THESIS REVIEWER'S REPORT

I. IDENTIFICATION DATA

Thesis title:	**The use of Radon transform in medicine and biology**
Author's name:	**Marija Pajdaković**
Type of thesis :	bachelor
Faculty/Institute:	Faculty of Electrical Engineering (FEE)
Department:	Department of Control Engineering
Thesis reviewer:	Doc. RNDr. Martin Bohata, Ph.D.
Reviewer's department:	Department of Mathematics FEE

II. EVALUATION OF INDIVIDUAL CRITERIA

Assignment **challenging**

How demanding was the assigned project?

The thesis deals with Radon transform which belongs to the area of functional analysis. This topic is not covered by undergraduate courses being taught at FEE CTU in Prague.

Fulfilment of assignment **fulfilled**

How well does the thesis fulfil the assigned task? Have the primary goals been achieved? Which assigned tasks have been incompletely covered, and which parts of the thesis are overextended? Justify your answer.

All goals of the thesis have been achieved. In particular, the Central Slice Theorem has been discussed in detail.

Methodology **correct**

Comment on the correctness of the approach and/or the solution methods.

Standard methods of mathematical analysis are used. The Fourier transform is introduced on the Schwartz space. However, many functions in the thesis do not belong to the Schwartz space. Therefore, it seems to me that it would be better to define the Fourier transform on the space of integrable functions.

Technical level **C - good.**

Is the thesis technically sound? How well did the student employ expertise in the field of his/her field of study? Does the student explain clearly what he/she has done?

Almost all chapters have good quality. Unfortunately, some proofs in the chapter on Fourier analysis contain errors (see the section "Additional commentary and evaluation"). On the other hand, I would like to point out that a deeper theory of Fourier transform is not easy especially for undergraduate students.

Formal and language level, scope of thesis **B - very good.**

Are formalisms and notations used properly? Is the thesis organized in a logical way? Is the thesis sufficiently extensive? Is the thesis well-presented? Is the language clear and understandable? Is the English satisfactory?

The thesis is written in English at very good level. The text is well structured and easy to read. I appreciate an effort to provide clear and detailed explanation of many concepts. I miss punctuation after some equations.

Selection of sources, citation correctness **A - excellent.**

Does the thesis make adequate reference to earlier work on the topic? Was the selection of sources adequate? Is the student's original work clearly distinguished from earlier work in the field? Do the bibliographic citations meet the standards?

The selected references are appropriate and their citations meet the usual standards in mathematics.

Additional commentary and evaluation (optional)

Comment on the overall quality of the thesis, its novelty and its impact on the field, its strengths and weaknesses, the utility of the solution that is presented, the theoretical/formal level, the student's skillfulness, etc.

Comments and some typos:

- Page 3: The wavelength range should correspond to the frequency range.
- Page 10: The formula for attenuation coefficient in Hounsfield units does not correspond to the values in Table 2.1. The fraction should be multiplied by 1000.
- Page 14: In (3.5), $\ell_{t,\theta}$ should be a set (see (3.4)). The same problem appears on the page 15.
- Page 17: The definition of line integral is not sufficient for our purposes. In the Radon transform, we would like to integrate over lines which are not curves of finite length.
- Page 27, Theorem 5.3.3: Statements 1 and 2 are special cases of the statement 3. Moreover, I would prefer to formulate linearity (statement 3) with complex scalars because Fourier transform lives naturally on a complex vector space. In the statement 6, it should be $\frac{1}{|k|^n}$ in place of $\frac{1}{|a|}$.
- Page 28: In the proof of the statement 4, I would omit the last equality in the first formula because we want to prove it. In the proof of the statement 6, a change of variables is not performed properly. According to the change of variables formula, we should have $\mathcal{F}\{f(k\boldsymbol{t})\} = \frac{1}{|k|^n}\int_{-\infty}^{\infty} f(\boldsymbol{u})e^{-\frac{i\langle \boldsymbol{u},\boldsymbol{\alpha}\rangle}{k}}\mathrm{d}\boldsymbol{u} = \frac{1}{|k|^n}\hat{f}\left(\frac{\boldsymbol{\alpha}}{k}\right)$.
- Page 30, line 6: $\frac{1}{\pi}$ should be $\frac{1}{\pi^n}$.
- Page 31, lines 2 and 3: We should integrate with respect to $u_1, \dots, u_n$ on the second line. There should not be multiplication by number 2 on the third line.
- Page 31: It would be worth to note that the norm in Remark 5.4.2 is Euclidean.
- Page 33: I do not understand Theorem 5.4.4 and its proof. The function $g(\boldsymbol{t})$ is equal to an integral which does not exist.
- Page 33, the formula for $\Delta_k(\boldsymbol{\omega})$: Type of the norm should be specified.
- Page 34, Theorem 5.4.5: There is no continuity assumption of h. Therefore, we cannot apply Theorem 5.4.3.
- Page 34, Theorem 5.4.6: Theorem does not hold without additional assumptions.
- Page 37: "$\mathcal{F}(\mathcal{R}f)$ live" should be "$\mathcal{F}(\mathcal{R}f)$ lives".
- Page 42: |0,2π) should be [0,2π). ρ should be in $(0, +\infty)$.
- Page 43, line 7: $e^{i\hat{r}\varphi\, cos(\varphi-\psi)}$ should be $e^{i\hat{r}\rho\, cos(\varphi-\psi)}$.
- Page 45: "have we" should be "we have".

III. OVERALL EVALUATION, QUESTIONS FOR THE PRESENTATION AND DEFENSE OF THE THESIS, SUGGESTED GRADE

The bachelor thesis under review is at very good level. The author clearly explains the use of Radon transform in medicine.

Questions:

- In the thesis, the Fourier transform is defined on the Schwartz space. Is every Schwartz function (i.e. function in the Schwartz space) integrable?
- Is it necessary to add three equations to the system (2.1)? Why do you not use the method of least squares?

The grade that I award for the thesis is **B - very good.**

Date: **2.6.2022** Signature: