10 | -11.1 | 14.9 |
| NV | 1 | 100 | 5.7 | -2.9 | 6.4 |
| Proportional | 1 | 100 | -6.8 | -14.3 | 15.8 |
| Proportional | 1.73 | 100 | -12.6 | -21.2 | 24.7 |
| X | 1 | 100 | 20.3 | 7.2 | 21.5 |
| X | 1.73 | 100 | -13.9 | -11.5 | 18.0 |

If the precision range is lowered to 10% of the relative error, then 67% of the axial strain estimates fall into this limit, and 44% in the case of the shear component.

The greatest error is 24.7% for the combined strain range.

The estimates by the proposed method were also compared with estimates of other methods found in the literature.

The comparison is made for 1070 steel, as it is the most frequently used material for the validations of the approximate methods. Moreover, all the approximations made on this material are based on the same experimental program of Barkey [6], so the predictions of several methods can also be compared.

In Table 3, the highlighted cells mark the lowest relative error for each loading case.

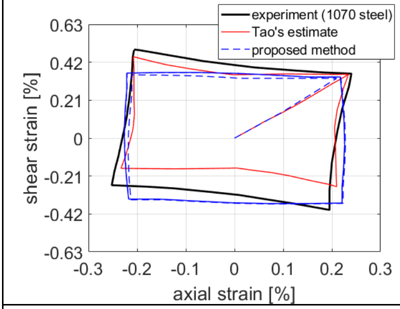
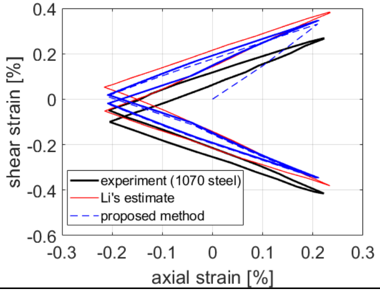
The proposed method provides the best predictions in 6 out of 13 cases.

**Table 3: Relative errors between measured and calculated combined strain ranges for selected methods for 1070 steel in percents.**

| path | $\sigma_{nom}$ [MPa]/ $\tau_{nom}$ [MPa] | Li et al. | Ince et al. | Tao et al. | proposed method |
|---|---|---|---|---|---|
| ksi | 258/168 | – | – | 10.4 | 3.1 |
| ksi | 296/193 | – | 9.9 | – | 10.0 |
| N | 258/168 | – | – | 9.4 | 1.6 |
| N | 296/193 | 7.9 | 13.1 | – | 11.7 |
| NV | 258/168 | – | – | 6.0 | 5.3 |
| NV | 296/193 | – | 6.3 | – | 9.9 |
| Proportional | 296/193 | – | – | 8.1 | 18.3 |
| Rotated V | 296/193 | 12.8 | – | – | 2.1 |
| S | 258/168 | – | – | 7.3 | 2.2 |
| S | 296/193 | 1.1 | 14.1 | – | 13.4 |
| Square | 296/193 | 8.7 | 11.9 | 15.5 | 19.9 |
| Square (clockwise) | 296/193 | 10.7 | 15.8 | – | 17.9 |
| V | 296/193 | 11.6 | – | – | 4.3 |

The greatest relative errors were made for the shear component of path Square. The lowest errors could be achieved by changing the value of the parameter $\mu_i$, but this would negatively affect the estimates for paths V and rotated V (Table 4).

**Table 4: Influence of parameter $\mu_i$ on estimates for 1070 steel.**

<table>
<tr><th>Path Square</th><th>Patrh Rotated V</th></tr>
<tr><td colspan="2">$\mu_i = 0.01$</td></tr>
<tr><td>
</td><td>
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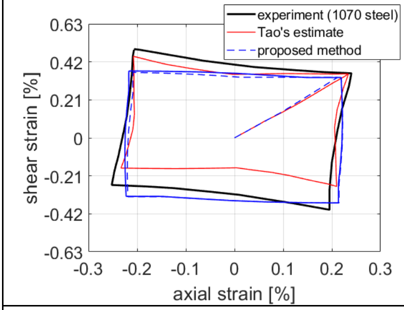
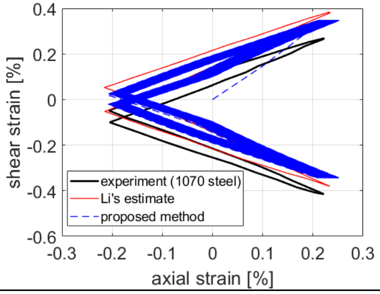
<tr><td>
</td><td>
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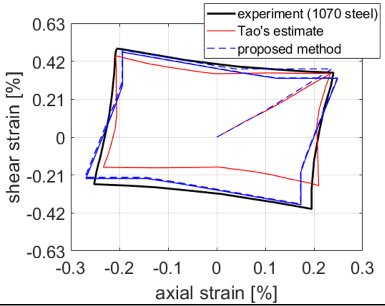
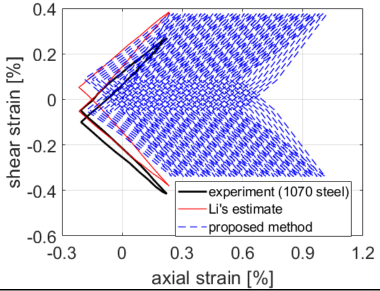
<tr><td colspan="2">$\mu_i = 1$</td></tr>
<tr><td>
</td><td>
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## 5. OUTCOMES

Experimental data on the notch tip strains are very limited to this day. The new experimental data obtained on 2124-T851 aluminum alloy specimens expand the set of currently available data. Responses to a wide variety of loading paths have been measured and presented. Path ``7'' represents an especially valuable addition as a path with a constant mean stress, since it allows to study the ratcheting effect and to tune the models' ratcheting parameters.

An overview of approximate methods for monotonic loading, as well as recommendations of which method to use, were published in [A5] and [A6].

As for the methods for cyclic loading, a code of the proposed approximate method written in the MATLAB programming language is available in Appendix C of the thesis and also in [A1]. Since details on combinations of notch correction methods and plasticity models represent a key value for researchers starting to deal with the stress-strain approximation methods, the published code allows researchers to better understand the method principle and recreate the method by themselves. The new method provides estimates with a relative error under 25% for many loading cycles without the need to carry out time-consuming elastic-plastic finite element analyses.

The code in its entirety can be used to obtain elastic-plastic stress-strain estimates. A stress history from the elastic solution at a critical location can be used as input for the code to estimate elastic-plastic stress-strain solution. That could speed up a design process or other stress assessment tasks.

# 6. CONCLUSIONS

The target of the thesis was to develop a novel pseudo curve based approximate method for calculating elastic-plastic stresses and strains at the notch tip under multiaxial cyclic loading condition. Each defined steps to achieve the target, as the target itself, was fulfilled:

1. A methodology on how to combine a notch correction and a plasticity model was developed. The implementation code, which is available in Appendix C of the doctoral thesis, provides a benefit to researchers dealing with the approximate methods for notch tip stress and strain calculation during the recreation process of the proposed method. The methodology and the implementation code were published in [A1].
2. A new and original approximate method for calculating the elastic-plastic stresses and strains at the notch tip under multiaxial cyclic loading was proposed and published in [A1]. Its novelty lies in the ability to incorporate three plasticity models, Abdel-Karim-Ohno, Ohno-Wang, and Chaboche's model, as its special cases. The approximate method provides results of competitive precision to other existing methods and allows fast estimates of stress and strain responses on cyclic multiaxial loading.
3. New and original experimental data of notch tip strains were measured on aluminum 2124-T851 specimens for a variety of loading paths. A total number of 18 experimental loading cases is presented in this work. Both frequently and infrequently used loading paths from literature were chosen. A loading path with a mean stress was included in the experimental program. The path provides a ratcheting response that is useful for studying its effects. In preceding research publications, only FE analyses of this specific path were used. To the author's knowledge, it is the first time an experimental notch strain response has been published for a loading path with a mean stress. The experimental results were also published in [A1].

## 7. FUTURE WORK

A future development in a practical way might be the implementation of the approximate method in an FE solver as a plugin. This would speed up the estimation process, as it would spare the time otherwise spent exporting stresses from elastic FE analyses. The accessibility directly from FE software would help the method to gain a wider use. The implementation would also allow to apply the method on all surface nodes of a model, which in turn would allow to study the method precision outside of a stress concentration region.

If higher precision needs to be achieved, a changeable value of the ratcheting parameter might be implemented. It should help capture ratcheting changes more accurately.

It is possible to expand the method by including the Calloch-Marquis non-proportional parameter, which would allow a non-proportional hardening to be taken into account, appearing, for example in the cases of stainless steels and coppers [20, 21].

Another way to improve the precision might be found by in-depth analyzes of differences between AKO and Jiang-Sehitoglu plasticity models, which showed itself as the most precise for the majority of its studied cases.

Finally, since thermo-mechanical loading is a common part of loading states and a need to calculate stresses and strains in such cases is as great as for mechanical loading under constant temperatures, the possibility to expand the method for thermo-mechanical loading might be investigated in the future.

## Publications of the author related to the topic of the thesis

A1. Lutovinov, M., Halama, R., Papuga, J., Bartošák, M., Kuželka, J. & Růžička, M. An Approximate Method for Calculating Elastic-Plastic Stress and Strain on Notched Specimens. *Materials* **15**, 22 pages. ISSN: 1996-1944. https://www.mdpi.com/1996-1944/15/4/*1432* (2022).

A2. Papuga, J., Vízková, I., Lutovinov, M. & Nesládek, M. Mean stress effect in stress life fatigue prediction re-evaluated. English. *MATEC Web of Conferences* **165**, 8 pages. https://www.matec-conferences.org/articles/matecconf/pdf/2018/24/matecconf_fatigue2018_10018.pdf (May 2018).

A3. Papuga, J., Kaľavský, A., Lutovinov, M., Vízková, I., Parma, S. & Nesládek, M. Evaluation of data sets usable for validating multiaxial fatigue strength criteria. English. *International Journal of Fatigue* **145**, 18 pages. https://www.sciencedirect.com/science/article/pii/S0142112320306253 (Apr. 2021).

A4. Papuga, J., Karkulín, A., Hanžl, O. & Lutovinov, M. Comparison of several methods for the notch effect quantification on specimens from 2124-T851 aluminum alloy. *Procedia Structural Integrity* **19**. Fatigue Design 2019, International Conference on Fatigue Design, 8th Edition, 405-414. ISSN: 2452-3216. https : // www .sciencedirect.com/science/article/pii/S2452321619305128 (2019).

A5. Lutovinov M.; Papuga, J. A *Verification of Methods for Calculating Notch Tip Stresses and Strains in 19th Workshop of Applied Mechanics* (CTU in Prague, 2015), 4 pages. ISBN: 978-80-01-05918-0.

A6. Lutovinov, M., Černý, J. & Papuga, J. A comparison of methods for calculating notch tip strains and stresses under multiaxial loading. *Frattura ed Integrita Strutturale* **38**, 237-243 (2016).

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## Publication Metrics

**Table 5: Citations based on Web of Science (https://www.webofscience.com/) and ResearchGate (https://www.researchgate.net/)**

| Publication | Number of citations based on | |
|---|---|---|
| | Web of Science | ResearchGate |
| A1 | - | - |
| A2 | 5 | 10 |
| A3 | 4 | 6 |
| A4 | 3 | 5 |
| A5 | - | - |
| A6 | 8 | 9 |

**Publication metrics according to Web of Science**

H-Index: 4

Total publications: 12

Sum of times cited: 28

Citing articles: 25

## Curriculum vitae

| | | |
|---|---|---|
| **Education:** | 2009-2013 | CTU in Prague, Bachelor's degree, Mechanical Engineering - Computer Aided Design |
| | 2013-2015 | CTU in Prague, Master's degree, Mechanical Engineering - Applied Mechanics |
| | 2015-2022 | CTU in Prague, Doctor of Philosophy - PhD, Mechanics of Solids, Deformable Bodies and Environment. Doctoral thesis title: “Approximate Methods for Calculating Notch Tip Strains and Stresses Under Multiaxial Cyclic Loading” |
| **Experience:** | 2013 | CTU in Prague, collection and processing of material data for the database of fatigue parameters FinLiv |
| | 2013-2015 | CTU in Prague/ VSB-Technical University of Ostrava, FinLiv database development, calculations and background analysis within a turbochargers assessment project |
| | 2016-2020 | CTU in Prague, FEA calculations within the international turbomachinery optimization projects Turboreflex and Flexturbine |
| | 2021-2022 | AKKA Czech Republic s.r.o., FEM analyst |
| **Awards:** | 2015 | Cena prof. Karla Spály - 2nd place, award for the best master thesis for the period from January to August 2015 at the Faculty of Mechanical University of Czech Technical University in Prague. |
| **Skills:** | | FEA, programming, ANSYS, Abaqus, Ansa, Meta, Medina, Femfat, Femsite, MATLAB, VBA |
| **Languages:** | | Czech, English, Russian |

## Resumé

| | | |
|---|---|---|
| **VŠ vzdělání:** | 2009-2013 | ČVUT, FS, Bakalářský studijní program: Strojírenství, Obor: Konstruování podporované počítačem |
| | 2013-2015 | ČVUT, FS, Navazující magisterský studijní program: Strojní inženýrství, Obor: Aplikovaná mechanika |
| | 2015-2022 | ČVUT, FS, Doktorský studijní program: Strojní inženýrství, Obor: Mechanika tuhých a poddajných těles a prostředí |
| **Pracovní zkušenosti:** | 2013 | ČVUT, FS, sběr a zpracování materiálových dat pro databáze únavových parametrů FinLiv |
| | 2013-2015 | ČVUT/ VŠB, práce na vývoji aplikace FinLiv, realizace výpočtů a podkladových analýz na turbodmychadlech |
| | 2016-2020 | ČVUT, práce na mezinárodních projektech Flexturbine a Turboreflex na vývoji turbín tepelných elektráren |
| | 2021-2022 | AKKA Czech Republic s.r.o., FEM výpočtář |
| **Ocenění:** | 2015 | Cena prof. Karla Spály, II. místo v soutěži o nejlepší diplomové práce v období leden a srpen 2015 na Fakultě strojní ČVUT v Praze. |
| **Dovednosti:** | | MKP, programování, ANSYS, Abaqus, Ansa, Meta, Medina, Femfat, Femsite, MATLAB, VBA |
| **Jazyky** | | Anglický, český, ruský |

## Summary

This work deals with approximate methods for calculating elastic-plastic stresses and strains on the surface of notched samples. In order to expand the range of currently available experimental notch strain response data, specimens manufactured from the 2124-T851 aluminum alloy were subjected to various multiaxial cyclic loading combinations. Then a new approximate method based on the Abdel-Karim-Ohno cyclic plasticity model was proposed. The results of the approximations were verified on own experimental results, as well as on experimental results available in the literature. A comparison with estimates by other methods was also made. The new method provides competitive results and a good correlation with the experimental data.

## Shrnutí

Tato práce se zabývá aproximačními metodami pro výpočet elasto-plastických deformací a napětí na povrchu kořene vrubu. Pro rozšíření současně dostupných experimentálních dat deformační odezvy v kořeni vrubu vzorky z hliníkové slitiny 2124-T851 byly zkoušeny různými kombinacemi axiální síly a kroutícího momentu. Poté byla navržena nová aproximační metoda založená na modelu cyklické plasticity Abdel-Karim-Ohno. Výsledky aproximačních výpočtů byly ověřené na vlastních experimentálních datech a experimentálních datech dostupných v literatuře. Bylo provedeno porovnání s výsledky výpočtů jiných aproximačních metod. Nová metoda poskytuje konkurenceschopné výsledky s dobrou shodou s experimentálními výsledky.